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Results in Mathematics

Equigeodesics on Generalized Flag Manifolds with $b_2(G/K) = 1$

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Abstract. In this paper we provide a characterization of structural equigeodesics on generalized flag manifolds with second Betti number $b_2(G/K) = 1$, and give examples of structural equigeodesics on generalized flag manifolds of the exceptional Lie groups F_4, E_6 and E_7 with three isotropy summands.

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1. Introduction

The orbits of the adjoint action of a semisimple compact Lie group define an important class of homogeneous manifolds called generalized flag manifolds. These manifolds were studied by many authors (see [5–8]).

Let $M = G/K$ be a homogeneous manifold with origin $o = eK$ (trivial coset) and $\mathfrak{g}$ be a G-invariant metric on M. A geodesic $\gamma(t)$ on G/K through the origin o is called homogeneous, if it is the orbit of a 1-parameter subgroup of G, that is,

$$\gamma(t) = (exptX) \cdot o,$$

where $X \in g$, and g is the Lie algebra of G.

In [3] the authors introduce the notion of homogeneous equigeodesics. A homogeneous equigeodesic is a homogeneous curve γ which is geodesic with respect to any G-invariant metric. Since the infinitesimal generator of a

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1-parameter subgroup is an element of the Lie algebra of G, it is natural to characterize the equigeodesics in terms of their infinitesimal generators. This allows us to use a Lie theoretical approach to study homogeneous geodesics on flag manifolds. The infinitesimal generator of an equigeodesics is called an equigeodesic vector. In [4] the authors have provided a characterization of all homogeneous equigeodesics with two isotropy summands.

Let $\mathfrak{k}$ and g be the Lie algebra of K and G, Π_K and Π be the simple root system of $\mathfrak{k}$ and g respectively, where $\Pi_K = \Pi - \{\alpha_{i_0}\}$ and where α_{i_0} is a simple root of g, and $\Pi_M = \Pi - \Pi_K = \{\alpha_{i_0}\}$. Let $\Gamma = \Gamma(\Pi)$ be the Dynkin diagram of the set of simple roots Π. By painting the vertice α_{i_0} black we obtain the painted Dynkin diagram of $M = G/K$.

An algebraic characterization of equigeodesic vectors in generalized flag manifolds is given in [3]. In [3] the authors provide a version of this formula (see Proposition 3.1) for equigeodesic vectors on generalized flag manifolds. Using this formula to determine whether a vector is equigeodesic is equivalent to solve an algebraic nonlinear system of equations whose variables are the components of the vector. However there exist some subspaces of the equigeodesic vectors, in these subspaces all the equigeodesic vectors are called structural equigeodesic vectors (see Definition 3.2).

The structural equigeodesic vectors are more treatable, since the geometric structure can be expressed in terms of Lie groups and algebras, root space decomposition, isotropy representation, etc.

In this paper we provide a characterization in terms of the equigeodesic vectors of homogeneous equigeodesics in generalized flag manifold G/K with second Betti number $b_2(G/K) = 1$ (see Lemma 3.5). We give a method (see Theorem 3.7) to find the structural equigeodesics associated to generalized flag manifolds with second Betti number $b_2(G/K) = 1$.

We explicitly describe the families of subspaces of which all elements are structural equigeodesic vectors on generalized flag manifolds associated to exceptional Lie groups F_4, E_6 and E_7 with three isotropy summands. Our results concern the generalized flag manifolds

$$F_4/SU(2) \times U(1) \times SU(3),$$
$$E_6/SU(3) \times U(1) \times SU(3) \times SU(2)$$

and

$$E_7/SU(5) \times U(1) \times SU(3).$$

The families of structural equigeodesic vectors are given in Tables 1, 2, and 3 respectively.

This paper is organized as follows: in Sect. 2 we recall some basic concepts about the geometry of flag manifolds. In Sect. 3 we focus on the case where the isotropy representation with second Betti number $b_2(G/K) = 1$, to find the structural equigeodesics associated to generalized flag manifolds with

Table 1. Structural equigeodesic vectors for $F_4/SU(2) \times U(1) \times SU(3)$

$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_{12}}$	$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{11}}$
$\mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_9}$	$\mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{11}}$
$\mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_6}$	$\mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_{12}}$
$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\alpha_1}$	$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\alpha_2}$
$\mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_4}$	$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\alpha_3}$
$\mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_5}$	$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\alpha_6}$
$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_7}$	$\mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_8}$
$\mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_9}$	$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_{10}}$
$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_{11}}$	$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_{12}}$
$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_6}$	$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_3}$
$\mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5}$	$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_{12}}$
$\mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_9}$	$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{11}}$
$\mathfrak{m}_{\gamma_1} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_{11}}$	$\mathfrak{m}_{\gamma_2} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{12}}$

second Betti number $b_2(G/K) = 1$ we give Theorem 3.7. In Sect. 4 we give the results about structural equigeodesic vectors on generalized flag manifolds associated to the exceptional Lie groups F_4, E_6 and E_7 with three isotropy summands.

2. Flag Manifolds

Let G be a compact connected simple Lie group and g be the corresponding Lie algebra. We denote by $g^{\mathbb{C}}$ the complexification of g and $Ad : G \to Aut(g)$ be the adjoint representation of G. A generalized flag manifold is a homogeneous space G/K where the isotropy subgroup K is the centralizer $C(S)$ of a torus S in G. If $S = T$ is a maximal torus then $K = C(S) = T$, and G/T is called a *full* flag manifold.

Let G/K be generalized flag manifold and $\mathfrak{k}$ be the Lie algebra of K. We denote by $o = eK$ the origin of the flag manifold (the identity coset of G/K). Since the Lie group G is simple and compact, the Cartan–Killing form $\langle\cdot,\cdot\rangle$ is non-degenerated and negative definite. Thus $Q(\cdot,\cdot) = -\langle\cdot,\cdot\rangle$ is an inner product. Let $\mathfrak{m} = \mathfrak{k}^{\perp}$ be the orthogonal complement of $\mathfrak{k}$ with respect to Q. Then the decomposition $g = \mathfrak{m} \oplus \mathfrak{k}$ is reductive, that is, $Ad(K)\mathfrak{m} \subset \mathfrak{m}$ and the tangent space at the origin $T_o(G/K)$ is identified with $\mathfrak{m}$.

We denote by $j : K \to Aut(\mathfrak{m})$ the isotropy representation of K on $\mathfrak{m}$. For a generalized flag manifold it is well known that the isotropy representation is completely reducible, that is,

$$\mathfrak{m} = \mathfrak{m}_1 \oplus \cdots \oplus \mathfrak{m}_s, \tag{1}$$

Table 2. Structural equigeodesic vectors for $E_6/SU(3) \times U(1) \times SU(3) \times SU(2)$

$\mathfrak{m}_{\gamma_1} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\alpha_{18}}$	$\mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\gamma_2} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{15}}$	$\mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8}$
$\mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{16}}$	$\mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\alpha_{18}}$	$\mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{18}}$	$\mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\alpha_{17}}$	$\mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8}$
$\mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\alpha_{17}}$	$\mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\alpha_{18}}$	$\mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{16}}$	$\mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_8}$
$\mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{18}}$	$\mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_7}$
$\mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\alpha_{17}}$	$\mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_8}$	$\mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_7}$
$\mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_8}$	$\mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7}$
$\mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7}$	$\mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\alpha_{18}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_8}$	$\mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8}$	$\mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_9}$
$\mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9}$	$\mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7}$
$\mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_9}$	$\mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_8}$
$\mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_7}$	$\mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{18}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_8}$

where each $\mathfrak{m}_i$ is an irreducible inequivalent component of the isotropy representation.

In this paper we consider equideosic vectors on flag manifolds G/K with $b_2(G/K) = 1$. Now we give a Lie theoretical description of the structure of flag manifolds with $b_2(G/K) = 1$.

Table 3. Structural equigeodesic vectors for $E_7/SU(5)\times U(1)\times SU(3)$

$\mathfrak{m}_{\gamma_1}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{27}}$
$\mathfrak{m}_{\gamma_2}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{29}}$
$\mathfrak{m}_{\gamma_3}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{28}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\gamma_4}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{28}}\oplus\mathfrak{m}_{\alpha_{29}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\gamma_5}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{28}}\oplus\mathfrak{m}_{\alpha_{29}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_1}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{28}}\oplus\mathfrak{m}_{\alpha_{29}}$
$\mathfrak{m}_{\beta_2}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{29}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_3}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_4}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_5}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{29}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_6}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{28}}\oplus\mathfrak{m}_{\alpha_{29}}$
$\mathfrak{m}_{\beta_7}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{29}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_8}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{28}}$
$\mathfrak{m}_{\beta_9}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{28}}$
$\mathfrak{m}_{\beta_{10}}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{22}}$
$\mathfrak{m}_{\beta_{11}}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{11}}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{26}}$
$\mathfrak{m}_{\beta_{12}}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_{12}}\oplus\mathfrak{m}_{\alpha_{14}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{29}}$
$\mathfrak{m}_{\beta_{13}}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{15}}\oplus\mathfrak{m}_{\alpha_{18}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{23}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{28}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_{14}}\oplus\mathfrak{m}_{\alpha_1}\oplus\mathfrak{m}_{\alpha_2}\oplus\mathfrak{m}_{\alpha_3}\oplus\mathfrak{m}_{\alpha_4}\oplus\mathfrak{m}_{\alpha_9}\oplus\mathfrak{m}_{\alpha_{10}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{22}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{26}}\oplus\mathfrak{m}_{\alpha_{28}}\oplus\mathfrak{m}_{\alpha_{29}}\oplus\mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\beta_{15}}\oplus\mathfrak{m}_{\alpha_5}\oplus\mathfrak{m}_{\alpha_6}\oplus\mathfrak{m}_{\alpha_7}\oplus\mathfrak{m}_{\alpha_8}\oplus\mathfrak{m}_{\alpha_{13}}\oplus\mathfrak{m}_{\alpha_{16}}\oplus\mathfrak{m}_{\alpha_{17}}\oplus\mathfrak{m}_{\alpha_{19}}\oplus\mathfrak{m}_{\alpha_{20}}\oplus\mathfrak{m}_{\alpha_{21}}\oplus\mathfrak{m}_{\alpha_{24}}\oplus\mathfrak{m}_{\alpha_{25}}\oplus\mathfrak{m}_{\alpha_{27}}\oplus\mathfrak{m}_{\alpha_{28}}$

TABLE 3. continued

$\mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{14}}$	$\mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{14}}$
$\mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{11}}$	$\mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{14}}$
$\mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{15}}$
$\mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{15}}$
$\mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{14}}$	$\mathfrak{m}_{\alpha_{10}} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{14}}$
$\mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{11}}$	$\mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{12}}$
$\mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{12}}$
$\mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{13}}$	$\mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{15}}$
$\mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{14}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_{18}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{13}}$
$\mathfrak{m}_{\alpha_{19}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_{20}} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{15}}$
$\mathfrak{m}_{\alpha_{21}} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{14}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_{22}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\beta_{14}}$
$\mathfrak{m}_{\alpha_{23}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{13}}$	$\mathfrak{m}_{\alpha_{24}} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{15}}$
$\mathfrak{m}_{\alpha_{25}} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{14}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_{26}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\beta_{14}}$
$\mathfrak{m}_{\alpha_{27}} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{15}}$	$\mathfrak{m}_{\alpha_{28}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{14}} \oplus \mathfrak{m}_{\beta_{15}}$
$\mathfrak{m}_{\alpha_{29}} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\beta_{14}}$	$\mathfrak{m}_{\alpha_{30}} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\beta_{14}}$
$\mathfrak{m}_{\gamma_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{15}}$	$\mathfrak{m}_{\gamma_1} \oplus \mathfrak{m}_{\beta_{14}} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{10}}$
$\mathfrak{m}_{\gamma_1} \oplus \mathfrak{m}_{\beta_{15}} \oplus \mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\alpha_{19}} \oplus \mathfrak{m}_{\alpha_{20}} \oplus \mathfrak{m}_{\alpha_{24}} \oplus \mathfrak{m}_{\alpha_{27}}$	$\mathfrak{m}_{\gamma_2} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\alpha_{19}} \oplus \mathfrak{m}_{\alpha_{22}} \oplus \mathfrak{m}_{\alpha_{26}} \oplus \mathfrak{m}_{\alpha_{29}}$
$\mathfrak{m}_{\gamma_2} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{14}} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\alpha_{21}} \oplus \mathfrak{m}_{\alpha_{25}}$	$\mathfrak{m}_{\gamma_2} \oplus \mathfrak{m}_{\beta_{13}} \oplus \mathfrak{m}_{\alpha_1} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{10}}$
$\mathfrak{m}_{\gamma_3} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\alpha_{13}} \oplus \mathfrak{m}_{\alpha_{20}} \oplus \mathfrak{m}_{\alpha_{22}} \oplus \mathfrak{m}_{\alpha_{24}} \oplus \mathfrak{m}_{\alpha_{26}} \oplus \mathfrak{m}_{\alpha_{30}}$	$\mathfrak{m}_{\gamma_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\alpha_{11}} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\alpha_{18}} \oplus \mathfrak{m}_{\alpha_{21}} \oplus \mathfrak{m}_{\alpha_{28}}$
$\mathfrak{m}_{\gamma_3} \oplus \mathfrak{m}_{\beta_{12}} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_6} \oplus \mathfrak{m}_{\alpha_8}$	$\mathfrak{m}_{\gamma_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\alpha_{20}} \oplus \mathfrak{m}_{\alpha_{22}} \oplus \mathfrak{m}_{\alpha_{29}} \oplus \mathfrak{m}_{\alpha_{30}}$
$\mathfrak{m}_{\gamma_4} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\alpha_{22}} \oplus \mathfrak{m}_{\alpha_{23}} \oplus \mathfrak{m}_{\alpha_{25}} \oplus \mathfrak{m}_{\alpha_{28}}$	$\mathfrak{m}_{\gamma_4} \oplus \mathfrak{m}_{\beta_{11}} \oplus \mathfrak{m}_{\alpha_2} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_9}$
$\mathfrak{m}_{\gamma_5} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\alpha_{27}} \oplus \mathfrak{m}_{\alpha_{29}} \oplus \mathfrak{m}_{\alpha_{30}}$	$\mathfrak{m}_{\gamma_1} \oplus \mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\beta_{14}} \oplus \mathfrak{m}_{\beta_{15}}$
$\mathfrak{m}_{\gamma_5} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\alpha_5} \oplus \mathfrak{m}_{\alpha_9} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\alpha_{20}} \oplus \mathfrak{m}_{\alpha_{22}} \oplus \mathfrak{m}_{\alpha_{23}} \oplus \mathfrak{m}_{\alpha_{25}} \oplus \mathfrak{m}_{\alpha_{28}}$	$\mathfrak{m}_{\gamma_2} \oplus \mathfrak{m}_{\beta_1} \oplus \mathfrak{m}_{\beta_4} \oplus \mathfrak{m}_{\beta_{13}}$
$\mathfrak{m}_{\gamma_5} \oplus \mathfrak{m}_{\beta_{10}} \oplus \mathfrak{m}_{\alpha_3} \oplus \mathfrak{m}_{\alpha_4} \oplus \mathfrak{m}_{\alpha_7} \oplus \mathfrak{m}_{\alpha_8} \oplus \mathfrak{m}_{\alpha_{12}} \oplus \mathfrak{m}_{\alpha_{15}} \oplus \mathfrak{m}_{\alpha_{16}} \oplus \mathfrak{m}_{\alpha_{17}} \oplus \mathfrak{m}_{\alpha_{20}} \oplus \mathfrak{m}_{\alpha_{22}}$	$\mathfrak{m}_{\gamma_3} \oplus \mathfrak{m}_{\beta_3} \oplus \mathfrak{m}_{\beta_6} \oplus \mathfrak{m}_{\beta_{12}}$
$\mathfrak{m}_{\gamma_4} \oplus \mathfrak{m}_{\beta_5} \oplus \mathfrak{m}_{\beta_8} \oplus \mathfrak{m}_{\beta_{11}}$	$\mathfrak{m}_{\gamma_5} \oplus \mathfrak{m}_{\beta_7} \oplus \mathfrak{m}_{\beta_9} \oplus \mathfrak{m}_{\beta_{10}}$

Let T be a maximal torus of G, and η be the Lie algebra of T. The complexification $\eta^{\mathbb{C}}$ is a Cartan subalgebra of $g^{\mathbb{C}}$. Let $\mathcal{R}$ be a root system of $(g^{\mathbb{C}}, \eta^{\mathbb{C}})$ and consider the root space decomposition

$$g^{\mathbb{C}} = \eta^{\mathbb{C}} \oplus \sum_{\alpha \in \mathcal{R}} g^{\mathbb{C}}_{\alpha}, \tag{2}$$

where $g^{\mathbb{C}}_{\alpha}$ denote the complex 1-dimensional root space.

Let $\mathcal{R}^+$ be a choice of positive roots and Π be corresponding set of simple roots. We fix once and for all a Weyl basis of $g^{\mathbb{C}}$ which amounts to take $E_\alpha \in g^{\mathbb{C}}_{\alpha}$ such that $Q(E_\alpha, E_{-\alpha}) = -1$, and $[E_\alpha, E_{-\alpha}] = -H_\alpha$, where $H_\alpha \in \eta^{\mathbb{C}}$ is determined by the equation $Q(H, H_\alpha) = \alpha(H)$, for all $H \in \eta^{\mathbb{C}}$. The vectors E_α satisfy the relation $[E_\alpha, E_\beta] = N_{\alpha,\beta} E_{\alpha+\beta}$ with $N_{\alpha,\beta} \in \mathbb{R}, N_{-\alpha,-\beta} = -N_{\alpha,\beta}$ and $N_{\alpha,\beta} = 0$ if $\alpha + \beta \notin \mathcal{R}$ (see [5, Chap. IX]).

Let $A_\alpha = E_\alpha - E_{-\alpha}$ and $B_\alpha = \sqrt{-1}(E_\alpha + E_{-\alpha})$. The vectors

$$A_\alpha, B_\alpha, \sqrt{-1}H_\beta, \quad (\alpha \in \mathcal{R}^+ \ \textit{and} \ \beta \in \Pi) \tag{3}$$

form a basis of g (compact real form of the Lie algebra $g^{\mathbb{C}}$).

For $\alpha \in \mathcal{R}^+$ let

$$\mathfrak{m}_\alpha = span_{\mathbb{R}}\{A_\alpha, B_\alpha\}, \tag{4}$$

be the real root space.

We have the following decomposition

$$g = \eta \oplus \sum_{\alpha \in \mathcal{R}^+} \mathfrak{m}_\alpha. \tag{5}$$

The next lemma gives us information about the Lie algebra structure of g.

Lemma 2.1. *The Lie bracket between the elements of (3) of g are given by*

$$\begin{aligned} [\sqrt{-1}H_\alpha, A_\beta] &= \beta(H_\alpha)B_\beta, & [A_\alpha, A_\beta] &= N_{\alpha,\beta}A_{\alpha+\beta} + N_{-\alpha,\beta}A_{\alpha-\beta}, \\ [\sqrt{-1}H_\alpha, B_\beta] &= -\beta(H_\alpha)A_\beta, & [B_\alpha, B_\beta] &= -N_{\alpha,\beta}A_{\alpha+\beta} - N_{\alpha,-\beta}A_{\alpha-\beta}, \\ [A_\alpha, B_\alpha] &= 2\sqrt{-1}H_\alpha, & [A_\alpha, B_\beta] &= N_{\alpha,\beta}B_{\alpha+\beta} + N_{\alpha,-\beta}B_{\alpha-\beta}. \end{aligned} \tag{6}$$

Since $\eta^{\mathbb{C}}$ is also a Cartan subalgebra of $\mathfrak{k}^{\mathbb{C}}$ (complexification of the Lie algebra of K), let $\mathcal{R}_K$ be the root system for $(\mathfrak{k}^{\mathbb{C}}, \eta^{\mathbb{C}})$ and let $\mathcal{R}_M = \mathcal{R} \backslash \mathcal{R}_K$. In a similar way, let $\mathcal{R}^+_K$ be a choice of positive roots and Π_K the corresponding set of simple roots for $\mathfrak{k}^{\mathbb{C}}$ and define $\mathcal{R}_M = \mathcal{R} \setminus \mathcal{R}_K$ and $\Pi_M = \Pi \setminus \Pi_K$ be the set of positive and simple complementary roots.

Let l be the rank of $\mathfrak{k}^{\mathbb{C}}$ and $\mu = \sum_{i=1}^{l} n_i\alpha_i$ be the highest root of $\mathcal{R}$, that is the unique root such that any root $\alpha = \sum_{i=1}^{l} c_i\alpha_i$ must satisfy $c_i \leq n_i$ for all i. The coefficients $n_i \in \mathbb{Z}$ are called *heights* of the simple root α_i.

We only consider the generalized flag manifolds corresponding to the Dynkin diagram $\Gamma = \Gamma(\Pi)$with one simple root painted black, thus a generalized flag manifolds has s isotropy summands if $\Pi_K = \Pi - \{\alpha_{i_0}\}$ and the simple root α_{i_0} has height s, that is $n_{i_0} = s$.

In order to describe the irreducible components $\mathfrak{m}_i(i = 1,\ldots,s)$, Let α_{i_0} be a simple root of height s and $\Pi_K = \Pi - \{\alpha_{i_0}\}$. For $n = 1,\ldots,s$ let

$$\mathcal{R}^+(\alpha_{i_0}, n) = \left\{ \alpha \in \mathcal{R}^+ : \alpha = \sum_{i=1}^{l} c_j\alpha_j, c_{i_0} = n \right\}, \tag{7}$$

and we define the subspaces $\mathfrak{m}_n$ of g by

$$\mathfrak{m}_n = \sum_{\alpha\in R^+(\alpha_{i_0},n)} \mathfrak{m}_\alpha. \tag{8}$$

Then $\mathfrak{m} = \mathfrak{m}_1 \oplus\cdots\oplus \mathfrak{m}_s$ and $\mathcal{R}^+_M = \mathcal{R}^+(\alpha_{i_0}, 1)\cup\cdots\cup \mathcal{R}^+(\alpha_{i_0}, s)$, each $\mathfrak{m}_n$ $(n = 1,\cdots,s\ )$ is an irreducible and inequivalent component of the isotropy representation, see [4].

Example 2.2. (Flag manifold of the exceptional Lie group G_2 with three isotropy summands). Let $\Pi = \{\alpha_1, \alpha_2\}$ be the simple roots of G_2 and $\mu = 2\alpha_1 + 3\alpha_2$ be the highest root. The Dynkin diagram for the Lie algebra of G_2 is

$$\circ_{\alpha_2} \Lleftarrow \circ_{\alpha_1}.$$

We describe the flag manifold associated $G_2/U(2)$ with $\Pi_K = \Pi - \{\alpha_2\}$. The painted Dynkin graph of $G_2/U(2)$ is

$$\bullet_{\alpha_2} \Lleftarrow \circ_{\alpha_1}.$$

Since the height of the root α_2 is 3, we have that $\mathfrak{m} = \mathfrak{m}_1 + \mathfrak{m}_2 + \mathfrak{m}_3$ and $\mathcal{R}^+(\alpha_2, 1) = \{\alpha_2, \alpha_1 + \alpha_2\}, \mathcal{R}^+(\alpha_2, 2) = \{\alpha_1 + 2\alpha_2\}$ and $\mathcal{R}^+(\alpha_2, 3) = \{\alpha_1 + 3\alpha_2, 2\alpha_1 + 3\alpha_2\}$.

3. Equigeodesics

Let $\mathfrak{g}$ be the invariant inner and B be the Ad-invariant scalar product on $\mathfrak{m}$. Then B is given by $B(X, Y) = Q(\Lambda X, Y)$, where the linear operator $\Lambda : \mathfrak{m} \to \mathfrak{m}$ is symmetric and positive with respect to the Cartan–Killing form of g. We will denote by Λ such invariant metric.

Let $\mathfrak{m} = \mathfrak{m}_1 + \cdots + \mathfrak{m}_s$ be a decomposition of $\mathfrak{m}$ into irreducible inequivalent components of the isotropy representation. A consequence of Schur's lemma is that $\Lambda|_{\mathfrak{m}_i} = \lambda_i Id|_{\mathfrak{m}_i}$ for $i = 1,\cdots,s$ and therefore any invariant scalar product has the form

$$B(X, Y) = \lambda_1 Q(X, Y)|_{\mathfrak{m}_1} + \cdots + \lambda_s Q(X, Y)|_{\mathfrak{m}_s},$$

where $\lambda_1 > 0,\ldots,\lambda_s > 0$. Therefore the set of invariant metrics can be parameterized by

$$\mathcal{M}^G = \{(\lambda_1,\ldots,\lambda_s) \in \mathbb{R}^s : \lambda_1 > 0,\ldots,\lambda_s > 0\}.$$

Let G/K be a generalized flag manifold. A curve of the form $\gamma(t) = (exptX)\cdot o$ is called an equigeodesic on G/K if it is a geodesic with respect to each invariant metric on G/K. The vector X is called equigeodesic vector.

The study of equigeodesics in generalized flag manifolds started in [3] with the description of equigeodesics on $SU(n)$-flags.

We have the following algebraic characterization of equigeodesic vectors.

Proposition 3.1. [3] *Let G/K be a generalized flag manifold and $X \in \mathfrak{m}$ be a nonzero vector. Then X is an equigeodesic vector if, and only if,*

$$[X, \Lambda X]_{\mathfrak{m}} = 0, \tag{9}$$

for each invariant metric Λ.

We remark that to solve Eq. (9) is equivalent to solve a nonlinear algebraic system of equations whose variables are the components of the vector X. Analysing the Lie bracket of the form $[A_\alpha, B_\beta]$, $[A_\alpha, A_\beta]$ and $[B_\alpha, B_\beta]$ described in Eq. (6) is clear that if the structural constants $N_{\alpha,\beta}$, $N_{-\alpha,\beta}$, $N_{\alpha,-\beta}$ vanish (e.g. if $\alpha \pm \beta \notin \mathcal{R}$) then these bracket also vanish and the system can be simplified. In some cases (depending just on the $\mathfrak{m}_i$-parts of X) the nonlinear system vanishes completely (i.e. the system is identically zero). This motivates the following definition:

Definition 3.2. An equigeodesic vector is said to be

(a) *structural:* if the algebraic system associated to Eq. (9) vanishes completely.

(b) Otherwise we call an equigeodesic vector *algebraic*, i.e. the coordinates of the vector X come from a solution of a (not identically zero) nonlinear algebraic system associated to Eq. (9).

Definition 3.3. An equigeodesic vector $X \in \mathfrak{m}$ is trivial if $X \in \mathfrak{m}_i$ for some i; otherwise is said to be nontrivial.

Remark 3.4. By definition trivial equigeodesic vectors are structural equigeodesic vectors.

We now focus on generalized flag manifolds with $b_2(G/K) = 1$. In this case the tangent space at the origin splits into $\mathfrak{m} = \mathfrak{m}_1 \oplus \cdots \oplus \mathfrak{m}_s$ and a vector $X \in \mathfrak{m}$ is written as $X = X_{\mathfrak{m}_1} + \cdots + X_{\mathfrak{m}_s}$ with $X_{\mathfrak{m}_i} \in \mathfrak{m}_i (i = 1, \cdots, s)$.

Lemma 3.5. *Let G/K be a generalized flag manifold with decomposition (1). A vector*

$$X = X_{\mathfrak{m}_1} + \cdots + X_{\mathfrak{m}_s} \in \mathfrak{m}$$

is equigeodesic if, and only if

$$[X_{\mathfrak{m}_i}, X_{\mathfrak{m}_j}] = 0, \tag{10}$$

where $1 \leq i, j \leq s$.

Proof. Let $\pi : g \to \mathfrak{m}$ be the projection onto $\mathfrak{m}$, then $[X, \Lambda X]_{\mathfrak{m}} = \pi([X, \Lambda X])$. If $X = X_{\mathfrak{m}_1} + \cdots + X_{\mathfrak{m}_s} \in \mathfrak{m}$ then we have

$$\begin{aligned}[X, \Lambda X]_{\mathfrak{m}} &= \pi([X, \Lambda X]) \\ &= \pi([X_{\mathfrak{m}_1} + \cdots + X_{\mathfrak{m}_s}, \Lambda(X_{\mathfrak{m}_1} + \cdots + X_{\mathfrak{m}_s})]) \\ &= \pi([X_{\mathfrak{m}_1} + X_{\mathfrak{m}_2} + X_{\mathfrak{m}_3}, \lambda_1 X_{\mathfrak{m}_1} + \cdots + \lambda_s X_{\mathfrak{m}_s}]) \\ &= (\lambda_s - \lambda_1)\pi[X_{\mathfrak{m}_1}, X_{\mathfrak{m}_s}] + (\lambda_{s-1} - \lambda_1)\pi[X_{\mathfrak{m}_1}, X_{\mathfrak{m}_{s-1}}] + \cdots \\ &\quad + (\lambda_2 - \lambda_1)\pi[X_{\mathfrak{m}_1}, X_{\mathfrak{m}_2}] \\ &\quad + (\lambda_s - \lambda_2)\pi[X_{\mathfrak{m}_2}, X_{\mathfrak{m}_s}] + (\lambda_{s-1} - \lambda_2)\pi[X_{\mathfrak{m}_2}, X_{\mathfrak{m}_{s-1}}] + \cdots \\ &\quad + (\lambda_3 - \lambda_2)\pi[X_{\mathfrak{m}_2}, X_{\mathfrak{m}_3}] + \cdots + (\lambda_s - \lambda_{s-2})\pi[X_{\mathfrak{m}_{s-2}}, X_{\mathfrak{m}_s}] \\ &\quad + (\lambda_{s-1} - \lambda_{s-2})\pi[X_{\mathfrak{m}_{s-2}}, X_{\mathfrak{m}_{s-1}}] + (\lambda_s - \lambda_{s-1})\pi[X_{\mathfrak{m}_{s-1}}, X_{\mathfrak{m}_s}] \\ &= (\lambda_s - \lambda_1)[X_{\mathfrak{m}_1}, X_{\mathfrak{m}_s}] + (\lambda_{s-1} - \lambda_1)[X_{\mathfrak{m}_1}, X_{\mathfrak{m}_{s-1}}] + \cdots \\ &\quad + (\lambda_2 - \lambda_1)[X_{\mathfrak{m}_1}, X_{\mathfrak{m}_2}] + (\lambda_s - \lambda_2)[X_{\mathfrak{m}_2}, X_{\mathfrak{m}_s}] \\ &\quad + (\lambda_{s-1} - \lambda_2)[X_{\mathfrak{m}_2}, X_{\mathfrak{m}_{s-1}}] + \cdots + (\lambda_3 - \lambda_2)[X_{\mathfrak{m}_2}, X_{\mathfrak{m}_3}] \\ &\quad + \cdots + (\lambda_s - \lambda_{s-2})[X_{\mathfrak{m}_{s-2}}, X_{\mathfrak{m}_s}] + (\lambda_{s-1} - \lambda_{s-2})[X_{\mathfrak{m}_{s-2}}, X_{\mathfrak{m}_{s-1}}] \\ &\quad + (\lambda_s - \lambda_{s-1})[X_{\mathfrak{m}_{s-1}}, X_{\mathfrak{m}_s}].\end{aligned}$$

□

According to [6] let $\{x_\alpha, \alpha \in \mathcal{R}; h_i, 1 \le i \le k\}$ be a Chevalley basis of g, then we have $[h_i, h_j] = 0, 1 \le i, j \le k; [h, x_\alpha] =< \alpha, \alpha_i > x_\alpha, 1 \le i \le k, \alpha \in \mathcal{R}; [x_\alpha, x_{-\alpha}] = h_\alpha$, where h_α is a $\mathbb{Z}$-linear combination of $h_1, h_2, \cdots h_k$; if α, β are independent roots, $\beta - r\alpha, \cdots, \beta + q\alpha$ the α-string through β, then $[x_\alpha, x_\beta] = 0$, if $q = 0$, while $[x_\alpha, x_\beta] = \pm(r+1)x_{\alpha+\beta}$ if $\alpha + \beta \in \mathcal{R}$.

Thus we get when $i < j$ if $i + j \le s$ then $[X_{\mathfrak{m}_i}, X_{\mathfrak{m}_j}] \in \mathfrak{m}_{i+j} \oplus \mathfrak{m}_{j-i}$, if $i+j > s$ then $[X_{\mathfrak{m}_i}, X_{\mathfrak{m}_j}] \in \mathfrak{m}_{j-i}$; when $i = j$ if $2i \le s$ then $[X_{\mathfrak{m}_i}, X_{\mathfrak{m}_i}] \in \mathfrak{k} \oplus \mathfrak{m}_{2i}$, if $2i > s$ then $[X_{\mathfrak{m}_i}, X_{\mathfrak{m}_i}] \in \mathfrak{k}$.

Since X is equigeodesic if, and only if $[X, \Lambda X] = 0$ for each invariant metric $\Lambda = \{\lambda_1, \ldots, \lambda_s\}(\lambda_1 > 0, \cdots, \lambda_s > 0)$ and it occurs if, and only if $[X_{\mathfrak{m}_i}, X_{\mathfrak{m}_j}] = 0$, where $1 \le i, j \le s$.

Remark 3.6. When $s = 2$ this gives Proposition 3.5 in [4] of L. Grama and C. Negreiros.
The next proposition provides a family of structural equigeodesic vectors on generalized flag manifolds G/K with s isotropy summands, which depends only on the Lie algebra structure of g.

Theorem 3.7. *Let G/K be a generalized flag manifold with $\mathfrak{m} = \mathfrak{m}_1 \oplus \cdots \oplus \mathfrak{m}_s$ and $\Pi_K = \pi - \{\alpha_{i_0}\}$. Let the positive roots $\mathcal{R}^+(\alpha_{i_0}, 1) = \{\beta_1^1, \ldots, \beta_{k_1}^1\}, \mathcal{R}^+(\alpha_{i_0}, 2) = \{\beta_1^2, \ldots, \beta_{k_2}^2\}, \ldots, \mathcal{R}^+(\alpha_{i_0}, s) = \{\beta_1^s, \ldots, \beta_{k_s}^s\}$. If*

$$\beta_{j_1}^{i_1} \pm \beta_{j_2}^{i_2} \pm \cdots \pm \beta_{j_l}^{i_l} \notin \mathcal{R},$$

where $(1 \leq i_1 < i_2 < \cdots < i_l \leq s, 2 \leq l \leq s; j_1 = 1, \ldots, r_1(r_1 \leq k_{i_1}); j_2 = 1, \ldots, r_2(r_2 \leq k_{i_2}); \cdots; j_l = 1, \ldots, r_l)(r_l \leq k_{i_l})$. *Then all vectors in the subspace* $\mathfrak{m}_{\beta_1^{i_1}} \oplus \cdots \oplus \mathfrak{m}_{\beta_{r_1}^{i_1}} \oplus \mathfrak{m}_{\beta_1^{i_2}} \oplus \cdots \oplus \mathfrak{m}_{\beta_{r_2}^{i_2}} \oplus \cdots \oplus \mathfrak{m}_{\beta_1^{i_l}} \oplus \cdots \oplus \mathfrak{m}_{\beta_{r_l}^{i_l}}$ *are structure equigeodesic vectors. Here* $\mathfrak{m}_{\beta_j^i}$ *is defined by (4).*

Proof. Let $X = X_{\mathfrak{m}_{i_1}} + X_{\mathfrak{m}_{i_2}} + \cdots + X_{\mathfrak{m}_{i_l}}$, where $X_{\mathfrak{m}_{i_1}} = a_1^{i_1} A_{\beta_1^{i_1}} + b_1^{i_1} B_{\beta_1^{i_1}} + \cdots + a_{r_1}^{i_1} A_{\beta_{r_1}^{i_1}} + b_{r_1}^{i_1} B_{\beta_{r_1}^{i_1}}$ represent its $\mathfrak{m}_{i_1}$ component, $X_{\mathfrak{m}_{i_2}} = a_1^{i_2} A_{\beta_1^{i_2}} + b_1^{i_2} B_{\beta_1^{i_2}} + \cdots + a_{r_2}^{i_2} A_{\beta_{r_2}^{i_2}} + b_{r_2}^{i_2} B_{\beta_{r_2}^{i_2}}$ represent its $\mathfrak{m}_{i_2}$ component, $\ldots, X_{\mathfrak{m}_{i_l}} = a_1^{i_l} A_{\beta_1^{i_l}} + b_1^{i_l} B_{\beta_1^{i_l}} + \cdots + a_{r_l}^{i_l} A_{\beta_{r_l}^{i_l}} + b_{r_l}^{i_l} B_{\beta_{r_l}^{i_l}}$ represents its $\mathfrak{m}_{i_l}$ component. Choosing a Weyl basis $E_\alpha \in g_\alpha^{\mathbb{C}}$ $(\alpha \in \mathcal{R})$ of $g^{\mathbb{C}}$ with

$$[E_\alpha, E_\beta] = \begin{cases} 0, & if \ \ \alpha + \beta \notin \mathcal{R}, \\ N_{\alpha,\beta} E_{\alpha+\beta}, & if \ \ \alpha + \beta \in \mathcal{R}, \end{cases}$$

where the constants $N_{\alpha,\beta}$ satisfy $N_{\alpha,\beta} = -N_{-\alpha,-\beta}$ and $N_{\beta,\alpha} = -N_{\alpha,\beta}$. Since $\beta_{j_1}^{i_1} \pm \beta_{j_2}^{i_2} \pm \cdots \pm \beta_{j_l}^{i_l} \notin \mathcal{R}(j_1 = 1, \ldots, r_1; j_2 = 1, \ldots, r_2; \cdots; j_l = 1, \ldots, r_l)$, we have $N_{\beta_{j_1}^{i_1}, \beta_{j_2}^{i_2}} = N_{-\beta_{j_1}^{i_1}, \beta_{j_2}^{i_2}} = N_{\beta_{j_1}^{i_1}, -\beta_{j_2}^{i_2}} = \cdots = N_{\beta_{j_{l-1}}^{i_{l-1}}, \beta_{j_l}^{i_l}} = N_{-\beta_{j_{l-1}}^{i_{l-1}}, \beta_{j_l}^{i_l}} = N_{\beta_{j_{l-1}}^{i_{l-1}}, -\beta_{j_l}^{i_l}} = 0$. Since X is a equigeodesics vector if, and only if

$$[X_{\mathfrak{m}_i}, X_{\mathfrak{m}_j}] = 0,$$

for $1 \leq i, j \leq s$. By direct computation, using the relations in (6), we know that the system of equations above vanishes, therefore the vector X is a structural equigeodesics vector. □

Remark 3.8. When $s = 2$ this is Proposition 3.8 in [4] of Grama and Negreiros.

Example 3.9. We describe the flag manifold $G_2/U(2)$ associated with $\Pi_K = \Pi - \{\alpha_2\}$. The painted Dynkin diagram is

$$\bullet_{\alpha_2} \Lleftarrow \circ_{\alpha_1}.$$

The highest root is $\mu = 2\alpha_1 + 3\alpha_2$, consider the positive roots $\mathcal{R}_M^+$, let $\beta_1 = \alpha_2, \beta_2 = \alpha_1 + \alpha_2; \gamma = \alpha_1 + 2\alpha_2; \xi_1 = \alpha_1 + 3\alpha_2, \xi_2 = 2\alpha_1 + 3\alpha_2$. We have $\mathcal{R}^+(\alpha_2, 1) = \{\beta_1, \beta_2\}, \mathcal{R}^+(\alpha_2, 2) = \{\gamma\}$ and $\mathcal{R}^+(\alpha_2, 3) = \{\xi_1, \xi_2\}$. Since $\beta_2 \pm \xi_1$ are not roots, the vectors in $\mathfrak{m}_{\beta_2} \oplus \mathfrak{m}_{\xi_1}$ are structural equigeodesic vectors.

4. Structural Equigeodesic Vectors on Flag Manifolds

In this section we give a family of structural equigeodesic vectors in some generalized flag manifolds of exceptional Lie groups. We classify the positive roots that satisfy the hypothesis of Theorem 3.7, for the flag manifolds $F_4/SU(2) \times U(1) \times SU(3), E_6/SU(3) \times U(1) \times SU(3) \times SU(2)$ and partial classification of such root spaces in the flag $E_7/SU(5) \times U(1) \times SU(3)$.

4.1. Structural Equigeodesic Vectors on $F_4/SU(2) \times U(1) \times SU(3)$

Let $\Pi = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ be a system of simple roots for F_4 such that the highest root is given by $\mu = 2\alpha_1 + 3\alpha_2 + 4\alpha_3 + 2\alpha_4$. The flag manifold $F_4/SU(2) \times U(1) \times SU(3)$ is determined by $\Pi_K = \Pi - \{\alpha_2\}$. In order to simplify the notation we denote a positive root by their coefficients, that is, if $\beta = x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 + x_4\alpha_4$ is a positive root, we write $\beta = (x_1, x_2, x_3, x_4)$.

Consider the positive roots

$$\begin{array}{llll}
\alpha_1 = (0,1,0,0), & \alpha_2 = (0,1,1,0), & \alpha_3 = (1,1,1,0), & \alpha_4 = (0,1,2,2), \\
\alpha_5 = (1,1,2,2), & \alpha_6 = (1,1,0,0), & \alpha_7 = (1,1,2,0), & \alpha_8 = (1,1,1,1), \\
\alpha_9 = (0,1,1,1), & \alpha_{10} = (0,1,2,1), & \alpha_{11} = (1,1,2,1), & \alpha_{12} = (0,1,2,0), \\
\beta_1 = (1,2,3,2), & \beta_2 = (1,2,2,2), & \beta_3 = (1,2,4,2), & \beta_4 = (1,2,2,0), \\
\beta_5 = (1,2,3,1), & \beta_6 = (1,2,2,1), & \gamma_1 = (1,3,4,2), & \gamma_2 = (2,3,4,2).
\end{array}$$

We have $\mathcal{R}^+(\alpha_2, 1) = \{\alpha_1, \cdots, \alpha_{12}\}, \mathcal{R}^+(\alpha_2, 2) = \{\beta_1, \cdots, \beta_6\}$ and $\mathcal{R}^+(\alpha_2, 3) = \{\gamma_1, \gamma_2\}$ and the irreducible components of the decomposition $\mathfrak{m} = \mathfrak{m}_1 + \mathfrak{m}_2 + \mathfrak{m}_3$ are given by

$$\mathfrak{m}_1 = \sum_{\alpha \in \mathcal{R}^+(\alpha_2,1)} \mathfrak{m}_\alpha, \quad \mathfrak{m}_2 = \sum_{\alpha \in \mathcal{R}^+(\alpha_2,2)} \mathfrak{m}_\alpha, \quad \mathfrak{m}_3 = \sum_{\alpha \in \mathcal{R}^+(\alpha_2,3)} \mathfrak{m}_\alpha.$$

Proposition 4.1. *The root spaces whose roots satisfy Theorem 3.7 are listed in Table 1 for the generalized flag manifold $F_4/SU(2) \times U(1) \times SU(3)$. In particular all vectors in these subspaces are structural equigeodesic vectors*

4.2. Structural Equigeodesic Vectors on $E_6/SU(3) \times U(1) \times SU(3) \times SU(2)$

Let $\Pi = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ be a system of simple roots for E_6 such that the highest root is given by $\mu = \alpha_1 + 2\alpha_2 + 2\alpha_3 + 3\alpha_4 + \alpha_5 + 2\alpha_6$. The flag manifold $E_6/SU(3) \times U(1) \times SU(3) \times SU(2)$ is determined by $\Pi_K = \Pi - \{\alpha_4\}$. As before, in order to simplify the notation we denote a positive root just by their coefficients.

Consider the positive roots

$$\begin{array}{llll}
\alpha_1 = (0,0,1,0,0,0), & \alpha_2 = (1,1,1,0,0,0), & \alpha_3 = (1,1,1,1,0,0), & \alpha_4 = (1,1,1,1,1,0), \\
\alpha_5 = (1,1,1,1,1,1), & \alpha_6 = (1,1,1,0,0,1), & \alpha_7 = (1,1,1,1,0,1), & \alpha_8 = (0,1,1,0,0,0), \\
\alpha_9 = (0,1,1,1,0,0), & \alpha_{10} = (0,1,1,1,1,0), & \alpha_{11} = (0,1,1,0,0,1), & \alpha_{12} = (0,1,1,1,0,1) \\
\alpha_{13} = (0,1,1,1,1,1), & \alpha_{14} = (0,0,1,1,0,0), & \alpha_{15} = (0,0,1,1,1,0), & \alpha_{16} = (0,0,1,1,1,1), \\
\alpha_{17} = (0,0,1,0,0,1), & \alpha_{18} = (0,0,1,1,0,1); & \beta_1 = (1,1,2,1,0,1), & \beta_2 = (1,2,2,1,0,1), \\
\beta_3 = (1,1,2,1,1,1), & \beta_4 = (1,1,2,2,1,1), & \beta_5 = (1,2,2,1,1,1), & \beta_6 = (1,2,2,2,1,1), \\
\beta_7 = (0,1,2,1,0,1), & \beta_8 = (0,1,2,1,1,1), & \beta_9 = (0,1,2,2,1,1); & \gamma_1 = (1,2,3,2,1,1), \\
\gamma_2 = (1,2,3,2,1,2). & & &
\end{array}$$

We have $\mathcal{R}^+(\alpha_4, 1) = \{\alpha_1, \ldots, \alpha_{18}\}, \mathcal{R}^+(\alpha_4, 2) = \{\beta_1, \ldots, \beta_9\}$, and $\mathcal{R}^+(\alpha_4, 3) = \{\gamma_1, \gamma_2\}$, and the irreducible components of the decomposition $\mathfrak{m} = \mathfrak{m}_1 + \mathfrak{m}_2 + \mathfrak{m}_3$ are given by

$$\mathfrak{m}_1 = \sum_{\alpha \in \mathcal{R}^+(\alpha_4,1)} \mathfrak{m}_\alpha, \quad \mathfrak{m}_2 = \sum_{\alpha \in \mathcal{R}^+(\alpha_4,2)} \mathfrak{m}_\alpha, \quad \mathfrak{m}_3 = \sum_{\alpha \in \mathcal{R}^+(\alpha_4,3)} \mathfrak{m}_\alpha.$$

Proposition 4.2. *Consider the generalized flag manifold $E_6/SU(3) \times U(1) \times SU(3) \times SU(2)$. The root spaces whose roots satisfy Theorem 3.7 are listed in Table 2. In particular all vectors in these subspaces are structural equigeodesic vectors.*

4.3. Structural Equigeodesic Vectors on $E_7/SU(5) \times U(1) \times SU(3)$

Let $\Pi = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\}$ be a system of simple roots for E_7 such that the highest root is given by $\mu = 2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 3\alpha_5 + 2\alpha_6 + \alpha_7$. The flag manifold $E_7/SU(5) \times U(1) \times SU(3)$ is determined by $\Pi_K = \Pi - \{\alpha_5\}$. As before, in order to simplify the notation, we denote positive roots just by their coefficients.

$$
\begin{aligned}
&\alpha_1 = (0,0,0,0,1,0,0),\ \alpha_2 = (0,0,1,1,1,0,0),\ \alpha_3 = (0,1,1,1,1,0,0),\\
&\alpha_4 = (0,1,1,2,1,0,0),\ \alpha_5 = (1,0,1,1,1,0,0),\ \alpha_6 = (1,1,1,1,1,0,0),\\
&\alpha_7 = (1,1,1,2,1,0,0),\ \alpha_8 = (1,1,2,2,1,0,0),\ \alpha_9 = (0,0,0,1,1,0,0),\\
&\alpha_{10} = (0,1,0,1,1,0,0),\ \alpha_{11} = (0,0,0,0,1,1,0),\ \alpha_{12} = (0,0,0,1,1,1,0),\\
&\alpha_{13} = (0,0,0,0,1,1,1),\ \alpha_{14} = (0,1,0,1,1,1,0),\ \alpha_{15} = (0,0,1,1,1,1,0),\\
&\alpha_{16} = (0,0,0,1,1,1,1),\ \alpha_{17} = (1,0,1,1,1,1,0),\ \alpha_{18} = (0,1,1,1,1,1,0),\\
&\alpha_{19} = (0,1,0,1,1,1,1),\ \alpha_{20} = (0,0,1,1,1,1,1),\ \alpha_{21} = (1,1,1,1,1,1,0),\\
&\alpha_{22} = (1,0,1,1,1,1,1),\ \alpha_{23} = (0,1,1,2,1,1,0),\ \alpha_{24} = (0,1,1,1,1,1,1),\\
&\alpha_{25} = (1,1,1,2,1,1,0),\ \alpha_{26} = (1,1,1,1,1,1,1),\ \alpha_{27} = (0,1,1,2,1,1,1),\\
&\alpha_{28} = (1,1,2,2,1,1,0),\ \alpha_{29} = (1,1,1,2,1,1,1),\ \alpha_{30} = (1,1,2,2,1,1,1);\\
&\beta_1 = (1,1,1,2,2,1,0),\ \beta_2 = (0,1,1,2,2,1,1),\ \beta_3 = (1,1,2,2,2,1,0),\\
&\beta_4 = (1,1,1,2,2,1,1),\ \beta_5 = (1,1,2,3,2,1,0),\ \beta_6 = (1,1,2,2,2,1,1),\\
&\beta_7 = (1,2,2,3,2,1,0),\ \beta_8 = (1,1,2,3,2,1,1),\ \beta_9 = (1,2,2,3,2,1,1),\\
&\beta_{10} = (1,2,2,3,2,2,1),\ \beta_{11} = (1,1,2,3,2,2,1),\ \beta_{12} = (1,1,2,2,2,2,1),\\
&\beta_{13} = (1,1,1,2,2,2,1),\ \beta_{14} = (0,1,1,2,2,2,1),\ \beta_{15} = (0,1,1,2,2,1,0);\\
&\gamma_1 = (2,2,3,4,3,2,1),\ \gamma_2 = (1,2,3,4,3,2,1),\ \gamma_3 = (1,2,2,4,3,2,1),\\
&\gamma_4 = (1,2,2,3,3,2,1),\ \gamma_5 = (1,1,2,3,3,2,1).
\end{aligned}
$$

We have $\mathcal{R}^+(\alpha_5, 1) = \{\alpha_1, \ldots, \alpha_{30}\}, \mathcal{R}^+(\alpha_5, 2) = \{\beta_1, \ldots, \beta_{15}\}$ and $\mathcal{R}^+(\alpha_5, 3) = \{\gamma_1, \ldots, \gamma_5\}$ and the irreducible components of the decomposition $\mathfrak{m} = \mathfrak{m}_1 + \mathfrak{m}_2 + \mathfrak{m}_3$ are given by

$$\mathfrak{m}_1 = \sum_{\alpha \in \mathcal{R}^+(\alpha_5,1)} \mathfrak{m}_\alpha, \quad \mathfrak{m}_2 = \sum_{\alpha \in \mathcal{R}^+(\alpha_5,2)} \mathfrak{m}_\alpha, \quad \mathfrak{m}_3 = \sum_{\alpha \in \mathcal{R}^+(\alpha_5,3)} \mathfrak{m}_\alpha.$$

Proposition 4.3. *A partial classification of the root spaces whose roots satisfy Theorem 3.7 are listed in Table 3 for the generalized flag manifold $E_7/SU(5) \times U(1) \times SU(2)$. In particular, all vectors in these subspaces are structural equigeodesic vectors.*

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