Mantle Flow and Determining Position of LAB Assuming Isostasy

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Abstract—The assumption of isostatic equilibrium is often used in geophysics; e.g., the depth of the lithosphere–asthenosphere boundary (LAB) is determined using data regarding the gravity field and topography, together with the assumption of isostasy. However, isostasy implies a hydrostatic state, which is contrary to the mantle convection hypothesis, which states that most of the mantle matter is in motion. We therefore discuss herein the question of when the assumption of isostasy can be used. It is suggested that isostasy may be used for parts of oceanic plates (except for subduction zones, hotspots, and oceanic ridges) and for many continental regions (except for postglacial regions and regions of intensive volcanic or tectonic activity). Moreover, using the results of a numerical model of convection and calculations of dynamic topography, it is shown that some generalization of isostasy is possible in the form of deep dynamic isostasy (DDI). It is also indicated that, for some regions without isostatic equilibrium (e.g., postglacial regions), it is possible to use the “isostatic” method with some corrections to the results.

Key words: Isostasy, mantle convection, lithosphere–asthenosphere boundary, plate tectonics.

1. Introduction

Lithosphere, asthenosphere, and the lithosphere–asthenosphere boundary (LAB) are important terms in the physics of the solid Earth, but their meanings are understood differently by different specialists (e.g., Jones et al. 2010; Grad et al. 2014; Czechowski and Grad 2015, 2018). We apply herein the meaning used in the theory of plate tectonics, where the lithosphere is a mechanically resistant layer divided into tectonic plates, i.e., units that can move relative to each other. The asthenosphere is a layer with lower effective viscosity (compared with the underlying mantle), separating the lithospheric plates from the

mantle below. Unfortunately, the movement of some plates is very slow and the real movement of the mantle material below the plates is often not known. Note also that the position of the LAB also depends on p – T conditions and rock composition (including water and CO₂ content) (e.g., Anderson 1995).

Isostasy is a state of gravitational equilibrium such that blocks of the lithosphere “float” in the asthenosphere at an elevation that depends on their thickness and density according to Archimedes’s law. The term “isostasy” was coined by Dutton in 1889. When a certain part of the Earth’s crust reaches the state of isostasy, it is said to be in isostatic equilibrium. Two principal models of isostasy are used: the Pratt–Hayford model and the Airy–Heiskanen model (e.g., Watts 2001). Some regions are not in isostatic equilibrium, e.g., postglacial regions which are still uplifting (like Fennoscandia) after melting of the ice cover.

Pekeris (1935) was probably the first to indicate that flow of matter in the mantle would cause deviations of the observed topography from that corresponding to isostatic equilibrium. These deviations are now known as dynamic topography. Numerical models of convection can give values of the dynamic topography using data independent of the assumption of isostasy. These models indicate that dynamic topography can reach a few kilometers (e.g., Richards and Hager 1984; Spasojevic and Gurnis 2012). However, this effect is rather overlooked in research using isostasy, because most such work considers regions without obvious signs of nonisostatic behavior.

The occurrence of mantle convection means that, for certain regions, the assumption of isostatic equilibrium is doubtful. The main goals of the present work are therefore to (1) examine the reliability of methods that assume isostasy and indicate regions

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where this assumption may lead to errors, (2) indicate ways to improve these methods, (3) examine the possibility of extending the applicability of the assumption of isostasy, and (4) describe future perspectives.

The remainder of this manuscript is organized as follow: The “classic” approach is presented in Sect. 2. Sections 3 and 4 present basic information on mantle convection and its effect in isostasy. The idea of deep dynamic isostasy is presented in the next section. Section 6 presents a method to improve the “classic” approach for determination of the depth of the LAB. Conclusions are given in Sect. 7. According to the best of the author’s knowledge, the main ideas and methods presented in Sects. 5 and 6 are published here for the first time.

2. Local Isostasy

The idea of local isostasy is based on the assumption that the upper layer of the Earth can be treated as a series of rigid columns (blocks) that float on a liquid layer. This means that the total mass above some level (known as a compensation level) is the same for each column. According to plate

tectonics (see Figs. 1 and 2 for some particulars of this theory), the asthenosphere is a layer with lower effective viscosity compared with the mantle below. Therefore, the assumption that the compensation level lies in the asthenosphere is the most reasonable.

Fullea et al. (2006) considered a simple 1 dimensional (1D) model of lithosphere. It includes the crust with density ρ_c (kg m^{-3}), the lithospheric mantle with density ρ_m , sea water with density ρ_w , and the asthenosphere with density ρ_a . E (m) is the elevation ($E > 0$ for the land and $E < 0$ for the sea-floor), z_c and z_L are the depths of the crust–mantle boundary (Moho) and LAB, respectively (with respect to sea level). L_0 (m) is the position of a hypothetical column composed of matter with the density of the asthenosphere ρ_a . Assuming an isostatic state, one can find that the elevation E is given by (e.g., Fullea et al. 2006)

$$E = \frac{z_c(\rho_m - \rho_c) - z_L(\rho_m - \rho_a) - \rho_a L_0}{(\rho_c - \rho_w)}, \quad (1)$$

where $\rho_w = 0$ is used for the land (i.e., if $E > 0$). For the simplified model of the lithosphere presented above, the geoid anomaly N (m) is given by (Fullea et al. 2006; Grinc et al. 2014)

Figure 1

Schematic representation of plate tectonics. Upward and downward convective currents are marked using large arrows

Figure 2

A few schemes of situations where local isostatic equilibrium is not satisfied; see text for explanation. Legend: 1, 2, 3—matter of lithosphere, 4—matter of asthenosphere, 5—matter of mantle below the asthenosphere. Arrows indicate motion of matter

$$N = -\frac{\pi G}{g} [\rho_w E^2 + \rho_c (z_c^2 - E^2) + \rho_m (z_L^2 - z_c^2) + \rho_a (z_{\max}^2 - z_L^2)] - N_0, \quad (2)$$

where N_0 is a constant, $g = 9.81 \text{ m s}^{-2}$, and $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Topography E and geoid data used in the equations can be taken from databases (e.g., Pavlis et al. 2012). Fullea et al. (2006) use Eqs. (1–2) to determine the positions of the Moho and LAB, assuming densities ρ_c , ρ_m , ρ_a , and the compensation level depth $z_{\max}$. If the position of the Moho is known, the system of Eqs. (1–2) can be used to determine the density ρ_m or ρ_a , instead of assuming their values. Note also that density is a function of temperature and pressure, according to the equation of state

$$\rho(p, T) = \rho(0, 0) (1 + p/K - \alpha(T - T_0)), \quad (3)$$

where $\alpha (\text{K}^{-1})$ is the coefficient of thermal expansion and $K (\text{Pa})$ is the bulk modulus. K is a function of density and of elastic properties of the rock. This means that the system of Eqs. (1–2) can also be used

to determine different thermal and/or elastic properties of the lithosphere (see Czechowski 2018).

Methods based on Eqs. (1–2) may seem like a silver bullet for many geophysical problems. Note, however, that such methods suffer from some drawbacks:

1. For local isostatic equilibrium, the lithosphere should behave like independent, separated columns (blocks) that float on a liquid layer. The detachment could be a result of faults. This is possible if the crust contains many preexisting faults. These preexisting zones of weakness could be reactivated under tectonic stresses (e.g., Turcotte and Schubert 2002, p. 74), enabling independent motion of the columns. However, verification of this assumption is often not possible (see Fig. 2a and Fullea et al. 2006; Grinc et al. 2014; Krysiniski et al. 2013, 2015; Czechowski 2018).
2. Using Eq. (2), gravitational anomalies resulting from the mass below the asthenosphere should be removed. It is often assumed that all long-

wavelength components of gravity variations result from deep sources (see Grinc et al. 2014). However, this assumption may not be correct in some regions (as the wavelength of gravitational anomalies depend on the shape of the source mass in addition to its depth; see Fig. 2b).

3. The mantle material takes part in the global convection. This prompts the questions of when and where the assumption of isostasy is justified. This is discussed in the following sections.

We consider herein mainly the simple models shown in Fig. 2d. Their compensation level lies in the asthenosphere. Situations with a deeper compensation level are discussed later. Note also the opposite possibility: some crustal layers could (like salt) behave like a fluid. In such places, the local compensation depth could lie in the crust. There are also models with a compensation level lying in the lithosphere (e.g., Krysinski 2009). They often give consistent results. This indicates that the assumption of a shallower level of compensation is often reasonable. This may be due to the fact that the LAB is moving down in time as a result of cooling of the lithosphere. Therefore, in the absence of intense erosion or significant tectonic movements, the isostatic equilibrium (in relation to the old compensation level) can be maintained for a long time.

3. Mantle Convection

Mantle convection is a thermal process operating in the mantle. Numerical modeling is an important method to investigate this process (e.g., Czechowski 2014; Czechowski and Leliwa-Kopystynski 2012, 2013). Thermal convection is a major engine of plate tectonics, being responsible for motion of lithospheric plates, the activity of hot plumes below hotspots, earthquakes, etc.

Consider the convection in a layer of thickness d . This process can be described by the following system of equations: the Navier–Stokes equation, the equation of thermal conductivity, the equation of continuity, and the equation of state (e.g., Czechowski 1993):

$$\rho_0 \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}, \quad (4a)$$

$$\rho_0 c \frac{DT}{Dt} = k \nabla^2 T + Q, \quad (4b)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (4c)$$

$$\rho = \rho_0(1 - \alpha T), \quad (4d)$$

where t (s), $\mathbf{v}$ (m s⁻¹), p (Pa), T (K), ρ (kg m⁻³), ρ_0 (kg m⁻³), α (K⁻¹), μ (Pa s), g (m s⁻²), c (J kg⁻¹ K⁻¹), Q (W m⁻³), and k (W K⁻¹ m⁻¹) denote time, velocity, pressure, temperature, density, reference density, coefficient of thermal expansion, viscosity, gravity, specific heat at constant pressure, efficiency of heat sources per unit volume, and coefficient of thermal conduction, respectively. Note that $\kappa = k/(\rho_0 c)$, where κ (m² s⁻¹) is the coefficient of thermal diffusivity. T , $\mathbf{v}$, p , and ρ can be determined from the system of Eqs. (4a–d). In this model, the parameters ρ_0 , α , μ , g , c , Q , and κ are assumed to be constant. Their individual values are not critical, because they enter into the equations only in the form of dimensionless numbers (as discussed below). Note that we use the Boussinesq approximation of the system of equations (see Czechowski 1993 for details).

The number of parameters can be reduced by using natural units (n.u.) for the problem. Let us define the following natural units of time, length, temperature, and pressure:

$$\tau = d^2/\kappa, \quad \lambda = d, \quad \Theta = Qd^2/(c\kappa\rho_0), \\ \sigma = \mu\kappa/d^2,$$

where d is the thickness of the liquid layer. The dimensional time and space coordinates are $t = t'\tau$, $x = x'\lambda$, $y = y'\lambda$, where t' , x' , y' are dimensionless time and space coordinates. In this case, dimensional differential operators can be expressed using the dimensionless (primed) differential operators as $D/Dt = (1/\tau)D'/Dt'$, $\nabla = (1/\lambda)\nabla'$, $\nabla^2 = (1/\lambda^2)\nabla'^2$. Now must express all other quantities in n.u. and substitute Eq. (4d) into the Navier–Stokes equation (for details, see e.g., Czechowski 1993, p. 173; Schubert et al. 2001). These operations reduce the number of parameters in the system to two dimensionless numbers: the Rayleigh number Ra and the Prandtl number Pr . Eventually, after dropping primes, we obtain the following dimensionless system:

$$\frac{1}{\text{Pr}} \frac{D\mathbf{v}}{Dt} = -\nabla p_{\text{nh}} + \nabla^2 \mathbf{v} + \text{Ra} T \mathbf{i}_r, \quad (5a)$$

$$\frac{DT}{Dt} = \nabla^2 T + 1, \quad (5b)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (5c)$$

where all quantities in Eq. (5) are given in n.u., p_{nh} is the nonhydrostatic part of the pressure, and $\mathbf{i}_r = \mathbf{g}/g$ is a unit vector directed in accordance with gravity. Ra and Pr are given by

$$\text{Ra} = \frac{\rho_0 \alpha g d^3 \Delta T}{\kappa \mu}, \quad (5d)$$

$$\text{Pr} = \frac{\mu}{\rho_0 \kappa}, \quad (5e)$$

where $\Delta T = \Theta = Qd^2/(c\kappa\rho_0)$. Note that the mantle convection (in the Earth and many other celestial bodies) is a solid-state convection; i.e., it operates in solid matter which behaves like a liquid for very slow processes. The effective viscosity of the Earth's mantle is very high, e.g., $\mu = \sim 10^{21}$ Pa s (see, e.g., Fjeldskaar 1994). This means that the Prandtl number is very large ($\sim 10^{23}$) and can be assumed to be infinite. That allows us to neglect the left-hand side in the Navier–Stokes equation. In the asthenosphere, the viscosity is somewhat smaller, so the Prandtl number may be 10–100 times smaller, although this does not change the conclusion regarding the left-hand side of Eq. (5a).

Eventually, the dimensionless form of the system of Eqs. (5) has only one dimensionless parameter, the Rayleigh number Ra, instead of the seven dimensional parameters ρ_0 , α , μ , g , c , Q , and κ . Note, however, that additional parameters are introduced by boundary and initial conditions, although they can also be expressed in dimensionless form. The dimensionless system is more general. Its solution is valid for any convection with the same values of the dimensionless parameters; i.e., the values of the seven dimensional parameters can be changed as long as the values of the dimensionless parameters remain the same. For orientation, we give here exemplary approximate (but realistic) values, assuming convection in the upper mantle (with $d = 680$ km and $\text{Ra} = 400,000$): $\alpha = 10^{-4} \text{ K}^{-1}$, $\mu = 10^{21} \text{ Pa s}$, $g = 10 \text{ m s}^{-2}$, $c = 1000 \text{ J kg}^{-1} \text{ K}^{-1}$, $Q = 2.75 \times 10^{-9} \text{ W m}^{-3}$, $\kappa = 10^{-6} \text{ m}^2 \text{ s}^{-1}$. Consequently, the same

solution may be used for some convection cells in the Earth's mantle, convection in the mantles of other terrestrial planets or icy satellites (e.g., Enceladus) (see Czechowski 2014; Czechowski and Leliwa-Kopystynski 2012, 2013).

The above system of equations can be used as the basis for a model of convection in the Earth's mantle. Note, however, that this model (and, in fact, also many other models of convection in the mantle) does not include the elastic or brittle properties of the lithospheric plates (see, e.g., Bürgmann and Dresen 2008 for more about rheology). These properties could be important factors, especially in subduction zones and for continental lithosphere. In a spreading center, the lithosphere is thin and hot, so its elastic properties are not very important (see, e.g., Lachenbruch and Morgan 1990).

4. Isostasy and Plate Tectonics

As mentioned above, the flow of matter in the mantle will cause deviations of the topography (known as dynamic topography) (e.g., Spasojevic and Gurnis 2012). Currently, based on theory and observations, it is widely accepted that intensive thermal convection operates in most of the mantle below the lithosphere.

Unfortunately, different models of mantle convection often give significantly different results concerning the magnitude of the dynamic topography (cf., e.g., Spasojevic and Gurnis 2012 and Conrad and Husson 2009). This is not strange, because the dynamic topography depends on many factors, while many properties of the mantle (e.g., its rheology) are not known with satisfactory accuracy. Fortunately, the differences between the results of such models mainly relate to the magnitude of the dynamic topography, while the distributions of positive and negative deviations of the large-scale dynamic topography is satisfactorily consistent.

According to present knowledge of mantle convection (including results of numerical models) and plate tectonics, lack of isostasy is expected in a few types of regions:

1. Subduction zones, where downward convection currents occur (Figs. 1, 2c), involving two

lithospheric plates (an upper horizontal one and a lower inclined one). A subduction zone can also include an oceanic trench, island arc, or mountain ranges. Satisfactory calculations of dynamic topography are extremely difficult here. This does not change the general conclusion that full isostatic equilibrium is unlikely in an active subduction zone.

2. Oceanic spreading centers, where the lithosphere is thin, but the upward flow of mantle material below the lithosphere causes large vertical forces that may not allow for isostasy (see, e.g., Spasojevic and Gurnis 2012 and Fig. 2e). Fortunately, the structure of the lithosphere and the position of the LAB are described well enough by the model of the cooling plate (e.g., Turcotte and Schubert 2002, p. 157).
3. Hotspots. These are areas located above hot mantle plumes. The plumes cause vertical forces that may not allow for isostasy. Plumes can have different sizes and intensities, originating at various depths (e.g., Foulger et al. 2000). Their effect on isostatic equilibrium can be moderate in some places, but large in others.
4. Postglacial regions, where the absence of isostatic equilibrium results from melting of the ice cover. The lack of isostasy here is not a result of mantle convection. During the last ice age, some regions (e.g., Fennoscandia, Canada) were covered by an ice sheet (~ 3 km thick 20,000 years ago). The weight of the ice layer caused the lithosphere to deform, forcing material in the asthenosphere to flow away from the loaded region (Fig. 2d). Deglaciation led to slow glacial isostatic adjustment. Presently, the uplift rates are ~ 1 cm year⁻¹ and the total uplift (from the end of deglaciation) could be a few hundred meters at the center of rebound. Note that the uplift of these regions leads to slow restoration of the isostatic equilibrium.
5. Some zones above horizontal currents in the asthenosphere. Such flow could be a result of forces of different origin, as discussed below.

Figure 2 presents schemes of situations where the simple approach based on Eqs. (1–2) could lead to incorrect conclusions. Figure 2a shows two different

lithospheric blocks connected together, which means that they cannot independently achieve isostasy. Eventually, an intermediate state will be established and Eqs. (1–2) will not be applicable to any of these blocks separately. This is the case in which the condition of applicability of the isostasy is not met (point 1 in Sect. 2). Unfortunately, the existence or absence of a fault between the blocks is often difficult to determine. In the case of block mountains (e.g., Sierra Nevada), faults separate them from surrounding regions. In the case of fold and thrust belts, such faults could be absent. In Fig. 2b, a horizontal, large-scale (lens-shaped) structure with different density in the crust introduces a long-wave gravitational anomaly. This anomaly may be (incorrectly) removed according to the typical criteria used in the “isostatic” method, which could lead to use of incorrect values of gravitational anomalies and ultimately incorrect determination of the LAB depth (point 2 in Sect. 2). Such a problem may be caused by an unrecognized large-scale layer of salt or another large-scale layer with different density in the lithosphere (e.g., Gao and Shen 2014).

Figure 2c presents the situation in subduction zones. A simple model used to derive Eqs. (1–2) cannot be applied for this complicated system with two moving lithospheric plates, two LABs, etc. (see, e.g., Czechowski and Grad 2018). Moreover, mass motion could introduce large dynamic topography. Figure 2d presents the flow in the asthenosphere during postglacial rebound: the matter flows towards the center of postglacial uplift; see Sect. 6 for more discussion. A situation typical of spreading centers (or hotspots) is shown in Fig. 2e. A large ascending current below the lithosphere could lead to large dynamic topography.

5. Idea of Deep Dynamic Isostasy

Let us consider mantle convection using the numerical model based on Eqs. (5a–5e). Figures 3, 4, and 5 present a two-dimensional model of two cells of mantle convection for Rayleigh number $Ra = 400,000$. This value corresponds to a small or medium-sized convection cell in the Earth’s mantle (e.g., the length of the cell could be approximately

Figure 3

Temperature distribution for convection with $Ra = 400,000$. Natural units are used. Upward and downward convective currents are marked

Figure 4

Distribution of stream function for convection given in Fig. 3. Natural units are used

from ~ 600 to ~ 1500 km and its thickness ~ 300 km to 700 km). Stress-free boundary conditions are used. The upper boundary is isothermal ($T = 0$), the lateral boundaries are adiabatic (zero heat flow), and the density of heat flow at the lower boundary is equal to 0.3 n.u. Steady-state convection

is calculated. We choose this model because the obtained convective cells exhibit all the basic properties of real cells in the mantle that are important for our discussion, and the calculations are not numerically very demanding. The finite difference method with upwind approximation of the advective term in

Figure 5

Vertical stress component on upper and lower boundary of convective cells given in Figs. 3 and 4. Natural units (n.u.) are used. The stresses correspond to the dynamic topography with amplitude of ~ 900 m (assuming the convective cells in the upper mantle), which means that the dynamic topography varies in the range from ~ -450 to $\sim +450$ m, and these values should be used for an approximate assessment of the shift of the LAB

the equation of the heat transfer (Eq. 5b) is used for discretization. The resulting system of equations is solved using the Chebyshev and minimal residual methods. The transformation from n.u. to SI units is quite simple. One just expresses the values of n.u. in SI units using the formula $\tau = d^2/\kappa$ for time units and $\Theta = Qd^2/(c\kappa\rho)$ for temperature units. The velocity unit is of course $d/\tau = \kappa/d$. Most of the parameters describe the properties of the rock of the mantle,

while some parameters describe the properties of the modeled convective cell.

Figure 3 shows the isotherms, i.e., contour lines connecting points having the same temperature. A large upwelling plume of hot matter is seen in the center of the figure, corresponding to the hot ascending current below the spreading center at an oceanic ridge (see also Fig. 1). Two currents of cold downward motion are found next to the lateral sides, corresponding to subduction zones. Note also the

upper thermal boundary layer, which corresponds to the lithosphere.

Figure 4 presents contours of constant stream function S , indicating the direction of flow. The velocity vector $\mathbf{v}$ is given by the following formulae:

$$v_x = -\partial S / \partial y \text{ and } v_y = \partial S / \partial x;$$

i.e., the value of the speed is inversely proportional to the distance between these lines.

Consider now the forces acting on the upper and lower boundary. Above the central part, i.e., above the hot rising current of low density, we observe the forces raising the surface (Fig. 5). These forces are responsible for the dynamic topography (see, e.g., Richards and Hager 1984; Spasojevic and Gurnis 2012). At the same time, the forces at the lower boundary (below the plume) act in the opposite direction, causing a depression of the lower boundary (Fig. 5). In this way, the surface upraising corresponds to the lowering of the lower boundary of the convective cell. Forces acting on the current of cold convection (which contains a sinking plate in the subduction zone) lead to opposite effects: The surface (i.e., ocean floor) is lowered (in the oceanic trench), while the lower boundary is raised (Fig. 5).

This means that the total mass between the deformed boundaries in a vertical column with a sinking plate is the same as the mass in the column with the hot plume. This conclusion can be regarded as an extension of isostasy and may be called “deep dynamic isostasy” (DDI). Compared with “classic” isostasy, the difference lies in the position of the compensation level, which for DDI is located at the bottom of the convective cell instead of the asthenosphere. For the whole mantle convection, this means the core–mantle boundary (CMB). For convection confined to the upper mantle, this may mean the compensation level at ~ 680 km. According to present knowledge, mantle convection extends over the whole mantle, but in some places (e.g., under small plates), it may be confined to the upper mantle only. For more discussion see, e.g., Foulger et al. (2000), Livermore et al. (2005), and Birkenmajer et al. (1990).

What are the benefits of using DDI? In contrast to the “classic” isostatic equilibrium, DDI is also satisfied if there is convection in the mantle. Therefore,

DDI can be used as a constraint, similar to “classic” isostasy. To use DDI, one does not have to know the details of the velocity field, but only the boundaries of the convective cell under consideration (in particular, the depth of the lower boundary of the cell). When using DDI, large-scale gravitational anomalies resulting from density anomalies above the compensation level should also be taken into account. Moreover, the compensation level for DDI is significantly deeper than in the “classic” method described in Sect. 2. This also means that one must make more assumptions about the structure of the considered layer. Eventually, the accuracy of results obtained using DDI could be worse than for the “classic” method.

The dynamic topography can also be used to apply some corrections to the “isostatic” method. Note that the dynamic topography is that part of the topography that is dynamically supported by the movement of mantle material rather than buoyancy. Assuming that the model given in Eqs. (1–2) is sufficiently realistic for the considered case, we introduce an appropriate correction to the topography given by Eq. (1). This correction of E will cause a shift of the LAB by a value depending on the densities in the system. For a small difference between the densities ρ_m and ρ_a , the shift in the LAB may be significant, being proportional to the dynamic topography multiplied by $\rho_c / (\rho_m - \rho_a)$. In their model, Fullea et al. (2006) used $\rho_m = 3245 \text{ kg m}^{-3}$, $\rho_a = 3200 \text{ kg m}^{-3}$, and $\rho_c = 2780 \text{ kg m}^{-3}$. For such values, the shift of the LAB exceeds ~ 60 times the value of the dynamic topography. In more advanced models (e.g., in the model considering the full equation of thermal conduction by Czechowski 2018), the shift in the LAB related to the dynamic topography may depend on the parameters of the system in a more complicated way. However, attention should be paid to the reliability problems of dynamic topographic values given by convective models (discussed in Sect. 4).

Let us now discuss the thermal regimes close to the upper boundary of the convective cell (Fig. 3). A few different regimes can be found, corresponding to different directions of motion: upwards, downwards, and horizontal. In vertical currents, two adiabatic regimes can be assumed (“cold” for downward

motion and “hot” for upward). The adiabatic gradient of temperature is rather moderate. In regions of horizontal motion (i.e., the upper thermal boundary layer), conduction is the dominant process of heat transfer, leading to a high vertical temperature gradient. Therefore, use of an adiabatic temperature gradient is justified only in areas where there are actually vertical convective currents or other vertical displacements of hot matter in the mantle (intrusions, diapirs). However, in other places, it is advisable to use a gradient corresponding to the thermal conductivity. In the case of old continental areas, this gradient may be small (similar to adiabatic).

The author is convinced that further development of models of convection in the mantle will lead to reliable three-dimensional models with sufficient resolution for regional problems (see, e.g., Levander and Miller 2012). Note also that it is necessary to include realistic rheological properties of the lithosphere in such models. It will then be possible to directly use the dynamic topography obtained from convective models. Better knowledge of the deep distribution of density will enable proper consideration of the gravitational field, without use of the simplified criterion with a spatial scale as a measure of the depth of field sources. Present advances in mantle convection models indicate that this will probably be achieved in a dozen years. Until then, we are doomed to use various simpler methods (e.g., isostasy, DDI, or methods presented in the next section).

6. Isostasy and Horizontal Motion in the Asthenosphere

The significant discrepancies between the dynamic topography given by various convection models suggest that they should be used with caution.

We also indicate worse accuracy when using DDI. Therefore, it is worth discussing the possibilities of extending the “classic” methods of determining the LAB (i.e., the methods presented in Sect. 2) for regions where deviations from isostasy are expected. We consider here horizontal motion in the asthenosphere as a cause of these deviations.

There are two main causes of horizontal flow in the asthenosphere. Such flow could be forced by the motion of oceanic plates. This could be modeled using Couette flow (see, e.g., Turcotte and Schubert 2002, p. 229). In this case, the state of isostasy may be approximately achieved, because the horizontal pressure gradient is low. This is the situation of oceanic plates (Figs. 1, 2e, 3). However, near spreading centers, there are wide ascending currents, which can lead to significant dynamic topography of the oceanic lithosphere (e.g., Spasojevic and Gurnis 2012). Consequently, Couette flow can be used as a model of the flow in some regions, but only outside oceanic ridges and subduction zones.

Horizontal flow in the asthenosphere may also be due to a horizontal pressure gradient. Consider the simple model of the asthenosphere as a liquid layer of thickness h with constant viscosity μ and a horizontal pressure gradient dp/dx . This situation is shown below the middle plate in Fig. 2d (where the flow is horizontal). In this case, the flow can be modeled by Poiseuille flow (see, e.g., Turcotte and Schubert 2002, p. 229). The solution of the Navier–Stokes equation (for such a situation) gives a horizontal velocity of (e.g., Turcotte and Schubert 2002, p. 229)

$$u(y) = \frac{1}{2\mu} \frac{dp}{dx} \left(y^2 - \frac{h^2}{4} \right), \quad (6)$$

where y is the distance from the center of the asthenosphere (see the coordinate system in Fig. 2d). The maximum velocity $U_{\max} = (h^2/8\mu)dp/dx$ occurs at $y = 0$. The corresponding pressure gradient is

$$\frac{dp}{dx} = \frac{8\mu}{h^2} U_{\max}. \quad (7)$$

This gradient depends on the effective viscosity and the thickness of the asthenosphere (Fig. 6). For $h = 10$ km, $\mu = 10^{20}$ Pa s, and $U_{\max} = 1$ cm year⁻¹, the pressure gradient in the asthenosphere is ~ 300 Pa m⁻¹. Over a distance of 100 km, this means a pressure change of $\sim 3 \times 10^7$ Pa, corresponding to a rock column of height $H = \sim 1000$ m. This is a significant value, indicating that calculations of the LAB position using Eqs. (1–2) will require significant correction (much larger than H). The correction depends on the density difference [see the discussion about the role of dynamic topography in

Figure 6

Pressure gradient versus viscosity in an asthenosphere of different thicknesses. The maximum velocity in the center of the asthenosphere is assumed to be 1 cm year^{-1}

Sect. 5, according to which the shift of the LAB exceeds the value of H by around 60 times, reaching $\sim 60 \text{ km}$ for $H = 1 \text{ km}$ for the parameter values used by Fullea et al. (2006)].

7. Conclusions

1. The theory of mantle convection indicates the possibility of significant dynamic topography in many regions of the Earth. Therefore, the role of the hypothesis of isostasy in geophysical research should be reconsidered.
2. Present models of mantle convection cannot give consistent values for the local dynamic topography, so simplified methods are still useful.
3. In Sect. 5, an extension of isostasy is introduced: “deep dynamic isostasy” (DDI). DDI is also satisfied if there is convection in the mantle, and it can be used as a constraint, similar to “classic” isostasy. Compared with “classic” isostasy, the difference lies in the position of the compensation

level, which for DDI is located at the bottom of the convective cell instead of the asthenosphere.

4. There are a few thermal regimes in a convection cell. In vertical currents, an adiabatic regime can be assumed. In the lithosphere, conduction is the dominant process (Sect. 5).
5. In some cases, “isostatic” methods can be improved by introducing corrections that take into account some effects of the flow in the mantle. A simple example is presented in Sect. 6.
6. In some cases, compensation levels could be assumed to lie in the lithosphere. It is suggested that, in specific situations, isostatic equilibrium (in relation to the old compensation level) can be maintained for a long time (Sect. 2).

This study indicates that “isostatic” methods (with suggested improvements such as DDI or methods presented in Sect. 6) could still be of significant value, therefore we plan to extend them. Presently, we are working on the application of these methods to specific geophysical problems. To do so, one must develop some standard procedures. To

make full use of the possibilities of the method described in Sect. 6, full data on the ascension of postglacial regions are necessary. One also needs a good model of the rheological properties of the crust and upper mantle. Then, using numerical methods, it becomes possible to calculate the 3 dimensional flow (using the equations of fluid motion, Eq. 4a, c) and thereby correct the results of the “isostatic” model. It is also worth improving the simple 1 dimensional model of the lithosphere discussed in Sect. 2, which could be done using methods presented in the papers of Czechowski (2018) or Jones et al. (2014). Developing standard procedures to use deep dynamic isostasy (DDI, Sect. 5) is even more complicated and will require a new approach to gravimetric data and use of data from seismic tomography to determine convective cell boundaries. This method will, however, open the possibility of researching deeper regions of the mantle than “classic” methods based on isostasy.

The results obtained using the discussed methods could be verified using other methods; e.g., the depth of the LAB could be verified using seismic methods (e.g., Grad et al. 2009, 2014; Levander and Miller, 2012). However, note that, according to Jones et al. (2010), the seismic LAB may sometimes not be the same as the LAB used in the theory of plate tectonics.

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