Book Review

Mathematical Problems in Meteorological Modelling by Bátka A., Csomós P., Faragó I., Horányi A., Szépszó G., Springer, Springer International Publishing Switzerland 2016, ISBN 978-3-319-40155-3

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The book contains the selected papers presented at the workshop entitled “Mathematical Problems in Meteorological Modelling” held in Eötvös Loránd University in Budapest, Hungary, in May 2014. The basic purpose of the workshop was to provide a wide forum for presentation and discussion of recent developments in mathematical modeling in meteorology, related to the advanced numerical methods, air-quality investigations, and the numerical weather prediction. The text is organized in three parts containing 12 chapters.

Part I, *Numerical Methods for Meteorological Problems*, is devoted to the presentation that the basic numerical techniques used to the atmospheric modeling. It starts with the chapter by A. Zlotnik and V. Gavrilin entitled *On a Conservative Finite-Difference Method for 1D Shallow Water Flows Based on Regularized Equations*. The governing system of equations can be deduced from the Navier–Stokes equations for an incompressible free surface fluid, supposing that the pressure field is near to hydrostatic equilibrium, and then, integrating vertically the whole system. It forms the first authoritative model for the analysis that the large-scale atmospheric processes. Next, the regularized 1D shallow water system of equations is introduced, to guarantee and satisfy the energy balance law. A three-point symmetric in space discretization scheme is developed, together with the proof that for this new discretization the pointwise energy balance law holds. The

numerical experiments for the explicit in time finite-difference scheme are presented and discussed. The computational results concern the flow in a channel with a dam break and the flow in a channel with a flat bottom except for the small hump of the parabolic shape in its middle part. The practical error estimation is also performed.

Chapter 2, *Discretization in Numerical Weather Prediction: A Modular Approach to Investigate Spectral and Local SISL Methods* by S. Caluwaerts et al., reviews the selected temporal and spatial discretization methods, used in numerical weather prediction models (NWP). An overview of discretization strategies used in some European NWP is given first. These include two main classes of time discretization schemes, namely the explicit and semi-implicit (SI) schemes, and three categories of spatial discretization schemes. Two SI family schemes are considered: the horizontally explicit vertically implicit scheme and a semi-Lagrangian (SL) approach for advection with a SI discretization called as SISL time discretization scheme. The spatial discretization schemes cover the finite-difference approximations, the finite-element method, and the finite volume approach. The next section concentrates on the spectral SISL discretization schemes, one of the main NWP approaches. The subsequent section concerns the Z-grid method as an attractive alternative to the conventional Arakawa A-grids, because of the Z-grid simulates the geostrophic adjustment process better than any of the A-systems. Some reflections and conclusions for future research are also presented.

Fractional calculus seems to be the adequate and promising tool to solve complex physical problems with memory effects and nonlocality. Chapter 3,

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Turbulence Modeling Using Fractional Derivatives by B. J. Szekeres, analyzes the two-dimensional turbulence model with assumption that the shear stresses are random variables with Lévy statistics. Numerical solutions of 2-D turbulent flow over a backward-facing step at various Reynolds numbers are presented, and the influence of two parameters, the fluid memory, and a stability parameter on the practical computations are discussed.

Chapter 4, *A Parallel Numerical Solution Approach for Nonlinear Parabolic Systems Arising in Air Pollution Transport Problems* by J. Karátson and B. Kovács, focuses on numerical treatment of complex problems appearing in environmental modeling. The time-dependent reaction–convection–diffusion transport system of equations, with the nonlinear coupling in the reaction term subjects to the mixed boundary conditions and localized interface conditions, is formulated first. Then, the approach based on operator preconditioning method is used, which consists of the four steps: time discretization (the implicit Euler method), space discretization (the finite-element method), outer iteration for the nonlinear systems (damped Newton’s method), and inner iteration for the linearized systems (preconditioned conjugate gradient method). The effectiveness of the method proposed here is illustrated by the simplified air pollution model involving the ten main pollutants.

Part II, *Air Quality Modelling*, presents certain numerical methods applied when solving the air pollution transport models. It begins with the chapter 5, *Eulerian and Lagrangian Approaches for Modelling of Air Quality* by Á. Leelőssy et al., devoted to the atmospheric dispersion models. Two basic groups of atmospheric transport equations are described: the Eulerian (deterministic) and the Lagrangian (stochastic) models. The main operational tool, the Danish Eulerian model (DEM), the large air pollution model for studying various phenomena connected with transport in the atmosphere in which the 56 chemical species are taken into account, is briefly discussed. Also, the recent UNI-DEM (the Unified Danish Eulerian model), which is able to treat 168 chemical contaminants, is mentioned. The term “Lagrangian” is referred to a wide range of models which simulate pollutant dispersion relative to a shifting reference frame. The basic

equations are the stochastic ordinary differential equations with the advection and of turbulent velocity field treated as the Markov process, described by the Langevin equation. The “puff” approach that merges Gaussian and Lagrangian turbulence description is also quoted in context of the HYSPLIT model (the hybrid single-particle Lagrangian integrated trajectory model).

Chapter 6, *Hydrodynamic Modeling of Industrial Pollutants Spreading in Atmosphere* by V. A. Prusov and A. Y. Doroshenko, deals with the numerical methods for effective description of propagation of atmospheric pollutants from towering stationary sources of emissions under real weather conditions. For the efficient description of forecasting regional meteorological processes, the “one-way interaction” approach implemented on embedded grid telescoping scheme is used. The proposed numerical method has been tested by the Hydrometeorological Centre of Ukraine and successfully applied in complex regional ecological–meteorological modeling in Ukraine.

The mathematical modeling of the environmental problems that are referred to the unbounded domains is particularly challenging task. Chapter 7, *Coordinate Transformation Approach for Numerical Solution of Environmental Problems* by T. Chernogorova et al., presents the special technique for the treatment of the point at infinity: coordinate transformation of the infinite domain onto a finite region. To obtain the numerical solution of the boundary-value problems for the advection–diffusion equation, the logarithmic transformation is chosen for one-to-one mapping of the computational domain. The well posedness of the convection–diffusion problems and the properties of its solutions are briefly discussed. The positive splitting numerical procedure for the solution of the 2D non-stationary model of air pollution is proposed, and for approximation of the problems, the finite volume difference schemes are suggested.

Chapter 8, *Impact of Climatic Changes on Pollution Levels* by Z. Zlatev et al., contains a terse discussion regarding the climate dynamics and of the pollution levels related to ozone in selected regions of Europe. The particular attention is given to the analysis of three climatic scenarios in the frame of the UNI-DEM, the advanced mathematical model used in

the computations. The model includes the following physical and chemical processes: the horizontal advection and horizontal diffusion, nonlinear chemical reactions and emission sources, dry and wet deposition, and vertical exchange processes. In sum, the discretization of this large-scale air pollution model leads to more than 142 million equations! The computer implementation of the calculations is briefly described including the appropriate splitting procedures together with the parallelization of the computational processes and the optimization of the cache memories. Some of the calculation results referring to the concentration of selected pollutants in several parts of Europe are presented and discussed.

Part III, *Meteorological Data Assimilation and Probabilistic Forecasting*, concentrates on numerical weather prediction, and the important related areas of data assimilation and predictability. Chapter 9, *An Invitation to Meteorological Data Assimilation* by Á. Bodó and P. Csomós, concerns the several recognized data assimilation methods used in NWP models, namely optimal interpolation, variational approaches, and Kálmán filter techniques. The initial section provides a brief overview of the key probabilistic tools needful later on.

Subsequently, the traditional optimal interpolation and the variational methods are discussed in one and more dimensions, respectively. The following section is the gentle introduction to the various Kálmán filter techniques, originally suggested by Kálmán in 1960 who obtained an equation for the time evolution of the error covariance matrix. Two spectacular examples are demonstrated, including the linear interaction model and the famous Lorenz system, which give an insight into how the parameters of the data assimilation methods affect the solution's precision. Some remarks regarding the nonlinear data assimilation for meteorological problems, i.e., the utilization of incremental variational analysis, and the particle filtering technique are also sketched.

Chapter 10, *Analysis of the Data Assimilation Methods from the Mathematical Point of View* by T. Szentimrey, focuses on the basic mathematical issues connected with the statistical approach to the data assimilation based on the Bayes estimation theory. A

unified and statistically consistent formulation of the problem is presented, and the use of the Bayesian cost function for data assimilation is suggested.

The predictability of weather and climate forecasts is connected with the inherent uncertainties in both initial conditions and model formulations. Chapter 11, *Ensemble Methods in Meteorological Modelling* by M. Szűcs et al., deals with an ensemble modeling approach to determine forecasts of future atmospheric states. The terse description of uncertainty sources in NWP models, that justifies the concept of probabilistic forecasts, is presented first. Then, the basic philosophy used for construction of the ensemble prediction system (EPS) is described. Usually, the EPS contains multiple (typically between 10 and 50) working NWP models, one starting with unperturbed initial weather conditions (the control forecasts) and others, with the perturbed initial conditions defined by adding small perturbations to the operational analysis for the respective time (1 day, for instance). Some examples of EPS are presented and discussed, including the ECMWF (European Centre for Medium-Range Weather Forecasts) EPS, the HMS (Hungarian Meteorological Service) EPS, as well as the Coupled Model Inter-comparison Projects dedicated to climate models. Also, the selected visualization methods facilitating the interpretation of EPS results (plume diagram, probabilistic maps, and the stamp diagram) are briefly explained.

Chapter 12, *Quantifying Sources of Uncertainty in Temperature and Precipitation Projections over Different Parts of Europe* by P. Szabó and G. Szépszó, offers a comprehensive overview of the basic sources of uncertainties arising in climate projections. These uncertainties in climate predictions emerge from three distinct sources: the first is the internal variability of the climate system; the second is the response (model) uncertainty, and the third is future anthropogenic activity. Results for different uncertainties are explored through temperature and precipitation time series, and are analyzed in quantitative way over various regions in Europe. The problem how to reduce the uncertainties via model improvement is stated, and two useful measures for

decision making and risk assessments, namely signal-to-noise ratio and time of emergence, are introduced and calculated.

Summarizing, this is a well written, absorbing, and highly informative book, especially for readers interested in the mathematical theory of climate and the numerical methods in atmospheric physics. It is of great value to researchers in applied mathematics, environmental fluid mechanics, and geophysics, particularly in atmospheric sciences. Also, this book

should be of interest to PhD/MD students but is probably too advanced for undergraduates.

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