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Book Review

Problems and Solutions: Nonlinear Dynamics, Chaos and Fractals by Willi-Hans Steeb, World Scientific, 2016; ISBN: 9813109920 (Hardcover)

Andrzej Icha[1]

Linear thinking has dominated science and technology for nearly several centuries. Nonlinear physics has been studied at least since the time of Henri Poincaré (1854–1912), but has grown emphatically relatively recently with the rapid manifestation of interest in chaotic dynamics and fractals. The book *Problems and Solutions* by W.-H. Steeb presents these ideas in an original manner, and promotes the use of methods of dynamical systems theory to understand, model and predict our nonlinear world. However, "it is trite but true that mathematics (physics) is learned by doing it, nor by watching other people do it" (Reed M., Simon B., *Methods of Modern Mathematical Physics*. I. Functional Analysis. Academic Press, INC, 1980, p. ix). Therefore, a unique feature of the book is its emphasis on examples, problems, solutions, and the formulation of separate problems to solve.

The book consists of 3 chapters. Each chapter begins with a "background" section. It is designed mainly to fix notation, definitions, conventions and to clarify what topics one should have studied before tackling the problems that follow.

Chapter 1, *One-Dimensional Maps*, starts with some basic terminology and notions used in the analysis of non- linear one-dimensional maps. It introduces the main definitions such as mapping, fixed points, periodic points, topologically conjugacy, as well as the key quantities to characterize the one-dimensional, chaotic systems; ergodicity, invariant measure, Liapunov exponents, topological entropy etc. After that the author presents the 82 problems with fully worked-out solutions of these problems. The following one-dimensional maps are considered: the logistic family maps, the Newton maps, the family of quadratic maps, the tent maps, the W-map, the Bernoulli map, the cubic map, the decimal map, and the Baker map. The various analytic maps are treated also, together with the invariant measure of the maps based on the Frobenius–Perron approach. Two interesting problems regard the Feigenbaum function (73), and the thermodynamics formalism for chaotic map (80). Several problems require the use of programming languages, i.e. C++, SymbolicC++, Maxima and R. Most of the programs are written so that they can be understood by beginners. For example, I have compiled the *.cpp programs under Windows 10 on a PC using a Dev-Cpp 5.11 TDM-GCC 4.9.2 (https://sourceforge.net/projects/orwelldevcpp/). In addition, the chapter includes sixty formulated problems to be solved by the reader.

Chapter 2 deals with *Higher-Dimensional Maps and Complex Maps*. The Hartman–Grobman and Hopf bifurcation theorems, two of the most powerful tools used in dynamical systems, are introduced first. The next section presents examples of two-dimensional maps such as Hénon map, Baker's transformation, the trace map, Arnold cat map, the whisker map, the butcher's map, and many others. A readable analysis of the initial value problem for the system of difference equations is performed also. This section also gives the examples of problems that should be solved using computer algebra systems (e.g. Fayer sequence (52)). The supplementary material (without solutions) contains 63 problems,

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[1] Institute of Mathematics, Pomeranian Academy in Słupsk, ul. Arciszewskiego 22c, 76–200 Słupsk, Poland. E-mail: majorana38@gmail.com

and includes, among others, the Kaplan–Yorke map, the quadratic Cremona map, the Tinkerbell map, a predator–prey model, the Lozi map, and the McMillian maps. Problem 50 is my favorite exercise of this section. It includes an interesting approach to the elliptic integral computation problem by means of arithmetic–geometric mean method. The next section discusses the complex maps (24 problems) and also consists of seven supplementary exercises, including the Ikeda laser map. The following section is devoted to the higher dimensional maps. Several of the problems concern the three-dimensional analytic maps. Besides, it is demonstrated how to calculate the weight matrix for the Hopfield network, the notion of the Grassmann product, the Denman–Beavers iteration for the square root of an $n \times n$ matrix and others. Thirteen additional problems concern, among them, hyperchaotic map, Fibonacci trace map, renormalization group analysis, and the various iterations procedures. The last section supplies a collection of problems in bitwise operations and string manipulations.

Chapter 3 addresses basic issues of *Fractals*. First, the notions of dimension are introduced together with the definition of hyperbolic iteration function system. The basic theorem, that characterizes such systems, is elucidated briefly. Then, the collection of 49 exercises from various fields of fractal geometry is presented. This interdisciplinary mosaic includes the Cantor set, the Koch snowflake, Sierpinski triangle, Sierpinski carpet, Mandelbrot set, Julia set, and others. Topics related to the fractional derivative of a function and the Misiurewicz points are treated also. Supplementary important problems include, among others, the Kronecker product for the construction of fractals, Weierstrass function, and a Jordan circle.

In all cases the problems are well posed and informative. None of these chapters will stand on their own as a source for a rigorous nonlinear dynamics lectures. A good textbook that describes the uses of dynamical systems approach in nonlinear science, with an emphasis on computations, is that of Steeb (Steeb W.-H., *The Nonlinear Workbook*. Sixth edition, World Scientific, 2015).

In sum, I highly recommend *Problems and Solutions* to anyone who wants to deeper learn about nonlinear dynamics, low-dimensional chaos and the elements of some of classical fractals.