Book Review

Chaos in Nature, by Christophe Letellier, World Scientific Series on Nonlinear Science, Series A, vol. 81, Series Editor Leon O. Chua, World Scientific, 2013; ISBN: 978-981-4374-42-2.

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Dynamical systems theory deals with the modeling of phenomena which change in time. Chaos theory explores the behavior of dynamical systems that are very sensitive to initial (or parameters) conditions. Because the irregularity is part of the intrinsic dynamics of the system, it denotes that such systems are not, in the long term, predictable.

Chaos in Nature is a highly informative, original and gentle introduction to chaos theory and its applications. The book opens with two forewords (Prof. O. E. Rössler 2004, and Prof. R. Gilmore 2012).

The book is divided into two parts. Part I, From Celestial Mechanics to Chaos, offers a terse historical survey of the main topics covered in the book and contains six chapters.

Chapter 1, The Laws of Dynamics, begins with the problems connected with the celestial mechanics, in order to indicate the significance of the three-body problem for the appearing the conception of the chaos. The following questions are briefly discussed in historical context: the origin of Kepler's description of planetary motion, the formulation of the law of gravitation, and Newton's approach to lunar dynamics.

The following chapter sketches the difficulties with calculating the motion of the Moon's apogee whereas the next, Simplification of the three-body problem, discusses the Earth–Moon system under the influence of the Sun.

For now, only three exact solutions are known for the three-body problem, all for the very specific configurations. Chapter 4, The Success of Celestial Mechanics, focuses on the spectacular achievements of the perturbation theory with emphasis on the role of the secular terms. As a result, two remarkable conclusions are stated: it is impossible to predict long time behaviour of planetary trajectories, and that these trajectories are very sensitive to initial conditions. It denotes that two necessary conditions for chaotic dynamics are fulfilled.

Chapter 5 tries to give an overview of the general methods that were proposed in order to deal with the restricted three-body problem. These include: the global approach initiated by G. W. Hill (1838–1914) and the qualitative dynamical systems theory by Henri Poincaré. Many conceptions of Poincaré are now fundamental and form effective tools for the characterization of chaotic motions, for example: Poincaré section, the first return map, homoclinic orbits, etc. H. Poincaré was probably the first who noticed that the growth of small perturbations is no longer a result of the methods of calculation but rather an immanent property of a certain class of systems.

The stability of the solar system is the theme of Chapter 6. Some historical remarks about the solar system stability are discussed first. Next, the method of investigation of the stability of planetary orbits in the Solar System, namely, the Kolmogorov–Arnold–Moser (KAM) theorem is outlined. In this context, the famous Hénon–Heiles system is presented and discussed. The recent works on numerical approaches to the problem are also briefly reviewed.

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A definite answer to the question of whether the solar system is stable or not is not known. “Thanks to the numerical experiments of the last two decades, we know now that the motion of the planets in the solar system is chaotic, which prohibits any accurate prediction of their trajectories beyond a few tens of millions of years” (Laskar J., *Is the Solar System stable?*, arXiv:1209.5996v1 (astro-ph.EP), 26 Sep 2012).

Part II, *Chaos in Nature: Properties and Examples*, is organized into nine chapters.

In the first chapter, the main differences between conservative and dissipative dynamical systems are explained. It contains a rather informative discussion of the dynamics of an ideal and damped pendulum from a phase-space and energy-conservation perspective. Also, two systems of two coupled oscillators are studied in some detail. In this context, the famous Duffing system is analysed, to illustrate the aperiodicity and sensitive dependence on initial conditions.

Chapter 2, *From Mathematics to Electronic Circuits*, is the longest in the book and includes an exhaustive historical introduction to electronic circuit problems in connection with the study of chaotic behavior. The essence of the classical book by A. A. Andronov et al., entitled *Theory of Oscillations* (1937), is briefly presented. Interesting material on vacuum tubes, valves, multivibrators, etc. is discussed next. The dynamical approach to the oscillators including Poincaré’s equation, Janet’s equation, Blondel’s equation, the van der Pol equation and some formulas for the multivibrators, is described in some detail. Also, an important contribution of German researchers to explain relaxation oscillations is presented.

In the context of the first computer calculations the so-called “Japanese attractor” discovered by Y. Ueda on November 27, 1961, is recalled. This “chaotic” attractor was observed by analog computer composed of operational amplifiers. The remaining themes address the Pikovski–Rabinovich autogenerator, a single-loop feedback system with various linear transfer functions and nonlinear feedback functions, and the chaotic attractors, investigated by Chua and co-workers. The chapter closes with a discussion of a chaotic thermionic diode demonstrating the relevance of the van der Pol equation to model such a device.

Chapter 3 familiarizes the reader with the concepts of chaotic behavior in a dissipative dynamical systems. One of the first and most famous equations regarding these topics, the Lorenz equations, are discussed. Lorenz’ magnificent contribution may be recapitulated as follows: he affirmed that extremely complicated behaviors could arise from relatively simple dynamical systems, and he found the property of very sensitivity to initial conditions (“butterfly effect”). He also opened the way to better understanding of turbulence phenomenon. Since the fundamental paper by E. N. Lorenz in 1963, Lorenz-like dynamics were identified in different physical systems as in hydrodynamics, magnetohydrodynamics, lasers, etc. Some remarks in this field are also presented.

Chapter 4 deals with the *Architecture of Chaotic Attractors*, with special emphasis on the Rössler system. Such systems appeared later than Lorenz’s equations and were introduced in the 1970s as prototype equations with the minimum components for continuous-time chaos, especially in chemical flows. The main contribution of O. E. Rössler to the chaos theory concerns the development of the constructive methods to find such a systems (see, e.g., Broer H., Takens F., *Dynamical Systems and Chaos*, Springer; p. 55, 2011).

Chapter 5 treats *Chemical Reactions*. The earliest observations are briefly mentioned, and the first model for self-sustained oscillating chemical reactions, developed by Lotka, is presented. The famous Belousov–Zhabotinsky reaction, which is the prototype of system for nonlinear chemical dynamics, is described. Experimental aspects of chaotic behavior in electrolytic reactions, which are electrochemical reactions that occurs between two electrodes in a cell, are also demonstrated.

The dynamical system approach to population evolution is presented in chapter 6. Some early papers regarding the study of population dynamics (Malthus’ and Verhulst’ models) are briefly mentioned. Next, the simple models of two interacting species, the Lotka–Volterra equations (L–V) and the predator–prey model, are described and analysed. Thereafter, the population models with three species are discussed, including the generalized L–V model with chaotic attractors, and model by Upadhyay et al.

Observational aspects of nonlinear population dynamics, based on the statistical analysis of time-series data, are presented also. Four decades of investigations suggest however, that ecological chaos is rare in natural systems, but the non-equilibrium paradigm in theoretical ecology is very important.

Chaos theory relating to life sciences was first researched in the early 1970s. Chapter 7 considers Chaos in Biology and Biomedicine. The glycolysis oscillations are described first, based on three-variable biochemical model. Different kinds of solutions are discussed, but experimental proofs of chaotic behavior remain an open question. Subsequently, the fluctuations in hematopoiesis reproduced by a delay model for these oscillations are analysed. Then, the cardiac arrhythmias, that are disturbances in the rhythm of the heart, are presented. In this context, a long and fascinating history of clinical electro-physiology is reviewed. Lastly, technical aspects of the cardio-respiratory devices, from the viewpoint of the dynamical system theory, are described.

Chapter 8, Chaotically Variable Stars, begins with a brief introduction of the history of how the variable stars were discovered and studied. The first differential models describing the luminosity of Cepheid variables, the Moore–Spiegel equations, a zonal model of pulsating star, the amplitude equations, etc. are presented. The variations of solar activity cycle are also studied, first in the historical context, and later on, by methods of dynamical systems theory. The model of solar activity by Knobloch et al. and the model by Aguirre et al. describing the long-term

evolution of sunspots, are presented and discussed. It should be stressed that an empirical identification of chaotic behavior in solar cycle is quite problematic in view of the quality of the available solar time series.

The final chapter presents a brief list of open questions on various aspects of chaos theory. Included are: a weakly dissipative dynamical systems, the notion of hyperchaos, toroidal chaotic attractors, and some remarks relating to epistemological significance of the term “chaos”.

Prof. O. E. RöSSLer, chaos theory expert, writes in his foreword to this unusual book: “It will probably become the Bible in the field”. The editorial work of the book presents a high level; figures and text are very readable and I found only a few minor misprints. Anyone teaching a modern course in low-dimensional dynamical systems should give it strong considerations. The knowledge of advanced mathematics and computational techniques is not a prerequisite for reading or understanding the matter of the book. It should be very useful for both students and active researchers in the area of dynamical systems in a broad range of applications. The particular part of the material should be also valuable to dynamicists working on subjects outside of their expertise.

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