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Book Review

"Geometric Mechanics. Part I: Dynamics and Symmetry", by Darryl D. Holm, second edition, Imperial College Press, 2011; ISBN 13: 978-1-84816-775-9 (pbk), USD 31.00

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This is an extraordinary textbook of an eminent applied mathematician, which should be read by everyone who aspires to the title of theoretical mechanic.

Mechanics, one of the most important branches of theoretical physics, has a tradition reaching more than 2,000 years. During the nineteenth and twentieth centuries, the modern formulations of mechanics were mainly developed by mathematicians and mathematical physicists. The mathematics of contemporary classical mechanics involves functional calculus, differential geometry and the theory of Lie groups and Lie algebras. The coordinate-independent formalism of differential geometry provides an uniform description for many physical objects such as, among others, *n*-particle systems, rigid bodies, perfect fluids, some rheological media, and electromagnetic, gravitational and quantum systems.

The book comprises six chapters and two appendices.

The first, 95-page-long chapter deals with the Fermat's principle for ray optics as an archetype of Lie symmetry reduction for various applications of geometric mechanics that are presented in this book. There is a careful treatment of Lagrangian and Hamiltonian formalisms, the notions of Lie groups and Lie algebras, symmetry reduction in the Hamiltonian description, quotient map, Poisson bracket, energy-Casimir stability, momentum maps, Lie–Poisson brackets, Nambu brackets, and orbit manifolds. These concepts are presented in enough detail and with relatively straightforward exercises and problems.

The second chapter provides a terse overview of Newtonian, Lagrangian and Hamiltonian mechanics of *n*-particle systems. Two classical, famous problems of geometric mechanics are described and discussed: rigid-body motion and the spherical pendulum.

In order to study the geometrical aspects of mechanics in more depth, the language of differential forms is needed, which is the theme of chapter 3. It contains necessary information about exterior calculus on symplectic manifolds, including tangent and cotangent bundles, canonical transformations, exterior calculus operations, Lie derivative of forms, etc. This formalism is illustrated for the perfect incompressible fluid flow. The formulation of Euler's equations in Hodge-star form in R^4 was the most interesting to me because I am a fluid dynamicist, but unfortunately I know these questions less well.

Oscillators and systems of coupled oscillators are perennially used to model various physical phenomena. The fourth chapter deals with the analysis of two coupled nonlinear oscillators. Such a system has a Hamiltonian structure and is used to investigate a single polarized optical laser pulse propagating as a travelling wave in an anisotropic, cubically nonlinear, lossless medium. The several orbit manifolds for *n*:*m* resonances are constructed, and the suitable Lie symmetry reductions for different *n*:*m* values are discussed in some detail. The fifth chapter explores the spring-pendulum (also known as swinging spring or elastic pendulum), a simple mechanical system with extremely complex dynamics. The analysis is made by a Lie symmetry

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reduction method, which through a suitable averaging technique, permits the reduction of the problem to an integrable Hamiltonian system. Generally speaking, this system is non-integrable and exhibits chaos (see e.g. Maciejewski *et al.*, *Non-integrability of the generalized spring-pendulum problem*, J. Phys. A: Math. Gen. *37* (2004) 2579–2597).

The last, sixth chapter focuses on the Maxwell-Bloch equations that describe the propagation of optical fields in a nonlinear, dispersive medium consisting of two-level atoms. As for the elastic pendulum, averaging the Lagrangian leads to the Lie symmetry needed for decreasing the order of the dynamics. As a consequence, the phase-space geometry and Hamiltonian structure of the invariant subsystem of the real Maxwell-Bloch equations are discussed. Let's notice that such equations can be received also, as the large Rayleigh number limit of the well-known Lorenz system.

Exercises and problems are a very valuable part of the book. Over 100 exercises (some with hints and answers) are placed in the text. Two appendices contain enhanced coursework, together with solutions and homework problems. Throughout the book it is clear that Prof. Holm always remembers students, which is great.

I warmly recommend that readers do the exercises and problems. It is trivial but true that theoretical mechanics is experienced by doing it, not by tracking how other people do it!

In conclusion, the book provides a very modern and elegant presentation of ideas and methods from geometric mechanics. The book's matter requires from the reader a moderate level of mathematical sophistication, including linear algebra, standard mathematical analysis, variational calculus and some familiarity with principles of theoretical mechanics. Undoubtedly, the book should be valuable not only for students but also for applied mathematicians, specialists in mechanics, as well as geophysicists and theoretical physicists.