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Can We Improve Estimates of Seismological Q Using a New "Geometrical Spreading" Model?

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Abstract—Precise measurements of seismological Q are difficult because we lack detailed knowledge on how the Earth's fine velocity structure affects the amplitude data. In a number of recent papers, Morozov (Geophys J Int 175:239–252, 2008; Seism Res Lett 80:5–7, 2009; Pure Appl Geophys, this volume, 2010) proposes a new procedure intended to improve Q determinations. The procedure relies on quantifying the structural effects using a new form of geometrical spreading (GS) model that has an exponentially decaying component with time, $e^{-\gamma t}$·γ is a free parameter and is measured together with Q. Morozov has refit many previously published sets of amplitude attenuation data. In general, the new Q estimates are much higher than previous estimates, and all of the previously estimated frequency-dependence values for Q disappear in the new estimates. In this paper I show that (1) the traditional modeling of seismic amplitudes is physically based, whereas the new model lacks a physical basis; (2) the method of measuring Q using the new model is effectively just a curve fitting procedure using a first-order Taylor series expansion; (3) previous high-frequency data that were fit by a power-law frequency dependence for Q are expected to be also fit by the first-order expansion in the limited frequency bands involved, because of the long tails of power-law functions; (4) recent laboratory measurements of intrinsic Q of mantle materials at seismic frequencies provide independent evidence that intrinsic Q is often frequency-dependent, which should lead to frequency-dependent total Q; (5) published long-period surface wave data that were used to derive several recent Q models inherently contradict the new GS model; and (6) previous modeling has already included a special case that is mathematically identical to the new GS model, but with physical assumptions and measured Q values that differ from those with the new GS model. Therefore, while individually the previous Q measurements have limited precision, they cannot be improved by using the new GS model. The large number of Q measurements by seismologists are sufficient to show that Q values in the Earth are highly laterally variable and are often frequency dependent.

Key words: Seismic attenuation, seismic amplitude, Q, 3D earth structure, focusing, multipathing.

1. Introduction

It is well known that precise measurements of the seismic quality factor, or Q, are difficult because they are conducted using seismic amplitudes, which are also affected by the Earth's complex velocity structures. In general the velocity structure is not yet known to the detail for a precise correction for its effects on amplitude. This fundamentally limits the precision of Q measurements. Nevertheless, there have been a large number of studies of seismic Q since the dawn of quantitative seismology. The quality of these studies has for several reasons improved over time. First, accumulations of amplitude data have enabled researchers to carefully screen data to avoid those contaminated by strong 3D structural effects (e.g., MITCHELL and XIE, 1994), and to average data that sample similar paths or regions to reduce errors caused by relatively rapid structural variations (YANG and FORSYTH, 2008; PRIETO *et al.*, 2009). Second, carefully designed measurement methods have been introduced to reduce measurement errors (e.g., CHUN *et al.*, 1987; MENKE *et al.*, 1995, ROMANOWICZ, 1998; XIE, 1998; WARREN and SHEARER, 2002). Third, our knowledge of the velocity structure has also steadily improved. At low frequencies it has become viable to account for the effects of laterally heterogeneous velocity structures in Q measurements. At high frequencies improved knowledge on velocity heterogeneity has lead to better estimates of uncertainty in Q measurements. Individual measurements of path-specific Q, or tomographic inversions of laterally varying Q models, are typically published with discussions of the measurement error or uncertainty (e.g., MITCHELL, 1995; XIE *et al.*, 2004; PHILLIPS *et al.*, 2005; ROMANOWICZ and MITCHELL, 2007; PASYANOS *et al.*, 2009). The vast Q measurements and tomographic Q models collectively reveal two main

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features of Q in the Earth's crust and mantle: it is highly laterally variable (by a factor of 10 as compared to the velocity variation which is typically less than 20–30%), and often frequency dependent (e.g., Der *et al.*, 1986, 1987; Anderson, 1989; Mitchell, 1991; Xie and Mitchell, 1990; Mitchell and Xie, 1994; Flanagan and Wiens 1994, 1998; Sobolev *et al.*, 1996; Xie, 1998; Xie *et al.*, 2004, 2006; Gung and Romanowicz, 2004; Selby and Woodhouse, 2002; Warren and Shearer, 2002; Shito *et al.*, 2004; Phillips *et al.*, 2005; Dalton and Ekstrom, 2006; Lekic *et al.*, 2009; Pasyanos *et al.*, 2009, also see summaries by Anderson, 1989; Flanagan and Wiens, 1994; Mitchell, 1995; Romanowicz, 1998; Romanowicz and Mitchell, 2007; Sato and Fehler, 2009). These features have led to inferences of variations of temperature and occurrence of melting in the crust and upper mantle, variations of pore fluid content and other forms of small-scale crustal structure heterogeneities. Improvement of Q models is currently an on-going process with the rapidly growing seismic data and computational power.

Citing the limitations in measurement precisions of seismological Q, Morozov (2008, 2009, 2010) proposes a new procedure for making Q determinations. This procedure is based on a theoretical modeling of velocity structural effects on seismic amplitudes, which includes an exponentially decaying form of geometrical spreading (GS), $e^{-\gamma t}$. Parameter "γ" is measured together with Q, thus setting a new set of free parameters in Q measurements. Using his new procedure and parameterization, Morozov has refit many sets of previously published data that were used to obtain frequency dependent Q models in earlier studies. While doing so he finds that all previously inferred frequency dependences disappear and Q is invariably frequency-independent. Moreover, the newly introduced free parameter γ absorbs much of the observed amplitude attenuation, causing the new Q measurements to be generally much higher and less laterally variable than the previous measurements. It is proposed that the value of "γ" correlates better than Q with tectonic environments. Morozov (2010) also dismisses the independent evidence provided by mineral physics experiments that show that intrinsic Q of mantle minerals are often frequency dependent. This dismissal is made despite the recent and significant improvements in the measurement conditions and precisions in laboratory studies (Jackson 2000, Jackson *et al.* 2004).

Are the new model, procedure and measured Q values by Morozov valid? Do they bring in new physical insight in Q studies while showing the existing knowledge on the values and the frequency dependence of Q grossly wrong? The answers to these questions are of obvious significance to the seismological community because in principle, Q is one of the two seismic parameters that can be measured using seismic data, and values of Q and its frequency dependence are strongly dependent on temperature, small-scale heterogeneity, and percentage of melting in the crust and mantle materials. Additionally, Q values are needed both for converting phase and group velocity measurements to body wave velocities (Liu *et al.*, 1976; Romanowicz, 1998), and for inferring temperatures using the latter velocities (Karato, 1993; Mcnamara *et al.*, 1997; Goes and Van Der Lee, 2002). Even for seismologists who measure seismic attenuation for practical purposes such as inferring source properties, monitoring underground explosions and predicting strong ground motions, it would be interesting to know whether or not seismic wave attenuation generally contains an exponentially decaying geometrical spreading, and if the large number of seismological Q studies, complemented by laboratory measurements, have only provided incorrect knowledge on the Q values and their frequency dependence.

This paper is motivated by searching for the answers of the above questions. In the following sections I will show that modeling seismic wave attenuation with an exponentially decaying geometrical spreading lacks a physical basis. The procedure and parameterization to measure Q based on such a modeling is effectively a curve-fitting of the frequency dependent attenuation data by a first order Taylor series expansion. Consequently, previously published high-frequency attenuation data that were fit by a power-law frequency dependence of Q are expected to be well fit by a first order expansion in the narrow frequency bands involved. For long-period fundamental mode surface waves, several recently published datasets cannot be fit by the first order expansion, thus fundamentally contradicting the new GS model. Recently measured intrinsic Q of mantle minerals in the laboratory at frequencies down to 1 Hz often exhibit frequency dependence. These measurements are free of velocity structural effects and are nearly impossible to reconcile with frequency independent total Q.

2. *Modeling of Seismic Amplitude Spectra*

If the source spectra and site response can be removed from seismic spectra recorded around an event elapse time (or wave travel time) t, the remaining spectra $A(f, t)$ contains only the path effects and can be generally expressed as (AKI and RICHARDS, 1980)

$$A(f,t) = P(f,t)e^{-\pi ft/Q(f)}, \tag{1}$$

where $P(f, t)$ is the effect of the Earth's velocity structure other than random scattering elaborated below, and $Q(f)$ is the total quality factor defined as

$$\Delta E/E = 2\pi/Q(f), \tag{2}$$

where ΔE is the loss of energy E over one wave length or period (KNOPOFF, 1964). Equation 1 is valid when the reciprocal of $Q(f)$ is small ($<<1$). This reciprocal can be expressed as

$$\frac{1}{Q(f)} = \frac{1}{Q_i(f)} + \frac{1}{Q_s(f)} \tag{3}$$

where $Q_i(f)$ is the intrinsic Q caused by the Earth's viscoelasticity (e.g., KNOPOFF, 1964; ANDERSON, 1989; see DAHLEN and TROMP, 1998 for a strict definition of anelasticity and viscoelasticity); $Q_s(f)$ is the scattering Q caused by the random scattering process in the Earth media (AKI, 1980).

2.1. *Intrinsic and Scattering Q*

While the intrinsic and random scattering losses of seismic wave energy can also be quantified by alternative parameters (XIE and FEHLER, 2009), Q is the most widely used physically based parameter by the seismological and mineral physics communities. The viscoelasticity that gives rise to $Q_i(f)$ is the property of general linear solids (BOLTZMANN, 1874; BORCHERDT, 2009). The temperature and pressure of the Earths' deep interior are in a favorable range to observe its viscoelasticity and finite $Q_i(f)$ (ANDERSON, 1989; KARATO and SPEZLER, 1990; RANALLI, 1995). At shallow depths microscopic pores and/or fractures also cause viscoelasticity of the media (e.g., CARCIONE *et al.*, 2007; LI *et al*, 2010).

The random scattering processes that cause finite $Q_s(f)$ occur for seismic waves that traverse sufficiently far to encounter numerous small-sized scatterers, so that a stochastic treatment is warranted for these processes. One of the best pieces of evidence that such random scattering occurs for seismic waves is the presence of long and prominent coda waves at seismic frequencies (AKI, 1980; LANGSTON, 1989; CAMPILLO and PAUL, 2003; SATO and FEHLER, 2009). The range of validity for statistical treatment of random scattering, in terms of sizes of heterogeneities and path lengths, have been discussed by numerous authors (AKI and RICHARDS, 1980; KENNETT, 1983; STEIN and WYSESSION, 2003). Figure 1 is adapted from AKI and RICHARDS (1980); it shows the effects of the Earth's velocity structure on seismic wave propagation with varying scale lengths of heterogeneity (a) and path lengths (L), both measured by the wave length (λ). Random scattering occurs in a large domain with a wide range of path lengths ($L > \lambda$), and a wide range of heterogeneity scale (a) that are grossly comparable, within about an order of magnitude, to the wave length (λ).

2.2. *Structural Effects Other than Random Scattering*

As shown in Fig. 1, for heterogeneous structures that are characterized by either very small, "microscopic" a ($a << \lambda$) or large a ($a \sim L$), there is virtually no wave energy conversion by scattering (the total $\Delta E/E < 0.1$) so the structural effects $P(f, t)$ are effectively caused by wave propagation in laterally uniform and vertically stratified (1D) reference Earth models. For moderately sized heterogeneities whose scale length a is smaller than L but significantly larger than λ, $P(f, t)$ are caused by complex processes such as diffraction, focusing and defocusing and multipathing. Distinction among these processes can be gradational (STEIN and WYSESSION, 2003). In this paper I will use "focusing and defocusing" to refer to the processes during which the whole or a portion of wavefront is distorted as compared to that in a uniform medium, but is spatially continuous and trackable over time (e.g., MORI and FRANKEL, 1992). Multipathing will be used to

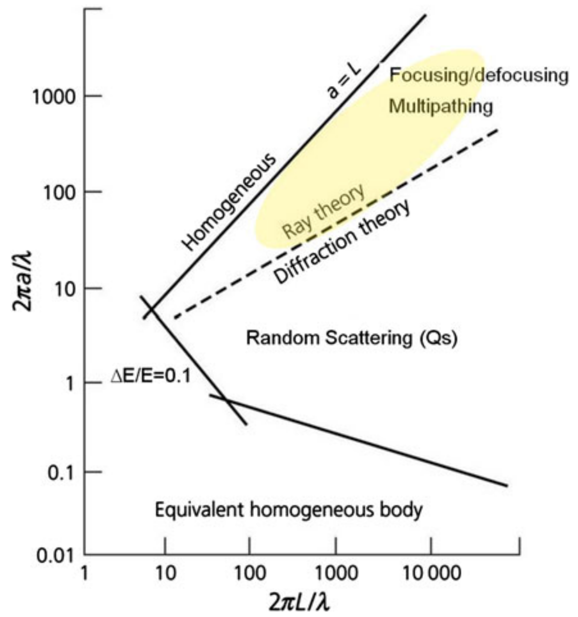

Figure 1
Classification of different wave propagation regimes in the $2\pi a/\lambda$–$2\pi L/\lambda$ diagram, where λ is wave number, L and a are propagation distance and heterogeneity size, respectively. "*FD*" and "*SE*" denotes numerical, finite difference and spectral element methods appropriate for simulating the propagation regime. ΔE is the total energy loss of the wave with initial energy E. (adapted after Aki and Richards, 1980, used with permission)

refer to processes in which wavefronts become spatially discontinuous and broken into multiple wave packets (e.g., Capon, 1970; Yomogida and Aki, 1985, 1987; Pavlis *et al.*, 1994; Ji *et al.*, 2005).

2.3. Geometrical Spreading Term

In uniform media structural effects $P(f, t)$ include that caused by wavefront expansion with time, known as the "geometrical spreading term" (GST, Aki and Richards, 1980). They also include effects of complex interactions and energy partitioning among various wave types along layer boundaries, which may cause inhomogeneous waves or leaked modes (Pilant, 1979; Aki and Richards, 1980). In heterogeneous media the portion of $P(f, t)$ caused by focusing and defocusing, as defined above with trackable wavefronts, are also known as GST (e.g., Cerveny *et al.*, 1974). The term GST has been loosely used for other portions of $P(f, t)$ (e.g., Yang *et al.*, 2007) but I will use it only in its strict definition in this paper. In general, $P(f, t)$ include both GST and the effects of complex multiple-wave interactions in layered, and/or heterogeneous media.

2.4. Approximate Amplitude Modeling and Measurement Error of Q

For a given amplitude dataset $A(f, t)$, accurate $Q(f)$ measurements require precisely calculated structural effects $P(f, t)$ in Eq. 1. Unfortunately, this is typically not possible because detailed knowledge of the heterogeneous velocity structure is not available. We can only estimate the portion of $P(f, t)$ in a crude, reference velocity structure, which in most cases is just a 1D structure. Denoting the portion of $P(f, t)$ associated with the reference structure by $P_0(f, t)$ and using it to approximate the full $P(f, t)$ in Eq. 1, we have

$$A(f,t) \approx P_0(f,t)e^{-\pi ft/Q(f)}. \tag{4}$$

Equation 4 is used to calculate reduced amplitude spectra,

$$A'(f,t) = A(f,t)/P_0(f,t) \approx e^{-\pi ft/Q(f)}, \tag{5}$$

which are used to estimate $Q(f)$. Denoting the error of the approximate modeling of $A(f, t)$ by Eq. 4 as

$$E_r(f,t) = P(f,t)/P_0(f,t), \tag{6}$$

a precise modeling by Eq. 1 can be rewritten as

$$A(f,t) = P_0(f,t)E_r(f,t)e^{-\pi ft/Q(f)}. \tag{7}$$

To first order the relative error of Q measurements, $\delta Q(f, t)/Q(f)$ caused by $E_r(f, t)$, is (Xie *et al.*, 2004)

$$\frac{\delta Q(f,t)}{Q(f)} = -\frac{Q(f)}{\pi tf}\ln|E_r(f,t) - 1|. \tag{8}$$

In practice, full isolation of the path effect for $A(f, t)$ is difficult so the error term $E_r(f, t)$ should also include uncompensated effects of source spectra and site response (Menke *et al.*, 2006). The error in Q measurements, $\delta Q(f, t)$, is reduced by using (a) data screening procedures to avoid small t values in Eq. 8, and to avoid visible contaminations by the processes of focusing/defocusing, multipathing and diffraction, (b) data averaging procedures to repeatedly sample similar paths to smooth out effects of these processes, and (c) measurement methods that are designed to reduce effects of these processes. Taken together, the large number of measurements of $Q(f)$ reveal that it is highly regionally variable and often frequency dependent (see Sect. 1).

3. The Proposed New "GS" Model and Q Measurement Procedure

With the good intention of reducing the measurement error for $Q(f)$, Morozov (2008, 2009, 2010) proposes an alternative procedure for Q measurements. The procedure is based on a new modeling of amplitude spectra that contains an exponentially decaying function as part of path effect $P(f, t)$. For clarity I rewrite the Morozov expression of path amplitude spectra corresponding to Eq. 1 above:

$$A(f,t) = G(f,t)e^{-\pi ft/Q_e}, \qquad (\mathrm{M}-1)$$

where letter "M" is used to label the equation to indicate it is from Morozov (2008, 2009, 2010). Note Morozov (2010, equation (2)) used term "$P(t, f)$" for my $A(f, t)$; that "$P(t, f)$" is not to be confused with my term $P(f, t)$ (Table 1). Term Q_e is called "effective Q", from "intrinsic dissipation and small-scale scattering" (Morozov, 2010) so it is just the conventionally defined $Q(f)$. The term $G(f, t)$ is called a "geometrical attenuation factor" (Morozov, 2008) or "geometrical spreading" in Morozov (2009, 2010). Careful reading of these references (see Appendix 1 for details) suggests that $G(f, t)$ is used to express the effect of Earth velocity structure, $P(f, t)$ in Eq. 1, rather than the strictly defined GST in the last section. I will refer to the $G(f, t)$ introduced by Morozov as the "geometrical spreading (GS) model". The common practice of replacing a true path effect by an approximate path effect (Eq. 4) is expressed by Morozov (2008, 2009, 2010) as

$$A(f,t) \approx G_0(f,t)e^{-\pi ft/Q_e}, \qquad (\mathrm{M}-2)$$

Mathematically, Eqs. M-1 and M-2 are identical to Eqs. 1 and 4 except new terms $G(f, t)$, $G_0(f, t)$ and Q_e are used to replace $P(f, t)$, $P_0(f, t)$, and $Q(f)$, respectively. So Eqs. 4 through 7 are also mathematically valid with Morozov's new terms:

$$A'(f,t) = A(f,t)/G_0(f,t) \approx e^{-\pi ft/Q_e} \quad \text{(for reduced amplitude)}; \qquad (\mathrm{M}-3)$$

$$E_r(f,t) = G(f,t)/G_0(f,t) \quad \text{(for error in path effect)}, \qquad (\mathrm{M}-4)$$

or

$$G(f,t) = E_r(f,t)G_0(f,t); \qquad (\mathrm{M}-4\mathrm{a})$$

and

$$A(f,t) = G_0(f,t)E_r(f,t)e^{-\pi ft/Q_e} \quad \text{(effect of error using } G_0(f,t)\text{)}. \qquad (\mathrm{M}-5)$$

It is then assumed that $G(f, t)$ and $G_0(t)$ are connected by the following relationship (equation 7 of Morozov, 2008 or (1) of Morozov, 2010)

$$G(f,t) = G_0(f,t)E_r(f,t) = G_0(f,t)e^{-\gamma t}, \qquad (\mathrm{M}-6)$$

where a new exponential term $e^{-\gamma t}$, known as "geometrical scattering" in Morozov (2008) and "residual GS" in Morozov (2010), is introduced to quantify the

Table 1

Comparison of some mathematical symbols in this paper and Morozov (2010)

Physical definition	Symbol in this paper	Symbol in Morozov (2010)
Path-isolated amplitude spectra	$A(f, t)$	$P(t, f)$[a]
Effects of velocity structure on amplitude (excluding random scattering)	$P(f, t)$	$G(t, f)$[b]
Simplified path effect in a reference structure	$P_0(f, t)$	$G_0(t, f)$[b]
Total Q	$Q(f)$	Q_e^c

[a] Morozov uses the argument order of "t, f" for all functions, whereas this paper uses the order of "f, t"

[b] Reffered to as "geometrical spreading" by Morozov (2010) but inferred to be identical to my $P(f, t)$

[c] Effectively set as a frequency-independent parameter by Morozov (2010)

error $E_r(f, t)$: $E_r(f, t) = e^{-\gamma t}$. This leads to a new form of Eq. M-1:

$$A(f,t) = G_0(f,t)e^{-(\gamma+\pi f/Q_e)t}. \quad (\mathrm{M}-7)$$

Based on the assumed Eqs. M-6 and M-7, Morozov (2008, 2009, 2010) proposes to set γ and Q_e as free parameters to be measured using the reduced amplitude data:

$$A'(f,t) = A(f,t)/G_0(f,t) = e^{-(\gamma+\pi f/Q_e)t}. \quad (\mathrm{M}-8)$$

The argument of the exponential function in the above equation is denoted as the "attenuation coefficient", $\chi(f)$:

$$\chi(f) = \gamma + \pi f/Q_e = \gamma + \kappa f \quad (\kappa = \pi/Q_e). \quad (\mathrm{M}-9)$$

Comparing Eq. M-8 with traditional Eq. 5, Morozov (2010) gives a conversion between his new free parameters and traditionally measured $Q(f)$:

$$\chi(f) = \gamma + \kappa f(\text{new}) = \pi f/Q(f)\ (\text{traditional}). \quad (\mathrm{M}-10)$$

In summary, the main difference between the modeling by Morozov for seismic amplitude and the traditional modeling is in his new GS model. First, his "$G(f, t)$" notation is used to represent the entire path effect $P(f, t)$, rather than the GST. Second and most important, Morozov assumes the error of approximating the whole path effect by that in a reference structure ($E_r(f, t)$ in Eq. 6 or M-4) can be quantified by the exponential function $e^{-\gamma t}$, known as "geometrical scattering" or "residual geometrical spreading". As elaborated in Appendix 1, Morozov (2008, 2009, 2010) does not give a physical basis for the exponential function $e^{-\gamma t}$. Therefore in the next section I will explore if such physical basis exists.

4. The Lack of a Physical Basis for the Exponential Geometrical Spreading

As discussed in Sect. 2, processes that contribute to the $E_R(f, t)$ (Fig. 1) include (a) focusing/defocusing associated with continuous wave fronts, (b) multipathing and diffraction that occur when a wavefront is broken, and (c) wave interactions in the vicinity of sharp boundaries. Morozov (2008, 2009, 2010) grossly refers to these processes as "geometrical scattering" or "residual GS". We now examine whether or not these processes can be generally expressed by a function $e^{-\gamma t}$.

4.1. Focusing and Defocusing

A simple example of focusing/defocusing, which occurs for wave propagation in a 2D plane, is shown in Fig. 2. This propagation is simulated using the finite difference method and resembles that of a surface wave from an explosive source to distances at which the curvature of the Earth can be ignored. Compared to the wave snapshot in a plane with a constant velocity (Fig. 2a), the snapshot in a plane with a simple linear velocity gradient (Fig. 2b) exhibits an amplitude focusing and defocusing in the slow-and fast-travel directions, respectively. I have calculated wave propagation in 3D media with and without a linear velocity gradient (not shown) and obtained similar effects of focusing and defocusing. Numerical calculations in Appendix 2 give analytical solutions of $P(t)$ (or $G(t)$) along the direction of the velocity gradient with focusing or defocusing (Eqs. 22 and 23 in Appendix 2), and the respective $E_r(t)$ (Eqs. 25, 26). These solutions are not of the form of $e^{-\gamma t}$. As elaborated in detail in Appendix 2, in addition to the scales of heterogeneity (a) and propagation length (L), our knowledge of the levels of lateral heterogeneity in the structure (less than about 30% for Earth media) also yields constraints on the implausibility of an exponential path effect $P(f, t)$ (or $G(f, t)$). The lessons learned from this much simplified case are that (1) an analytical form of $E_r(f, t)$ is available here and is not of exponential form, and (2) monotonic enhancement or decrease of amplitude, which is implied by an exponential $E_r(f, t)$, is unlikely to be sustainable over travel times (t) or distances that are large enough for $Q(f, t)$ measurements (see Eq. 8). This is because the Earth's velocity heterogeneity must alternate its sign from the mean velocity so that the velocities maintain a reasonable range (within $\sim\pm 30\%$).

I note that in Q measurements using long-period surface wave data, much improved laterally heterogeneous phase velocity models are nowadays

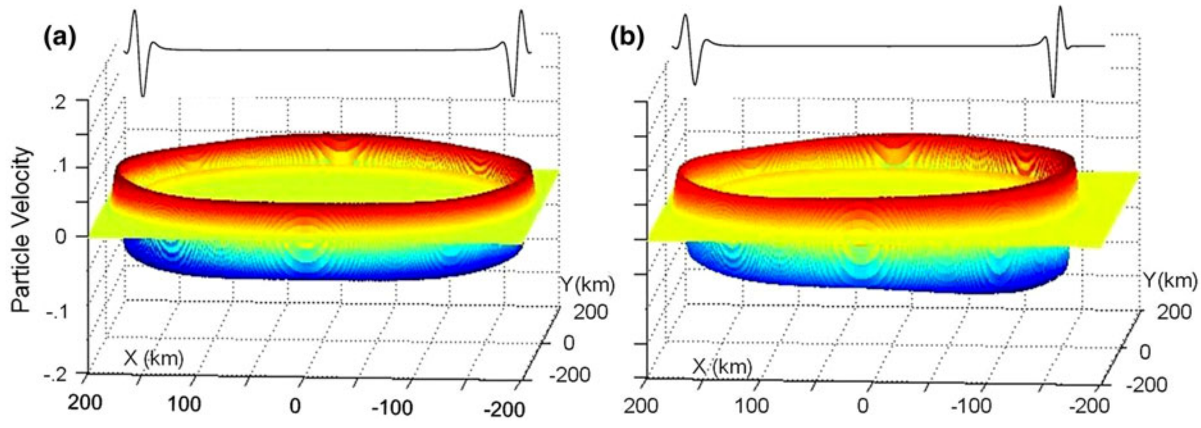

Figure 2
Snapshots of particle velocity in a 2D (x–y) plane caused by wave generated by a source located at the center, calculated at an event-elapse time of 48 s with a finite difference program by XIE and YAO (1988). A Gaussian-derivative source time function with a duration of about 6 s is used. The velocity model used in **a** has a constant velocity of 4.125 km/s. The model used in **b** has a velocity gradient of 0.00375 s^{-1} along the x direction, so that the velocity varies from 3.0 km/s at $x = -200$ km to 4.5 km/s at $x = 200$ km. Inset traces are particle velocities along the line of $y = 0$ km (the central line along the x direction), plotted to better show the effects of focusing/defocusing in case (**b**)

available, enabling first attempts to approximately calculate the focusing and defocusing effects (SELBY and WOODHOUSE, 2002; DALTON and EKSTROM, 2006; ROMANOWICZ, 2009). These calculations lead to improved Q measurements and tomographic Q models. In general, an exponential form of GS is neither physically adequate nor needed for numerical calculations.

4.2. *Multipathing*

There have been numerous observations of multipathing of long-period surface waves with array data (CAPON, 1970; PAVLIS *et al.*, 1994; JI *et al.*, 2005; TAPE *et al.*, 2010). In some cases when data sampling is dense and velocity heterogeneity is less drastic, 1D Q models under arrays have been developed by using a two plane-wave decomposition method (YANG and FORSYTH, 2008) with a simplified path effect $P(f, t)$. In general precise estimates of multipathing effects are difficult to obtain because, even in the best studied areas such as southern California, the 3D velocity models still lack the resolution and reliability needed for estimating multipathing $P(f, t)$ for Q studies (CHEN *et al.*, 2007; TAPE *et al.*, 2010). The difficulty is enhanced because multipathing is a wave interaction phenomena that can be fundamentally unstable (see e.g., JI *et al.*, 2005, Figures 1 through 5). This instability is similar to $P(f, t)$ caused by the triplication of upper-mantle P waves traveling in a 1D reference Earth structure, in which case the strictly defined GST can become singular (AKI and RICHARDS, 1980).

Because multipathing and diffraction are spatially localized phenomena (CAPON, 1970; YOMOGIDA and AKI, 1985, 1987; JI *et al.*, 2005), their effects cannot be adequately quantified by an exponential term $e^{-\gamma t}$. This is more easily seen when one converts the integrated time t in $e^{-\gamma t}$ to total distance traversed by the wave, L divided by average velocity v: the function then becomes $e^{-\gamma L/v}$. The exponential form will become adequate only after the distance L becomes large enough such that the wave has lost or gained energy by multiple occurrences of complications such as multipathing, diffractions and focusing/defocusing. At such distances the energy loss is not modeled by $P(f, t)$ (or $G(f, t)$). Rather, it is modeled statistically, using scattering Q in Eq. 3.

4.3. *Examples of Wave Interaction Along Structural Boundaries*

I considered many other observed seismic waves studied by several seismologists, myself and MOROZOV (2010), and find that for all but one wave. the exponential form of $E_r(f, t)$ lacks a physical basis.

The exceptional wave is the shear-coupled *PL* wave which systematically leaks energy with distance across the Moho interface (e.g., PILANT, 1979; AKI and RICHARDS, 1980; HELMBERGER and ENGEN, 1980). The path effect $P(f, t)$ does include an exponential decay, and this decay is already included in the approximate term $P_0(f, t)$ (or $G_0(f, t)$). The detail of $P(f, t)$ is sensitive to the details of velocities of P and S waves in the vicinity of Moho, therefore PL may not be a useful phase with which to study Q.

Many authors have numerically studied the attenuation of high-frequency Pn wave (e.g., CERVENY and RAVINDRA, 1971; MENKE and RICHARDS, 1980, 1983; SERENO and GIVEN, 1990; ZHU *et al.*, 1991; NOWACK and STACY, 2002; XIE, 1996, 2007; YANG *et al.*, 2007; MOROZOV, 2010). The path effect $P(f, t)$ for Pn is often referred loosely as "geometrical spreading" and is complex, being frequency and distance dependent. This occurs because the composition of Pn changes with distance and frequency. Pn is nearly a pure head wave at close-in (near regional) distances and becomes multiple, "whisper galley" refractions at larger distances. It is very sensitive to the details of velocity structure in the vicinity of Moho. Nevertheless, none of the analytical and numerical studies of Pn path effect $P(f, t)$ have resulted in a decay that is as fast as an exponential function.

From the discussions in this section, path effect $P(f, t)$ (or $G(f, t)$ in the notation of Morozov) caused by structural complications generally have no analytical solutions, and are too slow to be numerically fit by an exponential function. At sufficiently large distances the cumulative effects of wave-length scale velocity heterogeneities can effectively cause exponential decay of amplitudes, but this decay is already statistically quantified using scattering Q and cannot be counted again as a part of $P(f, t)$ (Eq. 1) or $G(f, t)$ (Eq. M-1).

5. The Fit of GS Model to Published Data

MOROZOV (2008, Table 2; 2010; Figures 3 through 6) succeeds in fitting several previously published datasets by his GS model with frequency-independent γ and Q_e. From these fittings he infers that the previously measured Q are biased, Q is frequency independent; and the new parameter γ is physically meaningful since it correlates with tectonic environments. Since the new GS model generally lacks a physical basis, none of those inferences is valid. In this section I will examine the reason why the new GS model can fit the previously obtained high-frequency data. I will also show that while MOROZOV (2010) successfully fits the long-period surface wave data of ANDERSON *et al.* (1965) by his GS model, other and newer surface wave datasets fundamentally contradict that model.

From Eqs. M-8 and M-9, the GS model for the reduced amplitude data is $e^{-\chi(f)t}$ where the "attenuation function" $\chi(f)$ is assumed to be a linear function of f given by (M-9). Therefore for a given set of reduced amplitudes, the fit for $\chi(f)$ under the GS model (assuming frequency-independent Q_e as advocated by Morozov) is effectively a first-order Taylor series expansion of $\chi(f)$; the fit is guaranteed at first-order precision. This fitting needs no physical meaning. Note that seismic wave spectra are typically narrow band (no wider than two decades of frequency), in which $\chi(f)$ is less likely to exhibit a significant non-linear (higher order) behavior, and more likely to be fit by the linear terms.

5.1. Fit to Short-Period Data

Numerous previous studies used high-frequency amplitude datasets to successfully fit a power-law frequency dependence of $Q(f)$ (for summaries, see ANDERSON, 1989; XIE and MITCHELL, 1990; MITCHELL, 1991, 1995)

$$Q(f) = Q_0 f^n, \tag{9}$$

where free parameters Q_0 and η are Q at 1 Hz and its power-law frequency dependence, respectively. The attenuation function $\chi(f)$ can be expressed in terms of these free parameters (Eq. M-10)

$$\chi(f) = \pi f^{1-\eta}/Q_0. \tag{10}$$

Figure 3 shows that in the typical frequency band covered by these previous data, the attenuation functions from a previously obtained power-law $Q(f)$ can be approximated well by linear trends that represent the new GS model. This is because the power-law functions have well-known long linear

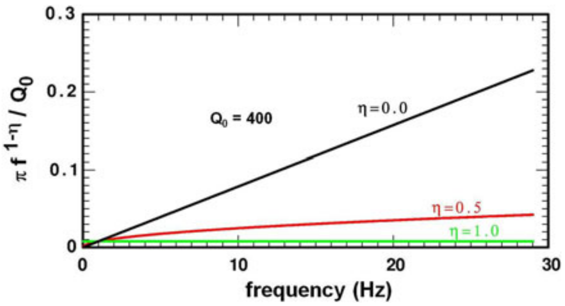

Figure 3
Examples of the attenuation function, $\chi(f) = \pi f / Q(f)$ versus frequency f constructed assuming $Q(f)$ follows a power law (Eq. 9), under which the quantity becomes $\pi f^{1-\eta}/Q_0$ (Eq. 10). In many published studies, a power-law $Q(f)$ with η between 0.0 and 1.0 were found to fit well regional and teleseismic wave amplitude data in high-frequency bands that are no wider than two decades (e.g., between 0.1 and 10 Hz or 1 and 30 Hz). Three curves with representative values of $\eta = 0.0$, 0.5 and 1.0 and a constant Q_0 of 400 are plotted; they can be viewed as examples of error-free data extracted from previous publications. A beginning frequency of 0.02 Hz is used to exclude lower frequencies at which the power laws may not hold. The new GS model by Morozov (2010) requires that $\chi(f)$ depends on f linearly (Eqs. M-9 and M-10 of this paper). As shown by the examples here, in the limited frequency bands the $\chi(f)$ constructed using power-law Q has little curvature and can indeed be refit by straight lines; the refit does not contradict a power-law $Q(f)$

tails. The narrow frequency band makes linear approximation of the tails very good, even for error-free data. There is a standing physical model to interpret power-law $Q(f)$, known as the "absorption band" model (e.g., Anderson, 1989). Linear fits to the tails do not invalidate the previously proposed power-law $Q(f)$ and the absorption band model, and do not bring in any physical meanings to the linear terms (γ and π/Q_e, both are frequency-independent).

5.2. Long-Period Surface Wave Data

For 1D Earth models, dispersion and amplitude decay (or $1/Q(f)$) of fundamental mode surface waves are related to layered velocity and $Q(f)$ values through complex, non-linear relationships (Anderson and Archambeau, 1964). Surface wave $Q(f)$ ($Q_R(f)$ and $Q_L(f)$ for Rayleigh and Love waves, respectively) are generally frequency-dependent because they sample depth-varying Q and velocity differently at each period. This phenomenon allows one to invert for layered Q structure (e.g., Anderson and Archambeau, 1964; Anderson *et al.*, 1965; Mitchell, 1995). Using his new GS model, Morozov (2010, Figure 6) converted the $Q_R(f)$ and $Q_L(f)$ data of Anderson *et al* (1965) to "attenuation coefficient" using $\chi(f) = \pi f / Q_R(f)$ or $\chi(f) = \pi f / Q_L(f)$ (Eq. M-10). These $\chi(f)$ values are then refit by the new GS model using (M-10), with frequency-independent γ and Q_e parameters. These parameters are then used to interpret the Earth structure in a totally different way than that used in traditional surface wave inversions.

Given the complex relationships between surface wave $Q(f)$ and the layered body wave Q and velocities, it is surprising that Morozov (2010) can fit well the $\pi f / Q_R(f)$ or $\pi f / Q_L(f)$ values in Anderson *et al.* (1965) by the linear relationship $\gamma + \pi f / Q_e$. To explore if the fit can be generally successful for the vast surface wave attenuation data in the numerous publications, I note that if the fit is valid we would have

$$\frac{\pi f}{Q_F(f)} = \gamma + \frac{\pi}{Q_e} f, \tag{11}$$

where subscript "F" is used to represent "R" or "L" for Rayleigh or Love waves, respectively. Replacing f by the inverse of period, $1/T$, Eq. 11 leads to a derivative

$$\frac{dQ_F(T)}{dT} = -\frac{\pi\gamma}{(\gamma T + \pi/Q_e)^2}, \tag{12}$$

which means that $Q_R(T)$ and $Q_L(T)$ would be monotonic functions of T, either decaying for a positive γ or increasing for a negative γ. Many published $Q_R(T)$ or $Q_L(T)$ models do not exhibit such monotonic behavior. For example, Figure 14 of Dalton and Ekstrom (2006) summarizes several 1D $Q_R(T)$ models. In that figure only one model (QM1) is monotonic with T. Other models vary with T in more complex manners and hence fundamentally contradict the GS model.

5.3. Comparison with Laboratory $Q_i(f)$ Measurements

Recently, substantial progress has been made in measuring intrinsic $Q_i(f)$ using samples of upper mantle materials, under realistically high temperature, pressure, and at frequencies down to 1 Hz (e.g., Jackson, 2000; Jackson *et al.*, 2004). For a wide range of temperature below the solidus and many samples with varying grain sizes, $Q_i(f)$ exhibits moderate power-law frequency dependence ($\eta \sim 0.3$). These measurements are not affected by any elastic structural effects on wave propagation, thus providing independent evidence of frequency dependent $Q_i(f)$ resulting from the viscoelasticity. From Eq. 3, the total $Q(f)$ must be frequency-dependent with a frequency-dependent $Q_i(f)$, unless $Q_s(f)$ has the same magnitude of $Q_i(f)$ but a frequency dependence that is exactly opposite to that of $Q_i(f)$. The latter scenario is very unlikely.

Morozov (2010) argues that laboratory measurements of $Q_i(f)$ are not relevant to the understanding of frequency dependence of seismic $Q(f)$, but his argument is based on an obsolete reference (Bourbie *et al.*, 1987) that summarized early-day acoustic wave measurements at ultra-high (KHz to GHz) frequencies, and at low temperatures and pressures-conditions that are indeed irrelevant to our debate.

6. Discussion

In this paper I have focused on whether or not there is a physical basis for the exponentially decaying GS model proposed by Morozov (2008, 2009, 2010). If this physical basis exists, it would be a new aspect of that model. The mathematical form of the Morozov model is actually not new because Dainty (1981) already proposed a special case under the traditional model, in which the reduced amplitude has a mathematical expression that is identical to Eqs. M-7 through M-10. For seismic waves between 1 and 30 Hz, Dainty (1981) assumed that the scatterers in the lithosphere are large ($a/\lambda > 2\pi$) and intrinsic $Q_i(f)$ is frequency-independent. In that case $Q_s(f)$ in Eq. 3 takes a special form

$$Q_s(f) = \frac{2\pi f}{gv}, \tag{13}$$

where g and v are medium turbidity and wave velocity, respectively. Equation 5 then becomes

$$A'(f,t) = \frac{A(f,t)}{P_0(f,t)} \approx e^{-\left(\frac{2\pi f}{Qi}+gv\right)t}. \tag{14}$$

This special case of the traditional model leads to an attenuation function of

$$\chi(f) = \frac{gv}{2} + \frac{\pi}{Q_i} f. \tag{15}$$

Equations 14 and 15 are mathematically identical to Eqs. M-8 and M-9, respectively. But unlike the new GS model, the form of $Q(f)$ proposed by Dainty (1981) was based on specific physical assumptions about the lithospheric heterogeneity, and had a stated range of applicability in terms of the data type and frequencies. Dainty (1981) physically predicted a frequency-dependent total $Q(f)$ and $Q_s(f)$, contrasting the frequency-independent total Q (denoted as Q_e) by Morozov (2008, 2009, 2010).

In this paper I have not discussed the path effect $P(f, t)$ for coda wave amplitudes, which were also refit by Morozov (2008, 2010). Multiple isotropic 3D (body wave) and 2D (surface wave) scatterings, in a uniform background structure, have closed form solutions that yield $P_0(f, t)$ or $G_0(f, t)$ (Zeng *et al.*, 1991; Sato, 1993). But exploration of the form of true $P(f, t)$ or $G(f, t)$ for coda waves requires further knowledge of statistical properties of scatterers, including their sizes, density, and angular distribution of scattering waves (i.e., the patterns of nonisotropic scattering by scatterers). These properties vary both laterally and with depth. Considering these complications, the length of this paper, and the limited further insight that could be gained, I chose not to discuss coda waves.

7. Conclusion

Viscoelasticity and small-scale heterogeneity of the Earth physically cause finite seismological Q, a quantity that is typically measured with a limited precision. This limitation is caused by a lack of knowledge of the Earth's larger-scale velocity structures. That knowledge is needed in order to assess their effects on seismic amplitude data. In individual studies, carefully chosen data screening, smoothing, and measurement methodologies have been used to avoid or reduce the unknown structural effects such as focusing, defocusing, multipathing and diffraction. Nevertheless, the larger number of Q measurements by seismologists using crustal and mantle seismic phases have revealed two robust features of Q: it is highly laterally variable, and often frequency dependent. These features are independently supported by recent laboratory measurements of intrinsic Q of mantle materials, conducted under realistic pressures and temperatures and at frequencies down to 1 Hz. The rapidly increasing body of data, improved velocity models and measurement methodologies will continuously bring in improvements of Q measurements.

A new "geometrical spreading" (GS) model for seismic amplitudes has recently been proposed by Morozov (2008, 2009, 2010) to improve Q measurements. The model contains an exponentially decaying term, $e^{-\gamma t}$, as part of the "geometrical spreading" or more generally, the unknown structural effects. Frequency independent γ and Q terms are set as free parameters in a new measurement procedure. Many published datasets were refit using the new procedure and parameters. These refits are used to infer that previous Q measurements by the community are systematically biased in their values and their frequency dependency. I find that the exponential term in the new GS model lacks a general physical basis. The fit of the new model to high-frequency data is effectively just a curve fitting with first-order Taylor series expansion. The fit to long-period surface wave data fail when multiple datasets, rather than a single dataset used by Morozov (2010), are examined. The new model, free parameters, and procedure prescribe an incorrect solution for improving Q measurements. They physically cause confusion, rather than providing new insight, of the Earth's Q structure.

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Appendix 1: The Intended Physical Meaning of the "Geometrical Spreading" Model by Morozov

The intended physical meaning of the GS model, quantified by $G(f, t)$ in Eq. M-1, which is taken from Morozov (2008; equations 7 and 8) and Morozov (2010, equation 1), is somewhat confusing. As stated in the main text, this GS model is used inconsistently with the traditionally physically defined geometrical spreading term caused by wavefront expansions. Morozov (2008) called the process causing $G(f, t)$ as "geometrical scattering" and phenomenologically described it as including "*all effects of background structures, such as ray bending and reflections,…*"; "*such effects are causal and may not be considered as scattering in other studies*". He also stated "*geometrical scattering is the process in which the wave energy is redistributed among the wave fronts*", which may be confused with any kind of elastic scattering. Subsequently Morozov (2010) stated that the variability of GS is caused by "*variations in crustal thickness, velocity gradients, layering, reflectivity, and other attributes of the lithospheric structure*". "*…phenomenological models* [of GS] *do not require detailed descriptions of the mechanisms of the wave processes but may be based on some general principles, such as the conservation of energy and time/spatial continuity*". Asked for a clarification, in revision of Morozov (2010; Igor Morozov, personal communication, November 2009) he adds "*I suggest: GS is the effect of the large scale, dissipation free structure on seismic amplitudes*". This last description of GS matches the conventionally defined path effect $P(f, t)$ in Sect. 2. The absence of any difference between Morozov's $G(f, t)$ and traditionally defined $P(f, t)$ is also seen by comparing Eq. M-1, taken from Eqs. 3 and 5 of Morozov (2008) and Eqs. 1 and 2 of Morozov (2010), with Eq. 1.

The assumed exponential portion of the GS model $e^{-\gamma t}$, called "geometrical scattering" (Morozov, 2008) and "residual geometrical spreading" (Morozov, 2010), is introduced without any physical reasoning in Morozov (2008, 2009) and in the first submission of Morozov (2010; Igor Morozov, written communication, November 2009). The key question we asked is (see Xie and Fehler, 2009) what is the physical basis by which to assume that, the path effects other than random scattering causes an exponential decay of amplitude with time? To address this, the revised version of Morozov (2010) stated "*one should not attribute excessive significance to the functional form of the amplitude correction factor* $e^{-\gamma t}$ *in equation (1)* [of Morozov, 2010; which is quoted as Eq. M-1 in Sect. 3]. *This* [GS] *model does not simu*late *any particular wave-spreading process, … an intractable problem. Mathematically, approximation (1)* [of Morozov, 2010 or (M-6) in this paper] *is suitable when* γt << *1, in which case other first-order approximations (such as* $G_0(1-\gamma t)$ *or* $G_0 t^{-\gamma t}$*) would work as well*". This new statement still offers no physical reasoning of why a drastically decaying $e^{-\gamma t}$ function is used for the "residual GS". Furthermore, its new condition for using the exponential function, given by

$$\gamma t \ll 1 \tag{16}$$

is not satisfied in any of the data analyzed by Morozov (2008; 2010). To see this we note condition (16) can be rewritten as

$$\Delta \ll V/\gamma \tag{17}$$

where Δ and V are the path length and wave velocity. The data plotted in Figure 1 of Morozov (2010) has a γ estimate of 0.008. Using a crustal shear velocity in Morozov (2010) of $V \sim 3.5$ km/s, condition (17) requires $\Delta <<$ 327.5 km, or $\Delta \sim 33$ km when the sign "<<" is used for meaning "smaller by an order of magnitude". The data plotted in the figure covers distances of up to 300 km which are much greater than 33 km, violating condition (17). Similarly drastic violations can be found for data plotted in Figures 2 through 4 of Morozov (2010), using his γ estimates of between 0.006 and 0.02, and V that is typically $\sim$3.5 km/s (data in Figure 5 of Morozov (2010) was used to infer for a virtually vanished $\gamma \sim 0$). The analysis of Lg and PNE data in Morozov (2008) violates condition (17) for the same reason. The analysis of coda waves in Morozov 2008 (Table 2) contradict condition (16) even more directly: for example, the data used by Mayeda *et al.* (1992) used t of up to 45 s; it is refit by a γ of 0.035 by Morozov (2008; Figures 3 and 4) which leads to a γt of $\sim$1.6.

Since imposing the new condition of Morozov (2010), quoted by (16) above, would make virtually all of the data analysis in Morozov (2008, 2010) invalid, I ignore that new condition in this paper. It remains that, no physical basis for the "residual GS" $e^{-\gamma t}$ is ever given by Morozov (2008, 2009; 2010). I devote a section of this paper to search for this physical basis by myself, and conclude it does not generally exist.

Appendix 2: Focusing and Defocusing Effects in a Structure with Lateral Velocity Gradient

Figure 2b in the main text demonstrates the focusing/defocusing effects with wave propagation in a 2D plane with a constant velocity gradient along the x direction. We can easily solve for the analytical form of the geometrical spreading effect along the line that parallels the x axis and includes the source (with $y = 0$ km; see the line of traces in Fig. 2). Propagation is fastest in the x direction and slowest in the $-x$ direction along this line, and the normal of the wave front is in-line. The geometrical spreading $G(x)$ along the line is

$$G(x) = \frac{1}{\sqrt{(|x|)}}. \tag{18}$$

Expressing the velocity field with gradient as

$$v(x) = v_0(1 + cx), \tag{19}$$

the travel time (or event-elapse time) can be expressed as function of x:

$$t = sign(x)\int \frac{\mathrm{d}x}{v(x)} = \frac{sign(x)}{v_0}\int \frac{\mathrm{d}x}{1+cx} = \frac{sign(x)}{v_0 c}\ln(1 + cx), \tag{20}$$

where $sign(x)$ is just the sign of x. Solving x in terms t we have

$$x = \frac{1}{c}\left(e^{sign(x)v_0 ct} - 1\right) \tag{21}$$

Substitute (21) into (18) we have

$$G(t) = \sqrt{\frac{c}{e^{v_0 ct} - 1}} \quad \text{for } x > 0. \tag{22}$$

and

$$G(t) = \sqrt{\frac{c}{1 - e^{-v_0 ct}}} \quad \text{for } x < 0. \qquad (23)$$

If one does not know the true velocity structure and assume a constant velocity, v_c, to calculate the geometrical spreading $P_0(t)$ or $G_0(t)$

$$G_0(t) = 1/\sqrt{v_c t}, \qquad (24)$$

then the error of $G_0(t)$ is

$$E_r(t) = \frac{G(t)}{G_0(t)} = \sqrt{\frac{c v_c t}{e^{v_0 ct} - 1}} \quad \text{for } x > 0, \qquad (25)$$

which monotonically decays with x because of defocusing, and

$$E_r(t) = \frac{G(t)}{G_0(t)} = \sqrt{\frac{c v_c t}{1 - e^{-v_0 ct}}} \quad \text{for } x < 0, \qquad (26)$$

which monotonically increases with $|x|$ because of focusing. A practical constraint on the t-range of validity of Eq. 19 (and therefore Eqs. 20 through 26 is that, the peak for velocity heterogeneity in the Earth does not exceed about 30%. This means along $x > 0$ direction where defocusing occurs, reasonable ranges of x and t are

$$x < 0.3/c, \qquad (27)$$

and

$$t < 1/v_0 c \ln(1 + 0.3). \qquad (28)$$

Using the c value of 1.0×10^{-3} and v_c of 3.75 km/s (the mean velocity in Fig. 2b), we obtain $x < 300$ km and $t < 69$ s, respectively. Figure 4 shows the defocusing $E_r(t)$ in this t range. A similar consideration along the $x < 0$ direction, where focusing occurs, lead to a range of validity of $x > -300$ km and $t < 96$ s, respectively. The defocusing $E_r(t)$ is also shown in Fig. 4 in the valid t range. Both focusing and defocusing $E_r(t)$ in Fig. 4 are expected to reverse trend beyond the cut-off t values because the monotonic velocity change to beyond 30% cannot be sustained in a reasonable structure. In comparison Fig. 4 also shows the exponential function of $e^{-\gamma t}$, proposed for $E_r(t)$ under the GS model (Morozov, 2008, 2010), with two end γ values of 0.006 and 0.02. Within the reasonable range of t values, the focusing and defocusing $E_r(t)$ depart from unity less than the $e^{-\gamma t}$ functions. Beyond the t ranges plotted, a physically reasonable $E_r(t)$ will be even more different than the $e^{-\gamma t}$ functions because focusing and defocusing are expected to reverse trends. Hence $e^{-\gamma t}$ are not adequate for quantifying the $E_r(t)$ calculated for defocusing and defocusing.

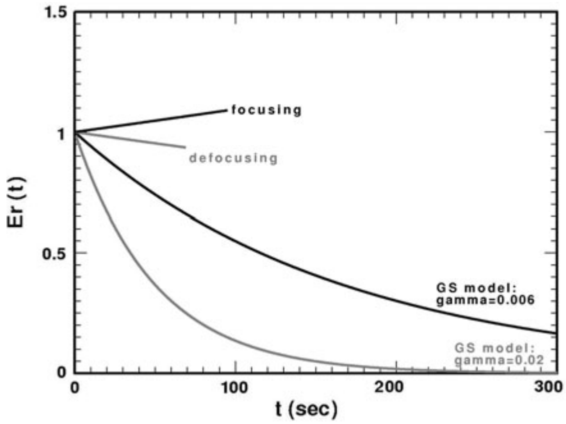

Figure 4
Errors in calculated path effect, $E_r(t)$, caused by unaccounted-for focusing and defocusing effects in a 2D gradient velocity structure (Fig. 2), given by Eqs. 25 and 26. The maximum time for the calculations correspond to distances at which a linear change of velocity reaches ±30% for defocusing and focusing, respectively. At greater distances the focusing and defocusing should alternate because the monotonic increase or decrease of velocity cannot be sustained. By comparison, the new GS model with γ values between 0.02 and 0.006 (Morozov, 2010) decay monotonically and drastically with time

The much simplified case considered in this appendix demonstrates that for focusing and defocusing phenomena, our knowledge of the peak values of Earth's velocity heterogeneity puts a constraint on whether the error of simplified path effect (P_0 or G_0) can be as drastic as that of an exponentially decaying function. In a more realistic structure the heterogeneity is neither linear nor monotonic, and the focusing/defocusing effects would alternate along path and compensate one another. The exponential functions would not generally be adequate for the cumulative focusing/defocusing effects until the propagation length, and the number of alterations, become so large that the random scattering regime (Fig. 1) is reached.

References

Aki, K. (1980), *Scattering and attenuation of shear waves in the lithosphere,* J. Geophys. Res. 85, 6496–6504.

Aki, K. and Richards, P.G. (1980) *Quantitative Seismology Theory and Methods,* W.H. Freeman, San Francisco, CA.

Anderson, D. L. (1989) *Theory of the Earth,* Blackwell Scientific Publications, Boston, 366 pp.

Anderson, D. L., Archambeau, C. B. (1964), *The Anelasticity of the Earth*, J. Geophys. Res., Vol. 69, No. 10, pp. 2071–2084.

Anderson, D. L., Ben-Menahem, A., and Archambeau, C. B. (1965), *Attenuation of seismic energy in the upper mantle*, J. Geophys. Res. 70, 1441–1448.

Boltzmann, S (1874), *Zur Theorie der elastischen Nachwirkung,* Kaiserl. Akad. Wiss. Wien Math.-Naturw. KL Sitzungsber. Math. Naturwiss Kl. Kaiserl Wiss. 70, 275–306.

Borcherdt, R. D. (2009), *Viscoelastic Waves in Layered Media,* Cambridge University Press, New York, 205 pp.

Bourbie, T., Coussy, O., and Zinsiger, B. (1987), Acoustics of porous media, Editions TECHNIP, France, 334 pp. ISBN 2710805168.

Campillo, M. and Paul, A. (2003), *Long-Range Correlations in the Diffuse Seismic Coda,* Science, 299(5606), 547–549, doi: 10.1126/science.1078551.

Capon, J. (1970), *Analysis of Rayleigh-wave multipath propagation at Lasa,* Bull. Seism. Soc. Am. 60, 1701–1731.

Carcione, J.M., Kosloff, D. and Kosloff, R. (2007), *Wave propagation simulation in a linear viscoelastic medium,* Geophys. J. Int., 95, 597–611.

Cerveny, V. and Ravindra, R. (1971), *Theory of Seismic Headwaves*, University of Toronto Press, Toronto.

Cerveny, V., J. Langer, and I. Psenik, (1974), *Computation of geometrical spreading of seismic body waves in laterally inhomogeneous media with curved interfaces,* Geophys. J., 38, 9–19.

Chen, P., Zhao, L. and Jordan, T.H., (2007), *Full 3D tomography for the crustal structure of the Los Angeles region*, Bull. Seism. Soc. Am., 97, 1094–1120.

Chun, K.Y., West, G.F., Kokoski, R.J. and Samson, C. (1987), *A novel technique for measuring Lg attenuation - Results from eastern Canada between 1-Hz to 10-Hz,* Bull. Seism. Soc. Am., 77, 398–419.

Dahlen, F.A. and Tromp, J. (1998), *Theoretical global seismology*, Princeton University Press, Princeton, NJ, 1025 pp.

Dainty, A.M. (1981), *A scattering model to explain seismic Q observations in the lithosphere between 1 and 30 Hz,* Geophys. Res. Lett. 8, 1126–1128.

Dalton, C.A. and Ekstrom, G. (2006), *Global models of surface wave attenuation,* J. Geophys. Res., 111, B05317, doi:10.1029/2005JB003997.

Der, Z. A., Lees, A. C., and Cormier, V. F. (1986), *Frequency dependence of Q in the mantle underlying the shield areas of Eurasia, Part III: the Q model,* Geophys. J. R. Astr. Soc. 87, 1103–1112.

Der, Z.A., Mcelfresh, T.W. and O'donnell, A. (1987), *An investigation of the regional variations and frequency dependence of anelastic attenuation in the mantle under the United States in the 0.5-4 Hz band*, Geophys. J. Royal Astro. Soc., 69, 67–99.

Flanagan, D.A. and Wiens, J. (1994), *Radial upper mantle attenuation structure of inactive back arc basins from differential shear wave measurements*, J. Geophys. Res., 99, 15,469–15,485.

Flanagan, D.A. and Wiens, J. (1998), *Attenuation of Broadband P and S Waves in Tonga: Observations of Frequency Dependent Q*, Pure and appl. Geophysics, 153, 345–375, doi:10.1007/s000240050199.

Gung, Y. and Romanowicz, B. (2004), *Q tomography of the upper mantle using three component long period waveforms*, Geophys. J. Int., 157, 813–830.

Goes, S. and Van Der Lee, S. (2002), *Thermal structure of the North American uppermost mantle inferred from seismic tomography,* J. Geophys. Res., 107, doi:10.1029/2000JB000049.

Helmberger, D.V. and G. R. Engen (1980), *Modeling the long-period body waves from shallow earthquakes at regional ranges*, Bull. Seis. Soc. Am., 70, 1699–1714.

Jackson, I, (2000), *Laboratory measurement of seismic wave dispersion and attenuation: recent progress,* in *Earth's Deep Interior: Mineral Physics and Tomography From the Atomic to the Global Scale*, AGU Monograph, 117, 265–289.

Jackson, I., Faul, U.H., Fitz Gerald, J.D. and Tan, B.H. (2004), *Shear wave attenuation and dispersion in melt-bearing olivine polycrystals: 1. Specimen fabrication and mechanical testing,* J. Geophys. Res., 109, B06201, doi:10.1029/2003JB002406.

Ji, C., Tsuboi, S., Komatitsch, D. and Tromp, J. (2005), *Rayleigh-wave multipathing along the west coast of North America*, Bull. Seis. Soc. Am., 95, 2115–2124.

Karato, S-I. (1993), *Importance of anelasticity in the interpretation of seismic tomography*, Geophys.Res. Lett., 20, 1623–1626, doi:10.1029/93GL01767.

Karato, S-I. and Spetzler, H.A. (1990), *Defect microdynamics in minerals and solid-state mechanisms of seismic wave attenuation and velocity dispersion in the mantle,* Rev. Geophys., 28, 399–421, doi:10.1029/RG028i004p00399.

Kennett, B.L.N. (1983), *Seismic Wave Propagation in Stratified Media*, Cambridge University Press, 1983 Cambridge, UK., 342pp.

Knopoff, L. (1964), *Q*, Rev Geophys. 2, 625–660.

Langston, C.A. (1989), *Scattering of long-period Rayleigh waves in Western North America and the interpretation of coda Q measurements,* Bull. Seism. Soc. Am., 79, 774–789.

Lekic, V., Matas, J. Panning, M. and Romanowicz, B. (2009), *Measurement and implication s of frequency dependence of attenuation,* Earth Planet. Sci. Lett., doi:10.1016/j.epsl.2009.03.030.

Li, J., Ma, G. and Zhao, J. (2010), *An equivalent viscoelastic model for rock mass with parallel joints,* J. Geophys. Res., 115, doi: 10.1029/2008JB006241.

Liu, H.P., Anderson, D.L., and Kanamori, H. (1976), *Velocity DIspersion due to Anelasticity: Implication for Seismology and Mantle Composition,* Geophys. J. R. Astro. Soc., 47, 41–58.

Mayeda, K., Koyanagi, S., Hoshiba, M., Aki, K. and Zeng, Y. (1992), *A comparative study of scattering, intrinsic, and coda Q-1 for Hawaii, Long Valley, and central California between 1.5 and 15.0 Hz,*, J. Geophys. Res., 97, 6643–6659.

Menke, W. and Richards, P.G. (1980), *Crust-mantle whispering gallery phases -a deterministic model of teleseismic Pn wave-propagation,* J. Gephys. Res., 85, 5416–5422.

Menke, W. and Richards, P.G. (1983), *The horizontal propagation of P-waves through scattering media—analog model studies relevant to long-range Pn propagation,* Bull. Seism. Soc. Am., 73, 125–142.

Menke, W., Levin, V. and Sethi, R. (1995), *Seismic attenuation in the crust at the mid- Atlantic plate boundary in south-west Iceland,* Geophys. J. Int., 122, 175–182.

Menke, W., Holmes, C.R. and Xie, J. (2006), On the Nonuniqueness of the Coupled Origin Time-Velocity Tomography Problem, Bull. Seis. Soc. Am., 96, 1131–1139; doi:10.1785/0120050192.

Mcnamara, D.E., Walter, W.R., Owens, T.J. and Ammon, C.J. (1997), *Upper mantle velocity structure beneath the Tibetan Plateau from Pn travel time tomography*, J. Geophys. Res., 102, 493–505.

Mitchell, B.J. (1991). *Frequency dependence of QLg and its relation to crustal anelasticity in the Basin and Range province,* Geophys. Res. Lett., 18, 621–624.

Mitchell, B.J., (1995), *Anelastic structure and evolution of the continental-crust and upper-mantle from seismic surface-wave attenuation,* Rev. of Geophys., 33, 441–462.

Mitchell, B.J. and Xie, J. (1994), *Attenuation of multiphase surface waves in the Basin and Range Province, Part III: Inversion for crustal anelasticity,* Geophys. J. Int., 116, 468–484.

Mori, J. and Frankel, A. (1992), *Correlation of P Wave Amplitudes and Travel Time Residuals for Teleseisms Recorded on the Southern California Seismic Network,* J. Geophys. Res., 97, B5, 6661–6674, doi:10.1029/91JB02578.

Morozov, I. (2008), *Geometrical attenuation, frequency dependence of Q, and the absorption band problem*, Geophys. J. Int., 175, 239–252.

Morozov, I. (2009), *Thirty years of confusion around "scattering Q"?*, Seism. Res. Lett., 80, 5–7.

Morozov, I. (2010), *On the Causes of Frequency-Dependent Apparent Seismological Q*, Pure and appl. Geophys., this volume.

Nowack, R.L. and Stacy, S. (2002), *Synthetic seismograms and wide-angle seismograms attributes from the Gaussian beam and reflectivity methods for models with interfaces and gradients*, Pure and appl. Geophys., 159, 1447–1464.

Pavlis, G. L., Alshukri, H., Mahdi, H., Repin, D., and Vernon, F.L. (1994), *JSP Arrays and Networks in Central Asia,* IRIS News Lett., XIII(2), 9–12.

Pasyanos, M.E., Matzel, E.M., Walter, W.R. and J. Rodgers, R.J. (2009), *Broad-band Lg attenuation modeling in the Middle East*, Geophys. J. Int., 177, 1166–1176, doi:10.1111/j.1365-246X.2009.04128.

Phillips, W.S., Hartse, H.E., and Rutledge, J.T. (2005), *Amplitude ratio tomography for regional phase Q,* Geophys. Res. Lett., 32, L21301, doi:10.1029/2005GL023870.

Pilant, W.L. (1979), *Elastic Waves in the Earth*, Elsevier Scientific Pub. Co. New York, 493 pp.

Prieto, G. A., Lawrence, J.F.and Beroza, G.C. (2009), *Anelastic Earth structure from the coherency of the ambient seismic field,* J. Geophys. Res., 114, B07303, doi:10.1029/2008JB006067.

Romanowicz, B. (1998), *Attenuation tomography of the Earth's Mantle: a review of current status,* Pure and appl. Geophys., 153, 257–272.

Romanowicz, B. A. (2009), *Progress and challenges in global mantle attenuation tomography,* (abs) Eos Trans. AGU, 90(52), Fall Meet. Suppl.

Romanowicz, B. and Mitchell, B., (2007)., *Q in the earth from crust to core,* in *Treatise on Geophysics,* I, 731–774, Elsevier, Amsterdam.

Ranalli, G. (1995). *Rheology of the Earth*, Chapman & Hall, 413 pp.

Sato, H. (1993), *Energy Transportation In One- and Two-Dimensional Scattering Media: Analytic Solutions of the Multiple Isotropic Scattering Model*, Geophys. J. Int. 112 141–146.

Sato, H., and Fehler, M. (2009), *Seismic Wave Propagation and Scattering in the Heterogeneous Earth*, Springer-Verlag Berlin Heidelburg, 308 pp.

Selby, N.D. and Woodhouse, J.H. (2002), *The Q structure of the upper mantle: Constraints from Rayleigh wave amplitudes*, J. Geophys. Res., 107, 2097, doi:10.1029/2001JB000257.

Sereno, T.J. and Given, J.W. (1990), *Pn attenuation for a spherically symmetric Earth model,* Geophys. Res. Lett., 17, 1141–1144.

Shito, A., Karato, S-I. and Park, J. (2004), *Frequency dependence of Q in Earth222s upper mantle inferred from continuous spectra of body waves*, Geophys. Res. Lett., 31, L12603, doi:10.1029/2004GL019582.

Sobolev, S.V., Zeyen, H., Stoll, G. and Werling, F. (1996), *Upper mantle temperatures from teleseismic tomography of French Massif Central including effects of composition, mineral reactions, anharmonicity, anelasticity and partial melt,* Earth and Planetary Science Lett. 139, 147–163, doi:10.1016/0012-821X(95)00238-8.

Stein, S. and Wysession, M. (2003), *An introduction to seismology, earthquakes, and earth structure,* Blackwell Publishing ltd., Malden, MA, 498 pp.

Tape, C., Liu, C., Maggi, A. and Tromp, J. (2010), *Seismic tomography of the southern California crust based on spectral-element and adjoint methods,* Geophys. J. Int. (2010) 180, 433–462 doi:10.1111/j.1365-246X.2009.04429.

Warren, L.M. and Shearer, P.M. (2002), *Mapping lateral variations in upper mantle attenuation by stacking P and PP spectra* J. Geophys. Res., 107, 2342, doi:10.1029/2001JB001195.

Xie, J. (1996), *Synthetic and observational study of Pn excitation and propagation in central Asia,* PHILLIPS Laboratory Scientific Report No. 1, PL-TR-96-2270, 24 pp., http://www.ldeo.columbia.edu/̃xie/xie96.pdf (last accessed July, 2010).

Xie, J. (1998), *Spectral inversion using Lg from earthquakes: Improvement of the method with applications to the 1995, western Texas earthquake sequence,* Bull. Seism. Soc. Am., 88, 1525–1537.

Xie, J. (2007), *Pn attenuation beneath the Tibetan Plateau,* Bull. Seism. Soc. Am., 97, 2040–2052, doi:10.1785/0120070016.

Xie, J. and Mitchell, B.J. (1990), *Attenuation of multiphase surface waves in the Basin and Range province, Part 1: Lg and Lg coda,* Geophysical J. Int., 102, 121–138.

Xie, J., Gok, R., Ni, J. and Aoki, A. (2004), *Lateral variations of crustal seismic attenuation along the INDEPTH Profiles in Tibet from Lg Q inversion,* J. Geophys. Res., 109, B10308, doi:10.1029/2004JB002988.

Xie, J., Wu, Z., Liu, R., Schaff, D., Liu, Y., and Liang, J. (2006), *Tomographic regionalization of crustal Lg Q in eastern Eurasia,* Geophys. Res. Lett., 33, L03315, doi:10.1029/2005GL024410..

Xie, J. and Fehler, M. (2009), *Comment on "Thirty Years of Confusion around Scattering Q" by Igor B. MOROZOV*, Seism. Res. Lett., 80, 646–647, doi:10.1785/gssrl.80.4.646.

Xie, X.B. and Yao, Z.X. (1988), *P-SV wave responses for a point source in two-dimensional heterogeneous media: finite-difference method,* Chinese J. Geophys., 31, 473–493.

Yang, X., Lay, T., Xie, X.-B., and Thorne, M. S. (2007), *Geometric spreading of Pn and Sn in a spherical Earth model,* Bull. Seis. Soc. Am., 97, 2053–2065.

Yang, Y., and Forsyth, D.W., (2008), *Attenuation in the upper mantle beneath Southern California: Physical state of the lithosphere and asthenosphere*, J. Geophys. Res., 113, B03308, doi:10.1029/2007JB005118.

YOMOGIDA, K., and AKI, K. (1985), *Waveform Synthesis of Surface Waves in a Laterally Heterogeneous Earth by the Gaussian Beam Method,* J. Geophys. Res., 90, 7665–7688.

YOMOGIDA, K., and AKI, K. (1987), *Amplitude and phase data inversions for phase velocity anomalies in the Pacific Ocean basin,* Geophys.J. Royal Astro. Soc. 88, 161–204.

ZENG, Y., SU, F., and AKI, K. (1991), *Scattering Wave Energy Propagation in a Random Isotropic Scattering Medium 1. Theory,* J. Geophys. Res., 96, 607–619, doi:10.1029/90JB02012.

ZHU, T.F., CHUN, K.Y. and WEST, G.F. (1991), *Geometrical spreading and Q of Pn waves: An investigative study in eastern Canada,* Bull. Seism. Soc. Am., 81, 882–896.

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