Ann. Henri Poincaré 15 (2014), 2441–2465

1424-0637/14/122441-25
published online December 18, 2013
DOI 10.1007/s00023-013-0307-z

Annales Henri Poincaré

Structural Stability for Two Convection Models in a Reacting Fluid with Magnetic Field Effect

Akil J. Harfash

Abstract. This paper deals with two fundamental models for convection in a reacting fluid and porous medium with magnetic field effect. We demonstrate that the solution depends continuously on changes in the chemical reaction and the electrical conductivity coefficients. The continuous dependence is unconditional in two dimensions but conditional in three dimensions.

1. Introduction

There has been much recent interest in obtaining stability estimates for solutions to physical problems in partial differential equations where changes in coefficients are allowed, or even the equations themselves change. This type of stability, which is often called structural stability to distinguish it from continuous dependence on the initial data, is studied for example in Ames and Payne [1–4], Franchi and Straughan [15–18], Lin and Payne [31–35], Payne and Song [41–43], and Payne and Straughan [45–49], Payne et al. [44], Straughan and Hutter [60], Harfash [24,26], and also occupies attention in the books of Bellomo and Preziosi [6], Ames and Straughan [5] and Straughan [58]. Such stability estimates are fundamental to analysing whether a small change in a coefficient or other data leads to a drastic change in the solution. A concrete example of structural stability, and in particular continuous dependence on modelling, is provided in the paper by Payne and Straughan [45], where it is shown how a solution to the Stokes equation for slow viscous flow approximates that to the Navier–Stokes equations. Thus, questions of continuous dependence on the model itself are fundamental and in many ways are as important as a study of stability itself.

The effect of the magnetic field on the onset of instability in fluid and porous medium layers and the effect of magnetic coefficient on the solution have received considerable attention. Harfash and Straughan [27] consider convec-

Birkhäuser

tion of a reacting solute in a viscous incompressible fluid occupying a horizontal plane layer subject to a vertical magnetic field. Then in [59], they study a model for Poiseuille flow instability in a porous medium of Brinkman type with magnetic field effect, in particular, they analyse the effect of slip boundary conditions on the onset of instability. Harfash [25] study the problem of convection in a variable gravity field with magnetic field effect using methods of linear instability theory and nonlinear energy theory. Then in [23], studies double-diffusive convection in a reacting fluid in the presence of a concentration field, a magnetic field, and thermal sources.

This paper continues the investigation of continuous dependence properties of models introduced by Harfash and Straughan in [23, 27]. The first model which is studied in this paper is the problem of convection with a dissolved reacting fluid layer and a vertically imposed magnetic field

$$v_{i,t} + v_j v_{i,j} = -p_{,\,i} + \upsilon \Delta v_i + g_i c + \mathbf{j} \times \mathbf{B}, \tag{1.1}$$

$$v_{i,i} = 0, \tag{1.2}$$

$$c_{,t} + v_i\, c_{,\,i} = D\Delta c - K_1 c, \tag{1.3}$$

$$v_i = 0, \quad \frac{\partial c}{\partial n} = 0, \quad \text{on} \quad \partial\Omega, \tag{1.4}$$

$$v_i(\mathbf{x}, 0) = \psi_i(\mathbf{x}), \quad c(\mathbf{x}, 0) = f_1(\mathbf{x}), \quad \text{in} \quad \Omega, \tag{1.5}$$

where $\mathbf{v}$ is the velocity vector, c is the concentration field, p is pressure field, D is the diffusion coefficient, g_i is the gravity vector, $\mathbf{B}$ is the magnetic induction field, $\mathbf{j}$ is the current and K_1 is the chemical reaction rate. This system holds on a bounded spatial domain Ω in $\mathbb{R}^3$ with boundary $\partial\Omega$ sufficiently smooth to allow applications of the divergence theorem. Standard indication notation is employed with Δ denoting the Laplacian. The functions ψ_i and f_1 are assumed to be smooth functions.

Recently, this model has received considerable interest owing to many real-life applications. The convective instability created by a top heavy layer of fluid containing a solute is one with many applications in atmospheric physics, oceanography, and in pollution where the solute can cover a city and linger for long periods of time. Models for such behaviour were developed by Franchi and Straughan [19] and they completed a detailed instability analysis of their highly nonlinear models. In a separate development, Hayat and Nawaz [28] studied stagnation point flow in a rotating frame for a fluid containing a reacting solute with a superimposed magnetic field acting. Since convection in chemically reacting fluids has been a topic of much recent interest, cf. Malashetty and Biradar [37], Rahman and Al-Lawatia [50], and electro-magnetic field effects on such processes have likewise attracted much attention, cf. Eltayeb et al. [11, 12], Kaloni and Mahajan [29], Maehlmann and Papageorgiou [38], Nanjundappa et al. [39], Reddy et al. [51], Shivakumara et al. [55, 56], Sunil et al. [62], thus we believe that a study of (1.1)–(1.5) is very important.

The second model which is studied in this paper is the problem of double diffusive convection with a dissolved reacting fluid layer and a vertically imposed magnetic field,

$$v_{i,t} + v_j v_{i,j} = -p_{,i} + \upsilon \Delta v_i + h_i T + g_i c + \mathbf{j} \times \mathbf{B}, \tag{1.6}$$
$$v_{i,i} = 0, \tag{1.7}$$
$$T_{,t} + v_i T_{,i} = K \Delta T, \tag{1.8}$$
$$c_{,t} + v_i c_{,i} = D\Delta c - K_1 c, \tag{1.9}$$
$$v_i = 0, \qquad \frac{\partial c}{\partial n} = 0, \qquad \frac{\partial T}{\partial n} = 0, \quad \text{on} \quad \partial\Omega, \tag{1.10}$$
$$v_i(\mathbf{x}, 0) = \psi_i(\mathbf{x}), \quad c(\mathbf{x}, 0) = f_1(\mathbf{x}), \quad T(\mathbf{x}, 0) = f_2(\mathbf{x}), \quad \text{in} \quad \Omega, \tag{1.11}$$

where $\mathbf{v}$ is the velocity vector, c is the concentration field, T is the temperature field, p is pressure field, D is the diffusion coefficient, $\mathbf{B}$ is the magnetic induction field, $\mathbf{j}$ is the current, K_1 is the chemical reaction rate and K is the thermal diffusivity. This system holds on a bounded spatial domain Ω in $\mathbb{R}^3$ with boundary $\partial\Omega$ sufficiently smooth to allow applications of the divergence theorem. The functions ψ_i, f_1 and f_2 are assumed to be smooth functions. In (1.6), we have employed a Boussinesq approximation in the sense that the density is linear in T and c so that the gravity term may be written as

$$-k_i g(\alpha T - \alpha_c c)$$

where α and α_c are the thermal and salt expansion coefficients respectively, and g is gravity, cf. Straughan [57] page 102. Then $g_i = g k_i \alpha_c$ and $h_i = -g k_i \alpha$ are gravity coefficients, where $k_i = (0, 0, 1)$.

The problem of double diffusive convection in fluid and porous media has attracted considerable interest during the last 50 years. This is because of its wide range of applications, for instant modeling geothermal reservoirs [9,22,54]. Bioremediation, where micro-organisms are introduced to change the chemical composition of contaminants, is a very topical area, cf. Celia et al. [7], Chen et al. [8], Suchomel et al. [61]. Contaminant movement or pollution transport is a further area of multi-component, flow in porous media which is of much interest in environmental engineering, cf. Curran and Allen [10], Ewing and Weekes [13], Franchi and Straughan [19]. Other very important and topical areas of double diffusive occur in oil reservoir simulation, e.g. Ludvigsen et al. [36], and salinization in desert-like areas, Gilman and Bear [21]. Solar ponds are a particularly promising means of harnessing energy from the Sun by preventing convective overturning in thermohaline system by salting from below, cf. Leblanca et al. [30] and Nie et al. [40].

To make the convective overturning instability problem tractable, we employ the quasi-static magnetohydrodynamics MHD approximation of Galdi and Straughan [20]. This assumes that the electric field, $\mathbf{E}$, may be derived from a potential $\mathbf{E} = -\nabla\chi$. The magnetic field $\mathbf{H}$ and the electric field satisfy Maxwell's equations, cf. Roberts [53], Fabrizio and Morro [14], so that

$$curl\, \mathbf{H} = \mathbf{j}, \qquad curl\, \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Here, $\mathbf{B} = \mu \mathbf{H}$ and then Galdi and Straughan [20] show that if the vertical component in the perturbed motion is zero in the limit magnetic Prandtl number $P_m = \nu/\eta \to 0$, where η is the resistivity, then $\mathbf{j} \times \mathbf{B}$ in equation (1.1) and (1.6) may be replaced by

$$\mathbf{j} \times \mathbf{B} = \sigma(\mathbf{v} \times \mathbf{B}_0) \times \mathbf{B}_0,$$

where σ is the electrical conductivity and $\mathbf{B}_0 = (0, 0, B_0)$ is a magnetic field with only the vertical component. This obviates the need to employ the full MHD equations which also involve an equation for the evaluation of the magnetic field, $\mathbf{H}$, cf. Rionero and Mulone [52].

In both models, we establish that the solution depends continuously on change in the chemical reaction and electrical conductivity coefficients. This is extremely important, because if a small change in a coefficient in an equation, or in the boundary data, or in the equations themselves, will induce a dramatic change in the solution and it may well say something about how accurate the model is as a vehicle to describe flow in fluid layer.

The plan of the paper is as follows. In Sect. 2, we study the continuous dependence for the model of convective motion with a dissolved reacting fluid layer. In Sect. 3, we investigate the continuous dependence for the model of double diffusive convection with a dissolved reacting fluid layer.

2. Continuous Dependence for the Problem of Convection with a Dissolved Reacting Fluid Layer and a Vertically Imposed Magnetic Field

Lemma 2.1. *If $c(\mathbf{x}, 0)$ and $T(\mathbf{x}, 0) \in L^\infty(\Omega)$, then*

$$\|c(\mathbf{x}, t)\|_\infty \leq c_\infty, \tag{2.1}$$

$$\|T(\mathbf{x}, t)\|_\infty \leq T_\infty, \tag{2.2}$$

where $c_\infty = \|c(\mathbf{x}, 0)\|_\infty$, $T_\infty = \|T(\mathbf{x}, 0)\|_\infty$.

Proof. Multiply (1.3) by c^{p-1} for $p > 1$ (where we assume the concentration is scaled to be non-negative, otherwise p is chosen as an even integer). Thus,

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_\Omega c^p \mathrm{d}x = -p(p-1) \int_\Omega c^{p-2} |\nabla c|^2 \mathrm{d}x - K_1 p \int_\Omega c^p \mathrm{d}x. \tag{2.3}$$

We may integrate this and drop non-positive terms on the right to deduce

$$\left\{ \int_\Omega c^p \mathrm{d}x \right\}^{1/p} \leq \left\{ \int_\Omega c_0^p \mathrm{d}x \right\}^{1/p}. \tag{2.4}$$

Let now $p \to \infty$ in (2.4) to find the desired result. Similar argument can be used to prove (2.2). □

Lemma 2.2. *If $c(\mathbf{x}, 0) \in L^2(\Omega)$, then*

$$\|c(\mathbf{x}, t)\|^2 \leq c_l, \tag{2.5}$$

where $c_l = \|c(\mathbf{x}, 0)\|^2$.

Proof. Multiply (1.3) by c and integrating over Ω, we have

$$\frac{\mathrm{d}}{\mathrm{d}t} \|c\|^2 = -2D\|\nabla c\|^2 - 2K_1\|c\|^2.$$

We may integrate this and drop non-positive terms on the right to deduce

$$\|c(\mathbf{x},t)\|^2 \le \|c(\mathbf{x},0)\|^2.$$

□

Lemma 2.3. *If $v_i(\mathbf{x},0) \in L^2(\Omega)$, then*

$$\|\mathbf{v}(\mathbf{x},t)\|^2 \le v_l, \tag{2.6}$$

where $v_l = (c_l + \|\mathbf{v}(\mathbf{x},0)\|^2)e^T$.

Proof. Multiply (1.1) by v_i and integrating over Ω. Using the Cauchy–Schwarz inequality, arithmetic-geometric mean inequality and drop a non-positive term on the right, we have

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{v}\|^2 \le \int_\Omega g_i v_i c \,\mathrm{d}x \le \|\mathbf{v}\|\|c\| \le \frac{1}{2}\|\mathbf{v}\|^2 + \frac{c_l}{2}.$$

We may integrate this, to get

$$\begin{aligned}\|\mathbf{v}\|^2 &\le c_l(e^t-1) + e^t\|\mathbf{v}(\mathbf{x},0)\|^2 \\ &\le (c_l + \|\mathbf{v}(\mathbf{x},0)\|^2)e^t \le (c_l + \|\mathbf{v}(\mathbf{x},0)\|^2)e^T.\end{aligned}$$

□

Lemma 2.4. *If $v_i(\mathbf{x},0) \in L^2(\Omega)$ and $c(\mathbf{x},0) \in L^2(\Omega)$, then*

$$\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x \le \frac{1}{\upsilon}\left[v_l^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2} + (v_l c_l)^{1/2}\right]. \tag{2.7}$$

Proof. Multiply (1.1) by v_i and integrating over Ω and drop a non-positive term on the right, we have

$$\int_\Omega v_{i,t}v_i\,\mathrm{d}x \le -\upsilon\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x + \int_\Omega g_i v_i c\,\mathrm{d}x.$$

Hence, employing (2.5) and (2.6) in the above inequality, we find, with use of the Cauchy–Schwarz inequality,

$$\begin{aligned}\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x &\le \frac{1}{\upsilon}\left[-\int_\Omega v_{i,t}v_i\,\mathrm{d}x + \int_\Omega g_i v_i c\,\mathrm{d}x\right] \\ &\le \frac{1}{\upsilon}\left[\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_i v_i\,\mathrm{d}x\right)^{1/2} + \left(\int_\Omega v_i v_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega c^2\,\mathrm{d}x\right)^{1/2}\right] \\ &\le \frac{1}{\upsilon}\left[v_l^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2} + (v_l c_l)^{1/2}\right].\end{aligned}$$

□

We now derive an a priori bound for $v_{i,t}$. Here, we have two different values for the bound of $v_{i,t}$. This is because we split the proof into two parts depending on the availability of the Sobolev inequalities. For two dimensions case, we have the following inequality [57]:

$$\int_\Omega v^4 \,\mathrm{d}x \le \left(\int_\Omega v^2 \,\mathrm{d}x\right)\left(\int_\Omega v_{,i}\, v_{,i} \,\mathrm{d}x\right), \tag{2.8}$$

while in three dimensions, we cannot use the above inequality, thus we will use the following inequality [57]:

$$\int_\Omega v^4 \,\mathrm{d}x \le \beta \left(\int_\Omega v^2 \,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_{,i}\, v_{,i} \,\mathrm{d}x\right)^{3/2}, \tag{2.9}$$

Lemma 2.5. *For two dimensions, if* $v_{i,t}(\mathbf{x},0) \in L^2(\Omega)$ *and* $c_{,t}(\mathbf{x},0) \in L^2(\Omega)$*, then*

$$\int_\Omega v_{i,t} v_{i,t} \,\mathrm{d}x \le v_{tl}(t), \tag{2.10}$$

where

$$v_{tl}(t) = \left(\frac{R_2\sqrt{\Phi(0)}}{R_1\sqrt{\Phi(0)}(e^{-R_2 t}-1) + R_2 e^{-R_2 t}}\right)^2,$$

$$\Phi = \Phi_v + \Phi_c, \quad \Phi_v = \int_\Omega v_{i,t} v_{i,t} \,\mathrm{d}x \quad \Phi_c = \int_\Omega c_{,t} c_{,t} \,\mathrm{d}x,$$

$$R_1 = \frac{v_l^{1/2}}{2\upsilon^2}, \quad R_2 = \frac{v_l^{1/2} c_l^{1/2}}{2\upsilon^2} + \frac{c_\infty^2}{2D} + 1.$$

Proof.

$$\begin{aligned}
\Phi_{c,t} &= 2\int_\Omega c_{,t} c_{,tt} \,\mathrm{d}x = 2\int_\Omega c_{,t}[-v_i\, c_{,\,i} + D\Delta c - K_1 c]_{,t} \,\mathrm{d}x \\
&= 2D\int_\Omega c_{,t}\Delta c_{,t} \,\mathrm{d}x - 2K_1\int_\Omega c_{,t} c_{,t} \,\mathrm{d}x - 2\int_\Omega c_{,t} v_{i,t} c_{,i} \,\mathrm{d}x - 2\int_\Omega c_{,t} v_i c_{,it} \,\mathrm{d}x \\
&\le -2D\int_\Omega c_{,it} c_{,it} \,\mathrm{d}x + 2\int_\Omega c_{,it} v_{i,t} c \,\mathrm{d}x \\
&= -2D\int_\Omega c_{,it} c_{,it} \,\mathrm{d}x + 2c_\infty\int_\Omega c_{,it} v_{i,t} \,\mathrm{d}x
\end{aligned}$$

$$\leq -2D\int_\Omega c_{,it}c_{,it}\,\mathrm{d}x + 2c_\infty\left(\int_\Omega c_{,it}c_{,it}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2}$$
$$\leq -2D\int_\Omega c_{,it}c_{,it}\,\mathrm{d}x + 2D\int_\Omega c_{,it}c_{,it}\,\mathrm{d}x + \frac{c_\infty^2}{2D}\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x.$$

Thus, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_c \leq \frac{c_\infty^2}{2D}\Phi_v. \tag{2.11}$$

Next, we will perform the same work for $\Phi_{v,t}$,

$$\begin{aligned}
\Phi_{v,t} &= 2\int_\Omega v_{i,t}v_{i,tt}\,\mathrm{d}x\\
&= 2\int_\Omega v_{i,t}\left[-v_jv_{i,j} - p_{,\,i} + \upsilon\Delta v_i + g_ic + \sigma b_0^2(k_iw - v_i)\right]_{,t}\,\mathrm{d}x\\
&= -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x - 2\int_\Omega v_{i,t}v_{j,t}v_{i,j}\,\mathrm{d}x\\
&\quad + 2\int_\Omega g_iv_{i,t}c_t\,\mathrm{d}x + 2\sigma b_0^2\int_\Omega (k_iw_{,t}v_{i,t} - v_{i,t}v_{i,t})\,\mathrm{d}x\\
&\leq -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x - 2\int_\Omega v_{i,t}v_{j,t}v_{i,j}\,\mathrm{d}x + 2\int_\Omega g_iv_{i,t}c_t\,\mathrm{d}x\\
&\leq -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x + 2\left(\int_\Omega v_{i,j}\,v_{i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega (v_{i,t}v_{i,t})^2\,\mathrm{d}x\right)^{1/2} + \Phi_v + \Phi_c.
\end{aligned}$$

where $\mathbf{k} = (0, 0, 1)$.

Now, the arithmetic-geometric mean inequality is used on the right-hand side together with the Sobolev inequality (2.8) and (2.7) to find

$$\begin{aligned}
\Phi_{v,t} &\leq -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x\\
&\quad + 2\left(\int_\Omega v_{i,j}\,v_{i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x\right)^{1/2} + \Phi_v + \Phi_c\\
&\leq \frac{1}{2\upsilon}\Phi_v\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x + \Phi_v + \Phi_c \leq \frac{1}{2\upsilon^2}\Phi_v\left[v_l^{1/2}\Phi_v^{1/2} + (v_lc_l)^{1/2}\right] + \Phi_v + \Phi_c.
\end{aligned} \tag{2.12}$$

Now, summing (2.11) and (2.12), we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v+\Phi_c) &\le R_1\Phi_v^{3/2}+R_2\Phi_v+\Phi_c\\ &\le R_1\Phi_v(\Phi_v+\Phi_c)^{1/2}+R_2(\Phi_v+\Phi_c)\\ &\le R_1\Phi_v(\Phi_v+\Phi_c)^{1/2}+R_1\Phi_c(\Phi_v+\Phi_c)^{1/2}+R_2(\Phi_v+\Phi_c)\\ \frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v+\Phi_c) &\le R_1(\Phi_v+\Phi_c)^{3/2}+R_2(\Phi_v+\Phi_c). \qquad (2.13)\end{aligned}$$

Integrating (2.13), we find the desired result (2.10) for two dimensions case. □

Lemma 2.6. *For three dimensions, if $v_{i,t}(\mathbf{x},0)\in L^2(\Omega)$ and $c_{,t}(\mathbf{x},0)\in L^2(\Omega)$, then*

$$\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x \le v_{tl}(t), \qquad (2.14)$$

where

$$v_{tl}(t)=\frac{R_4\Phi(0)}{R_3\Phi(0)(e^{-R_4t}-1)+R_4e^{-R_4t}}$$

$$\Phi=\Phi_v+\Phi_c,\quad R_3=\frac{27v_l\beta^4}{64v^5},\quad R_4=\frac{27v_lc_l\beta^4}{64v^5}+\frac{c_\infty^2}{2D}+1.$$

It is clear that for three dimensions the bound (2.14) is valid just for $t<\frac{1}{R_4}\ln(1+\frac{R_4}{R_3\Phi(0)})$, thus we have conditional continuous dependence in this case.

Proof. Using similar technique which is used for two dimensions case, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_c\le\frac{c_\infty^2}{2D}\Phi_v, \qquad (2.15)$$

and

$$\Phi_{v,t}\le -2v\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x+2\left(\int_\Omega v_{i,j}\,v_{i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega (v_{i,t}v_{i,t})^2\,\mathrm{d}x\right)^{1/2}+\Phi_v+\Phi_c.$$

Next, using the Sobolev inequalities (2.9), Young's inequality and (2.7) we derive

$$\begin{aligned}\Phi_{v,t}&\le -2v\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x\\ &\quad+2\beta\left(\int_\Omega v_{i,j}\,v_{i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/4}\left(\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x\right)^{3/4}+\Phi_v+\Phi_c\\ &\le\frac{27\beta^4}{128v^3}\Phi_v\left(\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x\right)^2+\Phi_v+\Phi_c\end{aligned}$$

$$\leq \frac{27\beta^4}{128\upsilon^5}\Phi_v[v_l^{1/2}\Phi_v^{1/2}+(v_l c_l)^{1/2}]^2+\Phi_v+\Phi_c$$
$$\leq \frac{27\beta^4}{64\upsilon^5}\Phi_v[v_l\Phi_v+v_l c_l]+\Phi_v+\Phi_c. \tag{2.16}$$

Now, summing (2.15) and (2.16), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v+\Phi_c)\leq R_3\Phi_v^2+R_4\Phi_v+\Phi_c\leq R_3(\Phi_v^2+2\Phi_v\Phi_c+\Phi_v^2)+R_4(\Phi_v+\Phi_c).$$

Thus, we deduce

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v+\Phi_c)\leq R_3(\Phi_v+\Phi_c)^2+R_4(\Phi_v+\Phi_c). \tag{2.17}$$

Upon integration of (2.17), we find the desired result (2.14) for three dimensions case. □

2.1. Continuous Dependence on σ

This section is devoted to establishing continuous dependence of the solution on σ. Let (v_{i1}, c_1, p_1) and (v_{i2}, c_2, p_2) be two solutions of (1.1)–(1.3) with the same data (1.4), (1.5), but with different electrical conductivity of the fluid σ_1 and σ_2. Now set

$$u_i = v_{i1}-v_{i2},\quad \phi = c_1-c_2,\quad \pi = p_1-p_2,\quad \sigma=\sigma_1-\sigma_2. \tag{2.18}$$

The difference in the two solutions (u_i, ϕ, π) then satisfies

$$u_{i,t}+v_{1j}u_{i,j}+u_jv_{2i,j} = -\pi_{,i}+\upsilon\Delta u_i+g_i\phi+\sigma b_0^2[(v_1\times k)\times k]_i$$
$$+\sigma_2 b_0^2[(u\times k)\times k]_i, \tag{2.19}$$
$$u_{i,i}=0, \tag{2.20}$$
$$\phi_{,t}+v_{1i}\,\phi_{,i}+u_i\,c_{2,i}=D\Delta\phi-K_1\phi, \tag{2.21}$$

with the boundary and initial conditions

$$u_i=0,\quad \frac{\partial\phi}{\partial n}=0,\quad \text{on}\quad \partial\Omega, \tag{2.22}$$
$$\theta(\mathbf{x},0)=0,\quad u_i(\mathbf{x},0)=0,\quad \text{in}\quad \Omega. \tag{2.23}$$

The proof of continuous dependence commences by multiplying (2.19) by u_i and integrating over Ω to find,

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{u}\|^2 = 2\int_\Omega u_iu_{i,t}\,\mathrm{d}x$$
$$= 2\int_\Omega u_i\left[-v_{1j}u_{i,j}-u_jv_{2i,j}-\pi_{,i}+\upsilon\Delta u_i+g_i\phi+\sigma b_0^2(k_iw_1-v_{1i})\right.$$
$$\left.+\sigma_2 b_0^2(k_iw-u_i)\right]\mathrm{d}x$$
$$\leq -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x-2\int_\Omega u_iu_jv_{2i,j}\,\mathrm{d}x+2\int_\Omega g_iu_i\phi\,\mathrm{d}x$$
$$+2\sigma b_0^2\int_\Omega(k_iw_1u_i-v_{1i}u_i)\,\mathrm{d}x$$

$$\leq -2\upsilon \int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x + 2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega (u_iu_i)^2\,\mathrm{d}x\right)^{1/2}$$
$$+2\left(\int_\Omega u_iu_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega \phi\phi\,\mathrm{d}x\right)^{1/2} + 4\sigma b_0^2 v_{l1}^{1/2}\left(\int_\Omega u_iu_i\,\mathrm{d}x\right)^{1/2},$$

where $v_{l1} = (c_{l1} + \|\mathbf{v}_1(\mathbf{x},0)\|^2)e^T, c_{l1} = \|c_1(\mathbf{x},0)\|^2$ and w, w_1 is the third component of the velocities u_i, v_{i1}, respectively. Using the Sobolev inequality (2.9), arithmetic-geometric mean inequality and (2.7), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{u}\|^2 \leq -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x$$
$$+ 2\beta\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega u_iu_i\,\mathrm{d}x\right)^{1/4}\left(\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x\right)^{3/4}$$
$$+\int_\Omega u_iu_i\,\mathrm{d}x + \int_\Omega \phi\phi\,\mathrm{d}x + \sigma^2 b_0^4 v_{l1} + \int_\Omega u_iu_i\,\mathrm{d}x$$
$$\leq \frac{27\beta^4}{128\upsilon^3}\|u\|^2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^2 + 2\|u\|^2 + \|\phi\|^2 + \sigma^2 b_0^4 v_{l1}$$
$$\leq \frac{27\beta^4}{128\upsilon^5}\|u\|^2\left[v_{l2}^{1/2}v_{lt2}(t)^{1/2} + (v_{l2}c_{l2})^{1/2}\right]^2 + 2\|u\|^2 + \|\phi\|^2 + \sigma^2 b_0^4 v_{l1}$$
$$\leq \frac{27\beta^4}{64\upsilon^5}\|u\|^2[v_{l2}v_{lt2}(t) + v_{l2}c_{l2}] + 2\|u\|^2 + \|\phi\|^2 + \sigma^2 b_0^4 v_{l1}, \tag{2.24}$$

where $v_{l2} = (c_{l2} + \|\mathbf{v}_2(\mathbf{x},0)\|^2)e^T, c_{l2} = \|c_2(\mathbf{x},0)\|^2$, the value $v_{tl1}(t)$ is equal to the value of $v_{tl}(t)$ which is defined in Lemmas 2.5 and 2.6 at the solution (v_{i2}, c_2, p_2). Next, multiply (2.21) by ϕ and integrate over Ω and using the Cauchy–Schwarz inequality, arithmetic-geometric mean inequality and (2.1), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\phi\|^2 \leq \frac{c_{2\infty}^2}{2D}\|u\|^2. \tag{2.25}$$

Let $R_5(t) = \frac{27\beta^4}{64\upsilon^5}[v_{l2}v_{lt2}(t)+v_{l2}c_{l2}] + \frac{c_{2\infty}^2}{2D} + 2$, and $R_6(t) = \int R_5(t)\mathrm{d}t$. Summing (2.24) and (2.24), we get

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\mathbf{u}\|^2 + \|\phi\|^2) \leq R_4(t)(\|u\|^2 + \|\phi\|^2) + \sigma^2 b_0^4 v_{l1}. \tag{2.26}$$

Integrating (2.26), we obtain the continuous dependence inequality on σ

$$\|\mathbf{u}\|^2 + \|\phi\|^2 \leq \sigma^2 b_0^4 v_{l1}\int_0^t e^{R_5(t)-R_5(s)}\mathrm{d}t. \tag{2.27}$$

2.2. Continuous Dependence on K_1

In this section, we demonstrate briefly how to establish a continuous dependence result for the chemical reaction rate K_1 in (1.1)–(1.3). Let (v_{i1}, c_1, p_1) and (v_{i2}, c_2, p_2) be two solutions of problem (1.1)–(1.3) for different chemical reaction coefficients K_{11} and K_{12}, respectively. Then, as previously, (u_i, ϕ, π) will solve the problem

$$u_{i,t} + v_{1j}u_{i,j} + u_j v_{2i,j} = -\pi_{,\,i} + \upsilon\Delta u_i + g_i\phi + \sigma b_0^2[(u\times k)\times k]_i, \tag{2.28}$$

$$u_{i,i} = 0, \tag{2.29}$$

$$\phi_{,t} + v_{1i}\,\phi_{,\,i} + u_i\,c_{2,i} = D\Delta c - K_{11}\phi - K_1 c_2, \tag{2.30}$$

subject to conditions

$$u_i = 0, \quad \frac{\partial\phi}{\partial n} = 0, \quad \text{on} \quad \partial\Omega, \tag{2.31}$$

$$\theta(\mathbf{x},0) = 0, \quad u_i(\mathbf{x},0) = 0, \quad \text{in} \quad \Omega. \tag{2.32}$$

Multiplying by u_i and integrating by parts over Ω, we find

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^2 &= 2\int_\Omega u_i u_{i,t}\,\mathrm{d}x \\
&= 2\int_\Omega u_i\left[-v_{1j}u_{i,j} - u_j v_{2i,j} - \pi_{,\,i} + \upsilon\Delta u_i + g_i\phi + \sigma b_0^2(k_i w - u_i)\right]\mathrm{d}x \\
&= -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x - 2\int_\Omega u_i u_j v_{2i,j}\,\mathrm{d}x \\
&\quad + 2\int_\Omega g_i u_i\phi\,\mathrm{d}x + 2\sigma b_0^2\int_\Omega (k_i w.u_i - u_i u_i)\,\mathrm{d}x \\
&\le -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x + 2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega (u_i u_i)^2\,\mathrm{d}x\right)^{1/2} \\
&\quad + 2\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega \phi^2\,\mathrm{d}x\right)^{1/2}.
\end{aligned}$$

Using the Sobolev inequality (2.9), Young's inequality and (2.7), we get

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{u}\|^2 \le{}& -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x + 2\beta\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/4} \\
&\times\left(\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x\right)^{3/4} + \int_\Omega u_i u_i\,\mathrm{d}x + \int_\Omega \phi^2\,\mathrm{d}x
\end{aligned}$$

$$\leq \frac{27\beta^4}{128\upsilon^3}\|\mathbf{u}\|^2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^2+\|\mathbf{u}\|^2+\|\phi\|^2$$
$$\leq \frac{27\beta^4}{128\upsilon^5}\|\mathbf{u}\|^2\left[v_{l2}^{1/2}v_{lt2}(t)^{1/2}+(v_{l2}c_{l2})^{1/2}\right]^2+\|\mathbf{u}\|^2+\|\phi\|^2$$
$$\leq \frac{27\beta^4}{64\upsilon^5}\|\mathbf{u}\|^2\left[v_{l2}v_{lt2}(t)+v_{l2}c_{l2}\right]+\|\mathbf{u}\|^2+\|\phi\|^2. \tag{2.33}$$

Next, multiply (2.30) by ϕ and integrate over Ω to obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\phi\|^2=(u_ic_2,\phi_{,i})-D\|\nabla\phi\|^2-K_{11}\|\phi\|^2-K_1(\phi,c_2). \tag{2.34}$$

Next, the Cauchy–Schwarz and arithmetic-geometric mean inequalities are employed and then drop a non-positive term on the right to see that

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\phi\|^2\leq\frac{c_{2\infty}^2}{2D}\|\mathbf{u}\|^2+\frac{c_{2\infty}^2}{2D}\|\phi\|^2+2DK_1^2. \tag{2.35}$$

Let $R_7(t)=(27\beta^4/64\upsilon^5)[v_{l2}v_{lt2}(t)+v_{l2}c_{l2}]+(c_{2\infty}^2/2D)+1$, and $R_8(t)=\int R_7(t)\mathrm{d}t$. Summing (2.33) and (2.35), we get

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\mathbf{u}\|^2+\|\phi\|^2)\leq R_6(t)(\|\mathbf{u}\|^2+\|\phi\|^2)+2DK_1^2. \tag{2.36}$$

An integration yields

$$\|\mathbf{u}\|^2+\|\phi\|^2\leq 2DK_1^2\int_0^t e^{R_8(t)-R_8(s)}\mathrm{d}t, \tag{2.37}$$

which is the desired continuous dependence result, thus the continuous dependence for u_i and ϕ follows from (2.37).

3. Continuous Dependence for the Problem of Double Diffusive Convection with a Dissolved Reacting Fluid Layer and a Vertically Imposed Magnetic Field

Lemma 3.1. *If* $T(\mathbf{x},0)\in L^2(\Omega)$, *then*

$$\|T(\mathbf{x},t)\|^2\leq T_l, \tag{3.1}$$

where $T_l=\|T(\mathbf{x},0)\|^2$.

Proof. The proof of this lemma follows directly using the same argument in Lemma 2.2. □

Lemma 3.2. *If* $v_i(\mathbf{x},0)\in L^2(\Omega)$, *then*

$$\|\mathbf{v}(\mathbf{x},t)\|^2\leq v_l, \tag{3.2}$$

where $v_l=(2c_l+2T_l+\|\mathbf{v}(\mathbf{x},0)\|^2)e^T$.

Proof. The first step involves multiplying (1.6) by v_i and integrating over Ω. Using the Cauchy–Schwarz inequality, arithmetic-geometric mean inequality and drop a non-positive term on the right, we have

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{v}\|^2 &= \int_\Omega g_i v_i c\,\mathrm{d}x + \int_\Omega h_i v_i T\,\mathrm{d}x \le \|\mathbf{v}\|\|c\| + \|\mathbf{v}\|\|T\| \\ &\le \frac{1}{2}\|\mathbf{v}\|^2 + c_l + T_l.\end{aligned}$$

We may integrate this, we get

$$\begin{aligned}\|\mathbf{v}\|^2 &\le (2c_l + 2T_l)(e^t - 1) + e^t\|\mathbf{v}(\mathbf{x},0)\|^2 \le (2c_l + 2T_l + \|\mathbf{v}(\mathbf{x},0)\|^2)e^t \\ &\le (2c_l + 2T_l + \|\mathbf{v}(\mathbf{x},0)\|^2)e^T.\end{aligned}$$

□

Lemma 3.3. *If* $v_i(\mathbf{x},0) \in L^2(\Omega)$ *and* $c(\mathbf{x},0) \in L^2(\Omega)$, *then*

$$\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x \le \frac{1}{\upsilon}\left[v_l^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2} + (v_l c_l)^{1/2} + (v_l T_l)^{1/2}\right]. \tag{3.3}$$

Proof. Multiply (1.6) by v_i and integrating over Ω and drop a non-positive term on the right, we have

$$\int_\Omega v_{i,t}v_i\,\mathrm{d}x \le -\upsilon\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x + \int_\Omega g_i v_i c\,\mathrm{d}x + \int_\Omega h_i v_i T\,\mathrm{d}x.$$

Hence, use (2.5), (3.1), (3.2) in this inequality together with the Cauchy–Schwarz inequality to arrive at

$$\begin{aligned}\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x &\le \frac{1}{\upsilon}\left[-\int_\Omega v_{i,t}v_i\,\mathrm{d}x + \int_\Omega g_i v_i c\,\mathrm{d}x + \int_\Omega h_i v_i T\,\mathrm{d}x\right] \\ &\le \frac{1}{\upsilon}\left[\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_i v_i\,\mathrm{d}x\right)^{1/2} + \left(\int_\Omega v_i v_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega c^2\,\mathrm{d}x\right)^{1/2}\right. \\ &\quad \left. + \left(\int_\Omega v_i v_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega T^2\,\mathrm{d}x\right)^{1/2}\right] \\ &\le \frac{1}{\upsilon}\left[v_l^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2} + (v_l c_l)^{1/2} + (v_l T_l)^{1/2}\right].\end{aligned}$$

□

Lemma 3.4. *For two dimensions, if* $v_{i,t}(\mathbf{x},0) \in L^2(\Omega)$ *and* $c_{,t}(\mathbf{x},0) \in L^2(\Omega)$, *then*

$$\int_{\Omega} v_{i,t}v_{i,t}\,\mathrm{d}x \leq v_{tl}(t), \tag{3.4}$$

where

$$\begin{aligned}
v_{tl}(t) &= \left(\frac{I_2\sqrt{\Phi(0)}}{I_1\sqrt{\Phi(0)}(e^{-I_2 t}-1)+I_2e^{-I_2 t}}\right)^2\\
\Phi &= \Phi_v+\Phi_c+\Phi_T, \quad \Phi_v = \int_{\Omega} v_{i,t}v_{i,t}\,\mathrm{d}x,\\
\Phi_T &= \int_{\Omega} T_{,t}T_{,t}\,\mathrm{d}x, \quad \Phi_c = \int_{\Omega} c_{,t}c_{,t}\,\mathrm{d}x,\\
I_1 &= \frac{v_l^{1/2}}{2\upsilon^2}, \quad I_2 = \frac{(v_lc_l)^{1/2}+(v_lT_l)^{1/2}}{2\upsilon^2}+\frac{T_\infty^2}{2K}+\frac{c_\infty^2}{2D}+2.
\end{aligned}$$

Proof. Firstly, we observe that

$$\begin{aligned}
\Phi_{c,t} &= 2\int_{\Omega} c_{,t}c_{,tt}\,\mathrm{d}x = 2\int_{\Omega} c_{,t}\left[-v_i\,c_{,\,i}+D\Delta c-K_1c\right]_{,t}\,\mathrm{d}x\\
&= 2D\int_{\Omega} c_{,t}\Delta c_{,t}\,\mathrm{d}x - 2K_1\int_{\Omega} c_{,t}c_{,t}\,\mathrm{d}x - 2\int_{\Omega} c_{,t}v_{i,t}c_{,i}\,\mathrm{d}x - 2\int_{\Omega} c_{,t}v_ic_{,it}\,\mathrm{d}x\\
&\leq -2D\int_{\Omega} c_{,it}c_{,it}\,\mathrm{d}x + 2\int_{\Omega} c_{,it}v_{i,t}c\,\mathrm{d}x\\
&= -2D\int_{\Omega} c_{,it}c_{,it}\,\mathrm{d}x + 2c_\infty\int_{\Omega} c_{,it}v_{i,t}\,\mathrm{d}x\\
&\leq -2D\int_{\Omega} c_{,it}c_{,it}\,\mathrm{d}x + 2c_\infty\left(\int_{\Omega} c_{,it}c_{,it}\,\mathrm{d}x\right)^{1/2}\left(\int_{\Omega} v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2}\\
&\leq -2D\int_{\Omega} c_{,it}c_{,it}\,\mathrm{d}x + 2D\int_{\Omega} c_{,it}c_{,it}\,\mathrm{d}x + \frac{c_\infty^2}{2D}\int_{\Omega} v_{i,t}v_{i,t}\,\mathrm{d}x,
\end{aligned}$$

thus we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_c \leq \frac{c_\infty^2}{2D}\Phi_v. \tag{3.5}$$

Similar argument can be applied for Φ_T to obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_T \leq \frac{T_\infty^2}{2K}\Phi_v \tag{3.6}$$

Next, we will preform similar work for $\Phi_{v,t}$,

$$\begin{aligned}
\Phi_{v,t} &= 2\int_\Omega v_{i,t}v_{i,tt}\,\mathrm{d}x \\
&= 2\int_\Omega v_{i,t}\left[-v_jv_{i,j} - p_{,\,i} + \upsilon\Delta v_i + g_ic + h_iT + \sigma b_0^2(k_iw - v_i)\right]_{,t}\mathrm{d}x \\
&= -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x - 2\int_\Omega v_{i,t}v_{j,t}v_{i,j}\,\mathrm{d}x + 2\int_\Omega g_iv_{i,t}c_t\,\mathrm{d}x \\
&\quad + 2\int_\Omega h_iv_{i,t}T_t\,\mathrm{d}x + 2\sigma b_0^2\int_\Omega(k_iw_tv_{i,t} - v_{i,t}v_{i,t})\,\mathrm{d}x \\
&\le -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x - 2\int_\Omega v_{i,t}v_{j,t}v_{i,j}\,\mathrm{d}x \\
&\quad + 2\int_\Omega g_iv_{i,t}c_t\,\mathrm{d}x + 2\int_\Omega h_iv_{i,t}T_t\,\mathrm{d}x \\
&\le -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x \\
&\quad + 2\left(\int_\Omega v_{i,j}\,v_{i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega (v_{i,t}v_{i,t})^2\,\mathrm{d}x\right)^{1/2} + 2\Phi_v + \Phi_c + \Phi_T.
\end{aligned}$$

Now, we use the Sobolev inequality (2.8), arithmetic–geometric mean inequality and (3.3), we have

$$\begin{aligned}
\Phi_{v,t} &\le -2\upsilon\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x + 2\left(\int_\Omega v_{i,j}\,v_{i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/2} \\
&\quad \times\left(\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x\right)^{1/2} + 2\Phi_v + \Phi_c + \Phi_T \\
&\le \frac{1}{2\upsilon}\Phi_v\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x + 2\Phi_v + \Phi_c + \Phi_T \\
&\le \frac{1}{2\upsilon^2}\Phi_v\left[v_l^{1/2}\Phi_v^{1/2} + (v_lc_l)^{1/2} + (v_lT_l)^{1/2}\right] + 2\Phi_v + \Phi_c + \Phi_T. \qquad (3.7)
\end{aligned}$$

Now, summing (3.5), (3.6) and (3.7), we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v + \Phi_c + \Phi_T) &\le I_1\Phi_v^{3/2} + I_2\Phi_v + \Phi_c + \Phi_T \\
&\le I_1\Phi_v(\Phi_v + \Phi_c + \Phi_T)^{1/2} + I_2(\Phi_v + \Phi_c + \Phi_T) \\
&\le I_1\Phi_v(\Phi_v + \Phi_c + \Phi_T)^{1/2} + I_1\Phi_c(\Phi_v + \Phi_c + \Phi_T)^{1/2}
\end{aligned}$$

$$+ I_1 \Phi_T (\Phi_v + \Phi_c + \Phi_T)^{1/2} + I_2(\Phi_v + \Phi_c + \Phi_T)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v + \Phi_c + \Phi_T) \leq I_1(\Phi_v + \Phi_c + \Phi_T)^{3/2} + I_2(\Phi_v + \Phi_c + \Phi_T). \tag{3.8}$$

Upon integration of (3.8), we find the desired result (3.4) for two dimensions case. □

Lemma 3.5. *For three dimensions, if* $v_{i,t}(\mathbf{x}, 0) \in L^2(\Omega)$ *and* $c_{,t}(\mathbf{x}, 0) \in L^2(\Omega)$, *then*

$$\int_\Omega v_{i,t} v_{i,t} \, \mathrm{d}x \leq v_{tl}(t), \tag{3.9}$$

where

$$v_{tl}(t) = \frac{I_4 \Phi(0)}{I_3 \Phi(0)(e^{-I_4 t} - 1) + I_4 e^{-I_4 t}}$$

$$\Phi = \Phi_v + \Phi_c + \Phi_T, \quad \Phi_v = \int_\Omega v_{i,t} v_{i,t} \, \mathrm{d}x,$$

$$\Phi_T = \int_\Omega T_{,t} T_{,t} \, \mathrm{d}x, \quad \Phi_c = \int_\Omega c_{,t} c_{,t} \, \mathrm{d}x,$$

$$I_3 = \frac{27 v_l \beta^4}{64 v^5}, \quad I_4 = \frac{27 \beta^4 (v_l c_l + v_l T_l)}{32 v^5} + \frac{T_\infty^2}{2K} + \frac{c_\infty^2}{2D} + 2.$$

It is clear that for three dimensions the bound (3.9) is valid just for $t < \frac{1}{I_4} \ln(1 + \frac{I_4}{I_3 \Phi(0)})$, thus we have conditional continuous dependence in this case.

Proof. Similar argument can be applied for three dimensions case to obtain

$$\frac{\mathrm{d}}{\mathrm{d}t} \Phi_c \leq \frac{c_\infty^2}{2D} \Phi_v, \tag{3.10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \Phi_T \leq \frac{T_\infty^2}{2K} \Phi_v \tag{3.11}$$

and

$$\Phi_{v,t} \leq -2v \int_\Omega v_{i,jt} v_{i,jt} \, \mathrm{d}x$$

$$+ 2 \left(\int_\Omega v_{i,j} \, v_{i,j} \, \mathrm{d}x \right)^{1/2} \left(\int_\Omega (v_{i,t} v_{i,t})^2 \, \mathrm{d}x \right)^{1/2} + 2\Phi_v + \Phi_c + \Phi_T.$$

Next, using the Sobolev inequality (2.9), Young's inequality and (3.3), we get

$$
\begin{aligned}
\Phi_{v,t} &\leq -2\upsilon \int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x + 2\beta\left(\int_\Omega v_{i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega v_{i,t}v_{i,t}\,\mathrm{d}x\right)^{1/4} \\
&\quad\times\left(\int_\Omega v_{i,jt}v_{i,jt}\,\mathrm{d}x\right)^{3/4} + 2\Phi_v + \Phi_c + \Phi_T \\
&\leq \frac{27\beta^4}{128\upsilon^3}\Phi_v\left(\int_\Omega v_{i,j}v_{i,j}\,\mathrm{d}x\right)^2 + 2\Phi_v + \Phi_c + \Phi_T \\
&\leq \frac{27\beta^4}{128\upsilon^5}\Phi_v[v_l^{1/2}\Phi_v^{1/2} + (v_l c_l)^{1/2} + +(v_l T_l)^{1/2}]^2 + 2\Phi_v + \Phi_c + \Phi_T \\
&\leq \frac{27\beta^4}{64\upsilon^5}\Phi_v[v_l\Phi_v + 2(v_l c_l + v_l T_l)] + 2\Phi_v + \Phi_c + \Phi_T. \qquad (3.12)
\end{aligned}
$$

Now, summing (3.10), (3.11) and (3.12), we obtain

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v + \Phi_c + \Phi_T) \leq I_3\Phi_v^2 + I_4\Phi_v + \Phi_c + \Phi_T \\
&\frac{\mathrm{d}}{\mathrm{d}t}(\Phi_v + \Phi_c + \Phi_T) \leq I_3(\Phi_v + \Phi_c + \Phi_T)^2 + I_4(\Phi_v + \Phi_c + \Phi_T).
\end{aligned} \qquad (3.13)
$$

Upon integration of (3.13), we find the desired result (3.9) for three dimensions case. □

3.1. Continuous Dependence on σ

In this section, we establish continuous dependence on the electrical conductivity coefficient σ. To do this, let (v_{i1}, T_1, c_1, p_1) and (v_{i2}, T_2, c_2, p_2) be solutions of (1.6)–(1.9) with the same boundary and initial conditions, but with different electrical conductivity coefficients σ_1 and σ_2. Now, we define

$$
u_i = v_{i1} - v_{i2}, \quad \theta = T_1 - T_2, \quad \phi = c_1 - c_2, \quad \pi = p_1 - p_2, \quad \sigma = \sigma_1 - \sigma_2, \qquad (3.14)
$$

Then, (u_i, θ, ϕ, π) is a solution of the problem

$$
\begin{aligned}
u_{i,t} + v_{1j}u_{i,j} + u_j v_{2i,j} &= -\pi_{,i} + \upsilon\Delta u_i + h_i\theta + g_i\phi + \sigma b_0^2[(v_1 \times k)\times k]_i \\
&\quad + \sigma_2 b_0^2[(u\times k)\times k]_i, \qquad (3.15) \\
u_{i,i} &= 0, \qquad (3.16) \\
\theta_{,t} + v_{1i}\,\theta_{,i} + u_i\,T_{2,i} &= K\Delta\theta, \qquad (3.17) \\
\phi_{,t} + v_{1i}\,\phi_{,i} + u_i\,c_{2,i} &= D\Delta\phi - K_1\phi, \qquad (3.18)
\end{aligned}
$$

subject to the boundary and initial conditions

$$
u_i = 0, \quad \frac{\partial\theta}{\partial n} = 0, \quad \frac{\partial\phi}{\partial n} = 0, \quad \text{on} \quad \partial\Omega, \qquad (3.19)
$$

$$
\theta(\mathbf{x},0) = 0, \quad \phi(\mathbf{x},0) = 0, \quad u_i(\mathbf{x},0) = 0, \quad \text{in} \quad \Omega. \qquad (3.20)
$$

The proof of continuous dependence commences by multiplying (3.15) by u_i and integrating over Ω to find,

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{u}\|^2 &= 2\int_\Omega u_i u_{i,t}\,\mathrm{d}x \\
&= 2\int_\Omega u_i\left[-v_{1j}u_{i,j} - u_j v_{2i,j} - \pi_{,i} + \upsilon\Delta u_i + g_i\phi\right. \\
&\quad \left. + h_i\theta + \sigma b_0^2(k_i w_1 - v_{1i}) + \sigma_2 b_0^2(k_i w - u_i)\right]\,\mathrm{d}x \\
&\leq -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x - 2\int_\Omega u_i u_j v_{2i,j}\,\mathrm{d}x + 2\int_\Omega g_i u_i\phi\,\mathrm{d}x \\
&\quad + 2\int_\Omega h_i u_i\theta\,\mathrm{d}x + 2\sigma b_0^2\int_\Omega (k_i w_1 u_i - v_{1i}u_i)\,\mathrm{d}x \\
&\leq -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x + 2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega (u_i u_i)^2\,\mathrm{d}x\right)^{1/2} \\
&\quad + 2\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega \phi^2\,\mathrm{d}x\right)^{1/2} \\
&\quad + 2\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega \theta^2\,\mathrm{d}x\right)^{1/2} + 4\sigma b_0^2 v_{l1}^{1/2}\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/2},
\end{aligned}
$$

where $v_{l1} = (2c_{l1} + 2T_{l1} + \|\mathbf{v}(\mathbf{x},0)\|^2)e^T, c_{l1} = \|c_1(\mathbf{x},0)\|^2, T_{l1} = \|T_1(\mathbf{x},0)\|^2$. Using the Sobolev inequality (2.9), arithmetic–geometric mean inequality and (3.3), we obtain

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^2 &\leq -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x + 2\beta\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/4} \\
&\quad \times\left(\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x\right)^{3/4} + 3\int_\Omega u_i u_i\,\mathrm{d}x + \int_\Omega \phi^2\,\mathrm{d}x + \int_\Omega \theta^2\,\mathrm{d}x + \sigma^2 b_0^4 v_{l1} \\
&\leq \frac{27\beta^4}{128\upsilon^3}\|\mathbf{u}\|^2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^2 + 3\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2 + \sigma^2 b_0^4 v_{l1} \\
&\leq \frac{27\beta^4}{128\upsilon^5}\|\mathbf{u}\|^2[v_{l2}^{1/2}v_{lt2}(t)^{1/2} + (v_{l2}c_{l2})^{1/2} + (v_{l2}T_{l2})^{1/2}]^2 \\
&\quad + 3\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2 + \sigma^2 b_0^4 v_{l1} \\
&\leq \frac{27\beta^4}{64\upsilon^5}\|\mathbf{u}\|^2[v_{l2}v_{lt2}(t) + v_{l2}c_{l2} + v_{l2}T_{l2}] + 3\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2 + \sigma^2 b_0^4 v_{l1},
\end{aligned}
\tag{3.21}
$$

where $v_{l2} = (2c_{l2} + 2T_{l2} + \|\mathbf{v}_2(\mathbf{x}, 0)\|^2)e^T, c_{l2} = \|c_2(\mathbf{x}, 0)\|^2, T_{l2} = \|T_2(\mathbf{x}, 0)\|^2$, the value $v_{tl1}(t)$ is equal to the value of $v_{tl}(t)$ which is defined in Lemmas 3.4 and 3.5 at the solution (v_{i2}, T_2, c_2, p_2).

Next, multiply (3.17) and (3.18) by θ and ϕ, respectively, and integrate over Ω and using the Cauchy–Schwarz inequality, arithmetic-geometric mean inequality, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\phi\|^2 \leq \frac{c_{2\infty}^2}{2D}\|\mathbf{u}\|^2. \tag{3.22}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\theta\|^2 \leq \frac{T_{2\infty}^2}{2K}\|\mathbf{u}\|^2. \tag{3.23}$$

Let $I_5(t) = (27\beta^4/64v^5)[v_{l2}v_{lt2}(t)+v_{l2}c_{l2}+v_{l2}T_{l2}]+(c_{2\infty}^2/2D)+(T_{2\infty}^2/2K)+3$, and $I_6(t) = \int I_5(t)\mathrm{d}t$. Summing (3.21), (3.22) and (3.23), we get

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2) \leq I_5(t)(\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2) + \sigma^2 b_0^4 v_{l1}. \tag{3.24}$$

Upon integration of (3.24), we arrive at the continuous dependence on σ inequality

$$\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2 \leq \sigma^2 b_0^4 v_{l1} \int_0^t e^{I_6(t)-I_6(s)}\mathrm{d}t. \tag{3.25}$$

Thus, (3.25) establishes the continuous dependence on the coefficient σ.

3.2. Continuous Dependence on K_1

In this section, we show that the solution of the problem (1.6)–(1.9) depends continuously on the coefficient K_1. Let us consider two solutions (v_{i1}, T_1, c_1, p_1) and (v_{i2}, T_2, c_2, p_2) of (1.6)–(1.9) and have the same initial and boundary data corresponding to two different nonzero values K_{11} and K_{12}. Set

$$\begin{aligned} &u_i = v_{i1} - v_{i2}, \quad \theta = T_1 - T_2, \quad \phi = c_1 - c_2, \quad \pi = p_1 - p_2, \\ &K_1 = K_{11} - K_{12}, \end{aligned} \tag{3.26}$$

so that (u_i, θ, ϕ, π) is a solution of the problem

$$u_i = -\pi_{,\,i} + v\Delta u_i + g_i\phi + \sigma b_0^2[(u \times k) \times k]_i, \tag{3.27}$$

$$u_{i,i} = 0, \tag{3.28}$$

$$\theta_t + v_{1i}\,\theta_{,\,i} + u_i\,T_{2,i} = K\Delta\theta, \tag{3.29}$$

$$\phi_t + v_{1i}\,\phi_{,\,i} + u_i\,c_{2,i} = D\Delta c - K_{11}\phi - K_1 c_2, \tag{3.30}$$

in $\Omega \times (0, \infty)$, and

$$u_i = 0, \quad \frac{\partial\theta}{\partial n} = 0, \quad \frac{\partial\phi}{\partial n} = 0, \quad \text{on} \quad \partial\Omega, \tag{3.31}$$

$$\theta(\mathbf{x}, 0) = 0, \quad \phi(\mathbf{x}, 0) = 0, \quad u_i(\mathbf{x}, 0) = 0, \quad \text{in} \quad \Omega. \tag{3.32}$$

The proof of continuous dependence commences by multiplying (3.27) by u_i and integrating over Ω to find ,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^2 &= 2\int_\Omega u_i u_{i,t}\,\mathrm{d}x\\
&= 2\int_\Omega u_i[-v_{1j}u_{i,j} - u_j v_{2i,j} - \pi_{,\,i} + \upsilon\Delta u_i + g_i\phi + h_i\theta + \sigma b_0^2(k_i w - u_i)]\,\mathrm{d}x\\
&= -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x - 2\int_\Omega u_i u_j v_{2i,j}\,\mathrm{d}x\\
&\quad + 2\int_\Omega g_i u_i\phi\,\mathrm{d}x + 2\int_\Omega h_i u_i\theta\,\mathrm{d}x + 2\sigma b_0^2\int_\Omega (k_i w.u_i - u_i u_i)\,\mathrm{d}x\\
&\le -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x + 2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega (u_i u_i)^2\,\mathrm{d}x\right)^{1/2}\\
&\quad + 2\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega \phi^2\,\mathrm{d}x\right)^{1/2} + 2\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega \theta^2,\mathrm{d}x\right)^{1/2}.
\end{aligned}$$

Using the Sobolev inequality (2.9), Young's inequality and (3.3), we have

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^2 \le{}& -2\upsilon\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x\\
&+ 2\beta\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^{1/2}\left(\int_\Omega u_i u_i\,\mathrm{d}x\right)^{1/4}\left(\int_\Omega u_{i,j}u_{i,j}\,\mathrm{d}x\right)^{3/4}\\
&+ 2\int_\Omega u_i u_i\,\mathrm{d}x + \int_\Omega \phi^2\,\mathrm{d}x + \int_\Omega \theta^2\,\mathrm{d}x\\
\le{}& \frac{27\beta^4}{128\upsilon^3}\|\mathbf{u}\|^2\left(\int_\Omega v_{2i,j}v_{2i,j}\,\mathrm{d}x\right)^2 + 2\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2\\
\le{}& \frac{27\beta^4}{128\upsilon^5}\|\mathbf{u}\|^2\left[v_{l2}^{1/2}v_{lt2}(t)^{1/2} + (v_{l2}c_{l2})^{1/2} + (v_{l2}T_{l2})^{1/2}\right]^2\\
&+ 2\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2\\
\le{}& \frac{27\beta^4}{64\upsilon^5}\|\mathbf{u}\|^2\left[v_{l2}v_{lt2}(t) + 2(v_{l2}c_{l2} + v_{l2}T_{l2})\right] + 2\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2.
\end{aligned} \tag{3.33}$$

Next, multiply (3.29) by θ and (3.30) by ϕ and integrate over Ω, respectively, and arithmetic-geometric mean inequality to obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\theta\|^2 \le \frac{T_{2\infty}^2}{2K}\|\mathbf{u}\|^2. \tag{3.34}$$

Similarly, by multiplying (3.30) by ϕ and integrate over Ω and using the Cauchy–Schwarz inequality, arithmetic-geometric mean inequality and drop a non-positive terms on the right, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\phi\|^2 \leq \frac{c_{2\infty}^2}{2D}\|\mathbf{u}\|^2 + \frac{c_{2\infty}^2}{2D}\|\phi\|^2 + 2DK_1^2. \tag{3.35}$$

Let $I_7(t) = (27\beta^4/64\upsilon^5)[v_{l2}v_{lt2}(t)+2(v_{l2}c_{l2}+v_{l2}T_{l2})]+(c_{2\infty}^2/2D)+(T_{2\infty}^2/2K)$ $+ 2$, and $I_8(t) = \int I_7(t)\mathrm{d}t$. Summing (3.33), (3.34), and (3.35), we get

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2) \leq I_7(t)(\|\mathbf{u}\|^2 + \|\phi\|^2) + 2DK_1^2. \tag{3.36}$$

An integration of (3.36) leads to

$$\|\mathbf{u}\|^2 + \|\phi\|^2 + \|\theta\|^2 \leq 2DK_1^2 \int_0^t e^{I_8(t)-I_8(s)}\mathrm{d}t. \tag{3.37}$$

We thus conclude that the nonzero solutions of double diffusive convection problem depend continuously on the effective chemical reaction coefficient.

The differences between the estimates (2.27) and (3.25) or between the estimates (2.37) and (3.37) are mainly in the coefficients involving the integrals from 0 to t. The same kind of continuous dependence is achieved in both cases. The situation would be very different if one were to consider stability problems where a fluid layer was heated below and simultaneously salted below.

Acknowledgements

The author would like to thank Prof. B. Straughan for his guidance.

References

[1] Ames, K.A., Payne, L.E.: Continuous dependence results for solutions of the Navier–Stokes equations backward in time. Nonlinear Anal. Theor. Meth. Appl. **23**, 103–113 (1994)

[2] Ames, K.A., Payne, L.E.: On stabilizing against modeling errors in a penetrative convection problem for a porous medium. Math. Models Meth. Appl. Sci. **4**, 733–740 (1994)

[3] Ames, K.A., Payne, L.E.: Continuous dependence results for a problem in penetrative convection. Q. Appl. Math. **55**, 769–790 (1997)

[4] Ames, K.A., Payne, L.E.: Asymptotic behavior for two regularizations of the Cauchy problem for the backward heat equation. Math. Models Meth. Appl. Sci. **8**, 187–202 (1998)

[5] Ames, K.A., Straughan, B.: Non-standard and improperly posed problems. In: Mathematics in Science and Engineering Series, vol. 194. Academic Press, New York (1997)

[6] Bellomo, N., Preziosi, L.: Modelling mathematical methods and scientific computation. CRC, Boca Raton (1995)

[7] Celia, M.A., Kindred, J.S., Herrera, I.: Contaminant transport and biodegradation. I. A numerical model for reactive transport in porous media. Water Resour. Res. **25**, 1141–1148 (1989)

[8] Chen, B., Cunningham, A., Ewing, R., Peralta, R., Visser, E.: Two-dimensional modelling of microscale transport and biotransformation in porous media. Numer. Methods Partial Differ. Equ. **10**, 65–83 (1994)

[9] Cheng, P.: Heat transfer in geothermal systems. Adv. Heat Transf. **14**, 1–105 (1978)

[10] Curran, M.C., Allen, M.B.: Parallel computing for solute transport models via alternating direction collocation. Adv. Water Resour. **13**, 70–75 (1990)

[11] Eltayeb, I.A., Hamza, E.A., Jervase, J.A., Krishnan, E.V., Loper, D.E.: Compositional convection in the presence of a magnetic field. I. A single interface. Proc. R. Soc. London A **460**, 3505–3528 (2004)

[12] Eltayeb, I.A., Hamza, E.A., Jervase, J.A., Krishnan, E.V., Loper, D.E.: Compositional convection in the presence of a magnetic field. I. Cartesian plume. Proc. R. Soc. London A **461**, 2605–2633 (2005)

[13] Ewing R.E., Weekes S.: Numerical methods for contaminant transport in porous media, vol. 202, pp. 75–95. Marcel Decker Inc., New York (1998)

[14] Fabrizio, M., Morro, A.: Electromagnetism of continuous media. Oxford University Press, Oxford (2003)

[15] Franchi, F., Straughan, B.: Continuous dependence on the body force for solutions to the Navier–Stokes equations and on the heat supply in a model for double-diffusive porous convection. J. Math. Anal. Appl. **172**, 117–129 (1993)

[16] Franchi, F., Straughan, B.: Continuous dependence on the relaxation time and modelling, and unbounded growth. J. Math. Anal. Appl. **185**, 726–746 (1994)

[17] Franchi, F., Straughan, B.: Spatial decay estimates and continuous dependence on modelling for an equation from dynamo theory. Proc. R. Soc. Lond. **A 445**, 437–451 (1994)

[18] Franchi, F., Straughan, B.: Structural stability for the Brinkman equations of porous media. Math. Methods Appl. Sci. **19**, 1335–1347 (1996)

[19] Franchi, F., Straughan, B.: A comparison of the Graffi and Kazhikhov–Smagulov models for top heavy pollution instability. Adv. Water Resour. **24**, 585–594 (2001)

[20] Galdi, G.P., Straughan, B.: Exchange of stabilities, symmetry and nonlinear stability. Arch. Rational Mech. Anal. **89**, 211–228 (1985)

[21] Gilman, A., Bear, J.: The influence of free convection on soil salinization in arid regions. Transp. Porous Media **23**, 275–301 (1996)

[22] Gricths, R.W.: Layered double-diffusive convection in porous media. J. Fluid Mech. **102**, 221–248 (1981)

[23] Harfash, A.J.: Magnetic effect on instability and nonlinear stability of double diffusive convection in a reacting fluid. Contin. Mech. Thermodyn. **25**, 89–106 (2013)

[24] Harfash, A.J.: Continuous dependence on the coefficients for double diffusive convection in Darcy flow with Magnetic field effect. Anal. Math. Phys. **3**, 163–181 (2013)

[25] Harfash, A.J.: Convection in a variable gravity field with magnetic field effect. (2013)

[26] Harfash, A.J.: Structural stability for convection models in a reacting porous medium with magnetic field effect. Ricerche Math. doi:10.1007/s11587-013-0152-x

[27] Harfash, A.J., Straughan, B.: Magnetic effect on instability and nonlinear stability in a reacting fluid. Meccanica **47**, 1849–1857 (2012)

[28] Hayat, T., Nawaz, M.: Unsteady stagnation point flow of viscous fluid caused by an impusively rotating disk. J. Taiwan Inst. Chem. Eng. **42**, 41–49 (2011)

[29] Kaloni, P.N., Mahajan, A.: Stability of magnetic fluid motions in a saturated porous medium. ZAMP **62**, 529–538 (2011)

[30] Leblanca, J., Akbarzadeha, A., Andrewsa, J., Lub, H., Goldingb, P.: Heat extraction methods from salinity-gradient solarponds and introduction of a novel system of heat extraction for improved efficiency. Solar Energy **85**, 3103–3142 (2011)

[31] Lin, C., Payne, L.E.: On the spatial decay of ill-posed parabolic problems. Math. Methods Appl. Sci. **3**, 563–575 (1993)

[32] Lin, C., Payne, L.E.: The influence of domain and diffusivity perturbations on the decay of end effects in heat conduction. SIAM J. Math. Anal. **25**, 1242–1258 (1994)

[33] Lin, C., Payne, L.E.: Structural stability for a Brinkman fluid. Math. Math. Methods Appl. Sci. **30**, 567–578 (2007)

[34] Lin, C., Payne, L.E.: Structural stability for the Brinkman equations of flow in double diffusive convection. J. Math. Anal. Appl. **325**, 1479–1490 (2007)

[35] Lin, C., Payne, L.E.: Continuous dependence on the Soret coefficient for double diffusive convection in Darcy flow. J. Math. Anal. Appl. **342**, 311–325 (2008)

[36] Ludvigsen, A., Palm, E., Mckibbin, R.: Convective momentum and mass transport in porous sloping layers. J Geophys. Res. B **97**, 12315–12325 (1992)

[37] Malashetty, M.S., Biradar, B.S.: The onset of double diffusive reaction convection in an anisotropic porous layer. Phys. Fluids **23**, 064102 (2011)

[38] Maehlmann, S., Papageorgiou, D.T.: Interfacial instability in electrified plane Couette flow. J. Fluid Mech. **666**, 155–188 (2011)

[39] Nanjundappa, C.E., Ravisha, M., Lee, J., Shivakumara, I.S.: Penetrative ferroconvection in a porous layer. Acta Mechanica **216**, 243–257 (2011)

[40] Nie, Z., Bu, L., Zheng, M., Huang, W.: Experimental study of natural brine solar ponds in Tibet. Solar Energy **85**, 1537–1542 (2011)

[41] Payne, L.E., Song, J.C.: Spatial decay estimates for the Brinkman and Darcy flows in a semi-infinite cylinder. Contin. Mech. Thermodyn. **9**, 175–190 (1997)

[42] Payne, L.E., Song, J.C.: Spatial decay for a model of double diffusive convection in Darcy and Brinkman flows. Z. Angew. Math. Phys. **51**, 867–880 (2000)

[43] Payne, L.E., Song, J.C.: Spatial decay bounds in double diffusive convection in Darcy flow. J. Math. Anal. Appl. **330**, 864–875 (2007)

[44] Payne, L.E., Song, J.C., Straughan, B.: Continuous dependence and convergence results for Brinkman and Forchheimer models with variable viscosity. Proc. R. Soc. Lond. **A 455**, 2173–2190 (1999)

[45] Payne, L.E., Straughan, B.: A Comparison of viscous flows backward in time with small data. Int. J. Nonlinear Mech. **24**, 209–214 (1989)

[46] Payne, L.E., Straughan, B.: Error estimates for the temperature of a piece of cold ice, given data on only part of the boundary. Nonlinear Anal. Theory Methods Appl. **14**, 443–452 (1990)

[47] Payne, L.E., Straughan, B.: Stability in the initial-time geometry problem for the Brinkman and Darcy equations of flow in porous media. J. Math. Pures Appl. **75**, 225–271 (1996)

[48] Payne, L.E., Straughan, B.: Analysis of the boundary condition at the interface between a viscous and a porous medium and related modelling questions. J. Math. Pures Appl. **77**, 317–354 (1998)

[49] Payne, L.E., Straughan, B.: Structural stability for the Darcy equations of flow in porous media. Proc. R. Soc. Lond. **A 454**, 1691–1698 (1998)

[50] Rahman, M.M., Al-Lawatia, M.: Effects of higher order chemical reaction on micropolar fluid flow on a power law permeable stretched sheet with variable concentration in a porous medium. Can. J. Chem. Eng. **88**, 23–32 (2010)

[51] Reddy, P.D.S., Bandyopadhyay, D., Joo, S.W., Sharma, A., Qian, S.: Parametric study on instabilities in a two-layer electromagnetohydrodynamic channel flow confined between two parallel electrodes. Phys. Rev. E **83**, 036313 (2011)

[52] Rionero, S., Mulone, G.: Nonlinear stability analysis of the magnetic Bénard problem through the Lyapunov direct method. Arch. Rational Mech. Anal. **103**, 347–368 (1988)

[53] Roberts, P.H.: An introduction to magnetohydrodynamics. Longman, London (1967)

[54] Rubin, H.: Effect of nonlinear stabilizing salinity profiles on thermal convection in a porous medium layer. Water Resour. Res. **9**, 211–248 (1973)

[55] Shivakumara, I.S., Ng, C.O., Nagashree, M.S.: The onset of electrothermoconvection in a rotating Brinkman porous layer. Int. J. Eng. Sci. **49**, 6464–663 (2011)

[56] Shivakumara, I.S., Lee, J., Ravisha, M., Raddy, R.G.: Effects of MFD viscosity and LTNE on the onset of ferromagnetic convection in a porous medium. Int. J. Heat Mass Transf. **54**, 2630–2641 (2011)

[57] Straughan, B.: The energy method, stability, and nonlinear convection. In: Series in Applied Mathematical Sciences, vol. 91, 2nd edn. Springer, Berlin (2004)

[58] Straughan, B.: Stability and wave motion in porous media. In: Series in Applied Mathematical Sciences, vol. 165. Springer, Berlin (2008)

[59] Straughan, B., Harfash, A.J.: Instability in Poiseuille flow in a porous medium with slip boundary conditions. Microfluid Nanofluid **15**, 109–115 (2013)

[60] Straughan, B., Hutter, K.: A priori bounds and structural stability for double diffusive convection incorporating the Soret effect. Proc. R. Soc. Lond. **A 455**, 767–777 (1999)

[61] Suchomel, B.J., Chen, B.M., Allen, M.B.: Network model of flow, transport and biofilm effects in porous media. Transp. Porous Media **30**, 1–23 (1998)

[62] Sharma Sunil, P., Mahajan, A.: Onset of Darcy-Brinkman double-diffusive convection in a magnetized ferrofluid layer using a thermal non-equilibrium model: a nonlinear stability analysis. J. Geophys. Eng. **7**, 417–430 (2010)

Akil J. Harfash
Department of Mathematical Sciences
University of Durham
Durham DH1 3LE, UK
e-mail: a.j.harfash@durham.ac.uk

Communicated by Nader Masmoudi.
Received: January 28, 2013.
Accepted: November 12, 2013.