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Tachyons in String Theory

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Abstract. We describe some of the recent developments in our understanding of the tachyon in string theory.

In this talk I shall try to address the following topics:

1. What are tachyons?
2. Dealing with tachyons in quantum field theory
3. Tachyons in string theory
4. Making sense of string theory tachyons
5. Possible application to cosmology
6. Outline of the derivation of various results
7. Summary and open questions

Let us begin with the question: What are tachyons? Historically tachyons were described as particles which travel faster than light. In modern days we think of tachyons as particles with negative mass^2, *i.e.* imaginary mass. Both descriptions sound equally bizarre. On the other hand tachyons have been known to exist in string theory almost since its birth, and hence we need to make sense of them.

Actually tachyons do appear in conventional quantum field theories as well. Consider, for example, a classical scalar field ϕ with potential $V(\phi)$. In p-space and 1-time dimension labeled by the time coordinate x^0 and space coordinates x^i $(1 \le i \le p)$ the lagrangian of the scalar field is:

$$L = \int d^p x [(\partial_0 \phi)^2 - \partial_i \phi \partial_i \phi - V(\phi)] \,. \tag{1}$$

Normally we choose the origin of ϕ so that the potential $V(\phi)$ has a minimum at $\phi = 0$. In this case quantization of ϕ gives a scalar particle of mass$^2 = V''(\phi)|_{\phi=0}$. This gives a positive mass^2 particle. But now suppose the potential has a maximum at $\phi = 0$. Then $V''(\phi)|_{\phi=0}$ is negative. Naive quantization will give a particle of negative mass^2. Thus we have a tachyon!

In this case however it is clear what we are doing wrong. When we identify $V''(0)$ as the mass^2 of the particle, we are making an approximation. We expand

$V(\phi)$ in a Taylor series expansion in ϕ, and treat the cubic and higher order terms as small corrections to the quadratic term. This is true only if the quantum fluctuations of ϕ around $\phi = 0$ are small. But for if $V(\phi)$ has a maximum at $\phi = 0$, then $\phi = 0$ is a classically unstable point. Hence we cannot expect the fluctuations of ϕ to be small. The remedy to this difficulty is to find the minimum ϕ_0 of the potential $V(\phi)$, and quantize the system around this point. More precisely this means that we can expand the potential around $\phi = \phi_0$, and treat the cubic and higher order terms in the expansion to be small. The mass^2 of the particle now can be identified as $V''(\phi_0)$. This is positive since $V(\phi)$ has a minimum at $\phi = \phi_0$. Hence the theory does not have tachyons.

Note that if there had been no reason to choose the origin of ϕ at zero, we could have defined a new field $\psi = \phi - \phi_0$ and expressed the potential as a function of ψ:

$$\widetilde{V}(\psi) = V(\psi + \phi_0)\,. \tag{2}$$

If from the beginning we worked with the field ψ then we would not have encountered the tachyon in the first place since $V''(\psi = 0)$ is positive. But there are often reasons why we choose the origin of field space in a specific way. For example, $\phi = 0$ could be a point with higher symmetry (*e.g.* $\phi \to -\phi$). This symmetry will be manifest in $V(\phi)$ but not so explicit in $\widetilde{V}(\psi)$. This leads to the phenomenon of spontaneous symmetry breaking. In such cases instead of having a single minimum, the potential has more than one minimum related by symmetry. *e.g.* if $V(\phi) = V(-\phi)$, then a minimum at ϕ_0 also means a minimum at $-\phi_0$.

If the potential has more than one degenerate minimum, we can consider field configurations where ϕ approaches different minimum in different regions of space. An example of such a field configuration is the domain wall, where we consider a field configuration where 1) ϕ depends on one spatial coordinate x^1, 2) as $x^1 \to \infty$, $\phi \to \phi_0$, 3) as $x^1 \to -\infty$, $\phi \to -\phi_0$, and 4) the total energy is minimized subject to these constraints. For this configuration the energy density is concentrated around $x^1 \simeq 0$. This gives rise to a 'codimension 1 defect'. For more complicated cases we can have more complicated defects (of higher codimension). Examples of such defects are vortices which are codimension 2 defects, t'Hooft Polyakov monopoles which are codimension 3 defects, etc. In general for a codimension k defect the energy density is localized around a subspace of dimension $(p - k)$.

The lessons learned from the field theory examples may be summarized as follows:

- Existence of tachyons in the spectrum tells us that we are expanding the potential around its maximum rather than its minimum.
- Associated with the existence of tachyons we often have spontaneous symmetry breaking and existence of defects.

We now turn to the discussion of tachyons in string theory. String theory contains infinite number of single 'particle' states, as if it is a field theory with infinite number of fields. But the conventional description of string theory is based on 'first quantized' formalism rather than a field theory. We take a string (closed or open) and quantize it maintaining Lorentz invariance. This gives infinite number of states characterized by momentum $\vec{p}$ and other discrete quantum numbers n. It turns out that the energy of the nth state carrying momentum $\vec{p}$ is given by $E_n = \sqrt{\vec{p}^2 + m_n^2}$, where m_n is some constant. Thus this state clearly has the interpretation of being a particle of mass m_n.

Quantization of some closed or open strings gives rise to states with negative m_n^2 for some n. This corresponds to a tachyon! For example, the original bosonic string theory formulated in (25+1) dimensions has a tachyon in the spectrum of closed strings. This theory is thought to be inconsistent due to this reason.

Superstring theories are free from closed string tachyons. But for certain boundary conditions, there can be tachyon in the spectrum of open strings even in superstring theories. Thus the question is: Does the existence of tachyons make the theory inconsistent? Or does it simply indicate that we are quantizing the theory around the wrong point? The problem in analyzing this question stems from the fact that unlike the example in a scalar field theory, the tachyon in string theory does not obviously come from quantization of a scalar field. Thus in order to understand the tachyon, we have to reconstruct the scalar field and its potential from the known results in string theory, and then analyze if the potential has a minimum.

It turns out that for open string tachyons we now know the answer in many cases. On the other hand, closed string tachyons are only beginning to be explored. Hence we shall focus mainly on open string tachyons in this talk.

There are five consistent, apparently different, superstring theories in 9-space and 1-time dimension. We shall focus on two of them, known as type IIA and type IIB string theories. Elementary excitations in this theory come from quantum states of the closed strings. But besides these elementary excitations these theories also contain 'composite' objects known as D-branes or more explicitly Dirichlet p-branes.

A Dp-brane is a p-dimensional objects. Thus for example D0-brane corresponds to a particle like object, a D1-brane corresponds to a string-like object, a D2-brane corresponds to a membrane like object and so on. But unlike the kinks and other defects in field theory which are associated with classical solutions of the equations of motion of the fields, D-branes are defined by saying what happens in their presence rather than by saying what they are. Consider, for example, a static flat Dp-brane in flat space-time, lying along a p-dimensional subspace. The definition of a Dp-brane is simply that fundamental strings can end on the p-dimensional hypersurface along which the D-brane lies. This has been illustrated in Fig.1

Quantum states of a fundamental open string with ends on a D-brane represent quantum excitation modes of the D-brane. D-branes need to satisfy various consistency requirements, and as a result D-branes for different p have different properties. For type IIA string theory, these properties are summarized as follows:

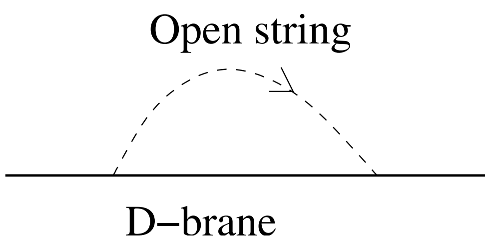

Figure 1: Fundamental strings (shown by dashed line) ending on a D-brane (shown by solid line).

1. For even p, Dp-branes are oriented and are known as BPS D-branes due to some special properties which they possess. For these branes, the mass per unit p-volume, also known as the tension $\mathcal{T}_p$ of the brane, is given by

$$\mathcal{T}_p = \frac{1}{(2\pi)^p g_s}\,, \tag{3}$$

in a unit in which the tension of the fundamental string is $\frac{1}{2\pi}$. We shall use this unit throughout this talk. g_s is a dimensionless constant known as the string coupling constant. We shall do all our analysis to lowest order in the perturbation expansion in g_s.

It turns out that all open string states on a BPS D-brane have mass$^2 \geq 0$. Hence there are no tachyons in the spectrum.

2. For odd p, the Dp-branes are unoriented (non-BPS). The tension $\widetilde{\mathcal{T}}_p$ of a non-BPS D-p-brane is given by,

$$\widetilde{\mathcal{T}}_p = \frac{\sqrt{2}}{(2\pi)^p g_s}\,. \tag{4}$$

Each such D-brane has one open string mode with mass$^2 = -\frac{1}{2}$. In other words, there is a tachyonic mode on each of these non-BPS D-p-branes.

For type IIB string theory the situation is reversed. There are now oriented (BPS) Dp-branes for odd p and unoriented (non-BPS) Dp-branes for even p. The results that we shall discuss will be valid both for type IIA and type IIB string theory. Whether we are talking about type IIA theory or type IIB theory should be understood from the context. For example, if we are refering to a non-BPS D-p-brane, then it should be understood that we are talking about type IIA theory if p is odd, and type IIB theory if p is even.

For oriented D-branes we define an anti-D-brane ($\bar{\text{D}}$-brane) to be a D-brane with opposite orientation It turns out that a coincident BPS Dp-brane $\bar{\text{D}}$-p-brane

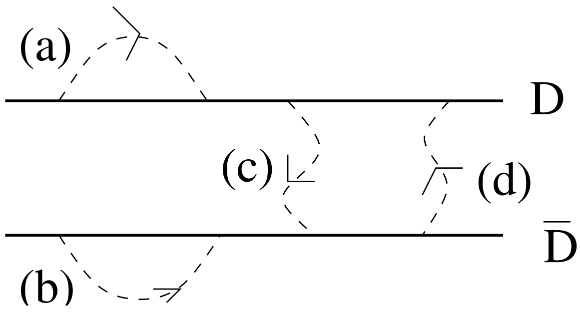

Figure 2: The tachyon on a Dp-$\bar{\text{D}}$-p-brane pair comes from open strings whose two ends lie on two different branes.

pair has two tachyonic modes, each of mass$^2 = -\frac{1}{2}$, from the open strings with one end on the brane and one end on the antibrane (sectors (c) and (d) in Fig.2).

Since string theory is formulated in a way that is different from a field theory, the method of analysis in string theory is very different from that in a field theory. Nevertheless it is useful to use the language of field theory to describe various situations in string theory. In particular, if we use the analogy with field theory origin of tachyons, then for a non-BPS D-p-brane, the dynamics of the single tachyonic mode should be described by a real scalar field T with negative mass^2 in p-space and one time dimensions. We shall refer to T as tachyon field. For the D-p-$\bar{\text{D}}$-p system, the dynamics of the pair of tachyonic modes should be described by a complex scalar field T with negative mass^2. The results in string theory can be stated *as if* the dynamics of the tachyon T is described by an effective potential $V_{eff}(T)$ or more generally an effective action $S_{eff}(T)$. This is what we shall do.

Let us begin by reviewing the properties of $S_{eff}(T)$ and $V_{eff}(T)$ which follow from simple considerations. First of all it is known that $S_{eff}(T)$ has simple symmetry properties. For example, for a non-BPS D-p-brane $S_{eff}(-T) = S_{eff}(T)$. On the other hand for a D-$\bar{\text{D}}$ system, $S_{eff}(e^{i\phi}T) = S_{eff}(T)$. The other property of that is obvious is that $V_{eff}(T)$ has a maximum at $T = 0$, since the field T is tachyonic.

The interesting questions to which we would like to know the answer are:

1. Does $V_{eff}(T)$ have a minimum?

2. If it does have a minimum, then what kind of mass spectrum do we get by quantizing the theory around the minimum?

3. Do we get topological defects involving the tachyon?

etc. It turns out that the answers to many of these questions are now known. These results can be summarized as follows:

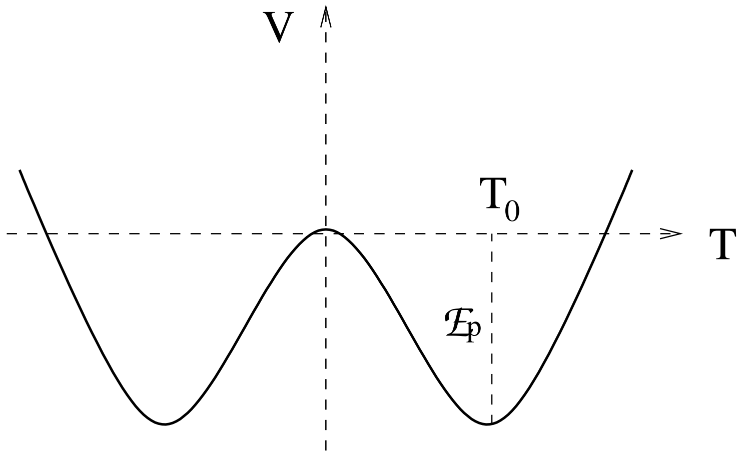

Figure 3: The tachyon potential on a non-BPS D-p-brane. The tachyon potential on a brane-antibrane system can be obtained by revolving this diagram around the vertical axis, so that we get a mexican hat potential.

1. $V_{eff}(T)$ does have a minimum at some value $|T| = T_0$. Furthermore, at this minimum[1, 2]
$$V_{eff}(T_0) + \mathcal{E}_p = 0\,, \tag{5}$$
where $\mathcal{E}_p$ denotes the total energy density of the original system. Thus $\mathcal{E}_p = \widetilde{\mathcal{T}}_p$ for a non-BPS Dp brane, and $\mathcal{E}_p = 2\mathcal{T}_p$ for Dp – $\bar{\mathrm{D}}p$ system. Thus at $|T| = T_0$ the total energy density vanishes identically. This situation has been illustrated in Fig.3.

2. $|T| = T_0$ configuration describes the closed string vacuum without any D-brane[1, 2]. Thus around this minimum there are no physical open string excitations. This is natural from the point of view of string theory, since the total energy vanishes at $T = T_0$, and hence we can identify this configuration as vacuum without any D-brane. Since there is no D-brane, there should be no open strings in the spectrum. However, this result is very surprising from the point of view of a normal field theory. Shifting the point around which we expand the potential can make a negative mass^2 state into a positive mass^2 state, but we do not eliminate the state altogether. On the other hand here expanding the action around the minimum of the potential not only gets rid of the original tachyon state, but also gets rid of the infinite number of other open string states which were present.

3. There are classical solutions of the equations of motion of T, representing lower dimensional D-branes [3, 4, 5, 6, 7, 8]. For example, on a non-BPS

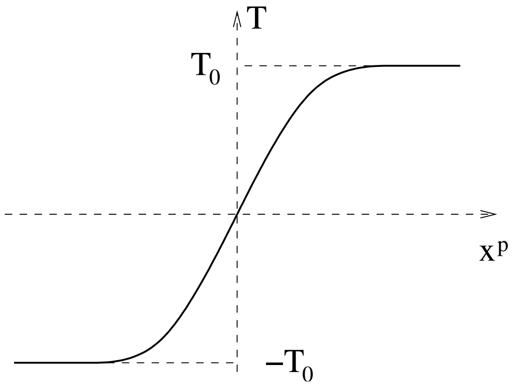

Figure 4: Tachyonic kink solution representing a BPS D-($p-1$)-brane.

Dp-brane, a kink represents a D-($p-1$)-brane. For this solution the energy density is localized around a codimension 1 subspace ($x^p = 0$) This looks like an ordinary kink solution in a field theory, but there is an important difference. In a conventional field theory, a defect lives on the space in which the field theory lives. Here, at the bottom of the potential, the object (original D-p-brane) whose dynamics the field theory describes disappears altogether. Nevertheless defects in the field can survive and describe non-trivial objects in the (9+1)-dimensional space-time in which full string theory lives.

There are also other more complicated examples of 'tachyonic defects'. For example, a vortex solution on a Dp-$\bar{\text{D}}p$ pair describes a BPS D-($p-2$)-brane [4]. On the other hand, a 't Hooft Polyakov monopole on a pair of coincident non-BPS Dp-branes describes a non-BPS D-($p-3$)-brane [7]. In this way all D-branes can be regarded as defects in the tachyon field living on D-branes of maximal dimension. This gives a more conventional description of D-branes as defects in the tachyon field. But more importantly this description gives a way to classify Dp-branes based on a branch of mathematics known as K-theory [5, 7]. Several new stable D-branes in various string theories have been discovered using this general scheme.

So far we have only described the properties of static solutions of the tachyon effective field theory. Let us now turn to dynamics, namely time dependent solutions of the equations of motion. In the case of a conventional scalar field, if we displace the field from its maximum and let it roll down the potential, the scalar field will oscillate about its minimum. Energy-momentum tensor $T_{\mu\nu}$ for this solution will have the form:

$$T_{00} = \mathcal{E}, \qquad T_{ij} = -p(x^0)\delta_{ij}, \qquad T_{i0} = 0\,. \tag{6}$$

Here $\mathcal{E}$ denotes the energy density, and remains constant due to energy conservation. p denotes the pressure, and will typically oscillate about an averge value (0 for a conventional scalar field) as the scalar field oscillates about its minimum. We can now ask: What happens if we displace the tachyon field on a D-$\bar{\text{D}}$ pair (or a non-BPS D-brane) and let it roll down the hill? It turns out that in this case the energy density remains constant as usual by energy conservation, but the pressure goes to zero asymptotically instead of oscillating about 0. Thus the final state is a gas of non-zero energy density and zero pressure [9, 10].

This seems a very strange result from the point of view of a conventional scalar field theory in which the action is given by the sum of a kinetic and a potential term. We can now ask if it is possible to write down an (unconventional) scalar field theory that can describe this apparently strange dynamics of the tachyon. It turns out that the tachyon dynamics near the minimum of the potential is describable in terms of a non-standard action for the tachyon field T [11]:

$$-\int d^{p+1}x\, V(T)\, \sqrt{1+\eta^{\mu\nu}\partial_\mu T\partial_\nu T}\,, \tag{7}$$

where η is the diagonal matrix with eigenvalues $(-1,1,1,\ldots 1)$, and $V(T)$ is the tachyon effective potential which in this parametrization has its minima at $T = \pm\infty$ and its maximum at $T=0$. For large T, $V(T)$ falls off as $e^{-\sqrt{2}T}$. This action as a candidate for tachyon effective field theory had been proposed earlier in [12, 13].

The classical dynamics of this system is best described in the Hamiltonian formalism. The Hamiltonian for this system is given by [14, 11]:

$$H = \int d^p x \sqrt{\Pi^2 + (V(T))^2}\, \sqrt{1+(\vec{\nabla}T)^2}\,, \tag{8}$$

where Π is the momentum conjugate to T. As the tachyon rolls down the potential hill, $V(T)\to 0$. Thus at late time we can ignore the $V(T)$ term in the Hamiltonian. It can be shown that in this limit the equations of motion derived from the Hamiltonian (8) are identical to the equations of motion of a pressureless non-interacting fluid (dust) with the identification that $|\Pi|\sqrt{1+(\vec{\nabla}T)^2}$ is interpreted as the energy density ρ of the dust, and $-\partial_\mu T$ is interpreted as the local $(p+1)$-velocity u_μ of the dust particle. Thus at late time the classical solutions in this field theory are in one to one correspondence with the configurations of a system of non-interacting dust. Since dust particles at rest correspond to a pressureless fluid, this automatically explains the result as to why the solutions describing a homogeneous rolling tachyon evolve into a system of zero pressure.

We should however add a caution that all the results quoted here are classical and are expected to be modified due to quantum corrections. Thus, for example, the pressureless gas obtained from the rolling tachyon may further decay into other states of the string. But as long as the coupling constant of the theory is small, we expect the life-time of the system to be large. This could be important in cosmology in the early universe, if brane-antibrane annihilation or decay of non-BPS D-branes played any role in the history of the universe.

Let us now briefly discuss a possible application of brane-antibrane annihilation process to inflationary cosmology. Typically inflation requires a (set of) scalar field(s) with reasonably flat potential near the top and a minimum where the potential vanishes. The question is: can we achieve this by a brane-antibrane pair or some variation of this configuration? The brane-antibrane pair has large energy near the top of the potential, but the potential is not flat near the top. Thus this system does not seem to be suitable for inflation. One possible way to achieve a flat potential is to separate the brane and the antibrane in space [15, 16]. To be more specific, consider (9+1) dimensional string theory with 6 of the dimensions wrapped into a compact space K. Take a D3-brane along the 3 non-compact directions, placed at a given point in K, and an $\bar{\text{D}}$3-brane along the same direction but placed at a different point of K. The D-$\bar{\text{D}}$ system has large energy density, but for sufficiently large separation there is no tachyon. They however have an attractive weak gravitational potential, and under its influence the brane and the anti-brane will slowly roll towards each other. From the point of view of a (3+1) dimensional observer, the separation between branes is interpreted as a scalar field ϕ. For large value of ϕ the potential is almost flat, but there is a small potential that drives ϕ towards smaller value. Thus if the universe started out with a large ϕ which then slowly decreased towards zero the universe will inflate during this phase. When ϕ becomes smaller than a critical value the tachyon develops and since the tachyon potential is very steep, the system quickly rolls down towards the minimum of the potential. In this scenario the tachyon dynamics is important for understanding end of inflation / reheating process. The details of this reheating process are still to be explored.

So far I have only mentioned various results, but not told you how to derive any of these results. Let me now say a few words about the various techniques which are used to derive these results. In stating the various results we have represented the tachyon by a scalar field. But we have one major problem, – that it is inconsistent to deal only with the tachyon and not take into accout its coupling with other fields which should represent the massive string states. Thus in order to study the classical dynamics of the tachyon field, we actually have to solve infinite number of equations involving infinite number of fields.

There are various approaches to this problem, but I shall mention only two of them. We can use an indirect approach where we use the fact that there is a one to one correspondence between solutions of equations of motion in string theory and two dimensional conformal field theories. In this approach we directly try to get a solution of the equations of motion (describing the defect solutions for example) by constructing the corresponding conformal field theory in two dimensions. This avoids the need to find the tachyon potential or its coupling to other fields. This procedure has been used to derive analytical results both for static and dynamical properties. In the direct approach (based on string field theory) [17, 18, 19, 20, 21, 22, 23] we take into accout the coupling of the tachyon to all the other fields and try to solve the coupled equations for all the fields using some approximation scheme, known as level truncation. In this scheme, we include only fields below a certain fixed mass (say M). This gives a finite number of fields, and the corresponding equations can be solved (numerically). Then we include more fields, with mass below M' ($M' > M$) and repeat the procedure. If the procedure converges as we go to larger and larger cut-off on the mass, then we are on the right track. So far this procedure has been used to study only the static properties of the tachyon. In these applications the results converge rapidly to the conjectured answers.

There are various other approaches all of which has been successful to various extents in studying the properties of the tachyon. I shall only list them here without giving any details:

1. Renormalization group flow [24]

2. Non-commutative geometry [25, 26]

3. Boundary string field theory [27, 28, 29]

etc.

I shall now summarize the talk by emphasizing once again the main points. As we have seen, we now have a good understanding of the physics of tachyons which arise from open strings living on unstable D-brane systems (non-BPS D-brane or D-$\bar{\text{D}}$ system). In these systems the minimum of the tachyon potential corresponds to total disappearance of the original brane. Nevertheless defects in the tachyon field can give rise to non-trivial D-branes which live not on the original brane but in the full (9+1)-dimensional space-time. Classical dynamics of the tachyon field near its minimum is represented by a non-standard field theory whose classical solutions represent configurations of non-interacting dust.

We conclude by listing one of the main open questions. Clearly we would like to know if we can make sense of closed string tachyon that appears in the original bosonic string theory. Existence of the tachyon is the only thing wrong with this theory, and hence by making sense of this tachyon we may make the theory consistent. For this we need to 1) establish the existence of the minimum of the potential, and 2) find an interpretation of the physics around this minimum. This is still an unsolved problem. However some progress has been made in understanding other kind of closed string tachyons which appear in superstring theories in non-trivial background [30]. In each case that has been understood, the minimum of the tachyon potential always corresponds to some kind of stable background. Thus the tachyon reflects the instability of the original background to decay into the new background. Success of this analysis raises hope that perhaps the tachyon in (25+1) dimensional bosonic string theory may also be understood in a similar manner.

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