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Erratum

Erratum to: On operators associated with tensor fields

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In this article, the following changes should be made.

1. The map ϕ_φ in Definition 2.1 should read $\phi_\varphi : \overset{*}{\Im}(M)|_{r+s>0} \to \Im(M)$ instead of $\phi_\varphi : \overset{*}{\Im}(M) \to \Im(M)$, and the property (e) appearing in Definition 2.1 should read

$$(e) \qquad \begin{aligned}(\phi_{\varphi X}\omega)Y &= (d(\imath_Y\omega))(\varphi X) - (d(\imath_Y(\omega\circ\varphi)))(X) + \omega((L_Y\varphi)X)\\ &= (\varphi X)(\iota_Y\omega) - X(\iota_{\varphi Y}\omega) + \omega((L_Y\varphi)X)\\ &= \phi_{\varphi X}\omega(Y) + \omega((L_Y\varphi)X)\end{aligned}$$

for all $\omega \in \Im_1^0(M)$ and $X, Y \in \Im_0^1(M)$, $\iota_Y\omega = \omega(Y) = \omega \overset{C}{\otimes} Y$, where $\phi_{\varphi X}\omega$ $(Y) = (\varphi X)(\iota_Y\omega) - X(\iota_{\varphi Y}\omega)$ is a notation only and it is not ϕ-operator because of $r = s = 0$.

2. Therefore Theorem 2.11 and its proof have to be replaced by

Theorem 2.11. *Let* $\omega \in \overset{*}{\Im}{}_s^0(M), s > 1$. *Then*

$$\begin{aligned}(\phi_{\varphi X}\omega)(Y_1,\dots,Y_s) &= (\varphi X)(\omega(Y_1,\dots,Y_s)) - X(\omega(\varphi Y_1,\dots,Y_s))\\ &\quad + \sum_{\lambda=1}^{s}\omega(Y_1,\dots,(L_{Y_\lambda}\varphi)X,\dots,Y_s) = \phi_{\varphi X}(\omega(Y_1,\dots,Y_s))\\ &\quad + \sum_{\lambda=1}^{s}\omega(Y_1,\dots,(L_{Y_\lambda}\varphi)X,\dots,Y_s),\end{aligned}$$

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where $\phi_{\varphi X}(\omega(Y_1,\ldots,Y_s)) = (\varphi X)(\omega(Y_1,\ldots,Y_s)) - X(\omega(\varphi Y_1,\ldots,Y_s))$ *is a notation only and it is not* ϕ*-operator.*

Proof. For simplicity, let $s=2$ and ω_{Y_1} be an 1-form such that $\omega_{Y_1}(Y_2) = \omega(Y_1,Y_2)$. By (e) of Definition 2.1 we have

$$\begin{aligned}(\phi_{\varphi X}\omega_{Y_1})Y_2 &= (\varphi X)(\omega_{Y_1}(Y_2)) - X(\omega_{Y_1}(\phi Y_2)) + \omega_{Y_1}((L_{Y_2}\varphi)X)\\ &= (\varphi X)(\omega(Y_1,Y_2)) - X(\omega(Y_1,\varphi Y_2)) + \omega(Y_1,(L_{Y_2}\varphi)X),\end{aligned}$$

from which, by virtue of

$$\begin{aligned}(\phi_{\varphi X}(\omega \overset{C}{\otimes} Y_1))Y_2 &= ((\phi_{\varphi X}\omega)\overset{C}{\otimes} Y_1 + \omega \overset{C}{\otimes} \phi_{\varphi X}Y_1)Y_2\\ &= (\phi_{\varphi X}\omega)(Y_1,Y_2) + \omega(\phi_{\varphi X}Y_1,Y_2),\end{aligned}$$

it follows that

$$\begin{aligned}(\phi_{\varphi X}\omega)(Y_1,Y_2) &= (\varphi X)(\omega(Y_1,Y_2)) - X(\omega(\varphi Y_1,Y_2))\\ &\quad + \omega(Y_1,(L_{Y_2}\varphi)X) + \omega((L_{Y_1}\varphi)X,Y_2).\end{aligned}$$

□

3. The properties (e_1) and (e_2) appearing in Definition 5.2 should read

$$\begin{aligned}(e_1)\quad (\phi_{\varphi,\tilde{\varphi}}\tilde{\omega})(\tilde{X},\tilde{Y}) &= (\phi_{\tilde{\varphi}}\tilde{\omega})(\tilde{X},\tilde{Y})\\ &= d((i_{\tilde{Y}}\tilde{\omega}))(\tilde{\varphi}\tilde{X}) - d(i_{\tilde{Y}}(\tilde{\omega}\circ\tilde{\varphi}))(\tilde{X}) + \tilde{\omega}((L_{\tilde{Y}}\tilde{\varphi})\tilde{X})\end{aligned}$$

and

$$\begin{aligned}(e_2)\quad (\phi_{\varphi,\tilde{\varphi}}\omega)(\tilde{X},Y) &= (\phi_{\varphi}\omega)(B\tilde{X},Y)\\ &= d((i_Y\omega))(\varphi(B\tilde{X})) - d(i_Y(\omega\circ\varphi))(B\tilde{X})\\ &\quad + \omega((L_Y\varphi)(B\tilde{X}))\end{aligned}$$

for any $\tilde{\omega} \in \Im_1^0(\tilde{M}),\ \omega \in \Im_1^0(M)$, respectively.

4. Similarly, the map ϕ_S in Definition 6.3 should read $\phi_S : {}^S\overset{*}{\Im}(M)|_{r+s>0} \to \Im(M)$, and the property (e) appearing in Definition 6.3 should read

$$\begin{aligned}(e)\qquad (\phi_S\omega)(X_1,X_2,Y) &= (d(\imath_Y\omega))(S(X_1,X_2))\\ &\quad - 2(d(\imath_Y(\omega\circ S)))(X_1,X_2)\\ &\quad + \omega(L_Y S)(X_1,X_2)\\ &= \phi_{S(X_1,X_2)}\omega(Y) + \omega(L_Y S)(X_1,X_2)\end{aligned}$$

for all $X_1,X_2,Y \in \Im_0^1(M)$, $\omega \in \Im_1^0(M)$, where $\imath_Y\omega = \omega(Y) = \omega\overset{C}{\otimes}Y$, $S(X_1,X_2) = -S(X_2,X_1)$, d is the exterior differentiation of $\imath_Y(\omega\circ S) \in \Im_1^0(M)$, $\phi_{S(X_1,X_2)}\omega(Y) = (d(\imath_Y\omega))(S(X_1,X_2)) - 2(d(\imath_Y(\omega\circ S)))(X_1,X_2)$ is a notation only and it is not ϕ-operator.

5. Also, the property (e) appearing in Definition 8.1 should read

$$(e) \qquad (\phi_{S(X_1,\dots,X_q)}\omega)Y = (d(\imath_Y\omega))(S(X_1,\dots,X_q)) - q(d(\imath_Y(\omega\circ S)))(X_1,\dots,X_q) + \omega((L_Y S)(X_1,\dots,X_q))$$

for all $X_1,\dots,X_q, Y \in \Im^1_0(M), q \geq 0$ and $\omega \in \Im^0_1(M)$, where $\imath_Y\omega = \omega(X), S$ is a vector-valued q-form, d is the exterior differentiation of $\imath_Y(\omega\circ S) \in \Lambda_{q-1}(M)$.

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