Correction

Correction to: Coupled Self-Organized Hydrodynamics and Stokes Models for Suspensions of Active Particles

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This note provides a list of errata and their correction for Reference [1].

- There is a typo in Eq. (5.12): the term in $(\tilde{\lambda} - 1)$ should come with a negative sign. This expression becomes:

$$\begin{aligned} & \left[i\rho_0(-\alpha + V_0 \cdot k) + \gamma|k|^2\rho_0 - \frac{(\tilde{\lambda} - 1)\tilde{b}\rho_0^2 k_0^2}{2|k|^2} \right] \bar{\Omega} \\ &= \left[\frac{1}{2} \frac{\rho_0}{|k|^2} \left(2\tilde{\lambda}k_0\bar{p} + \tilde{b}(\tilde{\lambda} + 1)(\rho_0\bar{k} + \bar{\rho}k_0) \right) - \frac{i}{\kappa}\bar{\rho} \right] k^\perp. \end{aligned} \quad (0.1)$$

- In the statement of Th. 5.1 (Linear Stability Analysis), we have that Eq. (5.5) is not true. As a consequence equation (5.6) is slightly modified into

$$\begin{aligned} D(\alpha, k) = & \left\{ \frac{\tilde{b}\rho_0}{2|k|^2} \left[\left(-4\tilde{\lambda} \frac{k_0^2}{|k|^2} + \tilde{\lambda} + 1 \right) (-\alpha + U_0 \cdot k) \right. \right. \\ & \left. \left. - c_1 k_0 \left(-2\tilde{\lambda} \frac{k_0^2}{|k|^2} + \tilde{\lambda} + 1 \right) \right] + \frac{i}{\kappa} c_1 \right\} (|k|^2 - k_0^2) \\ & - (-\alpha + U_0 \cdot k) \left[i(-\alpha + V_0 \cdot k) - \frac{(\tilde{\lambda} - 1)\tilde{b}\rho_0 k_0^2}{2|k|^2} + \gamma|k|^2 \right], \end{aligned} \quad (0.2)$$

when $\bar{k} \neq 0$. And Eq. (5.7) is valid when $\bar{k} \neq 0$. When $\bar{k} = 0$ we have a different scenario where $\bar{\rho} = 0$ and $\bar{\Omega} \neq 0$ arbitrary and

$$\alpha = V_0 \cdot k + i \left(\frac{(\tilde{\lambda} - 1)\tilde{b}\rho_0 k_0^2}{2|k|^2} - \gamma|k|^2 \right), \quad (0.3)$$

which gives only stable modes since $\tilde{\lambda} \in [-1, 1]$ and so $\text{Im}(\alpha) \leq 0$. (Finally, notice that, the parameter η in the original article does not play a role any more in this corrected version.)

To obtain these results we modify the proof of Th. 5.1 **Case B**, which we rewrite fully next (notice that the number of the equations that do not start by 0 refer to the number of the equations as they appear in the original article).

Proof. Case (B) Suppose that $k^\perp \neq 0$. Doing the inner product of Eq. (0.1) with k and using that $k^\perp \cdot k = |k|^2 - k_0^2$, the dispersion relation is given by (using Eq. (5.11)):

$$\begin{aligned} & \left\{ \tilde{b} \frac{\rho_0}{2|k|^2} \left[\rho_0 \bar{k} \left(-4\tilde{\lambda} \frac{k_0^2}{|k|^2} + \tilde{\lambda} + 1 \right) + \bar{\rho} k_0 \left(-\frac{2\tilde{\lambda} k_0^2}{|k|^2} + \tilde{\lambda} + 1 \right) \right] - \frac{i}{\kappa} \bar{\rho} \right\} (|k|^2 - k_0^2) \\ &= \left[i\rho_0(-\alpha + V_0 \cdot k) - (\tilde{\lambda} - 1) \frac{\tilde{b}\rho_0^2}{2} \frac{k_0^2}{|k|^2} + \gamma|k|^2 \rho_0 \right] \bar{k}, \end{aligned} \quad (0.4)$$

and from Eq. (5.10b) we have the relation

$$(-\alpha + U_0 \cdot k) \bar{\rho} + \rho_0 c_1 \bar{k} = 0. \quad (0.5)$$

Next we distinguish between the cases $\bar{k} \neq 0$ and $\bar{k} = 0$. Suppose that $\bar{k} \neq 0$, then from Eq. (0.5) we have that $-\alpha + U_0 \cdot k \neq 0$ and multiplying Eq. (0.4) by $-\alpha + U_0 \cdot k \neq 0$ and using Eq. (0.5), we get the dispersion relation in Eq. (0.2), after simplifying $\bar{k}$.

If, on the contrary, $\bar{k} = 0$, then one can check that $k^\perp \cdot \bar{\Omega} = 0$ (thanks to Eq. (5.10a)) and, therefore, they are normal. Since by assumption $k^\perp \neq 0$ from Eq. (0.1) we conclude that the coefficient in front of $k^\perp$ on the right hand side must be zero. Rewriting this term using that $\bar{k} = 0$ and Eq. (5.11) we have that

$$\bar{\rho} I = 0, \quad (0.6)$$

with

$$I := \tilde{b} \frac{\rho_0}{2|k|^2} k_0 \left(-\frac{2\tilde{\lambda} k_0^2}{|k|^2} + \tilde{\lambda} + 1 \right) - \frac{i}{\kappa}.$$

From here we deduce that $\bar{\rho} = 0$ because otherwise we should have that $I = 0$ but the imaginary part of I is non-zero, so $\bar{\rho} = 0$. In particular this implies that Eq. (0.5) is fulfilled and that $\bar{\Omega} \neq 0$ (otherwise we would have null perturbation). Since $\bar{\Omega} \neq 0$, from Eq. (0.1) again (remembering that $k^\perp \perp \bar{\Omega}$) we must have that the coefficient in front of $\bar{\Omega}$ is equal to zero. This gives the dispersion relation (0.3).

Now, we go back to the case $\bar{k} \neq 0$. To simplify the analysis we will restrict ourselves to the case where $k^\perp = k$, i.e. $k_0 = k \cdot \Omega_0 = 0$ and $\bar{k} \neq 0$. This implies, in particular, that $U_0 \cdot k = V_0 \cdot k = v_0 \cdot k$. With these considerations one can simplify the dispersion relation (0.2) into

$$\tilde{D}(\alpha, k) = 0,$$

where $\tilde{D}(\alpha, k)$ is given in Eq. (5.7). □

- There are two typos at the end of page 21: in the third line of **Case (A)** part b) should read $\text{Im}(\alpha)$ instead of $\text{Im}(\omega)$; and in the fifth line should read “ $\bar{\Omega}$ is arbitrary with $\bar{\Omega} \cdot \Omega_0 = 0$ ” rather than “ $\bar{\Omega}$ is arbitrary with $\bar{\Omega}, \Omega_0 \neq 0$ ”.

Compliance with ethical standards

Conflict of interest The authors declare they have no conflict of interest.

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Reference

- [1] Degond, P., Merino-Aceituno, S., Vergnet, F., Yu, H.: Coupled self-organized hydrodynamics and stokes models for suspensions of active particles. *J. Math. Fluid Mech.* **21**, 6 (2019). <https://doi.org/10.1007/s00021-019-0406-9>

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