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Hofer–Zehnder capacity of magnetic disc tangent bundles over constant curvature surfaces

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Abstract. We compute the Hofer–Zehnder capacity of magnetic disc tangent bundles over constant curvature surfaces. We use the fact that the magnetic geodesic flow is totally periodic and can be reparametrized to obtain a Hamiltonian circle action. The oscillation of the Hamiltonian generating the circle action immediately yields a lower bound of the Hofer–Zehnder capacity. The upper bound is obtained from Lu's bounds of the Hofer–Zehnder capacity using the theory of pseudo-holomorphic curves. In our case, the gradient spheres of the Hamiltonian H will give rise to the non-vanishing Gromov–Witten invariant.

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1. Introduction and main results. The notion of symplectic capacities was developed to investigate the existence of symplectic embeddings. As symplectomorphisms are always volume preserving, one could ask whether a symplectic embedding $M \hookrightarrow N$ exists if and only if there exists a smooth volume preserving embedding. The answer is no (in dimension larger than two) and was given by Gromov in 1985 with his non-squeezing theorem [6]. This means that there must be other global symplectic invariants than volume. A class of such invariants is given by symplectic capacities as introduced by H. Hofer and E. Zehnder in [7]. There, they constructed a special capacity, now known as the Hofer–Zehnder capacity, relating embedding problems with the dynamics on symplectic manifolds. Very importantly, its finiteness implies the existence of periodic orbits on almost all compact regular energy levels ([7, Ch. 4]).

Definition 1.1. Let (M, ω) be a symplectic manifold possibly with boundary ∂M. We call a smooth Hamiltonian function $H : M \to \mathbb{R}$ admissible if there

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exists a compact subset $K \subset M \setminus \partial M$ and a non-empty open subset $U \subset K$ such that

(a) $H|_{M\setminus K} = \max H$ and $H|_U = 0$,
(b) $0 \leq H(x) \leq \max H$ for all $x \in M$.

Denote by $\mathcal{H}(M)$ the set of admissible functions and by $\mathcal{P}_{\leq 1}(H)$ the set of non-constant periodic orbits with period at most one. The Hofer–Zehnder capacity of the symplectic manifold (M, ω) is then defined as

$$c_{\mathrm{HZ}}(M,\omega) := \sup\{\max H \mid H \in \mathcal{H}(M), \mathcal{P}_{\leq 1}(H) = \emptyset\}.$$

In this note, we consider the following setup. Let (Σ, g) be a closed oriented Riemannian surface of constant curvature κ. Denote by λ the pullback via the metric isomorphism $T\Sigma \cong T^*\Sigma$ of the canonical 1-form on $T^*\Sigma$. Further denote by $\sigma \in \Omega_2(\Sigma)$ the Riemannian area form. We now study for some $r > 0$ the disc tangent bundle

$$D_r\Sigma = \{(x, v) \in T\Sigma |\ g_x(v, v) < r^2\}$$

and equip it with the magnetically twisted symplectic form

$$\omega_s := \mathrm{d}\lambda - s\pi^*\sigma$$

for some real parameter $s \neq 0$. We call s the strength of the magnetic field. The most famous Hamiltonian on tangent bundles is the kinetic Hamiltonian

$$E : TM \to \mathbb{R},\ (x, v) \mapsto \frac{1}{2} g_x(v, v).$$

If $s = 0$, the associated Hamiltonian flow is the geodesic flow. In analogy, the Hamiltonian flow for $s \neq 0$ is called magnetic geodesic flow. Morally, turning on the magnetic term bends the flow lines, just like a charged particle moving on the surface under the influence of a magnetic field transversal to the surface. Our main (and only) theorem gives the value of the Hofer–Zehnder capacity for a certain range of s, r.

Theorem. *Let (Σ, g) be a closed oriented Riemannian surface of constant curvature κ. Denote by σ the corresponding area form and pick two real parameters $r > 0$ and $s \neq 0$ satisfying $s^2 + \kappa r^2 > 0$. Then for $\kappa \neq 0$,*

$$c_{\mathrm{HZ}}(D_r\Sigma, \mathrm{d}\lambda - s\pi^*\sigma) = \frac{2\pi}{\kappa}\left(\sqrt{s^2 + \kappa r^2} - |s|\right).$$

The capacity for $\kappa = 0$ is the limit

$$c_{\mathrm{HZ}}(D_r\Sigma, \mathrm{d}\lambda - s\pi^*\sigma) = \lim_{\kappa \to 0} \frac{2\pi}{\kappa}\left(\sqrt{s^2 + \kappa r^2} - |s|\right) = \frac{\pi r^2}{|s|}.$$

The theorem covers three types of surfaces: spheres ($\kappa > 0$), flat tori ($\kappa = 0$), and hyperbolic surfaces ($\kappa < 0$). The assumption $s^2 + \kappa r^2 > 0$ does not put any additional constraint on spheres and flat tori. For hyperbolic surfaces, it tells us to look at strong magnetic fields, i.e., $|s| > \sqrt{-\kappa}r$. The cases not covered are the sphere ($\kappa > 0$) and tori ($\kappa = 0$) with vanishing magnetic field ($s = 0$) and hyperbolic surfaces ($\kappa < 0$) with weak magnetic field. As shown in [3], the theorem (and the corollary) actually hold in the case of spheres with

vanishing magnetic field. For flat tori, the values where recently determined [1] to be twice the systol, in particular finite, but for hyperbolic surfaces even finiteness is unknown.
There are some modifications of the Hofer–Zehnder capacity we will also discuss. First one can look at this capacity with respect to a fixed free homotopy class of loops ν. We denote

$$\mathcal{P}_T(H;\nu) := \{\gamma \in C^\infty(\mathbb{R}/T\mathbb{Z}, M) \mid \dot\gamma(t) = X_H(\gamma(t)) \neq 0;\ [\gamma] = \nu\},$$

the set of non-constant T-periodic solutions to the Hamiltonian equations in the class ν and by $\mathcal{P}_{\leq T}(M,\nu)$ the set of non-constant periodic solutions in the class ν with period less or equal to T. The Hofer–Zehnder capacity with respect to this free homotopy class is defined to be

$$c^\nu_{\mathrm{HZ}}(M,\omega) := \sup\{\max H \mid H \in \mathcal{H}(M), \mathcal{P}_{\leq 1}(H;\nu) = \emptyset\}.$$

For $\nu = 0$, this is called π_1-sensitive capacity.
Second there is a relative version, considering only Hamiltonians vanishing on a subset $Z \subset M$ not touching the boundary. It is defined as follows. Denote by $\mathcal{H}(M,Z)$ the set of smooth functions satisfying a) and b) for an open neighborhood $U \supset Z$ and a compact set $K \subset M \setminus \partial M$ containing U, i.e., $U \subset K$. The relative Hofer–Zehnder capacity is then defined as

$$c_{\mathrm{HZ}}(M,Z,\omega) := \sup\{\max H \mid H \in \mathcal{H}(M,Z), \mathcal{P}_{\leq 1}(H) = \emptyset\}.$$

Observe that from the definitions it follows directly that

$$c_{\mathrm{HZ}}(M,Z,\omega) \leq c_{\mathrm{HZ}}(M,\omega) \leq c^0_{\mathrm{HZ}}(M,\omega).$$

Proving the theorem, it becomes clear that the Hamiltonian we use for the lower bound is actually vanishing on the zero-section, thus also bounds the relative capacity from below. In addition, the upper bound comes from pseudo-holomorphic spheres, therefore it detects contractible orbits which means we actually bound the π_1-sensitive capacity from above.

Corollary. *The π_1-sensitive and the relative Hofer–Zehnder capacity agree, i.e.,*

$$c_{\mathrm{HZ}}(D_r\Sigma,\Sigma,\omega_s) = c_{\mathrm{HZ}}(D_r\Sigma,\omega_s) = c^0_{\mathrm{HZ}}(D_r\Sigma,\omega_s).$$

This example also shows that the π_1-sensitive capacity is not continuous on all smooth families of domains bounded by smooth hypersurfaces, a question raised by Cieliebak, Hofer, Latschev, and Schlenk in [5, Prob. 7]. Indeed, for closed hyperbolic surfaces ($\kappa = -1$), we find that

$$c^0_{\mathrm{HZ}}(D_r\Sigma,\omega_1) = \begin{cases} 2\pi(1-\sqrt{1-r^2}) & \text{for} \quad r \leq 1, \\ \infty & \text{for} \quad r > 1, \end{cases}$$

is not continuous in r. It jumps precisely at the Mañé critical value [4, Sec. 5.2]. The value at $r = 1$ follows as the Hofer–Zehnder capacity is of inner regularity [7, Thm. 1, Ch. 3], hence lower semi-continuous in r. To see that the π_1-sensitive capacity for $r > 1$ is infinite, observe that any radial Hamiltonian that is constantly zero on $D_1\Sigma$ has no contractible periodic orbits, as curves in $\mathbb{CH}^1$ of constant geodesic curvature less than 1 are not periodic.

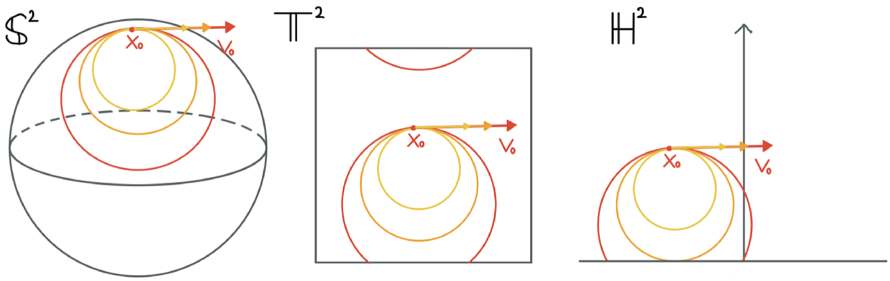

Figure 1. The picture shows families of geodesic circles. The vectors v_0 of different length indicate that the corresponding magnetic geodesic is a parametrized geodesic circle of geodesic curvature $\kappa_g = |s|/|v_0|$

Outline. In the second section, we modify the magnetic geodesic flow to obtain a semifree Hamiltonian circle action and in the third section, we use this circle action to prove the theorem and its corollary.

2. Magnetic geodesic flow. For the proof of our main theorem, the key ingredient is the fact that we can modify the magnetic geodesic flow to obtain a semifree Hamiltonian circle action. The construction of this circle action is done in this section.

We first work on the universal cover of Σ, hence $\mathbb{C}\mathrm{P}^1, \mathbb{C}^1, \mathbb{C}\mathrm{H}^1$ depending on the sign of the curvature. As for example shown in [2], magnetic geodesics on these spaces are curves of constant geodesic curvature $\kappa_g = \frac{|s|}{|v|}$. If R denotes the radius (with respect to the Riemannian metric g) of a geodesic circle, we know, using normal polar coordinates, that its circumference C and the geodesic curvature κ_g are

$$C = \frac{2\pi}{\sqrt{\kappa}} \sin(\sqrt{\kappa}R) = \frac{2\pi\sqrt{\kappa}^{-1}\tan(\sqrt{\kappa}R)}{\sqrt{1+(\tan(\sqrt{\kappa}R))^2}}, \tag{1}$$

$$\kappa_g = \frac{\sqrt{\kappa}}{\tan(\sqrt{\kappa}R)}. \tag{2}$$

Here we use the convention $\sqrt{-1} = i$ and the formulas $-i\sin(ix) = \sinh(x)$, $\cos(ix) = \cosh(x)$. Observe that in the hyperbolic case the geodesic curvature of geodesic circles can not be less than $\sqrt{|\kappa|}$. Indeed, curves of geodesic curvature less than $\sqrt{|\kappa|}$ do not close up. We therefore restrict to the regime of strong magnetic field, i.e., $s^2 + \kappa|v|^2 > 0$. See Fig. 1 for a visualisation.
From (2) and (1), we get

$$C = \frac{2\pi}{\kappa_g\sqrt{1+\kappa/\kappa_g^2}} = \frac{2\pi|v|}{\sqrt{s^2+\kappa|v|^2}},$$

where in the last step we inserted $\kappa_g = \frac{|s|}{|v|}$. Now, we conclude that the period is given by

$$T = \frac{C}{|v|} = \frac{2\pi}{\sqrt{s^2 + \kappa|v|^2}}.$$

In particular, the reparametrization $H = h \circ E$ with

$$h : \mathbb{R}_{\geq 0} \to \mathbb{R}, \quad h(E) = \frac{2\pi}{\kappa}\left(\sqrt{s^2 + 2\kappa E} - |s|\right),$$

induces a Hamiltonian S^1-action (of period $T = 1$). Note that the induced circle action is semifree as all, but the constant orbits, have period 1. If we now consider arbitrary Riemannian surfaces of constant curvature, it is a priori not clear that the induced circle action is still semifree. This is the statement of the following proposition.

Lemma 2.1. *Let (Σ, g, j) be a Riemann surface of constant sectional curvature κ. Then, for constants $s \in \mathbb{R} \setminus \{0\}$ and $r > 0$ satisfying $s^2 + \kappa r^2 > 0$, the Hamiltonian*

$$H : D_r\Sigma \to \mathbb{R}, \ (x, v) \mapsto \frac{2\pi}{\kappa}\left(\sqrt{s^2 + \kappa|v|^2} - |s|\right),$$

generates a semifree Hamiltonian circle action on the disc-subbundle $(D_r\Sigma, \omega_s)$ of the magnetically twisted tangent bundle.

Proof. If Σ is simply connected, we are done by the previous computations. If Σ is not simply connected, we know that $\kappa \leq 0$ and $\Sigma = \tilde{\Sigma}/\Gamma$ for a discrete subgroup Γ of isometries acting freely on the universal covering $\tilde{\Sigma} \in \{\mathbb{C}^1, \mathbb{CH}^1\}$. We need to make sure that the restriction of the projection $\tilde{\Sigma} \to \Sigma$ to any magnetic geodesic $\tilde{\gamma} \to \gamma$ is no covering of degree > 1. The prove goes by contradiction. Assume $\tilde{\gamma}$ was covering γ with some degree > 1, then there must be an element $g \in \Gamma$ that is a rotation around the center of $\tilde{\gamma}$. In particular, g fixes the center of $\tilde{\gamma}$, which yields a contradiction as Γ acts freely on $\tilde{\Sigma}$. $\square$

3. Proof of Theorem. In the previous section, we proved that for $r > 0$, satisfying $s^2 + \kappa r^2 > 0$, the Hamiltonian

$$H : D_r\Sigma \to \mathbb{R}; \ H(x, v) = \frac{2\pi}{\kappa}\left(\sqrt{s^2 + \kappa|v|^2} - |s|\right),$$

is well-defined, smooth, and generates a semifree S^1-action. We can now show that the oscillation of this Hamiltonian yields both a lower and an upper bound for the Hofer–Zehnder capacity and thus determines it.

Lower bound:
We modify the Hamiltonian H generating the circle action slightly so that it becomes admissible. This can be done with the help of a function $f : [0, \max H] \to [0, \infty)$ satisfying

$$\begin{aligned} &0 \leq f'(x) < 1, \\ &f(x) = 0 \ \text{ near } \ 0, \\ &f(x) = \max H - \varepsilon \ \text{ near } \ \max H. \end{aligned}$$

Then all solutions to the Hamiltonian system on $(D_r\Sigma, \omega_s)$ with Hamiltonian $\tilde{H} = f \circ H$ have period

$$T = \frac{1}{f'(E)} > 1.$$

Thus $\tilde{H}$ is admissible and we find the estimate

$$c_{\mathrm{HZ}}(D_r M, \omega_s) \geq \max(\tilde{H}) = \max(H) - \varepsilon, \qquad \forall \varepsilon > 0.$$

Upper bound:
The main abstract ingredient for the upper bound is a theorem by Lu [9, Thm. 1.10] in terms of (closed) Gromov–Witten invariants (see Appendix A). In order to obtain the right setup to apply Lu's theorem we compactify the disc tangent bundle using a Lerman cut [8].
Roughly speaking, for a Hamiltonian S^1-manifold (M, ω) with moment map $H : M \to \mathbb{R}$, the Lerman cut associates to a regular sub level set $\{H(x, v) < C\}$ a closed symplectic manifold $\left(\overline{\{H(x, v) < C\}}, \overline{\omega}\right)$ that contains $\{H(x, v) < C\}$ as an open dense symplectic submanifold and attaches the symplectic quotient $H^{-1}(C)/S^1$ at the boundary. For the precise definition of Lerman cuts, we refer to [8].

In our case, we do a Lerman cut with respect to the Hamiltonian H at the level set

$$\left\{ H(x, v) = \frac{2\pi}{\kappa} \left(\sqrt{s^2 + r^2} - |s| \right) \right\} = \{|v| = r\} = \partial D_r \Sigma$$

to compactify $(D_r\Sigma, \omega_s)$. The compactification $(\overline{D_r\Sigma}, \overline{\omega_s})$ is a closed symplectic 4-manifold with semifree Hamiltonian circle action. We denote its moment map by $\overline{H} : \overline{D_r\Sigma} \to \mathbb{R}$. The critical set, where the Hamiltonian $\overline{H}$ attains its minimum, corresponds to the zero-section and the critical set, where the Hamiltonian $\overline{H}$ attains its maximum, corresponds to the quotient $D_\infty := \partial D_r \Sigma / S^1$. Both are therefore of codimension two. Now if we choose a compatible S^1-invariant almost complex structure J, any gradient sphere $u(s, t) = \Phi^t_{\overline{H}} \gamma(s)$ is J-holomorphic and connects the two critical sets. Here $\Phi^t_{\overline{H}}$ denotes the Hamiltonian flow of $\overline{H}$ and γ is a non-constant gradient flow line with respect to the metric induced by J, i.e., $\dot{\gamma} = JX_{\overline{H}}$. The map $u : \mathbb{R} \times S^1 \to \overline{D_r\Sigma}$ indeed extends to the sphere as gradient flow lines connect critical points. By [10, Prop. 4.3], the 1-point Gromov-Witten invariant $\mathrm{GW}_A([pt.])$ in the class $A \in H_2(\overline{D_r\Sigma}, \mathbb{Z})$ of a gradient sphere ($A = [u]$) is non-vanishing. Further the homology class represented by the divisor D_∞ obtained from collapsing the boundary is proportional to the Poincaré-dual of $[\overline{\omega_s}]$. Thus

$$\mathrm{GW}_A([pt.], [D_\infty], [D_\infty]) = \mathrm{GW}_A([pt.]) \left(A \cdot [D_\infty]\right)^2 \neq 0.$$

This means we can apply a corollary A.1 of Lu's theorem, [9, Thm. 1.10] to obtain as upper bound

$$\begin{aligned} c_{\mathrm{HZ}}(D_r\Sigma, \mathrm{d}\lambda - s\pi^*\sigma) &= c_{\mathrm{HZ}}(\overline{D_r\Sigma} \setminus D_\infty, \overline{\omega_s}) \leq \overline{\omega_s}(A) = \int_{\mathbb{C}\mathrm{P}^1} u^*\overline{\omega_s} \\ &= \int_{-\infty}^{\infty} \mathrm{d}s \int_0^1 \mathrm{d}t\ \overline{\omega_s}(\partial_s u, \partial_t u) = \int_{-\infty}^{\infty} \mathrm{d}s \int_0^1 \mathrm{d}t\ \mathrm{d}\overline{H}(\dot\gamma(t)) \\ &= \max(\overline{H}) - \min(\overline{H}) = \frac{2\pi}{\kappa}\left(\sqrt{s^2+\kappa r^2} - |s|\right). \end{aligned}$$

Note that the first equality follows from the fact that the sub level set of H is symplectomorphic to the Lerman cut with the contracted boundary removed.

Other types of the Hofer–Zehnder capacity: Lu's theorem actually yields an upper bound for the π_1-sensitive Hofer–Zehnder capacity, as we are working with pseudoholomorphic spheres. Further the Hamiltonian $\tilde{H}$ used for the lower bound vanishes along the zero-section, thus $\max \tilde{H}$ also bounds the relative Hofer–Zehnder capacity from below. In total, we obtain

$$c_{\mathrm{HZ}}(D_r\Sigma, \Sigma, \omega_s) = c_{\mathrm{HZ}}(D_r\Sigma, \omega_s) = c^0_{\mathrm{HZ}}(D_r\Sigma, \omega_s).$$

As all orbits of $\tilde{H}$ are contractible, we can further conclude that $c^\nu_{\mathrm{HZ}}(D_r\Sigma, \omega_s) = \infty$ for any $\nu \neq 0$.

A. A corollary of Lu's theorem. In this appendix, we prove a corollary of Lu's theorem [9, Thm. 1.10] that is essential for finding the upper bound in the proof of our main theorem. We do not introduce the language of pseudo-symplectic capacities or Gromov–Witten invariants here and instead refer to [9] for the careful description of the setup. The notation for admissible functions is also the same as in Lu's article, only the notation for Gromov–Witten invariants differ, we replaced $\Psi_{A,g,m+2}$ with $\mathrm{GW}_{A,g,m+2}$.

Corollary A.1. *Let (M, ω) be a closed symplectic manifold of dimension $\dim M \geq 4$ and fix two disjoint closed*[1] *connected submanifolds $D_0, D_\infty \subset M$ of codimension at least two. Denote by $[D_0], [D_\infty] \in H_2(M, \mathbb{Q})$ the induced homology classes. Suppose there exists a homology class $A \in H_2(M; \mathbb{Z})$ for which the Gromov-Witten invariant*

$$\mathrm{GW}_{A,g,m+2}([D_0], [D_\infty], \beta_1, \ldots, \beta_m) \neq 0$$

for some homology classes $\beta_1, \ldots, \beta_m \in H_*(M; \mathbb{Q})$ and an integer $m \geq 1$. Then the relative Hofer–Zehnder capacity satisfies

$$c_{HZ}(M \setminus D_\infty, D_0; \omega) \leq \omega(A),$$

and if A is a spherical class, i.e., $g = 0$, then also

$$c^\circ_{HZ}(M \setminus D_\infty, D_0; \omega) \leq \omega(A).$$

Proof. We need to show that

$$\mathcal{H}_{\mathrm{ad}}(M \setminus D_\infty, D_0; \omega) \subset \mathcal{H}_{\mathrm{ad}}(M, \omega; [D_0], [D_\infty])$$

[1] Compact with no boundary!

because then

$$c_{HZ}(M \setminus D_\infty, D_0, \omega) \leq C^{(2)}_{HZ}(M, \omega; [D_0], [D_\infty]).$$

Indeed, as D_0, D_∞ are connected and of codimension at least two, the boundary of any small closed disc sub bundle $D_\varepsilon D_0, D_\varepsilon D_\infty$ of the normal bundle is connected and we can set $P = D_\varepsilon D_0$ and $Q = M \setminus \mathrm{Int}(D_\varepsilon D_\infty)$. For $\varepsilon > 0$ small enough, condition (1) is satisfied as D_0 and D_∞ are disjoint and compact. Now let $H \in \mathcal{H}_{\mathrm{ad}}(M \setminus \Sigma_\infty, \Sigma_0; \omega)$, then H vanishes on an open neighborhood of Σ_0 and constantly attains its maximum on a neighborhood of Σ_∞. In particular, for $\varepsilon > 0$ small enough, H also satisfies condition (2). Conditions (3) and (6) hold true per definition and (4) per construction. Condition (5) follows as M is compact, thus critical values can not accumulate. The π_1-sensitive claim follows analogously. □

Remark A.2. As finitely many points can be moved by Hamiltonian diffeomorphisms, i.e., $\mathrm{Ham}(M, \omega)$ is k-transitive, it follows that 2-point invariants yield upper bounds to the Hofer–Zehnder capacity of M and 1-point invariants yield upper bounds to the Hofer–Zehnder capacity of $M \setminus D_\infty$.

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