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Correction

Correction to: An elementary proof that the triharmonic Green function of an eccentric ellipse changes sign

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Correction to: Arch. Math. 107 (2016), 59–62
https://doi.org/10.1007/s00013-016-0909-z.

The note added in proof of [1] is false. The function

$$u(x,y) = \left(1 - x^2 - 144y^2\right)^m \left(1 - x + 200\,(1-x)^2 - 21y^2 - \varepsilon\right) \tag{1}$$

on $\Omega = \left\{(x,y) \in \mathbb{R}^2; x^2 + 144y^2 < 1\right\}$, with ε positive and small enough, produces a counterexample concerning the claim '$f \geq 0 \implies u \geq 0$' for

$$\begin{cases} (-\Delta)^m u = f & \text{in } \Omega, \\ \partial_\nu^k u = 0 \text{ for } 0 \leq k \leq m-1 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

only when $m = 2$ or $m = 3$. Indeed, $f = (-\Delta)^m u$ is positive while u changes sign for $m = 2, 3$. However, the function u fails to produce a counterexample for $4 \leq m \leq 8$ as claimed in the note, since both functions u and $(-\Delta)^m u$ change sign.

It seems possible to construct counterexamples for $m \geq 4$, but a straightforward generalisation from (1) is not obvious. For example, when $m = 4$, the following combination gives a counterexample on the same domain, again for small positive ε:

$$u(x,y) = \left(1 - x^2 - 144y^2\right)^4 \left(w(x,y)\left(\tfrac{1}{10} + w(x,y)\right) - \varepsilon\right) \tag{3}$$

The original article can be found online at https://doi.org/10.1007/s00013-016-0909-z.

with $w(x,y) = 1 - x + 200\,(1-x)^2 - 18y^2$ and

$$\begin{aligned} f(x,y) = {} & 690278247678673728 - 2802269782743715008\ x \\ & + 4266136671660720000\ x^2 - 2886586302976634880\ x^3 \\ & + 732441198825538560\ x^4 + 11199653433588055680\ y^2 \\ & - 22525165747433064960\ xy^2 + 11325749053509181440\ x^2y^2 \\ & + 3221858435872642560\ y^4 - 17406092393856\ \varepsilon. \end{aligned} \tag{4}$$

Some tedious computations show u, f satisfy (2) and, for ε small and positive, that f in (4) is positive while u in (3) changes sign on Ω.

Reference

[1] Sweers, G.: An elementary proof that the triharmonic Green function of an eccentric ellipse changes sign. Arch. Math. **107**, 59–62 (2016)

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