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# Corrigendum to "Note on zeros of the derivative of the Selberg zeta-function"

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**Abstract.** We correct constants in asymptotic formulas of our paper "Note on zeros of the derivative of the Selberg zeta-function", Arch. Math. 91 (2008), 238–246.

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In the paper we investigated asymptotic formulas related to the number of zeros of the derivative of the Selberg zeta-function on cocompact hyperbolic surface. Several constants in these formulas are wrong. The main problem was the constant $2\pi$ omitted in the formula of Proposition 8. All results were derived using this proposition. We thank Makoto Minamide for pointing out these mistakes. Here, we formulate our statements with corrected formulas and indicate other inaccuracies.

**Theorem 1.**

$$N_1(T) = \frac{\operatorname{area}(\Gamma\backslash H)}{4\pi}T^2 - \frac{\log N(P_{00})}{2\pi}T + o(T).$$

Formula (2.1) should be

$$N_0(T) = \frac{\operatorname{area}(\Gamma\backslash H)}{4\pi}T^2 + O\left(\frac{T}{\log T}\right).$$

**Theorem 2.**

$$\sum_{1<\gamma\le T}\left(\beta - \frac{1}{2}\right) = \frac{T}{2\pi}\log T + \frac{T}{2\pi}\log\frac{\operatorname{area}(\Gamma\backslash H)\sqrt{N(P_{00})}}{em_0\Lambda(P_{00})} + o(T).$$

**Theorem 4.**

$$\#\{t_j : 0 < t_j \le T\} = N_1\left(\frac{1}{2}, T\right) + \frac{T}{2\pi}\log N(P_{00}) + o(T).$$

*Here ordinates $t_j$ are counted without multiplicities.*

**Corollary 5.** *For any fixed* $\varepsilon > 0$,

$$\#\left\{\beta + i\gamma : Z'(\beta + i\gamma) = 0, \frac{1}{2} \le \beta < \frac{1}{2} + \frac{\log^{1+\varepsilon} T}{T}, 0 < \gamma \le T\right\} \sim N_1(T).$$

**Lemma 6.** *For* $t > 0$,

$$f(s) = \exp\left(2\pi i(g-1)\left(s - \frac{1}{2}\right)^2 + \frac{\pi i(g-1)}{6} + O\left(\frac{\sigma^2 + |\sigma t|}{e^{2\pi t}}\right)\right)$$

*uniformly in* $\sigma$.

Proof of Lemma 6 contains the obvious arithmetical mistake.

**Lemma 7.** *Let* $\varepsilon > 0$ *and* $\sigma_0 > 1$. *Then*

$$Z'(\sigma + iT) \ll \begin{cases} \exp(\varepsilon T) & \textit{for} \quad \frac{1}{2} \le \sigma \le \sigma_0, \\ \exp\left(\operatorname{area}(\Gamma\backslash H)\left(\frac{1}{2} - \sigma + \varepsilon\right) T\right) & \textit{for} \quad \sigma \le \frac{1}{2}. \end{cases}$$

In the proof of Lemma 7 the formula

$$\text{“}Z(\sigma + iT) \ll \exp\left(\left(\frac{1}{2} - \sigma + \varepsilon\right) T\right) \quad \text{for} \quad \sigma \le \frac{1}{2}\text{”}$$

should be changed to

$$\text{“}Z(\sigma + iT) \ll \exp\left(\left(\frac{1}{2} - \sigma + \varepsilon\right) \operatorname{area}(\Gamma\backslash H) T\right) \quad \text{for} \quad \sigma \le \frac{1}{2}\text{”}.$$

**Proposition 8.** *Let* $0 < a < 1/2$ *and* $2 < t_0 < 3$. *Then*

$$2\pi \sum_{\substack{\beta > -a \\ t_0 < \gamma \le T}} (\beta - a) = \left(\frac{1}{2} - a\right) \frac{\operatorname{area}(\Gamma\backslash H)}{2} T^2 + T\log T - T(\log(m_0 \Lambda(P_{00})) - a\log N(P_{00})) + T(\log(\operatorname{area}(\Gamma\backslash H)) - 1) + o(T).$$

In the proof of Proposition 8 formula (3.3) should be

$$I_1 = \left(\frac{1}{2} - a\right) \frac{\operatorname{area}(\Gamma\backslash H)}{2} T^2 + T\log T - T(\log(m_0 \Lambda(P_{00})) - a\log N(P_{00})) + T\left(\log \operatorname{area}(\Gamma\backslash H) - 1\right) + \int_{t_0}^{T'} \log|Z(1 - a - it)|dt + O(\log T).$$

**Proof of Theorem 2.** Remembering that $Z'(s)$ can have only finitely many nontrivial zeros in $\sigma < 1/2$, from Proposition 8 and Theorem 1 we obtain

$$\sum_{1<\gamma<T}\left(\beta-\frac{1}{2}\right)=\sum_{1<\gamma<T}(\beta-a)-\left(\frac{1}{2}-a\right)\sum_{1<\gamma<T}1$$
$$=\left(\frac{1}{2}-a\right)\frac{\operatorname{area}(\Gamma\backslash H)}{4\pi}T^2+\frac{T}{2\pi}\log T-\frac{T}{2\pi}(\log(m_0\Lambda(P_{00}))$$
$$-a\log N(P_{00}))+\frac{T}{2\pi}(\log(\operatorname{area}(\Gamma\backslash H))-1)$$
$$-\left(\frac{1}{2}-a\right)\left(\frac{\operatorname{area}(\Gamma\backslash H)}{4\pi}T^2-\frac{T}{2\pi}\log N(P_{00})\right)+o(T).$$

**Proof of Corollary 5.** Function $Z'(s)$ has only finitely many nontrivial zeros in $\sigma < 1/2$. Using Proposition 8 we see, that

$$\sum_{\substack{1<\gamma\le T\\ \beta\ge\frac{1}{2}+\frac{\log^{1+\varepsilon}T}{T}}}1\le\sum_{\substack{1<\gamma\le T\\ \beta\ge\frac{1}{2}+\frac{\log^{1+\varepsilon}T}{T}}}\frac{\beta-\frac{1}{2}}{\frac{\log^{1+\varepsilon}T}{T}}\ll\frac{T^2}{\log^{\varepsilon}T}=o(N_1(T)).$$

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