Correction to: The model companion of the class of pseudocomplemented semilattices is finitely axiomatizable

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In the paper “The model companion of the class of pseudocomplemented semilattices is finitely axiomatizable” a list of nine axioms, which are claimed to axiomatize the class of existentially closed pseudocomplemented semilattices, is given. After its publication an error in a key lemma was found by the reviewer of a subsequent paper. The error was rectified by strengthening one of these nine axioms and then proving that the resulting nine axioms do axiomatize this class, thereby proving the statement of the title. Here we explain the error, outline the correction and refer the reader to arXiv.org for a full revision of the published paper.

1. The error and its implication

In [1, Lemma 5.3] it is claimed that any of the axioms (AC1)–(AC4) and (EC1)–(EC5) holds in $\prod_{i \in I} \mathbf{P}_i$ if and only if it holds in every $\mathbf{P}_i$, where $(\mathbf{P}_i)_{i \in I}$ is a family of p-semilattices. This is not the case since e.g. (EC3) holds in $\mathbf{2} \times \widehat{\mathbf{A}}$ but does not hold in $\mathbf{2}$.

Therefore, the proof of the claim of the main theorem of the published paper, [1, Theorem 5.9], has to be carried out without using Lemma 5.3. Without Lemma 5.3 the subalgebra $\mathbf{S}$, as defined in the first bullet of [1, Section 5], cannot be assumed to be isomorphic to a subdirectly irreducible p-semilattice $\mathbf{2}$ or $\widehat{\mathbf{F}}_t$, $t \geq 1$. It may still be assumed to be isomorphic to a direct product $\mathbf{2}^u \times \prod_{i=1}^p \widehat{\mathbf{F}}_{f(i)}$ of such p-semilattices as is shown in the third paragraph of

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the sufficiency part of the proof of [1, Theorem 5.9]. In the revised version of the paper this statement is proved in an independent lemma, [3, Lemma 5.1], using [3, Lemma 4.4], which is added to the revised version of the paper.

As stated in [3, Lemma 5.1] of the revised paper we have the following situation: $\mathbf{S} \leq \mathbf{P} \leq \mathbf{Q}$, $\mathbf{S} \leq \mathbf{T} \leq \mathbf{Q}$,

$$\mathbf{S} \cong 2^r \times \prod_{i=1}^{s_1} \mathbf{F}_{f_1(i)} \times \prod_{i=1}^{s_2} \widehat{\mathbf{F}_{f_2(i)}} \quad (r, s_1, s_2 \in \mathbb{N}), \quad (1.1)$$

$$\mathbf{T} \cong 2^{r'} \times \prod_{i=1}^{s'} \widehat{\mathbf{F}_{g(i)}} \quad (r', s', g(i) \in \mathbb{N}), \quad (1.2)$$

where $\mathbf{S} \leq \mathbf{T}$ implies the numerical relations $r \leq r'$, $s_1 + s_2 \leq s'$, $1 \leq f_1(i) \leq g(i)$ ($1 \leq i \leq s_1$), $1 \leq f_2(i) \leq g(s_1 + i)$, ($1 \leq i \leq s_2$) and $1 \leq g(i)$ ($s_1 + s_2 + 1 \leq i \leq s'$).

2. The remedy

The subalgebra $\mathbf{S}$ not being subdirectly irreducible but a direct product as in (1.1) is dealt with by adding two lemmas, one semantic, the other syntactic. On the semantic side the following lemma must be added.

Lemma 5.2. *If $\mathbf{T} = 2^r \times \prod_{i=1}^q \widehat{\mathbf{F}_{g(i)}}$ with $1 \leq g(i)$ for $1 \leq i \leq q$, and $\mathbf{S} \leq \mathbf{T}$ such that $\mathbf{S} \cong 2^r \times \prod_{i=1}^p \mathbf{F}_{g(i)} \times \prod_{i=p+1}^q \widehat{\mathbf{F}_{g(i)}}$, $p \leq q$, then there is a sequence of subalgebras $\mathbf{T}_0, \dots, \mathbf{T}_p$ of $\mathbf{T}$ satisfying*

- $\mathbf{T}_0 = \mathbf{S}$,
- $\mathbf{T}_k \leq \mathbf{T}_{k+1}$ for $k = 0, \dots, p-1$,
- $\mathbf{T}_k \cong 2^r \times \prod_{i=1}^k \widehat{\mathbf{F}_{g(i)}} \times \prod_{i=k+1}^p \mathbf{F}_{g(i)} \times \prod_{i=p+1}^q \widehat{\mathbf{F}_{g(i)}}$, $0 \leq k \leq p$,
- $\mathbf{T}_p = \mathbf{T}$.

The second lemma to add is the syntactic counterpart of Lemma 5.2.

Lemma 5.8. *Let $\mathbf{P}$ and $\mathbf{Q}$ be p -semilattices with $\mathbf{Q}$ being an extension of $\mathbf{P}$, let $\mathbf{S} \cong 2^r \times \prod_{i=1}^p \mathbf{F}_{g(i)} \times \prod_{i=p+1}^q \widehat{\mathbf{F}_{g(i)}}$ be a finite subalgebra of $\mathbf{P}$ with $0 < p < q$, $g(i) \geq 1$ for $1 \leq i \leq q$. Furthermore, we assume that $\mathbf{T} \cong 2^r \times \widehat{\mathbf{F}_{g(1)}} \times \prod_{i=2}^p \mathbf{F}_{g(i)} \times \prod_{i=p+1}^q \widehat{\mathbf{F}_{g(i)}}$ is a finite subalgebra of $\mathbf{Q}$ that is an extension of $\mathbf{S}$. If $\mathbf{P}$ satisfies (AC1)–(AC4) and (EC1)–(EC5), then there is an extension $\mathbf{S}'$ of $\mathbf{S}$ in $\mathbf{P}$ satisfying $\mathbf{S}' \cong_{\mathbf{S}} \mathbf{T}$.*

To prove Lemma 5.8, a strengthened version of Axiom (EC3) is adopted. Not only is the existence of a proper dense element warranted, but for any skeletal element $b < 1$ the existence of a proper dense element d satisfying $b < d$ and $b^* \not\leq d$ is required. Thus, axiom (EC3) is now the sentence

$$(\forall b)(b < 1 \longrightarrow (\exists z)(b < d \ \& \ b^* \not\leq d)).$$

Furthermore, [1, Lemma 5.4] (corresponding to [3, Lemma 5.3] in the revised paper) and [1, Lemma 5.5] (corresponding to [3, Lemma 5.4] in the revised paper) have to be adapted:

- **Lemma 5.4:** In the published paper the chain of extensions starts with a subalgebra $\mathbf{S}$ of $\mathbf{T} \cong \mathbf{2}^r \times \prod_{i=1}^q \widehat{\mathbf{F}_{f(i)}}$ satisfying $\mathbf{S} \cong \widehat{\mathbf{F}_\ell}$, $0 \leq \ell \leq \max\{f(i) : 1 \leq i \leq q\}$. Now we have $\mathbf{S} \cong \mathbf{2}^r \times \prod_{i=1}^p \widehat{\mathbf{F}_{f(i)}}$, $p < q$.
- **Lemma 5.5:** In the published paper the chain of extensions starts with a subalgebra $\mathbf{S}$ of $\mathbf{T} \cong \mathbf{2}^{r'} \times \prod_{i=1}^q \widehat{\mathbf{F}_{f(i)}}$ satisfying $\mathbf{S} \cong \prod_{i=1}^q \widehat{\mathbf{F}_{f(i)}}$. The chain of extensions $\mathbf{T}_0, \dots, \mathbf{T}_{r'}$ can canonically be obtained by adjoining an explicit sequence of Boolean elements of $\mathbf{T}$. Without [1, Lemma 5.3] the chain to consider starts with $\mathbf{S} \cong \mathbf{2}^r \times \prod_{i=1}^q \widehat{\mathbf{F}_{f(i)}}$, $r < r'$. The chain is obtained by splitting Boolean atoms of $\mathbf{S}$ that are not an atom of $\mathbf{T}$.

The syntactic lemmas [1, Lemma 5.6], [1, Lemma 5.7] and [1, Lemma 5.8] (corresponding to lemmas [3, Lemma 5.8], [3, Lemma 5.10] and [3, Lemma 5.9], respectively) need not be adapted. They still prove that the extension steps leading from a subalgebra $\mathbf{S}$ of $\mathbf{T}$ in an extension $\mathbf{Q}$ of $\mathbf{P}$ as described in [3, Lemma 5.3], [3, Lemma 5.4] and [3, Theorem 5.12] can be carried out in $\mathbf{P}$ if $\mathbf{P}$ satisfies (AC1)–(AC4), (EC1)–(EC5). Axiom (EC3) is not used in the proofs of these lemmas.

Acknowledgements

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References

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