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Erratum

Erratum to "1-affine completeness of compatible modules"

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Abstract. We correct a theorem in the original paper concerning when a composition series of a compatible nearring module is a 1-affine complete chain.

The statement of [2, Theorem 4.1] is not correct because, contrary to item (2) of this theorem, a composition series which is a 1-affine complete chain may have factors of order 2 that are not R-isomorphic. Nevertheless, such factors are indeed R-isomorphic if the nearring R is an automorphism nearring of G such as $I(G)$ since the automorphisms generating R must act as the identity map on these factors. To see, however, that this does not hold in general if the automorphism nearring assumption is dropped, let G be the additive group of $\mathbb{Z}_4$ and $R = C_0(G)$. Then $G > \langle 2\rangle > 0$ is a 1-affine complete chain with $G/\langle 2\rangle$ and $\langle 2\rangle$ coprime by [2, Theorem 3.5] since $\langle 2\rangle$ is not contained in the sum of its maximal complements. A correct statement of the theorem is obtained by rewriting it in the following manner; the proof of this corrected result is obtained by making some modifications of the original one.

Theorem 4.1. *Suppose $G = G_0 > G_1 > \cdots > G_n = 0$ is a composition series of R-ideals of a faithful compatible R-module G of a nearring R with dccr. This series is a 1-affine complete chain if and only if $|G_i/G_{i+1}| = 2$ whenever G_i/G_{i+1} is a ring module and the following are satisfied for this factor:*

(1) *If G_i/G_{i+1} is coprime to G/G_i, then G_i/G_{i+1} is almost self-centralizing in G/G_{i+1}.*
(2) *If G_i/G_{i+1} is not coprime to G/G_i, then G_i/G_{i+1} is both contained in the sum of its maximal complements in G/G_{i+1} and a shift complete $R/\operatorname{Ann}_R(G/G_{i+1})$-ideal of G/G_{i+1}.*

Further, when G has a series satisfying these conditions, G and R are finite.

Proof. If all factors of the composition series that are ring modules have order 2, then G and hence R are finite, since minimal R-modules that are nonring modules are always finite. If the series is a 1-affine complete chain and G_i/G_{i+1} is a ring module coprime to G/G_i, then [2, Theorem 3.3] gives us $|G_i/G_{i+1}| = 2$

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and (1). If G_i/G_{i+1} is a ring module not coprime to G/G_i, then $|G_i/G_{i+1}| = 2$, since it will be R-isomorphic to some factor G_k/G_{k+1} with $k < i$ that is a ring module coprime to G/G_k and (2) follows from [2, Theorem 3.5] and [1, Proposition 5.2] or the note in the second sentence after [2, Theorem 3.5]. For the converse, use induction on n. As

$$\begin{aligned} R/\operatorname{Ann}_R(G/G_{n-1}) &\leq C_0(G)/\operatorname{Ann}_{C_0(G)}(G/G_{n-1}) \leq C_0(G/G_{n-1}) \\ &= R/\operatorname{Ann}_R(G/G_{n-1}) \end{aligned}$$

where the last equality holds by the induction hypothesis, we must have

$$R/\operatorname{Ann}_R(G/G_{n-1}) = C_0(G)/\operatorname{Ann}_{C_0(G)}(G/G_{n-1}). \tag{1}$$

If G_{n-1} is a nonring module, [2, Propositions 2.1 and 3.2] give us

$$|\operatorname{Ann}_R(G/G_{n-1})| = |\operatorname{Ann}_{C_0(G)}(G/G_{n-1})|.$$

This likewise holds if G_{n-1} is a ring module and G/G_{n-1} and G_{n-1} are coprime by [2, Propositions 2.2 and 3.2]. Finally, if G/G_{n-1} and G_{n-1} are not coprime, then G/G_{n-1} and G_{n-1} also are not coprime as $C_0(G)$-modules by [1, Lemma 2.1], since G_{n-1} does not have a unique maximal complement. Now, due to shift completeness, [2, Proposition 2.3] gives us $|\operatorname{Ann}_R(G/G_{n-1})| = |\operatorname{Ann}_{C_0(G)}(G/G_{n-1})|$ in this case as well. Thus, from equation (1), we have $|R| = |C_0(G)|$, and hence $R = C_0(G)$, completing our proof. $\square$

No changes are necessary in the discussion following [2, Theorem 4.1].

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