## Aequationes Mathematicae



## Correction to: One-sided invertibility of discrete operators with bounded coefficients

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## 6. Addendum

By the sufficiency in Theorem 3.12, the left (resp., right) invertibility of the operators  $A \in \{A_E, \tilde{A}_E, A_{E_m}, \tilde{A}_{E_m}\}$  on the spaces  $l^p$   $(1 follows from the invertibility of the operator <math>A^{\diamond}A$  (resp.,  $AA^{\diamond}$ ). The Fredholmness of such operators A is not necessary for their one-sided invertibility (for example, the right invertible operator  $A_{E_m} = E_m$  is not Fredholm for |m| > 1). On the other hand, the invertibility criterion for the operators  $B \in \{A^{\diamond}A, AA^{\diamond}\}$  presented in Theorem 3.15 is also valid if  $B \in \mathcal{W}_p$  and  $A \notin \mathcal{W}_p$ .

As a result, Theorems 4.4, 4.5, 5.4, 5.5 can be essentially reinforced by excluding the Fredholm condition for the operators  $A_E, \tilde{A}_E, A_{E_m}, \tilde{A}_{E_m}$ , respectively, and applying the invertibility criterion from Theorem 3.15.

The modified theorems have the following form.

**Theorem 6.1.** The operator  $A_E$  given by (4.2) and satisfying (4.3) is left invertible on the space  $l^p$  for  $p \in (1, \infty)$  if for  $B = A_E^{\diamond}A_E$  and all sufficiently large  $n \in \mathbb{N}$  the operators  $B_n^{\pm} = P_n^{\pm}BP_n^{\pm}$  are invertible on the spaces  $P_n^{\pm}l^p$ , respectively, and the  $(2n-1) \times (2n-1)$  matrix  $B_{n,0}$  defined for  $B = A_E^{\diamond}A_E$  by (3.33) is invertible. The operator  $A_E$  given by (4.2) and satisfying (4.3) and condition (A) is right invertible on the space  $l^p$  for  $p \in (1, \infty)$  if for  $B = A_E A_E^{\diamond}$  and all sufficiently large  $n \in \mathbb{N}$  the operators  $B_n^{\pm} = P_n^{\pm}BP_n^{\pm}$  are invertible on the space  $P_n^{\pm}l^p$ , respectively, and the  $(2n-1) \times (2n-1)$  matrix  $B_{n,0}$  defined

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for  $B = A_E A_E^{\diamond}$  by (3.33) is invertible. Under these conditions, one of the left (resp., right) inverses of the operator  $A_E$  is given by  $A_E^L = (A_E^{\diamond} A_E)^{-1} A_E^{\diamond}$  (resp., by  $A_E^R = A_E^{\diamond} (A_E A_E^{\diamond})^{-1}$ ).

**Theorem 6.2.** The operator  $\widetilde{A}_E$  given by (4.2) and satisfying (4.3) is left invertible on the space  $l^p$  for  $p \in (1, \infty)$  if for  $B = \widetilde{A}_E^{\diamond} \widetilde{A}_E$  and all sufficiently large  $n \in \mathbb{N}$  the operators  $B_n^{\pm} = P_n^{\pm} B P_n^{\pm}$  are invertible on the spaces  $P_n^{\pm} l^p$ , respectively, and the  $(2n-1) \times (2n-1)$  matrix  $B_{n,0}$  defined for  $B = \widetilde{A}_E^{\diamond} \widetilde{A}_E$  by (3.33) is invertible. The operator  $\widetilde{A}_E$  given by (4.2) and satisfying (4.3) and condition (B) is right invertible on the space  $l^p$  for  $p \in (1,\infty)$  if E is a permutation operator and for  $B = \widetilde{A}_E \widetilde{A}_E^{\diamond}$  and for all sufficiently large  $n \in \mathbb{N}$  the operators  $B_n^{\pm} = P_n^{\pm} B P_n^{\pm}$  are invertible on the spaces  $P_n^{\pm} l^p$ , respectively, and the  $(2n-1) \times (2n-1)$  matrix  $B_{n,0}$  defined for  $B = \widetilde{A}_E \widetilde{A}_E^{\diamond}$  by (3.33) is invertible. Under these conditions, one of the left (resp., right) inverses of the operator  $\widetilde{A}_E$  is given by  $\widetilde{A}_E^L = (\widetilde{A}_E^{\diamond} \widetilde{A}_E)^{-1} \widetilde{A}_E^{\diamond}$  (resp., by  $\widetilde{A}_E^R = \widetilde{A}_E^{\diamond} (\widetilde{A}_E \widetilde{A}_E^{\diamond})^{-1}$ ).

**Theorem 6.3.** Let  $p \in (1, \infty)$  and  $m \in \mathbb{Z} \setminus \{0\}$ . Then the slant-dominated discrete Wiener-type operator  $A_{E_m}$  is left (resp., right) invertible on the space  $l^p$  if for  $B = A_{E_m}^{\diamond} A_{E_m}$  (resp., for  $B = A_{E_m} A_{E_m}^{\diamond}$ ) and for all sufficiently large  $n \in \mathbb{N}$  the operators  $B_n^{\pm} = P_n^{\pm} B P_n^{\pm}$  are invertible on the spaces  $P_n^{\pm} l^p$ , respectively, and the  $(2n-1) \times (2n-1)$  matrix  $B_{n,0}$  given by (3.33) for B = $A_{E_m}^{\diamond} A_{E_m}$  (resp., for  $B = A_{E_m} A_{E_m}^{\diamond}$ ) is invertible. Under these conditions, one of the left (resp., right) inverses of the operator  $A_{E_m}$  is given by  $A_{E_m}^L =$  $(A_{E_m}^{\diamond} A_{E_m})^{-1} A_{E_m}^{\diamond}$  (resp., by  $A_{E_m}^R = A_{E_m}^{\diamond} (A_{E_m} A_{E_m}^{\diamond})^{-1}$ ).

**Theorem 6.4.** Let  $p \in (1,\infty)$  and  $m \in \mathbb{Z} \setminus \{0\}$ . Then the slant-dominated discrete Wiener-type operator  $\widetilde{A}_{E_m}$  is left (resp., right) invertible on the space  $l^p$  if for  $B = \widetilde{A}_{E_m}^{\diamond} \widetilde{A}_{E_m}$  (resp., for  $B = \widetilde{A}_{E_m} \widetilde{A}_{E_m}^{\diamond}$ ) and for all sufficiently large  $n \in \mathbb{N}$  the operators  $B_n^{\pm} = P_n^{\pm} B P_n^{\pm}$  are invertible on the spaces  $P_n^{\pm} l^p$ , respectively, and the  $(2n-1) \times (2n-1)$  matrix  $B_{n,0}$  given by (3.33) for  $B = \widetilde{A}_{E_m}^{\diamond} \widetilde{A}_{E_m}$  (resp., for  $B = \widetilde{A}_{E_m} \widetilde{A}_{E_m}^{\diamond}$ ) is invertible, where |m| = 1 in the case of right invertibility. Under these conditions, one of the left (resp., right) inverses of the operator  $\widetilde{A}_{E_m}$  is given by  $\widetilde{A}_{E_m}^L = (\widetilde{A}_{E_m}^{\diamond} \widetilde{A}_{E_m})^{-1} \widetilde{A}_{E_m}^{\diamond}$  (resp., by  $\widetilde{A}_{E_m}^R = \widetilde{A}_{E_m}^{\diamond} (\widetilde{A}_{E_m} \widetilde{A}_{E_m}^{\diamond})^{-1}$ ).

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