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Aequationes Mathematicae

Errata

Erratum to: "General solution of the 2-cocycle functional equation on solvable groups" [Aequationes Math. *73* (2007), 260–279]

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Corrections are needed for Theorem 5 and Corollary 9 in [1].

First, in Theorem 5 an additional condition is required concerning the subgroups of G. We must suppose that

$$K_i^{K_r \cdots K_{i+1}} \subseteq K_i \text{ for each } i = 1, \ldots, r-1. \tag{1}$$

This condition is necessary to guarantee that the first argument in each ψ_i term of the solution (17) in [1] belongs to the appropriate subgroup K_i. The condition (1) also ensures that the product $K_r \cdots K_{j+1}$ is for each $j = 1, \ldots, r-1$ a subgroup of G. Thus equations (18), (19), (20) in [1] make sense. It also follows that $K_j \cdots K_1$ is a normal subgroup of G for each $j = 1, \ldots, r-1$. Therefore $G \trianglerighteq G_{r-1} \trianglerighteq \cdots G_1 \trianglerighteq \{1\}$ is an invariant normal series for G, where $G_j := K_j \cdots K_1$ for each $j = 1, \ldots, r-1$, showing that G is solvable. Now a correct statement of Theorem 5 is as follows.

Theorem 5. *Let G be a group with abelian subgroups $K_1, \ldots, K_r$ satisfying condition* (1) *and such that each element $g \in G$ can be written uniquely as*

$$g = k_r \cdots k_1 \text{ for } k_j \in K_j \ (j = 1, \ldots, r).$$

Let V be a divisible abelian group with no 2-torsion. Then a map $F : G \times G \to V$ is a cocycle if and only if there exist a map $f : G \to V$, skew-symmetric bi-morphisms $\psi_i : K_i \times K_i \to V$ for $i = 1, \ldots, r$, and maps $\phi_j : K_j \times (K_r \cdots K_{j+1}) \to V$ for $j = 1, \ldots, r-1$ such that equations (17), (18), (19), *and* (20) *in* [1] *hold.*

Second, one term is missing from the general solution in Corollary 9. The correct statement of Corollary 9 requires the addition of a term $C(yw + (xy - z)v)$, where $C : R \to V$ is additive, in the general solution of the cocycle equation on the Heisenberg group over R. (It also should have been stated that R is commutative with 1.) The error occurs in the proof after M is decomposed into the sum $M(y, z, v, w) = \Psi_1(y, v) + B_2(y, w) + B_3(z, v) + B_4(z, w)$ of four bi-additive maps.

Since ψ_1 is a skew-symmetric bi-morphism, it follows that the maps Ψ_1, B_4 must be skew-symmetric, but this is not necessarily the case for the maps B_2, B_3. Rather they must satisfy $B_2(v,z)+B_3(z,v)=0$. Continuing from the equation $B_2(y,tv)+B_3(ty,v)=0$, putting $y=1$ there yields $B_3(t,u)=-B_2(1,tu)=:-C(tu)$ for some additive mapping $C:R\to V$. Thus $B_4=0$, $B_2(t,u)=-B_3(u,t)=C(tu)$, and the ψ_1 term boils down to $\psi_1((0,y,z),(0,v,w))=\Psi_1(y,v)+C(yw-zv)$. Hence $\psi_1(k_1^{l_2},l_1)=\psi_1((0,y,z-(x+u)y),(0,v,w-uv))=\Psi_1(y,v)+C(yw+(xy-z)v)$. The rest of the proof goes exactly as before.

Reference

[1] B. Ebanks, *General solution of the 2-cocycle functional equation on solvable groups*, Aequationes Math. *73* (2007), 260–279.

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