Aequationes Math. 76 (2008) 317–320
0001-9054/08/030317-4
DOI 10.1007/s00010-008-2938-6

Aequationes Mathematicae

Erratum to: "Special solutions of a general class of iterative functional equations" [Aequationes Math. *72* (2006), 269–287]

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The derivative of $T\phi(x)$ (with respect to x) given in equation (3.21) in our paper [1], is incorrect as it ignores the contribution of the term $\frac{\partial}{\partial x}\left[\phi_x^{-1}(y)\right]_{y=F(x)}$. This error has already crept in the earlier papers [2] and [3] by Si and Wang. However, our theorems are valid under additional assumptions. The following hypotheses are to be incorporated in Theorem 3.1 of [1]:

H-5: $|H_{ix}(x, y_1, \ldots, y_{n_i})| \leq L_i$ and $|H_{ix}(x, y_1, \ldots, y_{n_i}) - H_{iy}(y, y_1, \ldots, y_{n_i})| \leq L'_i|x-y|$, where $L_i,\ L'_i$ are nonnegative numbers for $i \in \mathbb{N}$.

H-6: $|H_{ix}(x, y_1, \ldots, y_{n_i}) - H_{ix}(x, \tilde{y_1}, \ldots, \tilde{y_{n_i}})| \leq \sum_{j=1}^{n_i} L'_{ij}|y_j - \tilde{y_j}|$, where L'_{ij} are nonnegative numbers for $i \in \mathbb{N}$, $j = 1, 2, \ldots, n_i$.

The corrected version of the main theorem now reads as:

Theorem 3.1*. *Let (λ_i) and (n_i) be sequences of nonnegative numbers and natural numbers respectively and $a_{ij} \in \mathbb{N}$ for $i \in \mathbb{N}$ and $j = 1, 2, ..., n_i$ with $\sum_{i=1}^{\infty} \lambda_i = 1$, $\lambda_1 > 0$ and $a_{11} = 1$. Let H_i be functions satisfying conditions* H-1 *to* H-6 *with $\eta,\ \delta > 0$ and nonnegative numbers $L_i,\ L'_i,\ L_{ij},\ L'_{ij},\ N_{ijs}$ and P_{ij} for $i \in \mathbb{N}$, $j = 1, 2, \ldots, n_i$ and $s = 1, 2, \ldots, n_i$. Assume further that*

(i) $M > 1$,

(ii) $L_0 = \sum_{i=1}^{\infty} \lambda_i L_i < \infty$,

(iii) $S_1 = \sum_{i=1}^{\infty} \sum_{j=1}^{n_i} \sum_{s=1}^{n_i} \lambda_i N_{ijs} M^{a_{ij}+a_{is}-2} < \infty$,

(iv) $S_2 = \sum_{i=1}^{\infty} \sum_{j=1}^{n_i} \sum_{k=a_{ij-2}}^{2(a_{ij}-2)} \lambda_i L_{ij} M^k < \infty$,

(v) $S_3 = \sum_{i=1}^{\infty} \lambda_i L'_i < \infty$,

(vi) $K_3 = \sum_{i=1}^{\infty} \sum_{j=1}^{n_i} \lambda_i P_{ij} M^{a_{ij}-1} < \infty$,

(vii) $K_7 = \sum_{i=1}^{\infty} \sum_{j=1}^{n_i} \sum_{s=1}^{a_{ij}-1} \lambda_i L'_{ij} M^{s-1} < \infty$

and

(viii) $K_0 > M^2 S_2$ *and* $\delta \geq \frac{K_1 L_0}{K_0}$ *where* $K_0 = \lambda_1 \eta$ *and* $K_1 = \sum_{i=1}^{\infty} \sum_{j=1}^{n_i} \lambda_i L_{ij} M^{a_{ij}-1}$.

Then for any given $F \in \mathcal{F}_\delta^1(I, K_0M - L_0, M^*)$*, the functional equation*

$$\sum_{i=1}^{\infty} \lambda_i H_i\big(x, f^{a_{i1}}(x), \ldots, f^{a_{in_i}}(x)\big) = F(x)$$

has a solution f *in* $\mathcal{R}^1(I, M, M')$*, where* $M' \geq \frac{M^*K_0 + M^2K_0S_1 + K_1S_3 + M(K_1S_4 + K_0K_3)}{K_0(K_0 - M^2S_2)}$ *and* $S_4 = \sum_{i=1}^{\infty}\sum_{j=1}^{n_i} \lambda_i L'_{ij} M^{a_{ij}-1}$.

Theorem 3.1* is proved using Lemma 3.2 together with the following corrected versions of Lemmas 3.3 and 3.4, viz. Lemma 3.3* and Lemma 3.4* respectively. It may be pointed out that Lemma 3.2 is valid under the hypotheses of Theorem 3.1*.

Lemma 3.3*. *Under the assumptions of Theorem 3.1*, for each* ϕ *in* $\mathcal{R}^1(I, M, M')$ *and* x, t *in* I *the map* $\phi_x(t)$ *from* I^2 *into* $\mathbb{R}$ *given by*

$$\phi_x(t) = \sum_{i=1}^{\infty} \lambda_i H_i\big(x, \phi^{a_{i_1}-1}(t), \ldots, \phi^{a_{in_i}-1}(t)\big)$$

is well-defined and satisfies the following:

(i) $\left|\frac{\partial}{\partial s}[{\phi_x}^{-1}(s)] - \frac{\partial}{\partial t}[{\phi_y}^{-1}(t)]\right| \leq \left(\frac{K_2L_0}{{K_0}^3} + \frac{K_3}{{K_0}^2}\right)|x-y| + \frac{K_2}{{K_0}^3}|s-t|$,

(ii) $\left|\frac{\partial}{\partial x}[{\phi_x}^{-1}(s)] - \frac{\partial}{\partial y}[{\phi_y}^{-1}(t)]\right| \leq \left(\frac{K_1S_3}{{K_0}^2} + \frac{K_1S_4L_0}{{K_0}^3} + \frac{L_0K_3}{{K_0}^2} + \frac{{L_0}^2K_2}{{K_0}^3}\right)|x-y| + \left(\frac{K_1S_4 + L_0K_2}{{K_0}^3}\right)|s-t|$,

where $L_0, K_0, K_1, K_3, S_1, S_2, S_3$ *and* S_4 *are as given in Theorem 3.1* and* $K_2 = S_1 + M'S_2$.

Lemma 3.4*. *Let* $g, h \in \mathcal{R}^1(I, M, M')$*. Under the assumptions of Theorem 3.1*, for* $x, y, s, t \in I$*, we have*

(i) $\|g_x - h_x\| \leq K_4\|g - h\|$,

(ii) $\left|\frac{\partial}{\partial s}[g_x(s)] - \frac{\partial}{\partial t}[h_x(t)]\right| \leq K_5\|g-h\| + K_6\|g'-h'\| + K_2|s-t|$,

(iii) $\left|\frac{\partial}{\partial x}[g_x(s)] - \frac{\partial}{\partial x}[h_x(t)]\right| \leq K_7\|g-h\| + S_4|s-t|$,

(iv) $\|g_x^{-1} - h_x^{-1}\| \leq \frac{K_4}{K_0}\|g-h\|$,

(v) $\left|\frac{\partial}{\partial s}[g_x^{-1}(s)] - \frac{\partial}{\partial t}[h_x^{-1}(t)]\right| \leq \left(\frac{K_5}{K_0^2} + \frac{K_2K_4}{K_0^3}\right)\|g-h\| + \frac{K_6}{K_0^2}\|g'-h'\| + \frac{K_2}{K_0^3}|s-t|$,

(vi) $\left|\frac{\partial}{\partial x}[g_x^{-1}(s)] - \frac{\partial}{\partial x}[h_x^{-1}(t)]\right| \leq \left(\frac{K_1K_7}{K_0^2} + \frac{S_4K_4}{K_0^3} + \frac{K_5L_0}{K_0^2} + \frac{K_2K_4L_0}{K_0^3}\right)\|g-h\| + \frac{L_0K_6}{K_0^2}\|g'-h'\| + \frac{1}{K_0^3}\{K_1S_4 + L_0K_2\}|s-t|$,

where $L_0, K_0, K_1, K_4, K_5, K_6, K_7, S_1, S_2, S_3$ *and* S_4 *are as in Theorem 3.1* and Lemma 3.2 and* $K_2 = S_1 + M'S_2$.

Theorem 3.5, Corollary 3.6 and Theorem 3.8 also require correction and their corrected versions are Theorem 3.5*, Corollary 3.6* and Theorem 3.8* respectively and are stated below. However, Corollary 3.7 requires no correction.

Theorem 3.5*. *In addition to the hypotheses of Theorem 3.1***, we suppose that the number* $\rho = \max\left\{\frac{K_4}{K_0} + \frac{K_1K_7}{K_0^2} + \frac{S_4K_4}{K_0^3} + \frac{MK_2K_4}{K_0^2} + \frac{MK_5}{K_0}, \frac{MK_6}{K_0}\right\}$ *is less than* 1*, where* $K_2 = S_1 + M'S_2$ *and* $K_0, K_1, K_4, K_5, K_6, K_7, S_1, S_2$ *and* S_4 *are as defined in Theorem 3.1** *and Lemma 3.2.*

Then for any $F \in \mathcal{F}_\delta^1(I, K_0M - L_0, M^*)$*, there is a unique function* f *satisfying the functional equation*

$$\sum_{i=1}^{\infty} \lambda_i H_i\big(x, f^{a_{i1}}(x), \ldots, f^{a_{in_i}}(x)\big) = F(x)$$

in $\mathcal{R}^1(I, M, M')$*. Further the solution* f *of the equation in* $\mathcal{R}^1(I, M, M')$ *continuously depends on the given function* F *in* $\mathcal{R}^1(I, K_0M - L_0, M^*)$*.*

Corollary 3.6* (the valid version of Corollary 3.6) corrects the main theorem due to Si and Wang [2]. For the statement of this corollary, we need the following hypotheses on the function H.

H-1$'$: $H \equiv H(x, y_1, \ldots, y_k)$ is a continuously differentiable function on I^{k+1} such that $H(a, \ldots, a) = a,\ H(b, \ldots, b) = b$.

H-2$'$: $0 < l \le H_{y_1}(x, y_1, \ldots, y_k)$ and $0 \le H_{y_j}(x, y_1, \ldots, y_k) \le L_j$, where $l,\ L_j$ are nonnegative numbers for $j = 1, 2, \ldots, k$, and H_{y_j} is the partial derivative of H with respect to the variable y_j for each $j = 1, 2, \ldots, k$.

H-3$'$: $|H_{y_j}(x, y_1, \ldots, y_k) - H_{y_j}(x, \tilde{y_1}, \ldots, \tilde{y_k})| \le \sum_{s=1}^{k} N_{js}|y_s - \tilde{y_s}|$, where N_{js} are nonnegative numbers for $j = 1, 2, \ldots, k$ and $s = 1, 2, \ldots, k$.

H-4$'$: $|H_x(x, y_1, \ldots, y_k)| \le L$ and $|H_x(x, y_1, \ldots, y_k) - H_y(y, y_1, \ldots, y_k)| \le L'|x - y|$ where $L,\ L'$ are nonnegative numbers.

H-5$'$: $|H_x(x, y_1, \ldots, y_k) - H_x(x, \tilde{y_1}, \ldots, \tilde{y_k})| \le \sum_{i=1}^{n_i} L'_j|y_j - \tilde{y_j}|$ where L'_j are nonnegative numbers for $j = 1, 2, \ldots, k$.

H-6$'$: $|H_{y_j}(x, y_1, \ldots, y_k) - H_{y_j}(y, y_1, \ldots, y_k)| \le P_j|x - y|$ where P_j are nonnegative numbers for $j = 1, 2, \ldots, k$.

Corollary 3.6*. *Let* $n_1, n_2, \ldots, n_k$ *be natural numbers with* $n_1 = 1$*, and let* $H(x, y_1, y_2, \ldots, y_k)$ *be a real function defined on* I^{k+1} *satisfying hypotheses* H-1$'$ *to* H-6$'$ *with* $l > 0$ *and nonnegative numbers* L, L', L_j, L'_j, P_j *and* N_{js} *for* $j = 1, 2, \ldots, k$ *and* $s = 1, 2, \ldots, k$*. Suppose that* $M > 1$ *and* $l > M^2S_2$ *and* $\delta \ge LK_1/l$ *where* $S_2 = \sum_{j=1}^{k}\sum_{s=n_j-2}^{2(n_j-2)} L_jM^s$ *and* $K_1 = \sum_{j=1}^{k} L_jM^{n_j-1}$*.*

Then for any given $F \in \mathcal{F}_\delta^1(I, lM - L, M^*)$*, the functional equation*

$$H(x, \phi^{n_1}(x), \ldots, \phi^{n_k}(x)) = F(x)$$

has a solution ϕ *in* $\mathcal{R}^1(I, M, M')$ *where* $M' \ge \frac{K_1L' + l(M^* + M^2S_1) + M(K_1S_3 + lK_3)}{l(l - M^2S_2)}$*,* $S_1 = \sum_{j=1}^{k}\sum_{s=1}^{k} N_{js}M^{n_j+n_s-2}$*,* $S_3 = \sum_{j=1}^{k} L'_jM^{n_j-1}$ *and* $K_3 = \sum_{j=1}^{k} P_jM^{n_j-1}$*.*

Theorem 3.8*. *In addition to the hypotheses of Theorem 3.1*, we suppose that* $F \in \mathcal{F}^1_\delta(I, K_0M - L_0, M^*)$ *where* $\delta \geq (\mu + \frac{L_0}{K_0})K_1$ *for some* $0 < \delta < 1$. *Then there is a solution for equation* (1.2) *in* $\mathcal{F}^1_\mu(I, M, M')$.

Since all the hypotheses of Theorem 3.1* are satisfied in both Examples 3.1 and 3.2, they do not need any alteration.

References

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Manuscript received: November 28, 2007 and, in final form, February 13, 2008.