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Correction

Correction to: On Locally $\mathcal{A}$-Convex Modules

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The original paper has been published inadvertently with an oversight concerning two remarks following Definitions 3.4 and 3.8 which are incorrect. We thank Professor Mart Abel for his comment which led us to correct these remarks. The remark following Definition 3.4, page 5, should state:

In terms of neighborhoods, this definition can be stated as follows: for each $W \in \mathcal{W}_0$, there exist V in $\mathcal{N}_0$ and $W' \in \mathcal{W}_0$ such that $V \cdot W' \subseteq W$.

Similarly, the remark following Definition 3.8, page 6, should state:

In terms of neighborhoods, this definition can be stated as follows: for each $W \in \mathcal{W}_0$, there exists $V \in \mathcal{N}_0$ such that $V \cdot W \subseteq W$.

With these corrections in mind, anything else in the paper is correct, except Proposition 4.16, page 16, which relates strict inductive limits with locally m-convex modules. Here we present a proposition which states that a strict inductive limit of locally m-convex modules is again a locally m-convex module. But we need the additional hypothesis that the algebra $\mathcal{A}$ is itself a strict inductive limit.

Proposition 4.16. *Let $\mathcal{A}$ be a locally convex algebra such that $\mathcal{A}$ is the strict inductive limit of a sequence of subalgebras $\{(\mathcal{A}_k, \sigma_k)\}_{k\in\mathbb{N}}$. Let (E, τ) be a locally convex space such that E is the strict inductive limit of the sequence of subspaces $\{(E_k, \tau_k)\}_{k\in\mathbb{N}}$, and each E_k is a locally m-convex left $\mathcal{A}_k$-module. Then E is a locally m-convex left $\mathcal{A}$-module.*

Proof. First, the action of $\mathcal{A}$ on E is clear. Namely, if $a \in \mathcal{A}$ and $x \in E$, then $a \in \mathcal{A}_r$ and $x \in E_s$ for some $r, s \in \mathbb{N}$. Take $k = \max\{r, s\}$ and therefore

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$a \in \mathcal{A}_k$ and $x \in E_k$. Define ax as it is defined in $E_{k.}$. The compatibility properties in the strict inductive limits ensure that this action is well defined and defines an $\mathcal{A}$-module structure on E.

Let us, as before, denote by $\mathcal{W}_0$ a fundamental system of neighborhoods of zero in E and by $\mathcal{N}_0$ a fundamental system of neighborhoods of zero in $\mathcal{A}$. Furthermore, let us denote by $\mathcal{W}_r$ and $\mathcal{N}_s$ fundamental systems of neighborhoods of zero in E_r and $\mathcal{A}_s$, respectively.

Take $W \in \mathcal{W}_0$ and, for each $k \in \mathbb{N}$, $W_k = j_k^{-1}(W) = W \cap E_k$. Note that $W_k \subseteq W_{k+1}$ for each k. Also, $\bigcup_{k\in\mathbb{N}} W_k = \bigcup_{k\in\mathbb{N}} (W \cap E_k) = W \cap \bigcup_{k\in\mathbb{N}} E_k = W \cap E = W$. Since E_k is a locally m-convex $\mathcal{A}_k$-module, for each $k \in \mathbb{N}$, there exists $V_k \in \mathcal{N}_k$ such that $V_k \cdot W_k \subseteq W_k$.

By properties of the strict inductive limit and since V_k is a neighborhood of zero in $\mathcal{A}_k$, there exists $V \in \mathcal{N}_o$ such that $V \cap \mathcal{A}_k = V_k$ for every k. As before, $V_k \subseteq V_{k+1}$ for every k and $\bigcup_{k\in\mathbb{N}} V_k = V$.

We claim that $V \cdot W \subseteq W$. So, take $z \in V \cdot W$. Then $z = ax$ where $a \in V_r$ and $x \in W_s$ for some $r, s \in \mathbb{N}$. Take $k = \max\{r, s\}$, therefore $a \in V_k$ and $x \in W_k$. So, $z = ax \in V_k \cdot W_k \subseteq W_k \subseteq W$ and the claim is proved. □

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