Correction

Correction to: A General Approach to Equivariant Biharmonic Maps

Stefano Montaldo and Andrea Ratto

Correction to: Mediterr. J. Math. 10 (2013), 1127–1139
<https://doi.org/10.1007/s00009-012-0207-3>

In this erratum first we amend the stability study of some proper biharmonic maps $\varphi_\alpha : T^2 \rightarrow S^2$ (Theorem 3.2 of [1]). We also correct the proof of a claim in Example 3.5 of [1], showing that biharmonic maps do not satisfy the classical Sampson's maximum principle for harmonic maps.

1. Equivariant biharmonic maps and applications

We use the notation of Example 3.1 of [1]. Theorem 3.2 of [1] is not correct and must be replaced by

Theorem 1.1. *Let $\varphi_\alpha : T^2 \rightarrow S^2$ be a proper biharmonic map as in equation (3.8)(ii) of [1]. Then φ_α is an unstable critical point.*

Proof. It suffices to prove that φ_α is unstable with respect to equivariant variations. To this purpose, we compute the second variation of the reduced bienergy functional (we denote by α^* the constant function $\alpha \equiv \pi/4$):

$$\begin{aligned} \nabla^2 E_2^\varphi(\alpha^*)(V, V) &= \frac{d^2}{dh^2} [E_2^\varphi(\alpha^* + hV)]|_{h=0} \\ &= \int_0^{2\pi} \frac{d^2}{dh^2} \left[(h\ddot{V})^2 + \frac{k^4}{4} \sin^2\left(\frac{\pi}{2} + 2hV\right) - h\ddot{V}k^2 \sin\left(\frac{\pi}{2} + 2hV\right) \right] \Big|_{h=0} d\theta \\ &= \int_0^{2\pi} \frac{d^2}{dh^2} \left[h^2\ddot{V}^2 + \frac{k^4}{4} \cos^2(2hV) - h\ddot{V}k^2 \cos(2hV) \right] \Big|_{h=0} d\theta \\ &= \int_0^{2\pi} [2\ddot{V}^2 - 2V^2k^4] d\theta. \end{aligned}$$

Work supported by P.R.I.N. 2015—Real and Complex Manifolds: Geometry, Topology and Harmonic Analysis—Italy and by Fondazione di Sardegna and Regione Autonoma della Sardegna (Project GESTA).

The original article can be found online at <https://doi.org/10.1007/s00009-012-0207-3>.

By taking $V \equiv 1$, we conclude from the last equality that φ_{α^*} is unstable, as required to end the proof. The case $\alpha \equiv 3\pi/4$ is analogous. $\square$

As a consequence of Theorem 1.1, Remark 3.3 of [1] should be deleted.

Next, we use the notation of Example 3.5 of [1]. The claim of Example 3.5 of [1], stating that biharmonic maps do not verify Sampson's maximum principle for harmonic maps, is correct. However, in order to prove it, we use the function

$$\alpha(r) = r e^{-\sqrt{\lambda} r}, \quad r \in \mathbb{R} \quad (1.1)$$

instead of the one which appeared in (3.21) of [1]. Indeed, the function in (1.1) admits a *strictly positive interior maximum point* at $r_0 = (1/\sqrt{\lambda}) > 0$. Thus, the image through φ_{α} of an open set $S^m \times (r_0 - \varepsilon, r_0 + \varepsilon)$ is contained in the concave side of $S = \partial B_{\alpha(r_0)}(O)$ provided that $\varepsilon > 0$ is small.

Acknowledgements

The authors wish to thank Prof. C. Oniciuc for pointing out the two errors in [1] which have been the object of the present erratum.

References

- [1] Montaldo, S., Ratto, A.: A general approach to equivariant biharmonic maps. *Med. J. Math.* **10**, 1127–1139 (2013)
- [2] Sampson, J.H.: Some properties and applications of harmonic mappings. *Ann. Sci. Écol. Norm. Sup. Sér.* **4**(11), 211–228 (1978)

Stefano Montaldo

Dipartimento di Matematica e Informatica

Università degli Studi di Cagliari

Via Ospedale 72

09124 Cagliari

Italia

e-mail: montaldo@unica.it

Andrea Ratto

Dipartimento di Matematica e Informatica

Università degli Studi di Cagliari

Viale Merello 93

09123 Cagliari

Italia

e-mail: rattoa@unica.it