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Correction

Correction to: Fractional Schrödinger Equation in Bounded Domains and Applications

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We take this opportunity to correct some errors in Theorems 1, 2 and 3 of the original article. We keep the notations given in original article.

Theorem 1 was formulated in original article as follows:

Theorem 1. *For every* $f \in \mathcal{C}^+(\partial D)$*, problem (9) admits one and only one nonnegative solution* u*. Furthermore,*

$$e^{-1}\, M_D^\alpha f \leq u \leq M_D^\alpha f \quad on \quad D.$$

The mistake is only in the left hand side of the above inequality. The corrected version runs as follows:

Theorem A. *For every* $f \in \mathcal{C}^+(\partial D)$*, problem (9) admits one and only one nonnegative solution* u*. Furthermore,*

$$C\, M_D^\alpha f \leq u \leq M_D^\alpha f \quad on \quad D, \tag{1}$$

where $0 < C < 1$ *is a constant depending on* α, μ, f *and* D*.*

Before going to the proof, we need the following preparatory lemma.

Lemma 1. *For every* $t \geq 0$*, let* $L_t := (I + tS)^{-1}S$*. For every integer* $n \geq 1$ *and every* $h \in \Lambda^+$*, we have*

$$\left(\frac{\partial}{\partial t}\right)^n L_t h = (-1)^n n! L_t^{n+1} h \tag{2}$$

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and

$$\left(\frac{\partial}{\partial t}\right)^n (I+tS)^{-1}h = (-1)^n n! L_t^n (I+tS)^{-1}h. \tag{3}$$

Proof. Let $h \in \Lambda^+$ and $n \geq 1$. Seeing that for every $t, t' \geq 0$,

$$(I+t'S)^{-1} - (I+tS)^{-1} = -(t'-t)(I+tS)^{-1}S(I+t'S)^{-1},$$

we immediately deduce that $L_{t'}L_t = L_t L_{t'}$ and

$$\frac{\partial}{\partial t}L_t h := \lim_{t' \to t} \frac{L_{t'}h - L_t h}{t'-t} = -L_t^2 h.$$

Recalling that, for $n \geq 2$,

$$L_{t'}^n - L_t^n = (L_{t'} - L_t)(L_{t'}^n - 1 + L_{t'}^{n-2}L_t + \cdots + L_{t'}L_t^{n-2}L_t^{n-1}),$$

we obtain that $\lim_{t' \to t} L_{t'}^n h = L_t^h$ and

$$\frac{\partial}{\partial t}L_t^n h = -nL_t^{n+1}h.$$

By induction, (2) follows. Moreover, since

$$I - tL_t = I - t(I+tS)^{-1}S = (I+tS)^{-1}(I+tS-tS) = (I+tS)^{-1},$$

we obtain that

$$\begin{aligned}\left(\frac{\partial}{\partial t}\right)^n (I+tS)^{-1}h &= -\left(\frac{\partial}{\partial t}\right)^n tL_t h \\ &= -t\left(\frac{\partial}{\partial t}\right)^n L_t h - n\left(\frac{\partial}{\partial t}\right)^{n-1} L_t h\end{aligned}$$

which implies using (2) that

$$\left(\frac{\partial}{\partial t}\right)^n (I+tS)^{-1}h = (-1)^n n! L_t^n (I+tS)^{-1}h.$$

This completes the proof of (3). □

Now, we are ready to prove the left inequality in (1).

Proof. Let $f \in \mathcal{C}^+(\partial D)$ and denote $\frac{M_D^\alpha f}{M_D^\alpha 1}$ by h. We recall from original article that the solution u of problem (9) is given by $u = M_D^\alpha 1(I+S)^{-1}h$. Let $x \in D$ and $\varepsilon \in]0,1[$ be fixed, and consider the function θ defined on $\mathbb{R}_+$ by

$$\theta(t) = \varepsilon + (I+tS)^{-1}h(x).$$

By [Lemma 3 in original article], the operators $(I+tS)^{-1}$ and L_t are nonnegative on Λ^+. This implies that $\theta > 0$ and, by (3), $(-1)^n\theta^{(n)} \geq 0$ for all integer n. So, by Bernstein's theorem, $\ln\theta$ is convex. In particular, $\ln\theta(1) \geq \ln\theta(\varepsilon) + (1-\varepsilon)\theta'(\varepsilon)/\theta(\varepsilon)$, or equivalently,

$$\theta(1) \geq \theta(\varepsilon)\, e^{(1-\varepsilon)\frac{\theta'(\varepsilon)}{\theta(\varepsilon)}}. \tag{4}$$

We have $\theta(\varepsilon) = \varepsilon + (I+\varepsilon S)^{-1}h(x)$ and, by (3), $\theta'(\varepsilon) = -L_\varepsilon(I+\varepsilon S)^{-1}h(x)$, whence $\lim_{\varepsilon \to 0}\theta(\varepsilon) = h(x)$ and $\lim_{\varepsilon \to 0}\theta'(\varepsilon) = -Sh(x)$. Hence, by letting ε tend 0 in (4), we obtain

$$(I+S)^{-1}h(x) \geq h(x)\, e^{-\frac{Sh(x)}{h(x)}}.$$

This completes the proof by taking $C := e^{-\|\frac{Sh}{h}\|}$. □

Theorem 3 in original article, which depends on the estimate (1), requires to be put right as well:

Theorem B. *it If* $1 \leq \gamma < \frac{2+\alpha}{2-\alpha}$ *then for every* $f \in \mathcal{C}^+(\partial B)$*, there exists a constant* $0 < C < 1$ *such that*

$$C\, M_B^\alpha f \leq M_B^{\alpha,\gamma} f \leq M_B^\alpha f.$$

The proof of Theorem B requires only to replace the constant e^{-1} in the proof of [Theorem 3 in original article] by the constant C given in (1).

At the beginning of the proof of [Theorem 2 original article], we used the passage

$$u \in C(B) \cap L^1(B) \Longrightarrow u \leq c\, \delta^{-1}.$$

But this is not true in general. One can, for example, take $u(x) = |x - x_0|^{1-d}$ for some $x_0 \in \partial B$. In fact, some supplementary condition on the solution u, under which the above passage holds true, is missing. Accordingly, the corrected version of [Theorem 2 in original article] should be reformulated as follows:

Theorem C. *For every* $1 < \gamma < 1 + \alpha$*, problem (16) has no nonnegative blow up solutions which behaves near the boundary* ∂B *like* $\delta^{-\beta}$ *for some real* β*.*

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