

Stephen R. Wassell | *The Mathematics of Palladio's Villas:
Workshop '98*

Stephen Wassell describes the the itinerary and goals of the 1998 workshop tours of the Villas of Andrea Palladio, and discusses the mathematical aspects of the Renaissance master's designs.

Introduction

The purpose of the 1998 workshop tour of Palladio's villas was to study the mathematical aspects of the designs of the villas *in situ* with scholars knowledgeable in the relationships between architecture and mathematics. This three-day research tour of eight of Palladio's villas took place 10-12 June 1998, following the conference Nexus '98: Relationships Between Architecture and Mathematics.¹ The participants of the workshop (Figure 1) were:

- Kim Williams, architect and author of *Italian Pavements: Patterns in Space*, organizer of the Nexus conferences, editor of the online *Nexus Network Journal*.
- Vera W. de Spinadel, Director of the Centre of Mathematics and Design of the Faculty of Architecture, Design and Urbanism at the University of Buenos Aires, Argentina, co-director of the Mathematics & Design conferences.
- David and Ruth Spieser, the former of whom is Professor Emeritus of the Catholic University of Louvain, Switzerland, as well as an expert on symmetry.
- Rocco Leonardis, architect, artist, and educator based in New York, expert on Italian art and culture.
- Michael Ostwald, Assistant Dean of the Faculty of Architecture at the University of Newcastle, Australia, and director of the architectural theory program there, who joined us for the first day.
- Susan Knight, translator/interpreter and art tour consultant working in Tuscany, who handled the logistics of the workshop tour.
- Stephen Wassell, author of this paper, director of the 1998 workshop tours of the villas of Palladio.

The workshop was a great success, thanks in large part to thorough planning by Kim and Susan. The villas visited during the 1998 workshop were: Villa Pisani Ferri De Lazara, Bagnolo di Lonigo; Villa Rotonda, Vicenza (Figure 2); Villa Valmarana Bressan, Monticello di Conto Otto; Villa Valmarana Malinverni, Lonedo di Lugo; Villa Barbaro, Maser; Villa Emo, Fanzolo; Villa Cornaro, Piombino Dese; Villa Foscari (La Malcontenta), Gambarare di Mira.

Workshop experiences

The workshop succeeded most immediately in providing the participants with the first-hand experience essential to fully appreciating Palladio's designs. More importantly, in my mind, relationships were formed that have led to continued discourse and collaboration. The discussions during, between, and after visitations to the villas involved issues crucial to understanding our own reactions to Palladio's work, by allowing us to challenge each other with deliberate, directly motivated queries. Much of the discussion has focused on the underpinnings of beauty, especially concerning the role of proportion, since this has been such a central theme in the literature on Palladio (further discussed below). Are certain proportions inherently more aesthetically pleasing than others are, and if so, why? While discussions of proportions produced as many questions as answers, consensus was reached on Palladio's brilliant use of geometric forms, both in two and three dimensions, and his artful use of symmetry as a consistent design principle. It was agreed that the sheer variety of designs found



Figure 1.
The participants of the 1998 workshop on the steps of the Villa Bisani, Bagnolo: first row (l-r), Rocco Leonardis, Michael Ostwald, Stephen Wassell; second row (l-r), David Speiser, Ruth Speiser, Countess Cornelia Ferri di Lazara, Susan Knight, Kim Williams, Vera Spinadel.



Figure 2.
The Villa Rotonda, showing the juxtaposition of geometric elements in the form of classical motifs, ordered by strict rules of symmetry.

in his architecture is further evidence of his genius. While Palladio certainly used various aspects of mathematics as design tools, there could be no mathematical formula that would capture his design process. Palladio's ability to shape spaces, individually as well as in combination (which inherently necessitates conceiving designs in the four dimensions of space-time), and then to articulate those spaces masterfully using classical forms, is what ultimately sets his work apart.

Workshop discussions

The discussions during the first workshop, and the collaborations that have developed since, center chiefly on the issue of proportion in Palladio's architecture. To best understand the goals of future workshops, it is necessary to review the rich literature concerning one of the most influential architects of the Renaissance. This thread of research was started, in this century at least, by Rudolf Wittkower, author of *Architectural Principles in the Age of Humanism*. In this 1949 classic, Wittkower makes a strong case that Renaissance architects such as Alberti and Palladio used proportions derived from music in their architecture; because such "harmonic proportions" are beautiful to the ear, it was believed they were part of a higher universal design, and thus equally beautiful to the eye. Wittkower traces the evolution of this idea back to the Pythagoreans through Plato, who "in his *Timaeus* explained that cosmic order and harmony are contained in certain numbers".²

Alberti, whose architectural treatise *De re aedificatoria* (first printed in 1486) was certainly known to Palladio, is quite explicit on the subject, stating for example, "The very same numbers that cause sounds to have that *concininitas*, pleasing to the ears, can also fill the eyes and mind with wondrous delight".³ Palladio, on the other hand, seems rather ambivalent on the subject. While in his 1567 opinion on a model for a new cathedral for Brescia he said, "[J]ust as the proportions of voices are harmony to the ears, so those of measurement are harmony to the eyes...."⁴ In his *Quattro Libri* (1570), which is much less theoretical than Alberti's treatise, Palladio makes no explicit mention of analogies to music, not even when he states, "There are seven types of room that are the most beautiful and well proportioned and turn out better: they can be made circular, though these are rare; or square; or their length will equal the diagonal of the square of the breadth; or a square and a third; or a square and a half; or a square and two-thirds; or two squares."⁵

Even though Palladio enters the harmonic proportion debate only implicitly through his oeuvre, Wittkower makes the argument that Palladio's circle of scholars and artists, which included such architectural theorists as Giangiorgio Trissino and Daniele Barbaro, would have embraced the "neo-Platonist" concept of harmonic proportions and would have influenced Palladio to use them in his architecture.

In the past two decades, architectural historians following in Wittkower's path (and thus often dubbed "the Wittkowerians") have sought to better evaluate Palladio's use of harmonic proportions by analyzing the dimensions of his designs as given in Book II of the *Quattro Libri*. Through their study of the 44 plans of Book II, Deborah Howard and Malcolm Longair conclude, for example, that about 2/3 (65-70%) of the dimensions given are "harmonic numbers", whereas only 45% would be harmonic if Palladio picked dimensions at

random.⁶ They also find that, while harmonic numbers are fairly spread out among the plans of Book II, those plans which are completely harmonic were all designed after about 1556, and for patrons who may have shared an interest in musical theory.

Branko Mitrović extends the analysis by considering heights of rooms as well as lengths and widths. He also gives a provocative explanation of how $\sqrt{2}:1$ (one of the seven proportions Palladio touts as “most beautiful”) can be viewed as a harmonic proportion since it corresponds to the augmented fourth of an equitempered scale, which was suggested by a contemporary of Palladio named Gioseffe Zarlino. Mitrović concludes that “Wittkower’s thesis...can be seen to be more consistent than it previously appeared.”⁷

In my mind, however, Mitrović makes a much more compelling pair of observations. First, he finds in Book II six unexplained (i.e., non-harmonic) proportions that approximate $\sqrt{3}:1$, the closest being the large corner rooms of Palladio’s most celebrated villa, the Villa Rotonda (Figure 2), which measure 26 by 15 (26/15 is within 0.07% of $\sqrt{3}$). Mitrović calls this proportion “triangulation” because $\sqrt{3}:1$ is the ratio of the longer leg to the shorter leg in a 30°-60°-90° triangle (half of an equilateral triangle). It should be noted that Alberti describes precisely this construction of $\sqrt{3}$ in his treatise, as well as the realization of both $\sqrt{3}$ and $\sqrt{2}$ as the diagonal of a cube and the diagonal of one of its faces, respectively (Figure 3). Perhaps Palladio was expressing a fondness for pure mathematics at least as much as an appreciation of harmonic proportions. The second, related, observation is equally compelling. Mitrović first reminds us that Palladio describes and illustrates geometric constructions for the arithmetic mean (Figure 4), the geometric mean (Figure 5) and the harmonic mean (Figure 6), which Palladio proposes for use as the heights of vaulted rooms (the length and width being the two extremes of which the mean is taken):

*[W]e should make use of each of these heights depending on which one will turn out well to ensure that most of the rooms of different sizes have vaults of an equal height and those vaults will still be in proportion to them, so that they turn out to be beautiful to the eye and practical for the floor or pavement which will go above them...*⁹

Again, the Villa Rotonda provides an example. Mitrović observes that the height of each of the four corner rooms is given by the arithmetic mean, $(26 + 15) / 2 = 20.5$, so that each has a height to width ratio of approximately 1.3666. Moreover, the adjacent four rooms (the only other primary rooms on the *piano nobile* other than the central domed room; see Figure 7), each of which measure 15 by 11, not only share a dimension with the corner rooms but are of proportion approximately 1.3636, which is within 0.2% of 1.3666. There is another crucial link between rooms that Mitrović fails to observe, however: the diameter of the central domed room is 30, which is the hypotenuse of a (virtual) 30°-60°-90° triangle having 26 and 15 as its two legs (Figure 8)! It seems highly unlikely that such closely interlocked mathematical puzzles would not be deliberate themes of Palladio’s design.

Unfortunately, one can only conjecture on Palladio’s intentions, given the lack of explicit references to music or mathematical theory in the *Quattro Libri*. It remains, then, to continue to gain evidence of Palladio’s motivations and intentions from clues implicit in his work or

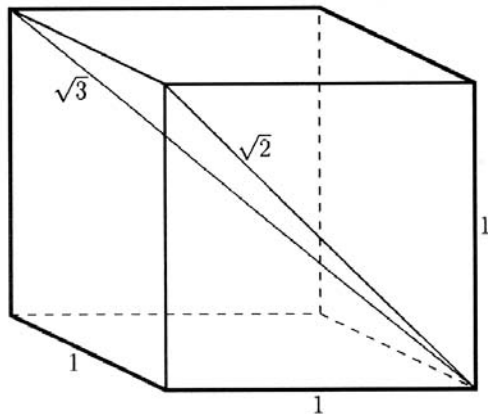


Figure 3.

Illustration of Alberti's description of a unit cube exhibiting the proportions $\sqrt{2}$ and $\sqrt{3}$, based on figure LXV from James Leoni's illustrated translation of Alberti's treatise, with the caption "a cube, showing two diagonals".

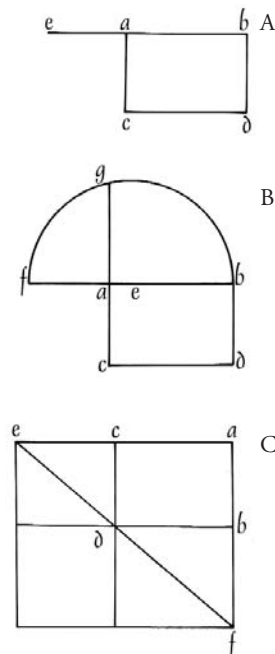


Figure 4.

Palladio's constructions for the arithmetic, geometric and harmonic means.

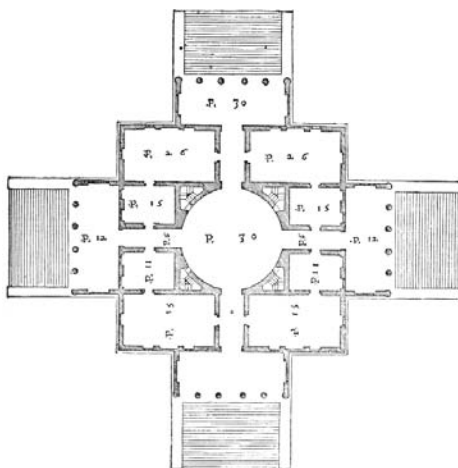


Figure 5.

The plan of the Villa Rotonda, *piano nobile*, from Andrea Palladio, *Quattro Libri*.

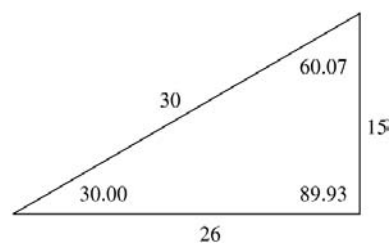


Figure 6.

The geometric relationship between the three dimensions (30, 26 and 15) specified in the *Quattro Libri* within the five primary rooms of the Villa Rotonda (angle measurements are given in degrees with two decimal places of accuracy).



Figure 7.

Window detail, Villa Rotonda. The proportion as measured from the photograph is almost precisely 2:1, but on-site measurements are necessary to ensure accuracy.

based on what we know of his life experiences.

Such evidence is at the heart of a collaborative effort involving four workshop participants at the 1998 workshop. The project is to translate into English and annotate Silvio Belli's 1573 treatise on proportion, *Della Proportione et Proportionalità*. A mathematician, engineer, and cofounder of the Olympic Academy, Belli was a close friend of Palladio for many years. Exploring Belli's ideas on proportion may provide important insight into Palladio's own mathematical thinking. Surprised upon finding no English translation of Belli's treatise, I had approached Kim Williams about this project just before the first workshop. She accepted the invitation to be the principal translator, while Vera W. de Spinadel, Rocco Leonardis, and I gladly agreed to provide constructive criticism in order to assist her in honing the translation and commentary.

Goals and agenda for future workshops

During future workshop visits to the Villa Rotonda (and possibly other villas), I will take the opportunity to measure the window and door openings in order to analyze their proportions. This was motivated by observations made during the first workshop. David Speiser noted that the iron grill in front of some of the windows at the Villa Rotonda seemed to suggest that a $\sqrt{3}$ proportion was employed (see Figure 7), though we were quickly able to deduce that the grill is not composed of equilateral triangles as it first appeared. Even so, this inspired me to photograph the window as carefully as possible in order to avoid distortion, and up to the accuracy of the photograph, the proportion of the window is 2:1 (Figure 9). Vera Spinadel suggested that

measuring the wall openings in Palladio's villas might uncover new evidence for our analysis of Palladio's use of proportion. One issue that will need to be addressed is how to relate measurements taken *in situ* with idealized dimensions from the *Quattro Libri*.¹⁰

Future workshops will provide the opportunity for gathering additional evidence. Of course, the most direct benefit of future workshops will again be the exposure of the participants to Palladio's exquisite designs, and to each other while experiencing those designs. The opportunity to measure the window and door openings of the Villa Rotonda is an added benefit. If these measurements prove to be useful for the analysis of Palladio's use of proportion, it would be most beneficial to conduct an exhaustive study of the wall openings of Palladio's villas. In any case, the future workshop, like its predecessor, is certain to be a wonderful learning experience for all involved.

Acknowledgments

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Notes

1. The 1998 workshop was partially supported by grant #9846 from the Graham Foundation for Advanced Studies in the Fine Arts.
2. Rudolf Wittkower, *Architectural Principles in the Age of Humanism*, p. 105.
3. Leon Battista Alberti, *De re aedificatoria (On the Art of Building in Ten Books)*, bk. ix, ch. v, p. 305. An understanding of Alberti's conception of *concinntas* may be gleaned from the following quote from bk. ix, ch. v, p. 302: "It is the task and aim of *concinntas* to compose parts that are quite separate from each other by their nature, according to some precise rule, so that they correspond to one another in appearance." See the glossary entry for this term, p. 421-422, for a more detailed discussion.
4. Bruce Boucher, *Andrea Palladio: The Architect in His Time*, p. 239.
5. Andrea Palladio, *I quattro libri dell'architettura (The Four Books on Architecture)*, bk. I, ch. xxi, p. 57.
6. Deborah Howard and Malcolm Longair, "Harmonic Proportion and Palladio's *Quattro Libri*," pp. 121ff. The idea that a single number can be harmonic--or not harmonic--is fallacious, as they essentially admit (p. 134), but they convincingly argue that their system of harmonic numbers is a valid tool by which to measure Wittkower's thesis vis-à-vis Palladio's plans.
7. Cf. Branko Mitrović, "Palladio's Theory of Proportions and the Second Book of the *Quattro Libri dell'Architettura*," This is well within the allowed error tolerance of many rigorous statistical experiments.
8. Andrea Palladio, *Quattro Libri*, bk. I, ch. xxiii, p. 59.
9. Deborah Howard and Malcolm Longair discuss the relative merits of measurements obtained from the *Quattro Libri* as opposed to those taken on site; see "Harmonic Proportion," p. 117-118.

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Stephen R. Wassell is an Associate Professor of Mathematical Sciences at Sweet Briar College in Virginia (USA). He received a Bachelor of Science degree in Architecture (1984), a Master's and a Ph.D. in Mathematics (1987 and 1990), and a Master's in Computer Science (1999), all from the University of Virginia. His primary research areas are now computer science and the relationships between architecture and mathematics. He designed and taught a course entitled "Math in Architecture" at Sweet Briar in the Winter Term of 1995. He presented his research on the mathematics of Palladio's villas at Nexus '98 and will present at Nexus 2000 on mathematics in art before the *Quattrocento* as a context for understanding Renaissance architecture.