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RESEARCH

El Escorial, Derand and Guarini: About Formal Control of Ribbed Spherical and Elliptical Vaults

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Abstract

Guarino Guarini's ribbed vaults have been connected with Islamic sources. However, this connection relies almost exclusively on plans and visual observations from the church floor. By contrast, the vertical layout, although clearly shown in Guarini's *Dissegni di architettura civile ed ecclesiastica,* and the intrados surface of the ribs have not been taken into account, except by Marco Boetti (Guarino Guarini, Umberto Allemandi, Torino, 2006). In this contribution, I will deal with some possible sources of Guarini's designs for the ribbed vaults of Sainte-Anne-la-Royale in Paris, Padri Somaschi in Messina, San Lorenzo in Turin and San Gaetano in Nice. I will take into account not only the plan layout of the ribs, but also their shape in space, and the nature of the intrados surface, analysing such possible sources as the stereotomic treatise of François Derand and several Spanish stonecutting manuscripts connected with El Escorial.

Keywords Guarino Guarini · Ribbed vaults · Elliptical arches · Ellipsoidal vaults · Cone developments

Introduction

The lost Church of Sainte-Anne-la-Royale in Paris, the extant one of San Lorenzo in Turin, and the unbuilt projects for Padri Somaschi in Messina and San Gaetano in Nice feature ribbed vaults with ribs that do not cross at the centre of the vault (Boetti 2006). Sainte-Anne-la-Royale was started in 1662, but funds were exhausted as early as 1666, and the project met with strong opposition from the French classicists, so it was left unfinished and finally demolished. There has been much discussion over the date of Padri Somaschi in Messina; recent studies place it around 1670–1672. As for San Lorenzo, the church proper was built between 1670 and 1680. A project for San

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Gaetano in Nice was prepared in 1675, but the church was not built (Coffin 1956; Tapié 1957: 227; Millon 2006: 10–13; Piccoli 2006; Piazza 2016).

Three books connected directly or indirectly with architecture were published under the name of Guarino Guarini. Only *Euclides adauctus* appeared in Guarini's lifetime (1671); it deals with geometry, but Treatise (that is, Chapter) XXIX covers orthographic projection, while Treatise XXXII deals with the development of surfaces into a plane and makes a passing remark to ashlar vaults.

The second book, *Disegni di architectura civile ed ecclesiastica* was published in 1686, shortly after Guarini's death in 1683. Basically, it includes drawings; the most relevant for our purposes bears an inscription attesting that Guarini himself furnished the drawings for the plates.

Finally, the well-known *Architettura Civile* was published in 1737 by Bernardo Vittone and the Theatines of Turin. Some of the plates for this book are reused from *Disegni …;* others are redrawn from *Disegni* or *Euclides adauctus*, but many of them are new. The same goes for the text, which includes some material taken from *Euclides adauctus* but also much new material. This is why I have relied on material from *Euclides adauctus* (1671) and the *Disegni* … (1686), rather than the better-known *Architettura Civile* (1737).

Vault Plans

The ribs in the four churches under analysis do not cross at the centre of the vault, a usual feature of Muslim domes (Schubert 1908: 308; Giedion [1941] 2008: 126–127). In the main dome and the lantern in San Lorenzo and some Islamic examples, such as the vaults over the *maqsura* in the mosque of Cordova, as well as the Qubba Ba'adiyn in Marrakesh, the geometry of the vault is based on an octagon, with ribs springing from each vertex of the octagon to both directions.

In the central dome of the Cordova *maqsura* and the lantern of San Lorenzo, the path of the ribs jumps from the initial vertex to the second next one, then to the fourth and the sixth one and then returns to the starting vertex (Figs. 1, 2). As a consequence, the ensemble of the ribs decomposes into two interlocking squares, a typical motif in Muslim architectural decoration.

By contrast, in the vaults in the lateral sections of the *maqsura* and the main dome in San Lorenzo, ribs leap to the third next vertex and then to the sixth and the ninth one, that is, the first one after the starting vertex. As a result, the path of the ribs does not reach the starting vertex until it has passed through all vertexes and does not decompose into simpler shapes (Figs. 3, 4).

A small detail in the top left corner of the drawings for Sainte-Anne-la-Royale in *Disegni* … depicts the rib layout, which is quite singular (Fig. 5).[1] The layout

[1] The *Disegni* … (Guarini 1686) include three series of drawings. The first one carries captions such as Lastra 1 Trattato III. The plan of Saint-Anne-la-Royale and the plan and section of San Lorenzo, included later on, belong to the second series, which simply carries a number from 2 to 27. Besides, there are some single drawings and a whole third series without numbering.

Fig. 1 Vault in the lantern of San Lorenzo in Turin, seen through the main dome. Image: author

is also based on an octagon, but at each vertex there is a podium providing place for two separate pairs of ribs. Each of these ribs jump to the second next podium, both left and right, so the scheme resembles generally the one in the lantern of San Lorenzo. However, since four ribs start from each podium, two heading to the left and two to the right, each of the rib sets is doubled, and the horizontal projection of each of these sets is a rectangle, rather than a square.

We may ask ourselves if this rib layout was part of the project for Sainte-Anne or a later elaboration for the *Dissegni* … Edoardo Piccoli (2006) has published a plan for Sainte-Anne in the Cotte section in the Bibliothèque Nationale in Paris which coincides essentially with the general plan in the *Dissegni*. This suggests that the detail plan for the ribs is part of the project of the church of the 1660s and not a re-elaboration dating from the 1680s.

Also, the vaults over the presbytery in San Lorenzo and the main one in Padri Somaschi are ribbed vaults with a hexagonal plan, with ribs jumping from the starting vertex to the second next one, a shape familiar to us as a Star of David or Seal of Solomon.

Finally, the rib vault projected for San Gaetano in Nice is based on a pentagon, and ribs jump to the second next vertex. However, since the number of vertexes of the pentagon is odd, rather than even, ribs do not finish their path until reaching again the original vertex, and the ensemble does not decompose into simpler shapes.

Fig. 2 Central dome in the *maqsura* of the Great Mosque of Cordova. Image: author

The traditional explanation for Islamic influence in Guarini relies on the hypothesis of a trip across Spain while planning or building Saint Mary of the Divine Providence in Lisbon. However, Joaquín Bérchez and Fernando Marías (2006: 495–496; see also Morrogh 2006) pointed out that such possible trip does not explain the issue, since the construction of the Lisbon church has been lately placed in the 1680s. It is also worthwhile to remark that only some of the later vaults designed by Guarini, the ones for the main dome and the lantern of San Lorenzo, resemble closely the vaults in Cordova and Marrakesh. Are there other plausible sources for Guarini's rib patterns?

Thomas Bradwardine's *Geometria Speculativa*, written in the fourteenth century and published in 1495, includes two stellated octagons (see Brigaglia et al. 2018). In the *Octogonus primi ordinis* each side of the octagon is extended until it meets the second next side; the result is exactly the same as the one in the *maqsura* in Cordova and San Lorenzo's lantern. In the *Octogonus secundi ordinis* the sides of the original octagon are protracted until each side meets the third next side, just as the vaults at both sides of the Cordova *maqsura* and the main dome in San Lorenzo (Fig. 6).

The layout of the vault over the presbytery in San Lorenzo and Padri Somaschi coincides again with Bradwardine's *Exagonus primis ordinis*, resulting from the extension of each side of a hexagon until it meets the prolongation of the second next side. Also, the one for San Gaetano in Nice matches Bradwardine's *Pentagonus primi ordines* (Figs. 7, 8).

Fig. 3 Vaults over the lateral sections of the *maqsura* in the Great Mosque of Cordova. Image: author

Thus, Bradwardine or other treatises dealing with stellated octagons and hexagons, such as the one by Charles de Bovelles (1557: 56r, 58v, 59r), are a possible source for the planar layout of the ribs in San Lorenzo. This does not leave Islamic sources completely out of the picture, since there is an earlier study on star polygons in Adelard of Bath's Latin translation of Euclid's *Elements*, from the Arabic text by al-Ḥajjāj ibn Yūsuf ibn Maṭar (Brigaglia 2018). Also, the elevation layout and the intrados surfaces in Turin have little in common with these Islamic vaults, as we will see next, and thus the similarities between San Lorenzo and Muslim vaults are restricted to the planar layout.

Rib Shapes

In the section of the Church of San Lorenzo included in *Dissegni* … some ribs are placed obliquely, as a result of the octagonal geometry of the main dome, and thus are deformed in the elevation. However, two ribs are parallel to the projection plane of the elevation, and thus are shown in true form in the elevation. This makes clear that all ribs in the main dome are raised arches (Figs. 9, 10). The reader may ask whether the actually built ribs in San Lorenzo are raised or round. Ugo Quarello (2009: 380) published the surveys for a nineteenth century restoration of the church; in particular, a drawing of a plan and a cross-section shows the oval profile of the

Fig. 4 Main dome in San Lorenzo in Turin. Image: author

ribs. In the section of the unbuilt Church of Padri Somaschi in Messina (Guarini 1686, unnumbered plate) there are no arches shown in true form, but their span can be measured in the plan and their rise in the elevation, leading to the same conclusion. By contrast, the arches in the vaults over the *maqsura* of Cordova and the bays at both sides of the *maqsura*, are not raised, but round (Fig. 11).[2]

Raised elliptical or oval arches, in contrast with surbased or pointed ones, are not frequent.[3] François Derand, a Jesuit mathematician and architect, considered an essential source for Guarini's stereotomy by La Greca (Guarini [1737] 1968: 289–290, note 1) and Borin,[4] includes them in his *Architecture des voûtes* (1643: 319–335), in particular for rectangular groin vaults (Fig. 12). In this kind of vaults, if all perimetral arches are round ones, the keystones of the short side arches are placed below the keystones of the long side ones, and the simple usual solution for groin vaults, involving cylindrical surfaces, cannot be used.

[2] Available at https://www.academiacolecciones.com/arquitectura/arquitectura-al-andalus.php?pag=2&orden=3&direccion=0#&gid=1&pid=AA-101_41

[3] The reader may object to my indiscriminate use of 'oval' and 'elliptical'. However, Derand includes in his treatise (1643, 294) a 'compas à ovale; ou pour former des ellipses'.

[4] See Paolo Borin's Ph. D. dissertation, 'La geometria come strumento totale … per lo studio dell'opera di Guarino Guarini' presented at Università IUAV di Venezia in 2019: 89-94. A different opinion has been posited by Taraldsen (2023).

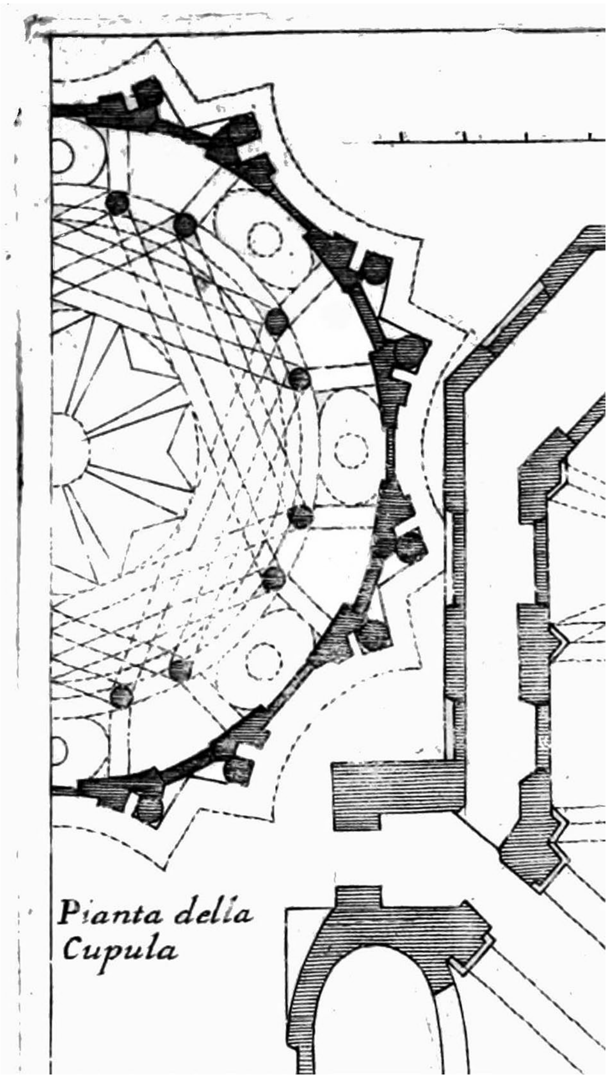


Fig. 5 Plan of the Church of Sainte-Anne-la-Royale. Detail of the main vault. Image: Guarini 1686: 9

Derand solved this problem in the Parisian Jesuit Church of Saint-Paul-Saint-Louis laying out the short side arches as raised oval ones (Fig. 13). All this had a precedent in the Church of the Jesuit college of La Flêche, built by Étienne Martellange, where Derand had taught Mathematics (Moisy 1950).

Derand (1643: 16–17) used also oval arches when illustrating a buttress-dimensioning rule, which was quite popular from the 16th through the eighteenth

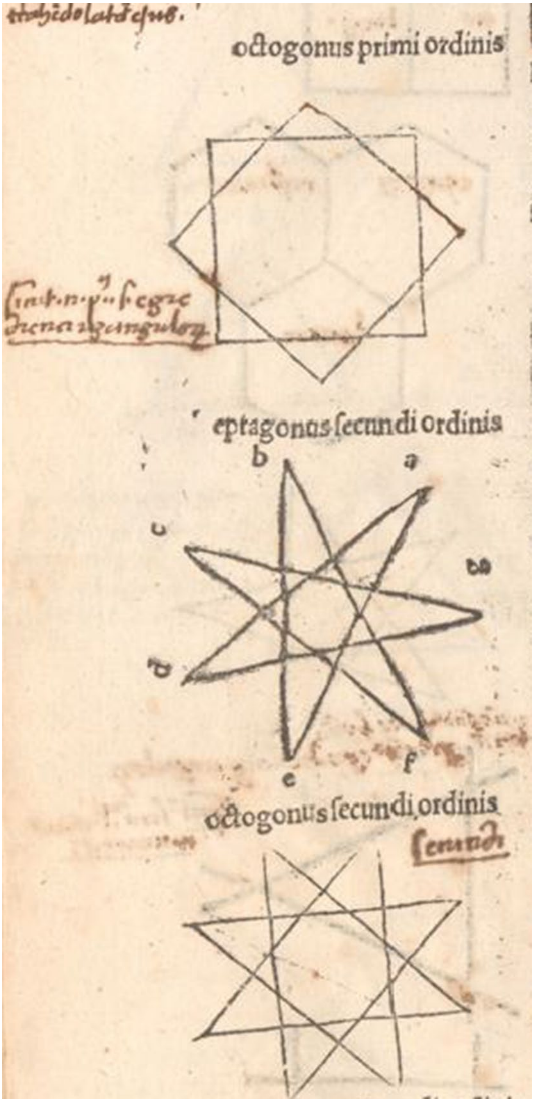


Fig. 6 *Octogonus primi ordinis* and *Octogonus secundi ordinis*. Image: Bradwardine [c. 1350] 1495

century. In order to compute the required thickness for the buttresses of an arch of any shape, he divides the length of the intrados of the arch in three equal parts, draws a line passing by the division point and the springing of the arch, and extends it, going further than doubling its length. Next, he draws a semicircle with its centre at the springing of the arch passing through the division point, until it meets the extended segment. Of course, the resulting point is placed at a

Fig. 7 Vault over the presbytery of San Lorenzo in Turin. Image: author

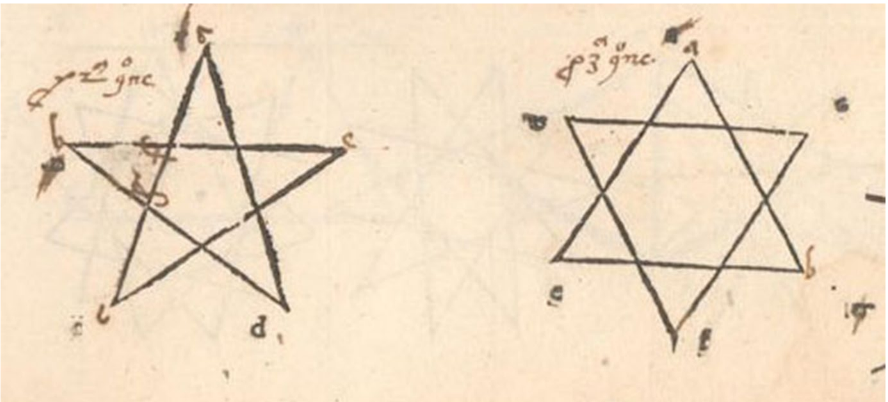


Fig. 8 *Pentagonus primi ordines* and *Exagonus primi ordinis.* Image: Bradwardine [c. 1350] 1495

distance from the springing that equals the distance between the springing and the division point. Once this is done, he draws a vertical line passing through the intersection of the semicircle and the extended segment; the distance between

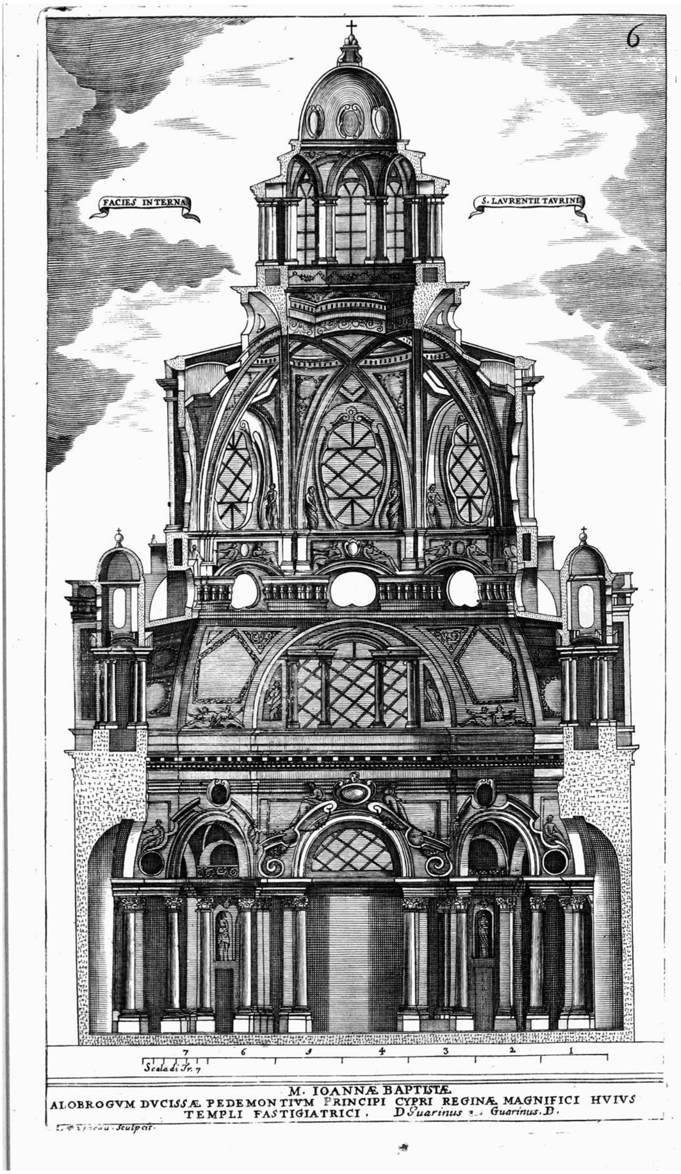


Fig. 9 Section of San Lorenzo in Turin. Image: Guarini 1686: 6

this vertical line and another vertical line drawn through the springing gives the required buttress thickness (Fig. 14).

Derand applies the procedure to round, raised oval, segmental, and pointed arches, implying that segmental arches exert greater thrust that other types, since

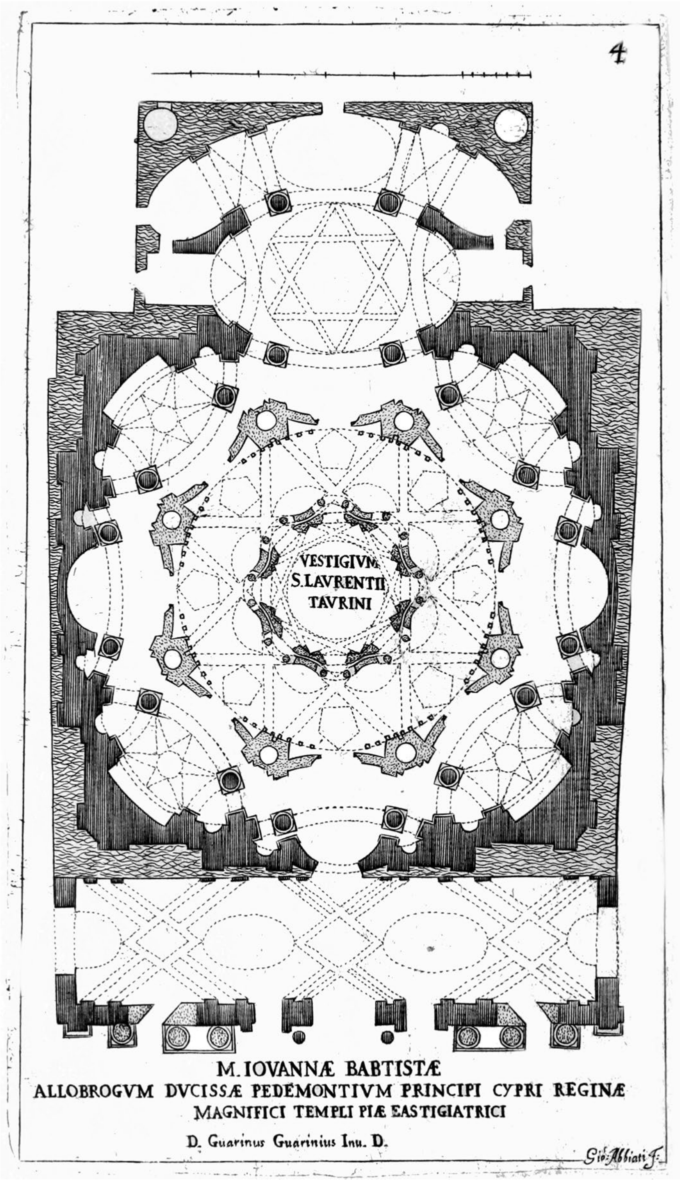


Fig. 10 Plan of San Lorenzo in Turin. Image: Guarini 1686: 4

they need thicker buttresses; round arches exert lesser thrust, but pointed and raised oval arches exert even less thrust. All this suggests that Derand was thinking about raised oval arches as a classical alternative to pointed arches.

Fig. 11 Section of the domes over the *maqsura* in the Great Mosque of Cordova. Image: Photogrammetric survey by Antonio Almagro

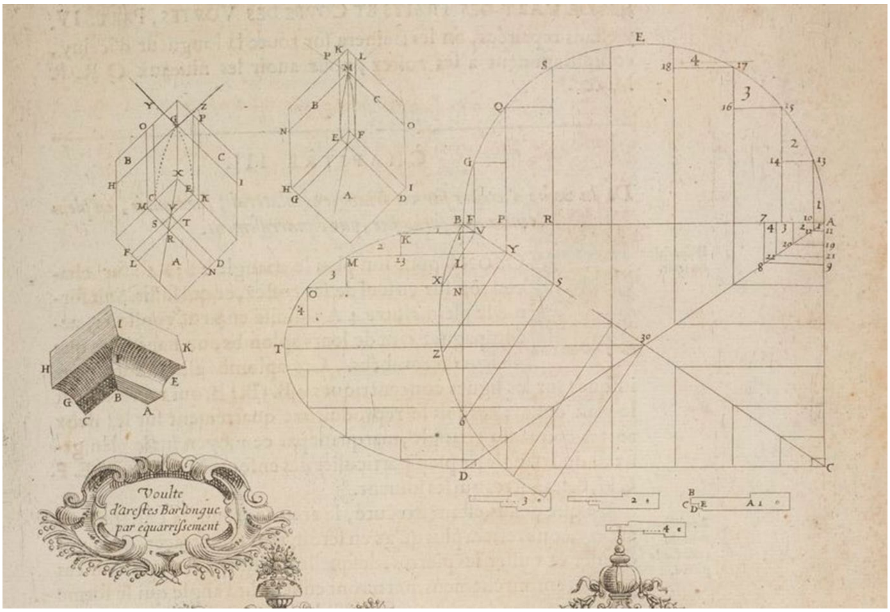


Fig. 12 Rectangular groin vault. Image: Derand 1643: 331

Fig. 13 Rectangular groin vaults in the Church of Saint-Paul Saint-Louis in Paris, built by François Derand. Image: author

There are earlier Spanish examples for this unusual shape. Raised oval or elliptical arches are used in the Escorial complex in the corridors leading to the main dome, a secluded place (Bustamante 1994: 491–493). These corridors were not used the monks, the king, or the court; they were meant for maintenance purposes. So, we may assume that the election of the oval or elliptical shape for these arches and vaults had an experimental rather than practical purpose (Fig. 15).

Is there a direct connection between El Escorial, a sober, practical Counter-reformation construction and Guarini's complex mathematical scheme? As pointed out by Susan Klaiber (2006: 329–330), both El Escorial and San Lorenzo in Turin were built as reparations for the burning of the collegiate Church of Saint Quentin, a city in Picardy, in 1557, by an army commanded by Emmanuele Filiberto of Savoy serving the interests of Philip II of Spain. El Escorial was built at a relatively fast pace and finished in 1584. By contrast, the Savoyard rulers started building a Church for San Lorenzo but, later on, the regency of Christine

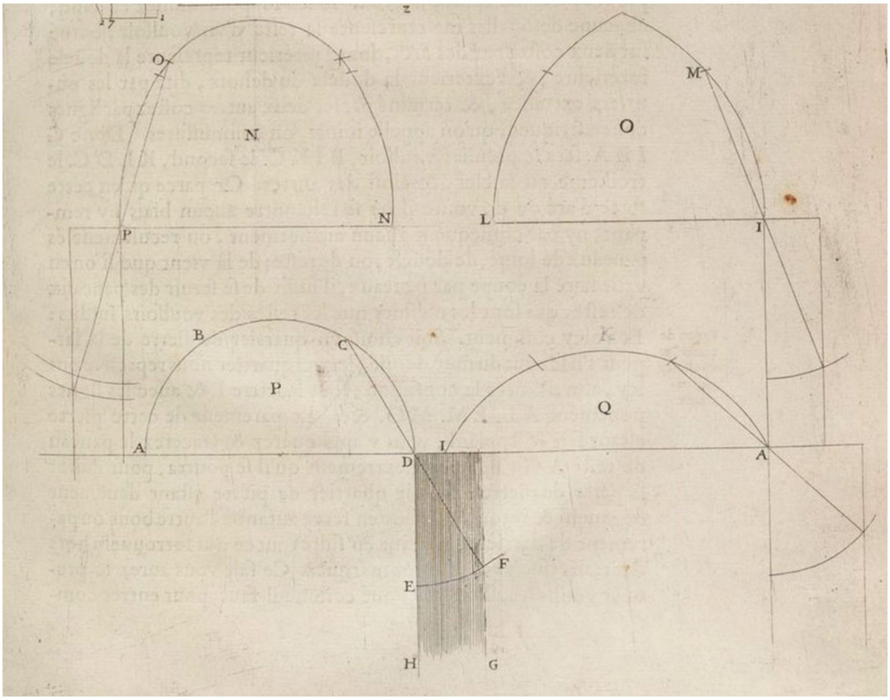


Fig. 14 Buttress-dimensioning rule. Image: Derand 1643: 17

of France did not foster the commemoration of a victory of Spanish and Savoyard troops over a French army. Thus, the building of a new Church of San Lorenzo was protracted until 1670. It was finished in 1680 according to Guarini's design, although the façade was built later with a different design (Millon 2006: 10; Klaiber 2006: 330).

However, Klaiber did not identify much significant architectural connections between El Escorial and San Lorenzo in Turin, other than the central layout of both churches. There is, however, an indirect connection between the Escorial and Guarini, which, at the end, may prove more relevant for our purposes. Derand's buttress dimensioning rule is explained, in a slightly different form, in a stonecutting manuscript by Ginés Martínez de Aranda ([c. 1600] 1986). He simply draws the vertical line passing through the division point, measures the distance from the intersection point to the springing and takes this distance as the required buttress thickness (Fig. 16). Thus, his method is conceptually clearer, although drawing the vertical line as a perpendicular to the springing line may require more effort than simply drawing the line connecting the division point with the springing.

Martínez de Aranda illustrates the buttress-dimensioning rule with round, segmental and pointed arches, but he does not use oval arches. However, this author

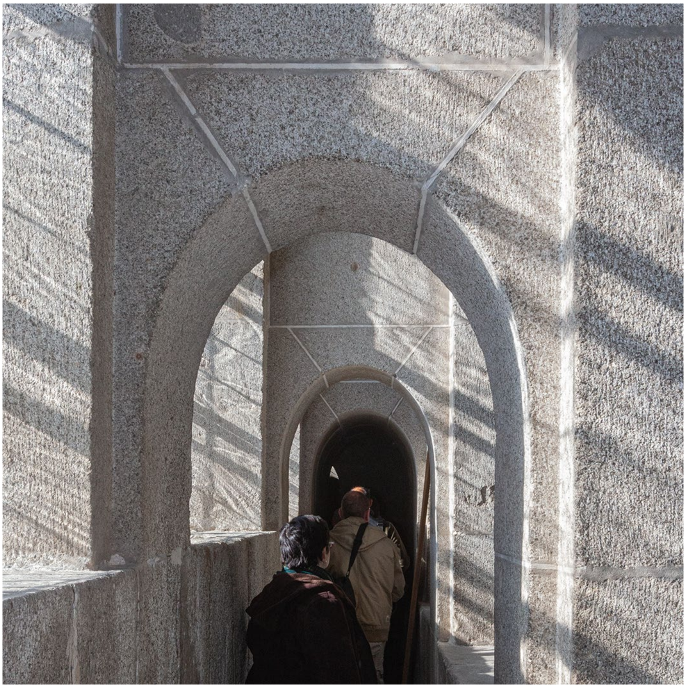


Fig. 15 Oval arches in the corridors leading to the exterior of the main dome in El Escorial main church. Image: author

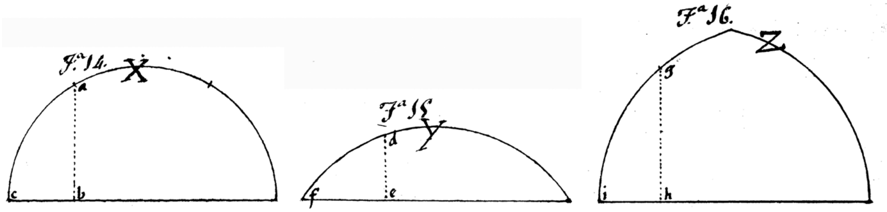


Fig. 16 Buttress-dimensioning rule. Image: Martínez de Aranda [c. 1600] 1986: 5–6

had used surbased and raised oval arches in the reconstruction of the old Cathedral in Cádiz, after the partial destruction brought about by the English attack of 1596, in order to adjust to the different spans between walls and columns in the damaged church.

Also, in his manuscript, Martínez de Aranda presented a generalisation of a procedure for the construction of elliptical arches put forward by Serlio (1545: 13v–14r) and Alonso de Vandelvira ([c. 1580] 1977: 18v). It is interesting to point out that Serlio stresses the flexibility of this construction; it can be used to construct

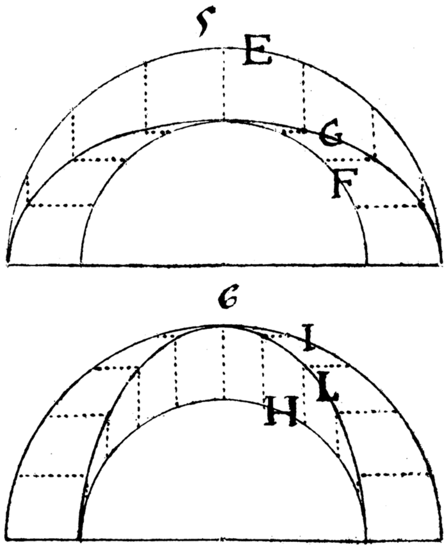


Fig. 17 Surbased and raised oval arches. Image: Martínez de Aranda [c. 1600] 1986: 3

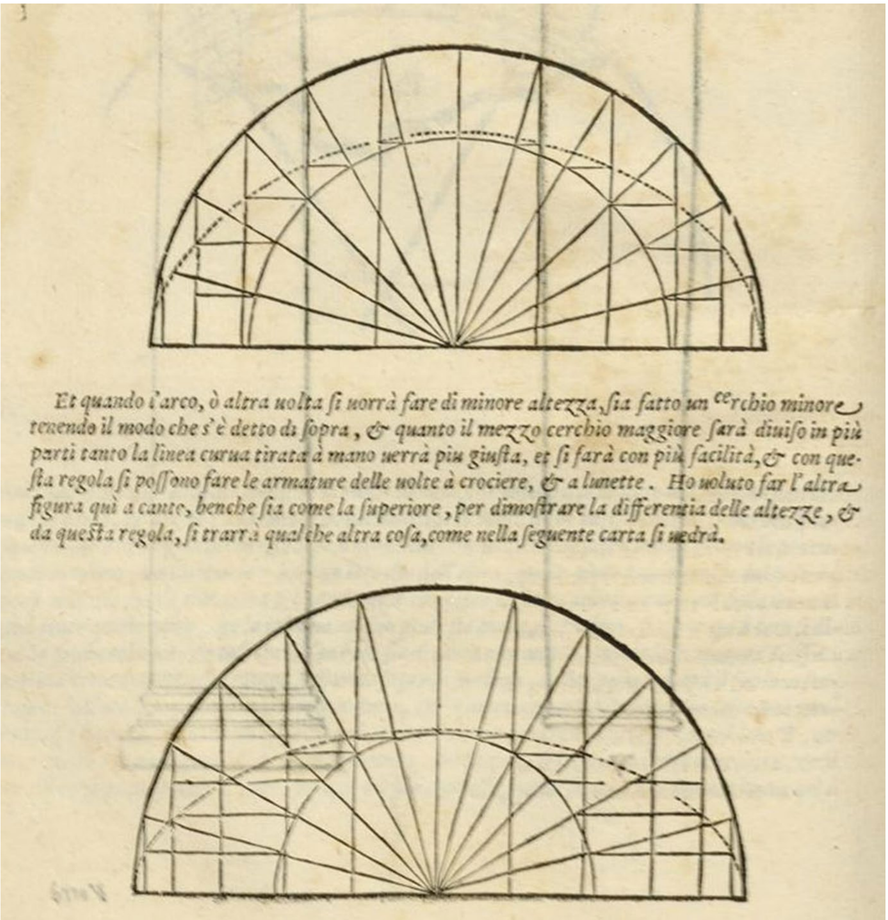
an oval arch with any span and rise; significantly, he includes two arches with different heights, but both are surbased. This suggests that Serlio was not thinking about raised arches; otherwise, he would have included a raised arch to show the flexibility of his procedure (Figs. 17, 18).

Maybe the reader will ask how these ideas reached Guarini. Again, it seems that the lines of influence may pass through El Escorial. Alonso de Vandelvira's manuscript was known in El Escorial, since he had lent a copy to Juan de Valencia, a master in Herrera's entourage. As for Martínez de Aranda, he was surely in contact with Vandelvira and with Cristóbal de Rojas, the author of the first Spanish treatise on fortification (1598), a by-product of the lessons that he had taught in the Academy of Mathematics under the supervision of Juan de Herrera.

Also, according to Rodríguez Gutiérrez de Ceballos (1966: 288–291, 295–299), Herrera exerted a strong influence on Giuseppe Valeriano, the designer of the Collegio Romano, the main centre of Jesuit learning in Rome, together with Bartolomeo Ammannati. Thus, the influence of El Escorial may have reached Derand through Jesuit circles and from Derand may have passed to Guarini.

Rib Intrados Surfaces and Intersections

Another detail sets San Lorenzo vaults apart from Islamic ones. As Marco Boetti pointed out, the intrados surface of many Guarini vaults, including San Lorenzo, is not cylindrical, as in Islamic vaults, but it seems to be conical, spherical, or



Et quando l'arco, ò altra uolta ſi uorrà fare di minore altezza, ſia fatto un cerchio minore tenendo il modo che s'è detto di ſopra, & quanto il mezzo cerchio maggiore ſarà diuiſo in più parti tanto la linea curua tirata à mano uerrà piu giuſta, et ſi farà con più facilità, & con queſta regola ſi poſſono fare le armature delle uolte à crociere, & a lunette. Ho uoluto far l'altra figura quì a canto, benche ſia come la ſuperiore, per dimoſtrare la differentia delle altezze, & da queſta regola, ſi trarrà qualche altra coſa, come nella ſeguente carta ſi uedrà.

Fig. 18 Surbased oval arches. Image: Serlio 1545: 13v–14r

ellipsoidal. As a result, the intersections between intrados surfaces are smooth (Fig. 19). By contrast, their counterparts in Islamic examples show hard and neat folds and creases (Fig. 20). This is clear in the vault of the *maqsura*, its lateral spaces and the Capilla de Villaviciosa in Cordova; it shows also in Christian adaptations of the Cordova scheme, ranging from Armenian monasteries to Spanish churches in Torres del Río, Almazán or the crossing towers in the cathedrals of Saragossa and Tarazona (Peropadre 1993: 38–39).

However, a number of problems must be taken into account in connection with this issue. First, the sphere is not a developable surface. Thus, Renaissance architects and builders substituted cone frustums for spherical rings when preparing templates to dress the stones of spherical vaults. This furnishes a pretty good approximation

Fig. 19 Main dome in San Lorenzo in Turin. Detail of rib intersections. Image: author

to the spherical surface, provided that the thickness of the spherical ring is small, compared with the radius of the sphere; this condition is usually met in ashlar courses (Rabasa 1996, 2003).

Where did Guarini take this idea from? Derand (1643: 354–357) offers an efficient procedure to control the form of the voussoirs of hemispherical domes, in particular their intrados surfaces, substituting cone frustums for spherical rings, which represent vault courses (Fig. 21). In the standard solution, courses are horizontal; however, Derand (1643: 360–381) explains a number of alternative solutions with vertical courses following triangular, square, rectangular, or pentagonal layouts, again substituting cone frustums for spherical rings (Fig. 22). Guarini (1671: 588–590) offers quite similar solutions for complex divisions of the sphere (Fig. 23). However, these methods are not Derand's inventions. The idea of substituting cones for spherical rings seems to have originated at the same time in Spain and France during the sixteenth century, since it is used in a full-size drawing in the rooftops of Seville cathedral dated in in 1543–1544 or presented in the treatise by Philibert de l' Orme (Ruiz de la Rosa 2011; De l'Orme 1567: 111v–112v, 114v–115r, 116r–118v).

However, the vaults in San Lorenzo and the ones included in the *Dissegni* … for Sainte-Anne-la-Royal, Padri Somaschi and San Gaetano (Guarini 1686: 4, 6, 9, 11, 12, unnumbered plate) are not continuous but ribbed. Derand (1643: 392–395) includes a ribbed vault in his treatise, solving it by the usual method in Early Modern

Fig. 20 Vault over the first bay of the central nave of Al-Hakem II in the Great Mosque of Cordova. Detail of rib intersections. Image: author

rib vaults, which involves depicting the ribs in true shape and size, as opposed to orthogonal projection (Rabasa 1996), but he introduces some improvements to this method, using implicitly the notion of tangency. In particular, he states that 'you should place its centre at PN, diameter of the diagonal ribs, so the liernes meet at a more pleasant junction with the keystone'.[5] The drawing shows that Derand is placing the centre of the lierne at a vertical line passing through the keystone of the vault (Fig. 24). Thus, the end radii of both liernes are coincident; as a result, the tangent, will be the same, guaranteeing the 'pleasant junction'. Derand does not use the word 'tangency', since he is writing for stonecutters, but it is clear that his method involves a substantial improvement over sixteenth-century procedures. Hernán Ruiz ([c. 1550] 1974: 46v; see Rabasa 1996: 427–429) tackled this problem by trial and error. However, Derand's use of tangency guarantees continuity between ribs in the same plane, but cannot be directly applied to ribs in different planes.

[5] 'faisant neanmoins que le centre d'iceluy marqué T, se trouve dans PN, diametre des diagonales prolongé; & ce afin que les arcs des dites liernes, se trouvent d'une plus agreable rencontre, avec la clef P'.

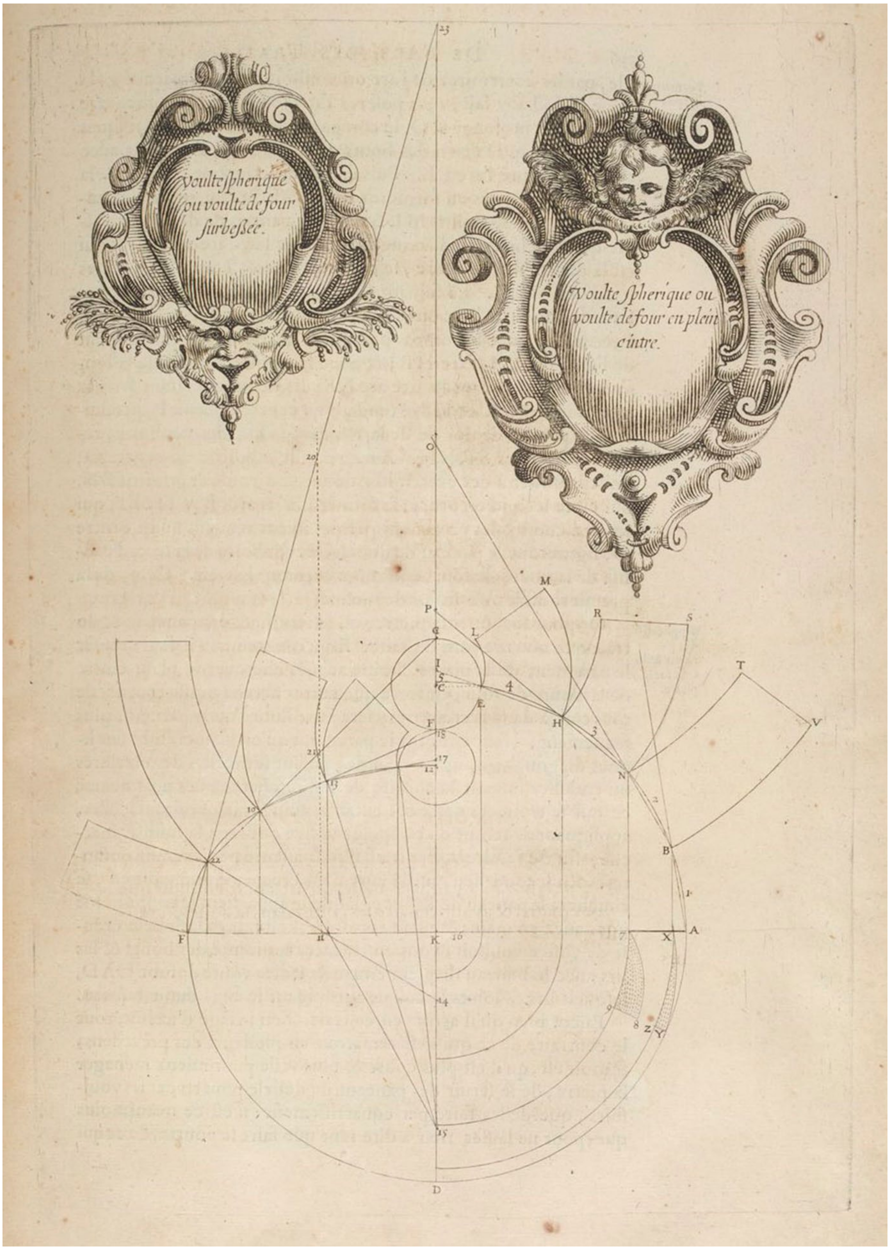


Fig. 21 Left, templates for a surbased elliptical vault with horizontal courses. Right, templates for a hemispherical vault with horizontal courses. Image: Derand 1643: 355

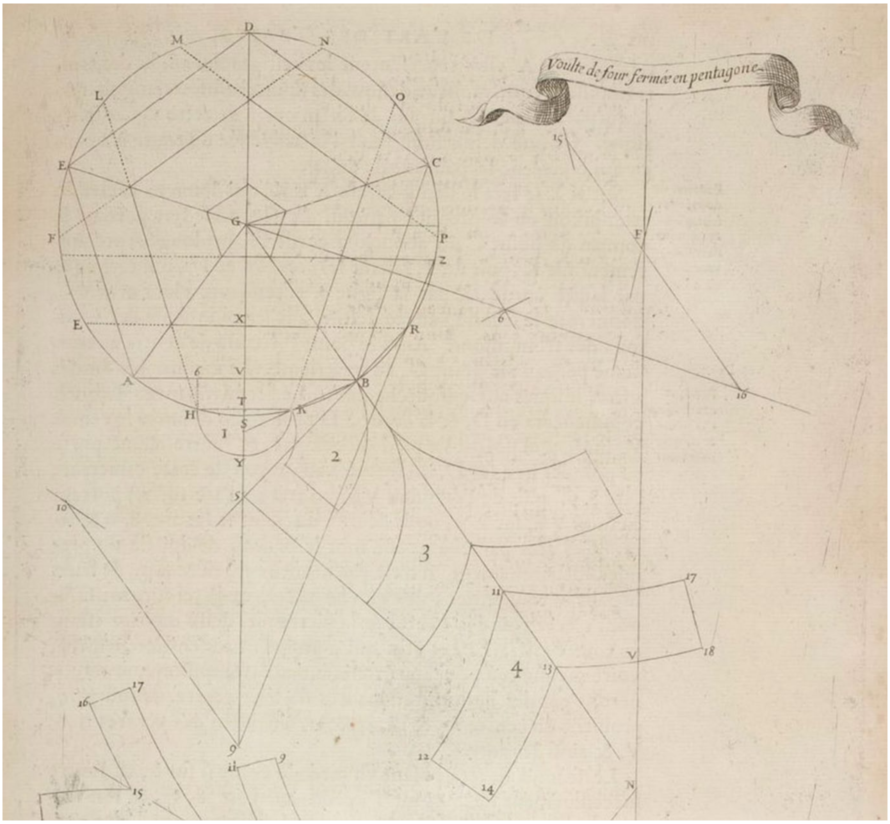


Fig. 22 Templates for a hemispherical vault with vertical courses laid out in the shape of pentagons. Image: Derand 1643: 361

Startingly, the cone-development method was applied to control the shape of ribs (Vandelvira [c. 1580] 1977: 94v–97r). Further, the personal notebook of Alonso de Guardia uses the cone development method to get templates for diagonal ribs—which of course pass through the centre of the vault—and tiercerons—which do not (Fig. 25).

Other details must be taken into account. First, the intrados surface of the ribs in the main vault of San Lorenzo and the project for Padri Somaschi seems to be ellipsoidal. Derand (1643: 358–359) uses the cone-development method for the surbased oval vault, cutting an ellipsoid of revolution by horizontal planes, and obtaining circular sections that can be approximated by ordinary cones, whose developments are simple (Fig. 21). However, he also offers (Derand 1643: 398–400) an alternative solution based on the simpler squaring method, with an

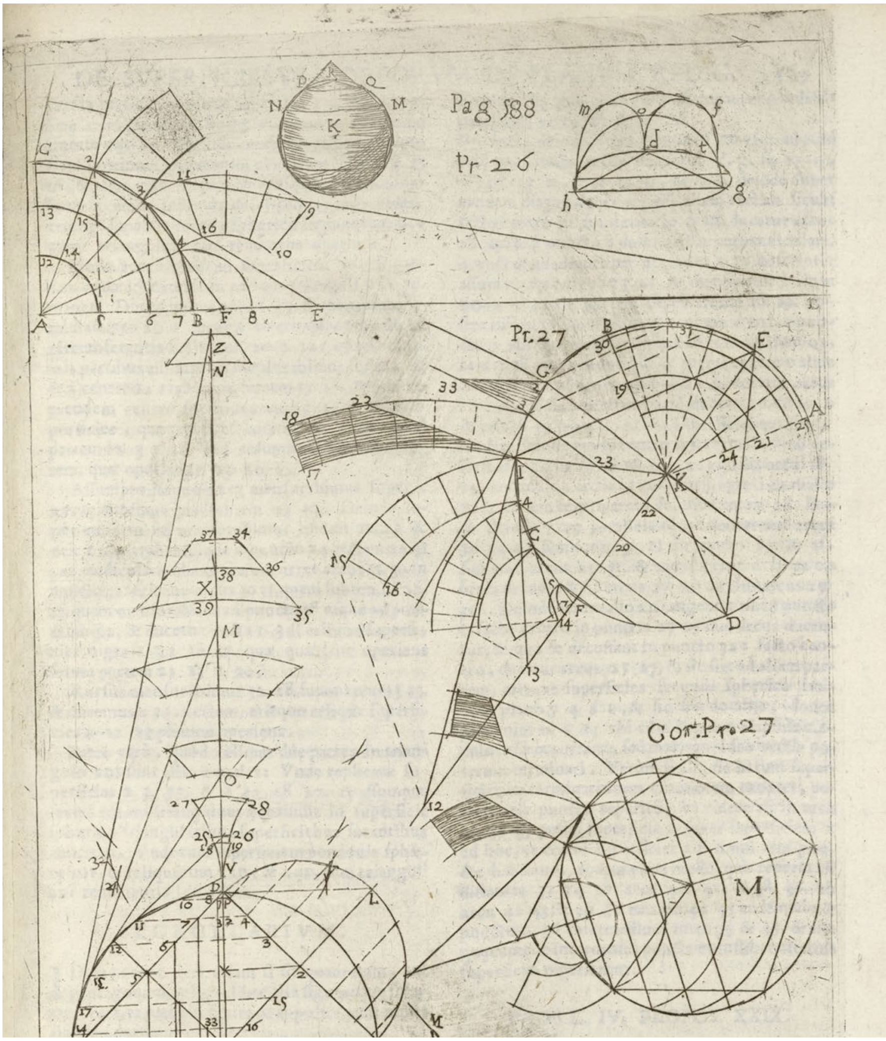


Fig. 23 Templates for hemispherical and other vaults. Image: Guarini 1671: 588

ingenious variant: the whole scheme can be rotated and transformed in a raised oval vault (Fig. 26).

Vandelvira ([c. 1580] 1977: 71v–78r) had followed a similar path in the late sixteenth century. He presents six different oval vaults in his manuscript. The first one ([c. 1580] 1977: 71v–72r) is actually raised: the section along the larger axis

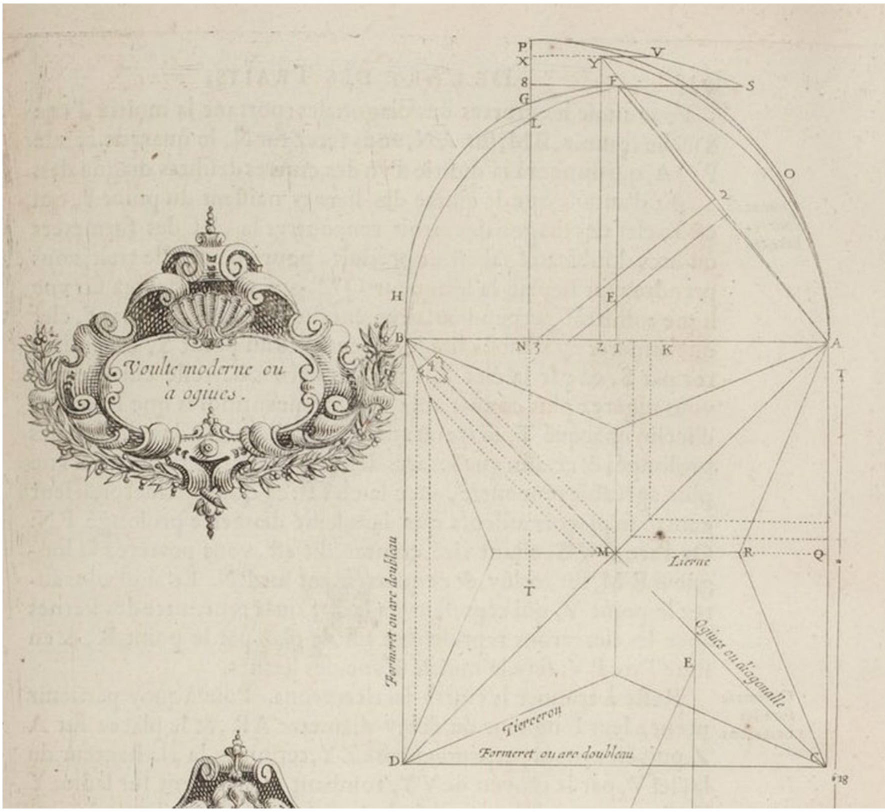


Fig. 24 Formal control of a rib vault. Image: Derand 1643: 393

of the oval or elliptical surface is circular, and thus the cross-section along the lesser axis is a raised oval, since both sections must have their uppermost points at the same height. In this case, he uses vertical semicircular courses parallel to the long axis, so he can easily apply the cone development method. However, in another example (c. 1585: 73v–74r) courses are horizontal and oval. It seems that he does not know a procedure to develop a cone with elliptical cross-sections, and thus he applies triangulation in order to construct templates for the intrados voussoirs of the vault (Fig. 27).

Guarini was surely aware of these procedures, either directly or indirectly. In fact, he included two propositions relevant for these problems in treatise XXXII of *Euclides adauctus* (1671: 594–595; see Bianchini 2008: 163–166 for

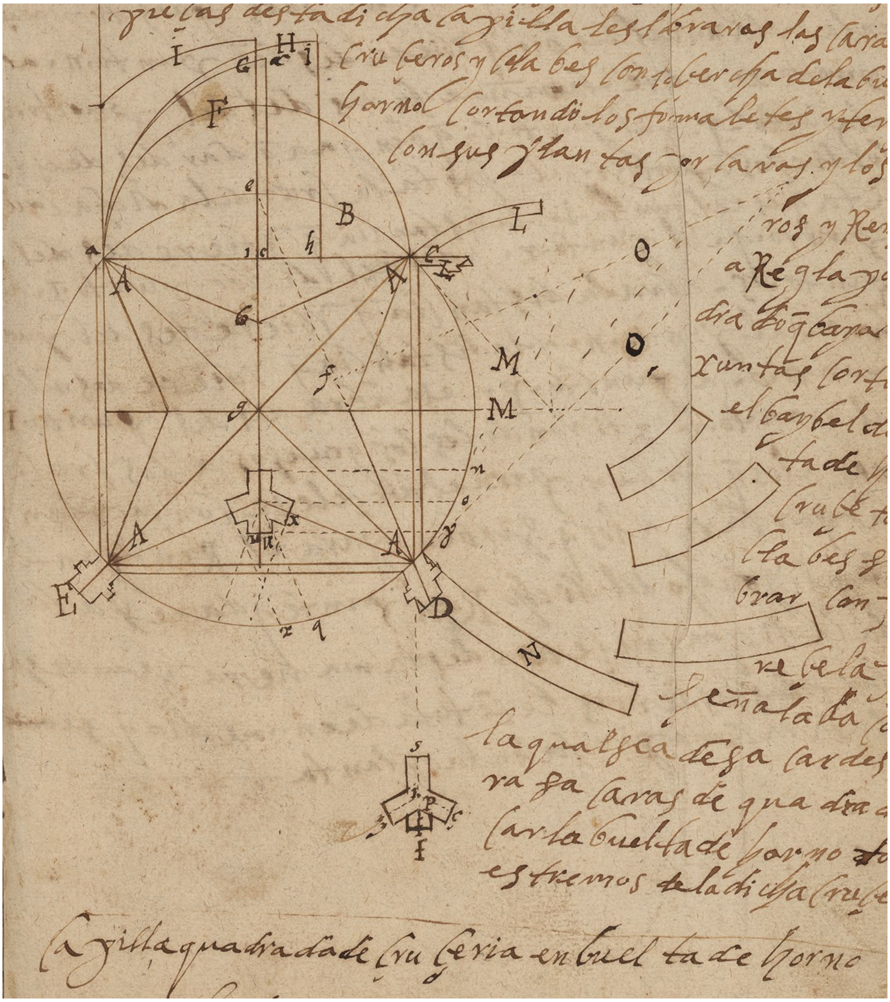


Fig. 25 Cone developments applied to diagonal and tierceron ribs. Image: Alonso de Guardia. 'Rasguños de arquitectura y cantería'. Ms. c. 1600. Madrid, Biblioteca Nacional, ER/4196

a translation from Latin into Italian). Proposition 39 deals with the computation of the oblique sections of the ellipsoid. Given the length of both semiaxes of the resulting ellipse, Guarini applies an affine transformation to a circle in order to construct the resulting section, which is of course an ellipse. Guarini's method (Fig. 28) furnishes the position of the key points in the resulting ellipse, with the same procedure that stonemasons used to locate voussoir corners (Martínez de Aranda [c. 1600] 1986: 3).

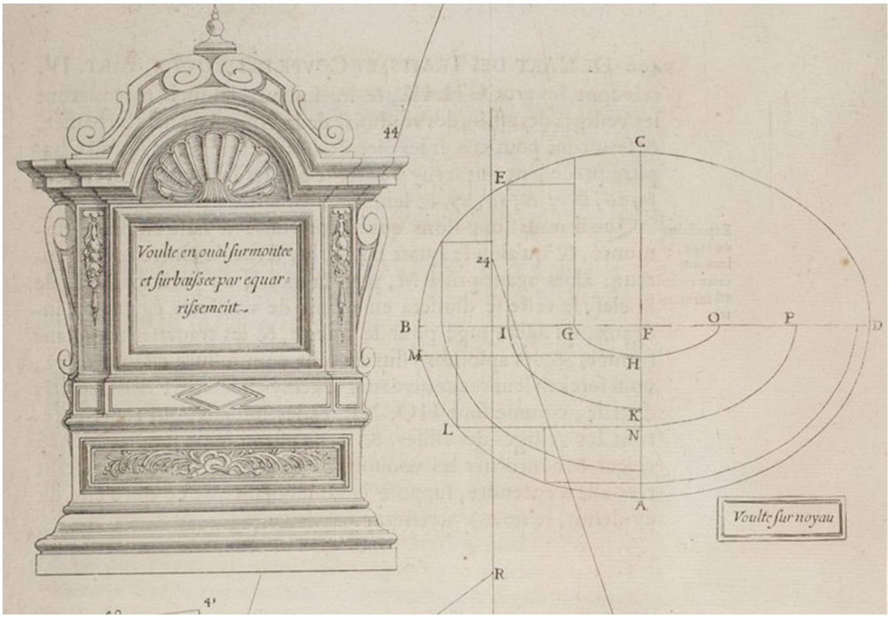


Fig. 26 Surbased oval vault dressed by squaring. The scheme can be reversed and applied to a raised vault. Image: Derand 1643: 399

Next, in proposition 40, Guarini develops the part of the ellipsoidal surface enclosed between two of these sections, that is, an ellipsoidal ring. In order to do so, he measures the distances between the points he has marked in the elliptical sections by means of proposition 39. Again, the result strongly resembles stonecutting procedures, in particular the gores used by Vandelvira ([c. 1580] 1977: 63r, 64r, 74r, 75r) and Derand (1643: 303, 307, 317), akin to the ones employed in the construction of globes. However, there is a significant difference. Derand applies this technique to symmetrical squinches, not to vaults proper, getting symmetrical gores. Vandelvira applies it to spherical and oval vaults, constructing both symmetrical gores and asymmetrical ones. Guarini, of course, gets strikingly distorted gores for his oblique ellipsoidal rings (Fig. 29).

Conclusion

The planar layouts of the Turin main dome and lantern closely resemble Islamic ones, in particular the ones over the *maqsura* in the mosque of Cordova. However, in the presbytery in San Lorenzo, the projects for Padri Somaschi and

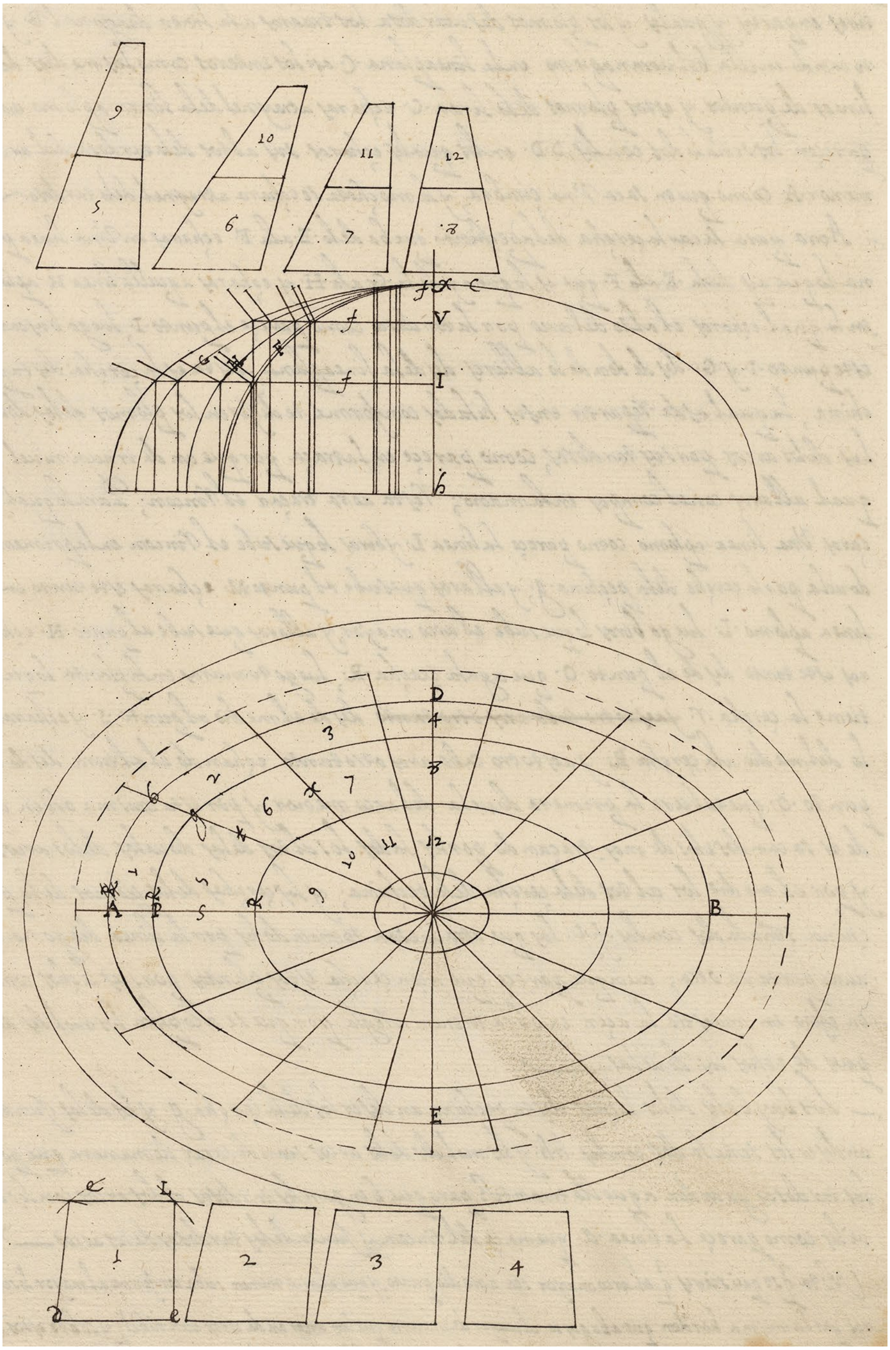


Fig. 27 Formal control of a vault with horizontal oval courses by triangulation. Image: Alonso de Vandelvira, 'Libro de trazas de cortes de piedras'. Copy by Felipe Lázaro de Goiti, 1646. Biblioteca Nacional, Madrid, Ms. 12.719

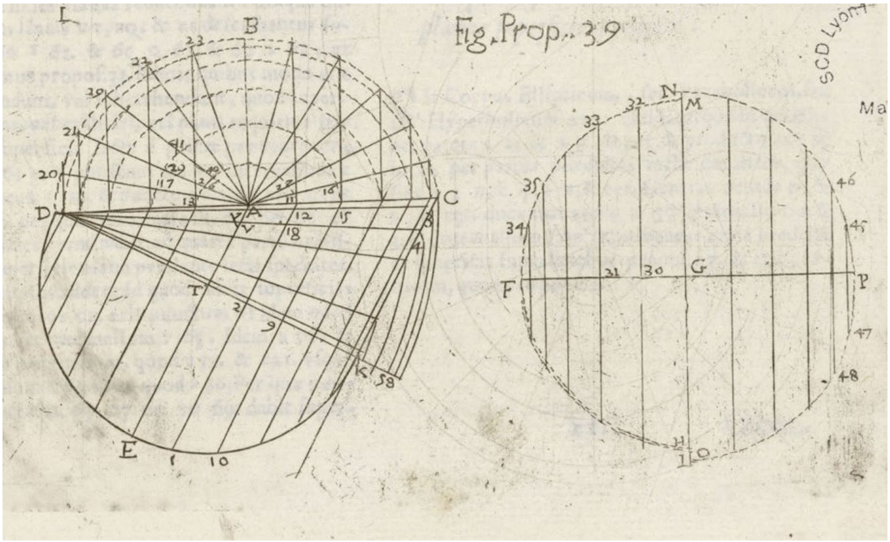


Fig. 28 Construction of oblique sections of the ellipsoid. Image: Guarini 1671: 592

San Gaetano, and, more importantly, Saint-Anne-la-Royale, which predates San Lorenzo, the similarity with Islamic built vaults is only generic. Thus, rather than direct observation of actual Islamic vaults by Guarini, a transmission through al-Ḥajjāj ibn Yūsuf ibn Maṭar, Adelard of Bath, Bradwardine or other sources seems more likely.

We should also take into account that, even in San Lorenzo, the elevation layout and the intrados surface of the ribs depart clearly from these particular Islamic examples. The raised arches seem to derive from Derand, although there are a number of precedents in the corridors in the Escorial, Cádiz old cathedral, and the Jesuit college of La Flêche. Stonecutting writers, including Derand, used conical developments for spherical and even ellipsoidal vaults; also, Vandelvira and Guardia applied this method to ribbed vaults. However, neither Derand nor Vandelvira used the cone development method when they dealt with oval or elliptical courses; in this case, they resorted to enclosing solids or triangulation. Guarini went a step further and explained an abstract, as opposed to empirical, method for the development of orthogonal or oblique sections of the ellipsoid in his *Euclides adauctus* (1671: 594–595). Quite probably, he used this procedure in the ribbed raised elliptical vaults of San Lorenzo, and he was planning to use it in the unbuilt Church for Padri Somaschi in Messina.

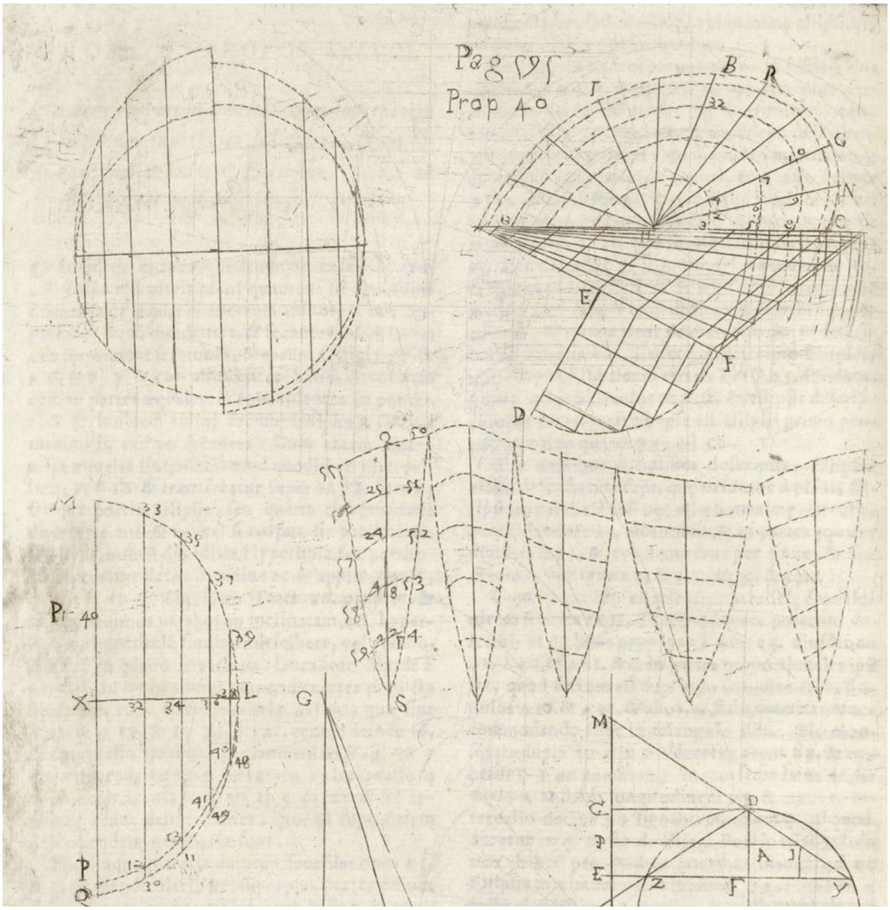


Fig. 29 Development of oblique rings of the ellipsoid. Image: Guarini 1671: 594

All this suggests that Guarini was directly or indirectly aware of the procedures used by Vandelvira and Derand to control the shape of ellipsoidal vaults and elliptical arches, but he refined them, progressing from empirical, artisanal methods to a mathematical approach.

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