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Drawing, Shapes and Parametric Design of Rod and Node Architectural Structures

Ornella Zerlenga[1] · **Vincenzo Cirillo**[1] · **Margherita Cicala**[1]

Abstract

This research focuses on extensible structures composed of rods and nodes, which have been developed since the mid-twentieth century. A geometric, mathematical, and graphical analysis of simple double-rod scissor structures, and then investigates the three-rod and node structures of architect E. P. Piñero, using digital and parametric modeling.

Keywords Extensible structures · Rods and nodes · Parametric design · Geometric shapes

Introduction

Since ancient times man has needed to build architectural elements characterized by: mobility, transformability, assembly and disassembly. This need derives from nomadic and warrior peoples who, out of necessity, erected tents, bridges and towers made from rigid and transformable grids. Among these kinematic structures are 'extensible' ones: a complex of building elements that move from a closed, cohesive configuration to an 'extended'-self-supporting one.

B. Fuller, J.W. Schweller, G. Bell, K. Wachsman, E.P. Pinero, and F. Escrig are some architects who experimented with 'transformable' structures in the second half of the 20th century.

The aim of the paper is to analyze the structure and 'primary' geometries of double-rod scissor structures, and then those with rods and nodes by Emilio Pérez

✉ Margherita Cicala
 margherita.cicala@unicampania.it

 Ornella Zerlenga
 ornella.zerlenga@unicampania.it

 Vincenzo Cirillo
 vincenzo.cirillo@unicampania.it

1 Department of Architecture and Industrial Design, University of Campania Luigi Vanvitelli, Via San Lorenzo, 81031 Aversa, Italy

Piñero (1935–1972) through the graphical analysis of a patent by the Spanish architect (Piñero 1961), with the dual methodology: digital modeling as a tool for investigation and visualization of geometries; parametric design software for movement analysis.

The Scissor System for Extensible Structures

Architectural structures made by rods and nodes correspond in figurative terms to geometric relationships through the intersection of the fundamental geometric entities of line and point. This theoretical condition appears applied in constructivist architecture and in modern and contemporary architecture. The resulting works turn out to be in their form defined by Kandinsky as pure or abstract in space (Kandinsky 1926) and which, in becoming matter, take on, according to the artist, a conspicuous character generated by a line-force application energy on a nodal point (Fig. 1a).

These structures are based on the principle of repetition of a geometric starting module, the joining of which, allows different movements of the final structure. The transition from single units to a group determines the formulation of the extensible system and, where the movement takes place through skillful geometric-mathematical control (Calvo López and Sanz Alarcón 2011). Their analysis, therefore, requires knowledge and control tools through parametric modeling systems for the investigation of shape, number of rods and nodes, directions, and degrees of movement.

Extensible structures have two different geometric configurations: one extended (open) and one retracted (closed) (Fig. 1b on the left). The transition from open to closed configuration can be made by sliding (translation), rotating, bending, or by a combination of the same. The important requirements for the perfect fabrication of this structure are ease of fabrication and structural stability over large areas where
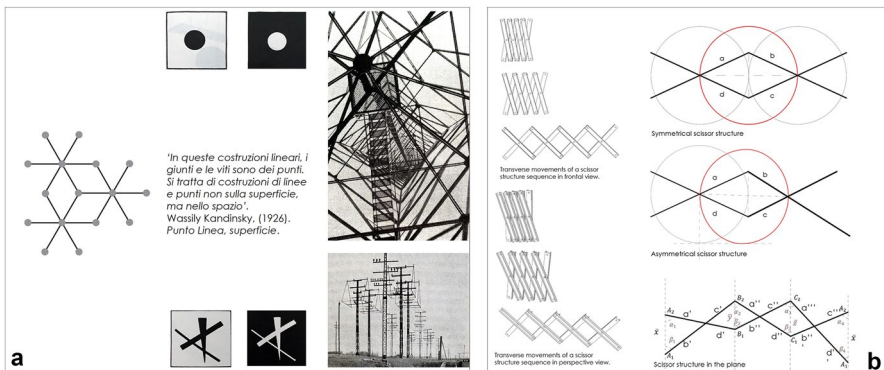
Fig. 1 a Link between geometric entities and rod and node structures in space according to Wassily Kandinsky; **b** scissor structures: transverse movement of a sequence of scissor structure in frontal and perspective view (left); symmetrical, asymmetrical and in-plane scissor structure (right). (Graphic image processing by M. Cicala)

they will assume an open configuration that also serves, at times, as a cover for the space.

The most common type of extensible structure is the scissor type with double rods, which allows for foldable membranes on rigid supports. Each rod consists of three restraints, an upper and a bottom one, determining the continuity of the structure and, finally, a joining bond between the two rods. The union of multiple elementary scissor structures determines the configuration of modular one-degree-of-freedom structures consisting of the angle of rotation around the middle hinge. As for the node, however, three different configurations and types of ties can be obtained: translation, polar, angled (Fig. 2c).

According how the rods are arranged, different interior geometries are generated in relation to movement, which are defined by regular or irregular quadrilaterals or deltoids: regular quadrilaterals for linear deployment; deltoids with two sets of consecutive congruent sides for variable curvature deployment; irregular quadrilaterals for curvature systems that can't flexed (Torres 2013) (Fig. 2a).

These structures joined sequentially together enable the composition of extensible flat structures of any shape provided there is a compatibility relationship between the bars. For the structure to be flat the condition must be: $c/a = d/b$; for it to be collapsible must be: $c + d = e + e = 2e$ (Fig. 1b on the right). Thus the 'scissors' all extend along the direction of development.

The analysis of the kinematic mechanism of double-rod scissor structures follows the geometric analysis of the isosceles triangles that are formed, which will, again, turn into rhombs in case of a multi-stage scissor mechanism both horizontally and vertically (Cheng et al. 2014). Given the scissor structure (Fig. 3) that has $\theta' = \theta''$ and DOC determining an isosceles triangle, we will have that $l_{co} = l/2 = l_{od}$, in a given length of L on which h will depend. Applying the Pythagorean Theorem on the basic triangle will have:
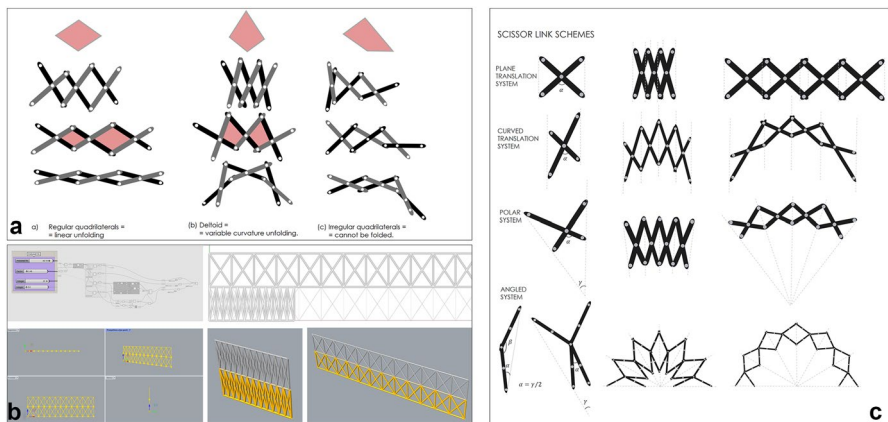
Fig. 2 a Geometric definition of scissors generating linear and curved course and related internal geometries generated; **b** parametric construction of a simple scissor structure in *Grasshopper for Rhino*; **c** the different patterns of scissor bonds: plane, curved, polar and angled translation system. (Parametric modeling and graphic image processing by M. Cicala)
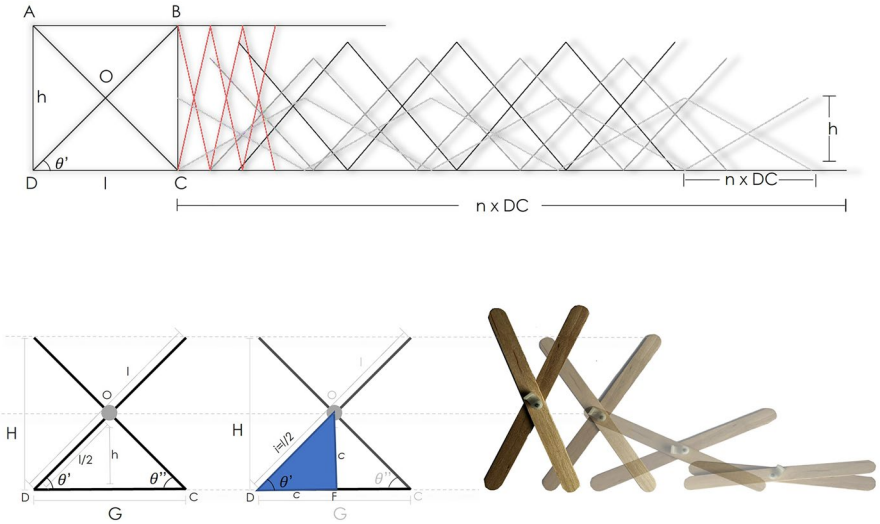
Fig. 3 Linear transverse motion analysis of a scissor structure with related geometric construction. (Graphic image processing by M. Cicala Graphic)

$$\left(\frac{l}{2}\right)^2 = h^2 + \left(\frac{S}{2}\right)^2$$

$$\frac{l^2}{4} = h^2 + \frac{S^2}{4}$$

$$h = \sqrt{\frac{l^2}{4} - \frac{S^2}{4}}$$

From here, considering θ' and given the ratio of the cathetus opposite the angle α to the hypotenuse, we will have:

$$sen\theta = \frac{h}{i}$$

$$h = sin\theta x i$$

$$i = \frac{l}{2} h = sen\theta \frac{l}{2}$$

This formula demonstrates the dependence relationship between the length, height and angle of rotation, respectively, where it is shown that h varies in relation to the change in the angle between the cathetus and the hypotenuse (Fig. 3).

Shape Grammar and Parametric Analysis of Rod and Node Structures by E. P. Piñero

Emilio Pérez Piñero is a precursor of scissor structures. From 1961 he dedicated himself to the development of flat grids and domes for temporary pavilions (Escrig 1993) (Fig. 4a). Geometrically, the insertion of an additional bar in the central node is the first significant transition from double scissor structures to Piñero's extensible structures, referring back to the mechanism of the 'hunter's chair' (Fig. 4b).

In their movement, these modules can be bent and unfolded, either for a simple structure or composed of several modules. Through the unfolding of a straight module that results in a flat grid, with a change in the lengths of the top and bottom sections of the bars, a module is obtained that generates a curved surface.

Principal among the architect's patents is *Patent US3185164* of 1961. The folding three-dimensional reticular structure of this is given by a system of rods connected through pivoting joints. These rods can be distributed in three-dimensional space by deploying until their connections result in an easily transportable closed compact bundle of straight lines.

For this condition to happen, design components assume considerable importance. The rods, nodes, joints, and possible connections between them are the basis of the geometric configurations of opening and closing.

Regarding nodes, we can consider two types: closed and open, both connecting between modules of structures and with a 6 mm thickness. Then there are the 'three nodes' that consist of a central node and three rods, in addition to anchor bolts, the distance between the rods is given by angles of 120°. Also, there are the 'four knots' formed by four rods anchored by bolts whose distance gives angles of 90° (Figs. 4c; 5a, b).

For rods, the composition of which is based on the decomposition of basic polyhedra, these can be three or four noded, which must be placed in the equidistant direction for flat structures and, 'free', (not equal) for curved structures (Fig. 5d).
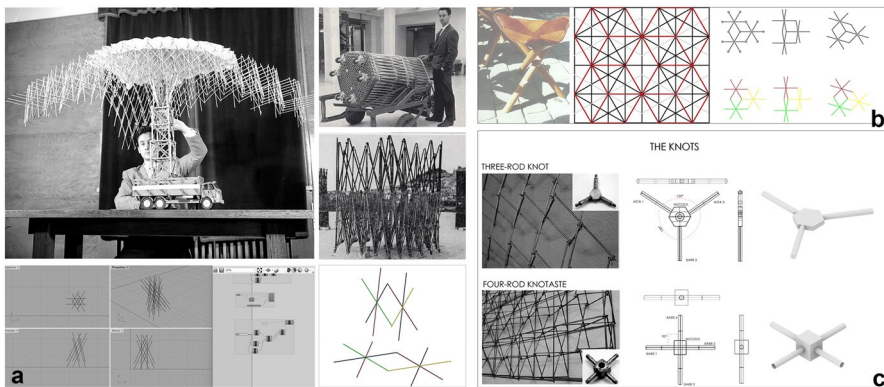
Fig. 4 a Summary images of Piñero's extensible structures, from physical models to digital parameterization; **b** from the three-rod 'hunter's chair' to Piñero's rod and joint meshes; **c** detail drawings of the types of nodes patented by the architect. (Modeling and graphic image processing by M. Cicala)
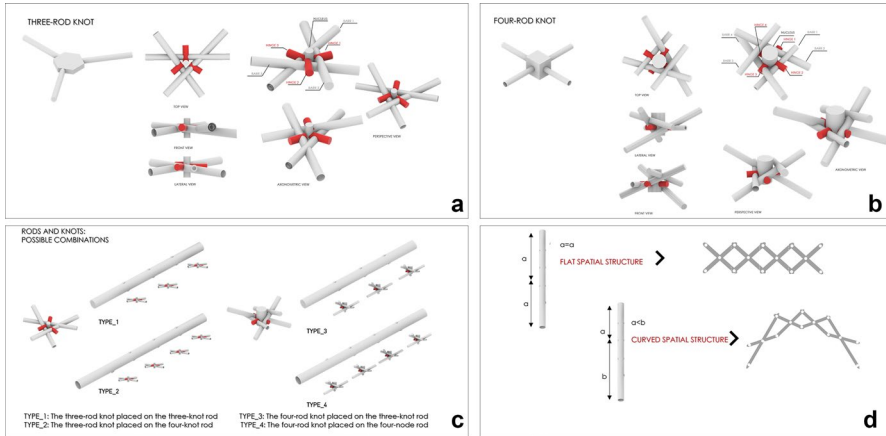
Fig. 5 Views of three-dimensional models Piñero's patent. **a** and **b** the two types of nodes at three and four rod knots; **c** three-dimensional views of the possible combinations of rods and nodes, patented by the architect, **d** three-dimensional views of the flat and curved spatial structures. (Modeling and graphic image processing by M. Cicala)

From the different combinations of rod types it's possible to generate simple, double, multiple, semi-cylindrical, and hemispherical structures (Fig. 5c).

The system of Piñero's basic module is given by three rods articulated in a central joint inscribed in a prism given by bases of equilateral triangles (De Miguel 1972; Ródenas-López et al. 2018), that is, the two Platonic solids (the regular tetrahedra) whose intersection of one of the vertices flows into the central node of the module of the structure. In contrast, the second system has four rods that meet in the central pivot inscribed in square-based prism, that is, two regular pentahedra. The condition in which scissor structures are simply inscribable in a prism and, do not generate solid geometries, occurs when a simple scissor system is placed along the surfaces of the sides of the prism and articulated along the midpoint. This condition will be successive to Piñero and occur in F. Escrig's structures (Valcárcel 2019).

For the movement investigation of the architect's basic three-rod module, the use of modeling in a digital environment is insufficient except for the geometric knowledge of the individual components (Fig. 6a). Therefore, it was necessary to use parametric modeling with Grasshopper for Rhino software (Fig. 2b) and, specifically, through the Kangaroo plug-in (Figs. 6b, c), which can manage by algorithm the interactions and dynamism of the module components.

This operation allowed us to define the movements of the rods and nodes in different variables and study the various geometries of the modules in relation to the rods and source nodes.

There are two parameters that affect the geometric characteristics of the structure under open or closed conditions: the number of rods encountered in an assemblage and the equality and inequality relationship of the distances of each intermediate coupling of each rod (*Piñero* 1961). The first case influences the geometric shape of the configuration in the plane: if three rods concur at each conjunction, the
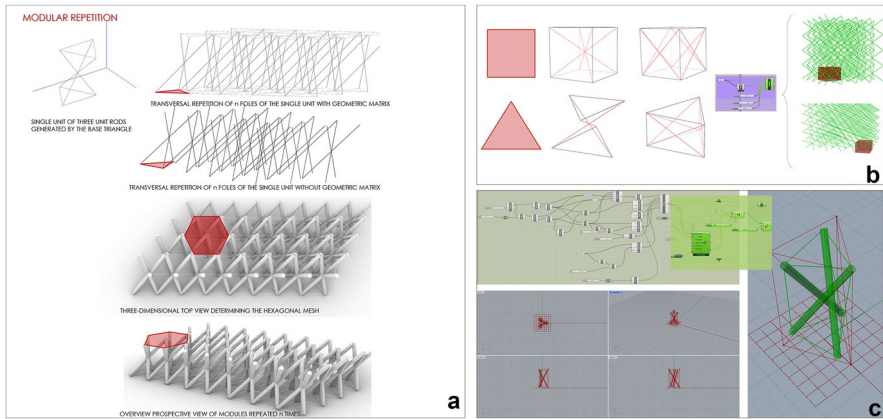
Fig. 6 **a** Geometry of Piñero's patented three-rod base module geometry and modeling with repetition of the base module in the plane; **b** Piñero's and Escrig's base modules processed by *Grasshopper for Rhino* algorithm and respective modular repetition; **c** Piñero's base module processed by *Grasshopper for Rhino* through the use of the *Kangaroo* plug-in whose algorithm is reported (Simple and parametric modeling and, graphic image processing, by M. Cicala)

contour explained will take the shape of a hexagon (Fig. 6); if four rods concur, the configuration will be quadrangular (Fig. 6b). In the second parameter, if the intermediate rod ratio is equidistant from the other two extremes (top and bottom), the structure has no curvature. This means that overall and ignoring the height, the structure is flat, that is, the series of repeated modules with equal characteristics belong to parallel planes. In opposition, if the intermediate coupling is not equidistant to the other two, the open structure defines a curved configuration, that is, the rod couplings lie on one of the three curved surfaces.

Conclusions

From simple scissor lattice structures, to the Egyptian chair, to the hunter's chair, as examples of extensible structures, the progress these have made has been considerable. The aim here was to highlight how much the apparent simplicity of these structures springs from and results in different geometric conformations derived from mathematical calculations.

The grammatical and syntactic study of the forms of the scissor structures and, of Piñero's extensible structures, turns out here to be a first cognitive approach oriented to a subsequent application of these extensible structures not used as an autonomous element of new construction, but as an architectural support of cultural heritage enhancement. Specifically, for the enhancement of slender architectures (Zerlenga et al. 2021), this topic is part of a broader research aimed at enhancing the value of contemporary Neapolitan bell towers, built before anti-seismic regulations and, therefore, subject to architectural vulnerability in case of natural disasters.

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Ornella Zerlenga   is Ph.D. in "Rilievo e Rappresentazione del Costruito" (Building Surveying and Representation) in 2004. Her Academic Career is at University of Campania "Luigi Vanvitelli" in "Drawing" (S.D.S ICAR/17 "Disegno"—08/E1): on 2000, Academic Researcher; on 2011, Associate Professor; since 2011, Full Professor. She was President of the Bachelor Degree courses in "Design and Communication" and "Fashion Design", and Master Degree course in "Architecture". She is the Rector's delegate for e-learning. Since 2015, she was deputy Director of the Department of Architecture and Industrial Design. Since December 2020, she is Director of the Department of Architecture and Industrial Design of the University of Campania "Luigi Vanvitelli". She was honoured several prizes and awards, during her academic career: 1989, "Luigi Vagnetti" Prize of the "Unione Italiana per il Disegno" (UID); 1992, "Storia del Disegno di Architettura" Prize (History of Architectural Drawing) of the "XY dimensioni del disegno" Scientific Journal; 2008, "Targa d'Argento" Award of the "Unione Italiana per il Disegno"; 2013, "Ianuarius" Award of the Treasure of San Gennaro Museum (Naples); 2022, Best Paper Award at 43th UID International Congress (paper on "The sound of light. New narrations for the bell tower of Santa Chiara in Naples", Experimentation session). She is a member of the teaching staff of the PhD in "Architecture, Industrial Design and Cultural Heritage" (University of Campania, and National PhD Program on "Heritage Science"). Since 2017, she teaches in the courses on "Didactics of Technical Drawing" and "Didactics of Technology", as part of the "24 CFU training course" (Italian Legislative Decree no. 59, art. 5; 2017, 13 April). She was member of funded PRIN in 2009, 2007, 2005, 2003, 2002, 2000. She is Group Leader of the Unit of University of Campania in funded research project "PREVENT. Integrated PRocedure for assEssing and improVing the resiliENce of existing masonry bell Towers at territorial scale" (funded for € 261.000,00); P.I. prof. Gianfranco De Matteis; partners: Ministero dei Beni e delle Attività Culturali; SUNTA s.r.l.; PROTOM GROUP Spa; VIRTO360. She is the coordinator of the research group "Knowledge, valorisation and digital communication of cultural heritage" at the Department of Architecture and Industrial Design of the University of Campania. She is founding partner of the "Eikonocity. History and Iconography of European Cities and Sites" Journal (University of Naples Federico II). She is member of the scientific board of the "Centro Interdipartimentale di Ricerca sull'Iconografia della Città Europea" (CIRICE, University of Naples Federico II). Since 2016, she is director of the editorial collection "Themes and frontiers of knowledge and design", La Scuola di Pitagora publishing. She is member of the scientific committee of the Journals: Diségno; XY digitale; DisegnareCon; Eikonocity; IMG-Journal; and editorial collections; national and international congress and expositions. Since 2013, she is member of the Technical-Scientific Committee of the "Unione Italiana per il Disegno" (UID); and since 2019, she is its Bursar.

Vincenzo Cirillo   graduated with honours in Architecture in 2013 (U.E.) at the University of Campania *Luigi Vanvitelli*. In 2019 he is Ph.D. and *Doctor Europeaus* in *ArchitectureIndustrial Design and Cultural Heritage* with a doctoral thesis in the discipline of representation (ICAR/17) entitled*Reflections and suggestions between geometry and form. The Neapolitan staircases of the Eighteenth century*. From 2019 to 2021 he is Research Fellow at the University of Campania *Luigi Vanvitelli* (SSD ICAR/17) with a research program entitled*Eighteenth-century celebrations in Naples. The design of ephemeral architectures*. From 2021 he is Assistant Professor at the University of Campania *Luigi Vanvitelli* (SSD ICAR/17). His research interests include the geometric analysis of architectural space, the digital representation of architecture and the environment, the graphical analysis of design and visual communication projects. He has participated in numerous international conferences, published in scientific journals and monographs. He has more than 80 peer-reviewed publications.

Margherita Cicala   is architect. She graduated in Architecture in 2016 (U.E.) at the University of Campania *Luigi Vanvitelli*. From 2019, she is Ph.D. student in *ArchitectureIndustrial Design and Cultural Heritage* with an interdisciplinary thesis on the methods of architectural survey of bell towers in Campania of the contemporary age and on the techniques of structural enhancement in steel and knots in cases of critical vulnerability. She is member of the Research Group *Knowledge, valorisation and digitization of cultural heritage* of the Department of Architecture and Industrial Design at University of Campania "Luigi Vanvitelli".