Reading Errors in Sundials with Incorrect Hour Lines: The Seventeenth-Century Sundial in Lerma

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Abstract The restoration project of a seventeenth century sundial at Lerma (Spain) with incorrect engraved marks for the hour lines and devoid of its gnomon raised the issue of whether it is possible to predict what might be the reading errors when providing a new gnomon for it. Moreover, the analysis of possible slight changes in gnomon's orientation and/or geometrical features to get better precisions was an appealing challenge. We must consider that Lerma's case is not unique: templates were used in the past to engrave marks in numerous dials, and small differences in orientation were not taken into account. Differences in latitude under 0.5° were also neglected, by rounding off to integer values. Although Lerma's sundial has been our guiding thread and its results are presented in detail, the method can be extrapolated to calculate reading errors for any dials with wrong marks for orientation or latitude different from the actual ones.

Keywords Sundials · Gnomons · Solar hours

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Introduction

The sundial located on the main façade of Colegiata de San Pedro, Lerma, Spain (Figs. 1, 2 and 3), is currently devoid of its gnomon and a restoration project is currently being considered for it. Its hour and half-hour lines are engraved in a limestone dial placed on a buttress. A vertical slit in the line of the XII hour suggests that the original caster was a triangular plate, perpendicular to the dial. However, remnants of mortar indicate that a shoddy repair with a steel bar and a supporting stick was attempted in the nineteenth or twentieth century. The analysis of the engraved hour lines indicates that the dial is correctly marked for its latitude and for a wall perfectly oriented southwards (Figs. 3 and 4). However, the southern façade, though perfectly vertical (not *inclined*¹), is rotated 6.6° westwards. The angle between the direction vector of the dial's plane and the South direction, called the wall declination, δ_w (Fig. 5), is taken conventionally as positive measured from the South eastwards (Soler 1997), so we have, $\delta_w = -6.6^\circ$. Therefore, its marks should have been those of Fig. 6.² This geometrical issue will lead to reading errors when a new gnomon is placed, so it is convenient to quantify them. These kinds of error in the marks were not infrequent in sundials on walls slightly rotated from South: templates and pre-calculated tables for dials facing exactly South were often used, and fractions of degrees of latitude were not taken into account. In the case of Lerma's dial, however, only the orientation is wrong: the difference between 42° , the latitude considered, and 42.03° , the actual latitude, is absolutely negligible.

It is not only reading errors that must be calculated for a gnomon presumably similar to the original historic one: with the help of an imaginary group of correct marks, slightly different suitable orientations and angular values for the new gnomon will be tested in order to minimize, if possible, the magnitude of errors. Therefore, the procedure developed in this paper will be useful not only for Lerma's sundial but also for any vertical sundial with mistaken hour lines. First of all, we'll analyse as an example the intervals of the day in which Lerma's dial is operative. Next, we'll develop the geometrical derivations to check different choices for its gnomon, showing the results both numerically and graphically according to the different calculation options. Finally, extrapolation to predict errors in all vertical sundials with problems of orientation or latitude will be stated. Errors in inclined sundials will not be analysed.

Time Intervals in Which the Dial is Operational

Analysing the time intervals when a sundial receives the Sun rays directly throughout the year will allow us to limit the study of errors to those hours in which it is operational in order to prevent obtaining nonsensical numerical solutions.

¹ An inclined sundial is also known as reclined. These terms are used for dials located in walls contained in a plane that is not perfectly vertical.

² Theoretical hour lines graph can be easily obtained from available applications such as "Shadows" (Blateyron 2017), "Orologio Solari" (Casalegno 2017), etc.

Fig. 1 The Colegiata de Lerma and the position of the sundial

Fig. 2 The sundial's location Image: Authors, after (Cervera Vera 1981) and (SIGPAC 2017)

Lerma's sundial will be our guiding thread. Of course, as Albert Waugh says, “no vertical sundial can ever catch the sun's rays for more than 12 of the 24 h in a day” (1973, p. 83). Our first input data will be δ_w and the latitude φ of an observer located exactly at the same point that the dial. This point is called V in Fig. 7. It must be remarked that all the calculations will be made for local solar time, without any corrections due to equation of time or to daylight saving time. It makes no sense for the aim of this paper to make any comparisons with the hours we can read in our wristwatches.

Fig. 3 The existing dial, without gnomon. Remnants of an attempt of repair can be seen

Fig. 4 Theoretical hour lines for $\varphi = 42.03^\circ$, wall facing South ($\delta_w = 0^\circ$)

Fig. 5 Definition of the wall declination, δ_w , as commonly accepted in gnomonics

Fig. 6 Hour lines for $\varphi = 42.03^\circ$, wall facing South-West ($\delta_w = -6.6^\circ$)

Fig. 7 Celestial sphere from the observer (point V). Interference of the wall at the instants of sunrise and sunset

Daylight Intervals due to the Dial's Orientation

If we represent the celestial sphere in orthographic projections centred at an observer at point V (the sundial's location, as stated before), we can see the interference of the wall with the Sun at sunrise and sunset due to its orientation (Fig. 7). We will assume some simplifications: solar declination³ δ as invariable throughout the day, and absence of atmospheric refraction. Instants of sunrise and sunset will be those when the centre of the Sun is placed on the horizon line.

In order to perform the calculations, we shall use the well-known astronomical formulae for sun's azimuth's and elevation (Meeus 1998):

Sun elevation $\hat{e}$:

$$\sin \hat{e} = \cos \omega \cos \delta \cos \varphi + \sin \varphi \sin \delta. \quad (1)$$

with: ω = sun's hour angle, φ = observer's latitude (for Lerma's sundial, $\varphi = 42.03^\circ$), δ = sun's declination

Solar azimuth γ (measured from North and clockwise):

$$\cos \gamma = \frac{\sin \delta - \sin \varphi \sin \hat{e}}{\cos \varphi \cos \hat{e}} \quad (2)$$

At sunrise and sunset, $\hat{e} = 0$, so, from Eq. 1:

$$\cos \omega = -\tan \varphi \tan \delta \quad (3)$$

And from Eq. 2:

$$\cos \gamma = \frac{\sin \delta}{\cos \varphi} \quad (4)$$

The wall azimuth of Lerma's sundial can be taken as $\gamma = 90^\circ - \delta_w = 96.6^\circ$ at sunrise and $\gamma = 90^\circ - \delta_w + 180^\circ = 276.6^\circ$ at sunset, so from Eq.4 we can easily obtain δ , taking into account that in this case $\varphi = 42.026944^\circ$.

Therefore,

$$\delta_1 = -4.898^\circ \text{ (sunrise)} \quad \delta_2 = 4.898^\circ \text{ (sunset)}.$$

If we have a look at any almanac for 2017 or, better, if we use the NOAA on-line application (NOAA 2017), dates for these declinations can be obtained. Other interactive almanacs, such as *Multiyear Interactive Computer Almanac* (U.S. Naval Observatory 2005), can also be helpful. The inputs are noon for Lerma (longitude 3.76°W), at 12:00 local time, and Time Zone + 1.

According to NOAA application for 2017: δ_1 corresponds to 8 March and 5 October ($\delta < 0$), and in these dates the sun's path cuts the base of the wall at B in the instant of sunrise; δ_2 corresponds to 2 April and 10 September ($\delta > 0$), and in these dates, the sun's path intersects the wall's basis at A (Fig. 7) in the sunset (Fig. 7). Therefore, in the intervals from 1 January to 8 March and from 5 October to 31 December, the observer located at point V on the dial can see the sunrise, and

³ Solar declination is called α in some books on sundials. We prefer the astronomical notation δ despite its resemblance with δ_w : α is used in astronomy for right ascension.

the same observer can see the sunset in the intervals from 1 January to 2 April and from 10 September to 31 December.

Local solar times (H_s)⁴ for sunrise and sunset can be obtained from the expression:

$$\omega = (H_s - 12) \cdot 15$$

which we transform for our calculations as:

$$H_s = 12 + \frac{\omega}{15} \quad (5)$$

Therefore, taking into account Eq. 3,

$$H_s = 12 + \arccos(-\tan \varphi \tan \delta) \quad (6)$$

For the dates when sunrise or sunset cannot be seen from the observer, we can calculate the instants of Sun's disappearing behind the wall using the astronomical expression:

$$\tan \gamma = \frac{\sin \omega}{\sin \varphi \cos \omega - \tan \delta \cos \varphi} \quad (7)$$

where γ is the azimuth of the wall's plane. Note that⁵

$$\gamma = \delta_w + 90^\circ$$

Equation 7 is commonly used in astronomy. It is quoted in (Meeus 1998), (Duffie and Beckman 2013), (Blanco-Muriel et al. 2001), and others, and is extensively analysed in (Sproul 2007). In this case, we know γ , so starting from the previous Eq. 7, we can transform it to directly obtain ω :

$$\omega = \arccos\left(\frac{\tan^2 \gamma \sin \varphi \cos \varphi \tan \delta \pm \sqrt{\tan^2 \gamma \sin^2 \varphi - \tan^2 \gamma \tan^2 \delta \cos^2 \varphi + 1}}{\tan^2 \gamma \sin^2 \varphi + 1}\right) \quad (8)$$

For each δ we obtain two values of ω , that is, ω_1 and ω_2 . Therefore, the hours of sunrise and sunset will be again:

$$H_s = 12 - \frac{1}{15} \omega_1 \text{ and } H_s = 12 + \frac{1}{15} \omega_2$$

The obtained values are shown in Table 1 and Table 2, where decimal fractions of hours are used instead of the *hh.mm.ss* format. According to the Tables, the earliest solar hour in the morning would correspond to 6.321 (6:19 AM), and the later one at 18.325 (6:19 PM), values that allow us to limit our later analysis of errors. Values are shaded for those dates in which the solar path intersects the plane of the wall. The difference in the dates of the shaded values in the Tables is due to the orientation of the dial. Note that, for the same date, it may be the case that we apply the method based on Eq. 6 for sunrise and that based on Eq. 8 for sunset.

⁴ H_s is commonly named *LST* in astronomy books.

⁵ This expression matches with the principle that the Sun is behind the dial if its azimuth differs from the dial's declination by more than 90° (Oglesby 2004).

Table 1 Earliest hours of direct sunlight on the dial

Earliest hours of sunlight over the sundial. Colegiata de Lerma												
date	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	7.497	7.065	6.450	6.997	7.806	8.411	8.504	8.033	7.253	6.410	6.902	7.413
2	7.490	7.045	6.427	7.026	7.830	8.424	8.497	8.011	7.225	6.381	6.923	7.424
3	7.482	7.025	6.403	7.054	7.854	8.435	8.490	7.989	7.198	6.353	6.944	7.435
4	7.474	7.005	6.380	7.082	7.878	8.447	8.481	7.966	7.170	6.325	6.964	7.445
5	7.466	6.985	6.356	7.110	7.902	8.457	8.472	7.943	7.143	6.295	6.985	7.455
6	7.457	6.964	6.332	7.138	7.926	8.467	8.463	7.920	7.115	6.318	7.005	7.464
7	7.447	6.943	6.309	7.166	7.949	8.476	8.452	7.897	7.087	6.341	7.025	7.472
8	7.437	6.922	6.308	7.194	7.972	8.485	8.441	7.873	7.059	6.364	7.045	7.480
9	7.427	6.901	6.336	7.222	7.995	8.493	8.430	7.849	7.031	6.388	7.064	7.488
10	7.415	6.880	6.365	7.250	8.017	8.501	8.418	7.825	7.004	6.411	7.083	7.495
11	7.404	6.858	6.394	7.278	8.039	8.508	8.406	7.801	6.976	6.434	7.102	7.501
12	7.392	6.837	6.423	7.305	8.061	8.514	8.393	7.777	6.948	6.457	7.121	7.507
13	7.379	6.815	6.452	7.333	8.083	8.519	8.379	7.752	6.919	6.480	7.140	7.512
14	7.366	6.793	6.481	7.360	8.104	8.524	8.365	7.727	6.891	6.503	7.158	7.517
15	7.353	6.771	6.509	7.388	8.124	8.528	8.350	7.702	6.863	6.526	7.176	7.521
16	7.339	6.748	6.538	7.415	8.145	8.532	8.335	7.677	6.835	6.549	7.193	7.525
17	7.324	6.726	6.567	7.442	8.165	8.535	8.319	7.651	6.807	6.572	7.210	7.528
18	7.310	6.704	6.596	7.469	8.184	8.537	8.303	7.625	6.779	6.594	7.227	7.530
19	7.294	6.681	6.625	7.496	8.204	8.539	8.287	7.600	6.750	6.617	7.244	7.531
20	7.279	6.658	6.654	7.523	8.222	8.539	8.269	7.574	6.722	6.640	7.260	7.533
21	7.263	6.635	6.682	7.549	8.241	8.540	8.252	7.548	6.694	6.662	7.276	7.533
22	7.246	6.613	6.711	7.576	8.259	8.539	8.234	7.521	6.665	6.685	7.292	7.533
23	7.230	6.590	6.740	7.602	8.276	8.538	8.216	7.495	6.637	6.707	7.307	7.532
24	7.213	6.567	6.769	7.628	8.293	8.536	8.197	7.469	6.608	6.729	7.322	7.531
25	7.195	6.543	6.797	7.654	8.310	8.534	8.178	7.442	6.580	6.751	7.336	7.529
26	7.178	6.520	6.826	7.680	8.326	8.530	8.158	7.415	6.552	6.773	7.350	7.526
27	7.160	6.497	6.855	7.705	8.341	8.526	8.138	7.388	6.523	6.795	7.363	7.523
28	7.141	6.474	6.883	7.731	8.356	8.522	8.118	7.361	6.495	6.817	7.377	7.519
29	7.123		6.912	7.756	8.371	8.517	8.097	7.334	6.466	6.838	7.389	7.515
30	7.104		6.940	7.781	8.385	8.511	8.076	7.307	6.438	6.860	7.401	7.510
31	7.084		6.969		8.398		8.055	7.280		6.881		7.504

Table 2 Latest hours of direct sunlight on the dial

Last hours of sunlight over the sundial, Colegiata de Lerma												
date	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	16.503	16.935	17.550	18.283	17.502	16.896	16.803	17.274	18.055	17.799	17.098	16.587
2	16.510	16.955	17.573	18.306	17.477	16.884	16.810	17.296	18.082	17.775	17.077	16.576
3	16.518	16.975	17.597	18.254	17.453	16.872	16.818	17.319	18.110	17.752	17.056	16.565
4	16.526	16.995	17.620	18.225	17.429	16.861	16.826	17.341	18.137	17.729	17.036	16.555
5	16.534	17.015	17.644	18.197	17.405	16.850	16.835	17.364	18.165	17.705	17.015	16.545
6	16.543	17.036	17.668	18.169	17.382	16.840	16.845	17.387	18.192	17.682	16.995	16.536
7	16.553	17.057	17.691	18.141	17.358	16.831	16.855	17.410	18.220	17.659	16.975	16.528
8	16.563	17.078	17.715	18.113	17.335	16.822	16.866	17.434	18.248	17.636	16.955	16.520
9	16.573	17.099	17.739	18.085	17.313	16.814	16.877	17.458	18.276	17.612	16.936	16.512
10	16.585	17.120	17.762	18.057	17.290	16.807	16.889	17.482	18.288	17.589	16.917	16.505
11	16.596	17.142	17.786	18.030	17.268	16.800	16.902	17.506	18.265	17.566	16.898	16.499
12	16.608	17.163	17.810	18.002	17.246	16.794	16.915	17.531	18.242	17.543	16.879	16.493
13	16.621	17.185	17.834	17.974	17.225	16.788	16.928	17.556	18.219	17.520	16.860	16.488
14	16.634	17.207	17.857	17.947	17.204	16.783	16.943	17.580	18.196	17.497	16.842	16.483
15	16.647	17.229	17.881	17.920	17.183	16.779	16.957	17.606	18.173	17.474	16.824	16.479
16	16.661	17.252	17.905	17.892	17.163	16.775	16.972	17.631	18.149	17.451	16.807	16.475
17	16.676	17.274	17.929	17.865	17.143	16.772	16.988	17.656	18.126	17.428	16.790	16.472
18	16.690	17.296	17.952	17.838	17.123	16.770	17.004	17.682	18.103	17.406	16.773	16.470
19	16.706	17.319	17.976	17.812	17.104	16.769	17.021	17.708	18.080	17.383	16.756	16.469
20	16.721	17.342	18.000	17.785	17.085	16.768	17.038	17.734	18.056	17.360	16.740	16.467
21	16.737	17.365	18.024	17.758	17.067	16.768	17.055	17.760	18.033	17.338	16.724	16.467
22	16.754	17.387	18.047	17.732	17.049	16.768	17.073	17.786	18.010	17.315	16.708	16.467
23	16.770	17.410	18.071	17.706	17.031	16.769	17.092	17.812	17.986	17.293	16.693	16.468
24	16.787	17.433	18.095	17.679	17.014	16.771	17.111	17.839	17.963	17.271	16.678	16.469
25	16.805	17.457	18.118	17.654	16.998	16.774	17.130	17.865	17.939	17.249	16.664	16.471
26	16.822	17.480	18.142	17.628	16.982	16.777	17.149	17.892	17.916	17.227	16.650	16.474
27	16.840	17.503	18.166	17.602	16.966	16.781	17.169	17.919	17.893	17.205	16.637	16.477
28	16.859	17.526	18.189	17.577	16.951	16.785	17.190	17.946	17.869	17.183	16.623	16.481
29	16.877		18.213	17.552	16.936	16.791	17.210	17.973	17.846	17.162	16.611	16.485
30	16.896		18.236	17.527	16.922	16.796	17.231	18.000	17.822	17.140	16.599	16.490
31	16.916		18.260		16.909		17.253	18.027	17.799	17.119		16.496

Shadow Cast by Architectural Elements and Nearby Trees

Close to our sundial there are several elements that cast their shadow on it in the early hours of the morning, and these hours vary throughout the year. These elements are a wall (oriented approximately North–South), almost perpendicular to the main façade, and a row of trees (see Figs. 1 and 2).

The solar dial is located 3 m from the ground level. The maximum height from the upper level of the nearby wall related to the dial’s level is 8.25 m and the minimum height is 4.60 m. On the same reference plane, the trees have a maximum height of about 6 m. These values, and their approximate distances to the sundial in the horizontal plane, serve us to draw a graph including the skyline of architectural elements and trees. It is useful to place this skyline on an graph of sunshine for our latitude, which we can also obtain through the aforementioned sources (Blateyron 2017; Casalegno 2017), or different on-line applications (Sun Earth Tools 2017), taking into account that we must consider solar local hours. In Fig. 8 we see the graph with the superimposed profile, which we can use to graphically obtain the values to correct the sunshine hours.

Table 3 takes into account the values graphically obtained for sunrise with the help of Fig. 8. This Table 3 now replaces Table 2. Table 1 has no modifications.

Based on Table 1 and Table 3, we can obtain the graphic of Fig. 9, where theoretical values of sunrise are dashed and the corrected ones due to the surrounding architectonic elements are marked as a solid line.

Derivation Procedure for the Calculation of Errors

Now we are going to analyse the possible errors of reading for different hypothesis related to the orientation and angular features of a gnomon. Our calculations will take Fig. 10 as the starting point.

Fig. 8 Sunlight graph with the upper lines of trees and adjacent wall superimposed

Table 3 Earliest hours of direct sunlight on the dial corrected as a function of adjacent wall and trees

First hours of sunlight over the sundial. Colegiata de Lerma.(Corrected by architectonical landscape)												
date	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	8.304	7.984	7.473	6.997	7.806	8.411	8.504	8.033	7.253	7.260	7.803	8.228
2	8.297	7.967	7.454	7.026	7.830	8.424	8.497	8.011	7.225	7.277	7.820	8.236
3	8.291	7.949	7.434	7.054	7.854	8.435	8.490	7.989	7.198	7.295	7.837	8.243
4	8.284	7.931	7.414	7.082	7.878	8.447	8.481	7.966	7.170	7.313	7.854	8.251
5	8.277	7.914	7.395	7.110	7.902	8.457	8.472	7.943	7.143	7.331	7.871	8.259
6	8.270	7.896	7.375	7.138	7.926	8.467	8.463	7.920	7.115	7.348	7.888	8.266
7	8.263	7.879	7.355	7.166	7.949	8.476	8.452	7.897	7.087	7.366	7.905	8.274
8	8.256	7.861	7.336	7.194	7.972	8.485	8.441	7.873	7.059	7.384	7.922	8.281
9	8.249	7.843	7.316	7.222	7.995	8.493	8.430	7.849	7.031	7.402	7.939	8.289
10	8.242	7.826	7.296	7.250	8.017	8.501	8.418	7.825	7.004	7.419	7.956	8.297
11	8.236	7.808	7.277	7.278	8.039	8.508	8.406	7.801	7.024	7.437	7.973	8.304
12	8.229	7.791	7.257	7.305	8.061	8.514	8.393	7.777	7.045	7.455	7.990	8.312
13	8.222	7.773	7.237	7.333	8.083	8.519	8.379	7.752	7.051	7.473	8.007	8.319
14	8.215	7.756	7.218	7.360	8.104	8.524	8.365	7.727	7.056	7.490	8.024	8.327
15	8.208	7.738	7.198	7.388	8.124	8.528	8.350	7.702	7.062	7.508	8.041	8.334
16	8.201	7.720	7.179	7.415	8.145	8.532	8.335	7.677	7.067	7.526	8.058	8.342
17	8.194	7.703	7.159	7.442	8.165	8.535	8.319	7.651	7.073	7.544	8.075	8.350
18	8.187	7.685	7.139	7.469	8.184	8.537	8.303	7.625	7.078	7.561	8.092	8.357
19	8.181	7.668	7.120	7.496	8.204	8.539	8.287	7.600	7.084	7.579	8.109	8.365
20	8.174	7.650	7.100	7.523	8.222	8.539	8.269	7.574	7.089	7.597	8.126	8.372
21	8.167	7.630	7.079	7.549	8.241	8.540	8.252	7.548	7.095	7.615	8.143	8.380
22	8.160	7.611	7.058	7.576	8.259	8.539	8.234	7.521	7.100	7.632	8.160	8.373
23	8.142	7.591	7.037	7.602	8.276	8.538	8.216	7.495	7.118	7.650	8.168	8.366
24	8.125	7.571	7.016	7.628	8.293	8.536	8.197	7.469	7.135	7.667	8.175	8.359
25	8.107	7.552	6.996	7.654	8.310	8.534	8.178	7.442	7.153	7.684	8.183	8.353
26	8.090	7.532	6.975	7.680	8.326	8.530	8.158	7.415	7.171	7.701	8.190	8.346
27	8.072	7.512	6.954	7.705	8.341	8.526	8.138	7.388	7.189	7.718	8.198	8.339
28	8.054	7.493	6.933	7.731	8.356	8.522	8.118	7.361	7.206	7.735	8.206	8.332
29	8.037		6.912	7.756	8.371	8.517	8.097	7.334	7.224	7.752	8.213	8.325
30	8.019		6.940	7.781	8.385	8.511	8.076	7.307	7.242	7.769	8.221	8.318
31	8.002		6.969		8.398		8.055	7.280		7.786		8.311

Fig. 9 Variation along the year of earliest and latest hours of direct sunlight on the dial. Dashed lines are those for sunrise without surrounding elements' correction

Fig. 10 Angular relationships between a generic real gnomon OA and an imaginary gnomon O'A

In Fig. 10, a generic incorrect gnomon is shown as OA. The lower end of the gnomon is the point A, whose shadow will be cast on the wall. Now, let O'A be an imaginary correctly-oriented gnomon whose lower end is the same point A. It will be contained on a vertical plane following the North–South direction. The angle between this gnomon and the horizontal plane will be φ , the latitude of the observer. If we call A' the shadow of the point A on the wall, the line OA' will allow us to read the faulty hour on the original dial, and the line O'A' will correspond to the actual hour on our imaginary dial related to the imaginary gnomon. Usually, either

OB or OA are known. For our derivations, we will consider OA as known. Other inputs will be δ_w , φ , φ' and μ , the angle between a vertical plane perpendicular to the wall and the vertical plane containing the gnomon (Fig. 10). In Lerma, as stated above, $\varphi = 42.03^\circ$, and $\delta_w = -6.6^\circ$.

From Fig. 10, we can obtain the values η and θ , which we'll use later:

$$\eta = \arcsin(\cos \varphi \cos \delta_w) \quad (9)$$

$$\theta = \arccos\left(\frac{\sin \varphi}{\cos \eta}\right) \quad (10)$$

Also:

$$g = \frac{OA \cdot \cos \varphi' \cos \mu}{\cos \delta_w \cos \varphi} \quad (11)$$

Now, let us look at Fig. 11, in which A' (shadow of the point A) is shown. Lines for solar hours of noon in the real and imaginary dials, H_s and H'_s , are shown; both of them are vertical. Vectors $\rho = OA'$ and $\rho' = O'A'$ are the shadows of real and imaginary gnomons and let us read the solar hour related to the real and imaginary dials, respectively.

Coordinates of O' related to O are:

$$O'_x = OA \cdot \sin \varphi' - g \cdot \sin \varphi \quad (12)$$

$$O'_y = OA \cdot \cos \varphi' \sin \mu + g \cdot \cos \varphi \sin \delta_w \quad (13)$$

For solar hour H'_s in the imaginary dial, we will have an hourly angle ω' related with that according to:

$$H'_s = 12 + \frac{\omega'}{15}$$

Which we can write as:

$$\omega' = (H'_s - 12) \cdot 15$$

Fig. 11 Geometrical polar coordinates of A' (shadow of A) in real and imaginary dials

According to (Soler 1997), the angle between the vertical straight line on O' (that is to say, the line for noon ($H_s' = 12:00$) in the imaginary dial) and the straight line of shadow $O'A'$ is:

$$\cot \tau' = \frac{\cos \delta_w}{\cos \varphi \tan \omega'} - \sin \delta_w \tan \varphi \quad (14)$$

According to the same author, the distance $O'A'$ will be:

$$\rho' = \frac{g \cdot \cos \delta}{\cos \delta \cos(\theta - \tau') \cos \eta - \sin \delta \sqrt{1 - \cos^2(\theta - \tau') \cos^2 \eta}} \quad (15)$$

η , θ and g are those values calculated in Eq. 9, Eq. 10 and Eq. 11.

If we relate the coordinates of A' to the primitive dial's origin, O :

$$A'_x = \rho' \cos \tau' + O'_x \quad (16)$$

$$A'_y = \rho' \sin \tau' + O'_y \quad (17)$$

Therefore, the angle τ on the primitive dial can be obtained from:

$$\tan \tau = \frac{A'_y}{A'_x} \quad (18)$$

To obtain ω we can start again from Eq. 14, but related to the new obtained values. Therefore, taking into account that the primitive dial's marks were drawn considering an erroneous $\delta_w = 0^\circ$ instead of $\delta_w = -6.6^\circ$

$$\cot \tau = \frac{\cos \delta_w}{\cos \varphi \tan \omega} - \sin \delta_w \tan \varphi = \frac{1}{\cos \varphi \tan \omega}$$

That is to say:

$$\omega = \arctan\left(\frac{\tan \tau}{\cos \varphi}\right)$$

And, according to it, the hour on our primitive dial will be:

$$H_s = 12 + \frac{\omega}{15}$$

Finally, the advance or delay between the actual solar hour and the hour read on our incorrect dial will be, measured in minutes:

$$\Delta H_s = (H_s - H'_s) \cdot 60 \quad (19)$$

Results for Different Hypotheses

The process we have just gone through allow us to graphically represent different hypothesis for the restitution of the gnomon. It is interesting to remark that the significant values are the angular ones, and variations in the numerical value of OA have no influence in the calculation process. OA can be taken as $OA = 1$ or other arbitrary values in order to perform the calculations for example, via spreadsheet.

Table 4 Values of delays throughout the year in minutes for different combinations of φ' and μ in the new gnomon for Lerma's sundial

Hs	6.4	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	18.2
$\varphi' = 42.03^\circ \mu = 0^\circ$														
1-Jan			21.11	22.78	24.21	25.29	25.93	26.02	25.51	24.37	22.65			
1-Feb			20.08	21.27	22.31	23.12	23.62	23.73	23.39	22.59	21.34			
1-Mar		18.07	18.52	18.99	19.46	19.88	20.20	20.36	20.31	20.00	19.45	18.69		
20-Mar		17.50	17.34	17.27	17.31	17.45	17.65	17.87	18.03	18.10	18.06	17.92	17.72	
1-Apr		17.14	16.58	16.16	15.94	15.91	16.05	16.30	16.61	16.92	17.21	17.45	17.66	17.70
1-May			14.79	13.58	12.74	12.33	12.33	12.70	13.37	14.25	15.27	16.37		
1-Jun				11.70	10.44	9.77	9.70	10.17	11.10	12.38	13.92	15.63		
21-Jun				11.32	9.97	9.25	9.16	9.66	10.64	12.01	13.65	15.48		
1-Jul				11.42	10.10	9.39	9.31	9.80	10.76	12.11	13.73	15.52		
1-Aug			14.30	12.86	11.86	11.35	11.32	11.73	12.50	13.53	14.75	16.09		
1-Sep			16.01	15.34	14.92	14.76	14.86	15.14	15.57	16.06	16.58	17.10	17.61	
22-Sep		17.49	17.31	17.23	17.26	17.40	17.60	17.81	17.98	18.06	18.04	17.91	17.72	
1-Oct	17.71	17.76	17.87	18.04	18.27	18.54	18.80	18.98	19.05	18.95	18.68	18.26		
1-Nov			19.68	20.68	21.57	22.27	22.73	22.85	22.58	21.90	20.84			
1-Dec			20.91	22.49	23.85	24.88	25.49	25.58	25.10	24.02	22.40			
21-Dec			21.19	22.90	24.36	25.47	26.12	26.21	25.68	24.51	22.76			
$\varphi' = 42.03^\circ \mu = 6.6^\circ$														
1-Jan			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17			
1-Feb			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17			
1-Mar		17.14	14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75		
20-Mar		17.14	14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75	17.60	
1-Apr		17.14	14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75	17.60	17.70
1-May			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75		

Table 4 continued

Hs	6.4	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	18.2
1-Jun				9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75		
21-Jun				9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75		
1-Jul				9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75		
1-Aug			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75		
1-Sep			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75	17.60	
22-Sep		17.14	14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75	17.60	
1-Oct	17.71	17.14	14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17	15.75		
1-Nov			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17			
1-Dec			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17			
21-Dec			14.37	9.99	5.27	1.60	0.00	0.79	3.65	7.78	12.17			
$\phi' = 40.00^\circ \mu = 0^\circ$														
1-Jan			26.26	28.50	29.45	28.96	27.15	24.40	21.25	18.28	15.89			
1-Feb			24.43	26.27	26.95	26.36	24.63	22.16	19.49	17.12	15.42			
1-Mar		19.69	21.67	22.95	23.25	22.52	20.94	18.91	16.93	15.45	14.74	14.84		
20-Mar		18.14	19.58	20.43	20.47	19.66	18.21	16.53	15.07	14.23	14.24	15.08	16.50	
1-Apr		17.15	18.24	18.83	18.71	17.86	16.51	15.04	13.91	13.48	13.94	15.23	17.08	17.49
1-May			15.11	15.11	14.63	13.72	12.60	11.66	11.30	11.79	13.26	15.56		
1-Jun				12.42	11.72	10.79	9.86	9.31	9.49	10.63	12.78	15.79		
21-Jun				11.88	11.13	10.19	9.31	8.84	9.12	10.39	12.69	15.84		
1-Jul				12.03	11.29	10.35	9.46	8.97	9.22	10.46	12.71	15.82		
1-Aug			14.24	14.08	13.52	12.59	11.55	10.76	10.60	11.34	13.07	15.65		
1-Sep			17.25	17.65	17.40	16.53	15.25	13.95	13.07	12.94	13.72	15.33	17.51	
22-Sep		18.11	19.53	20.38	20.41	19.60	18.15	16.47	15.03	14.21	14.23	15.08	16.52	
1-Oct	17.69	18.84	20.52	21.56	21.71	20.94	19.43	17.59	15.90	14.77	14.46	14.97		

Table 4 continued

Hs	6.4	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	18.2
1-Nov			23.71	25.41	25.99	25.35	23.66	21.31	18.81	16.68	15.24			
1-Dec			25.91	28.07	28.97	28.46	26.67	23.97	20.91	18.05	15.80			
21-Dec			26.40	28.68	29.65	29.17	27.36	24.58	21.40	18.37	15.93			
$\varphi' = 40.00^\circ \mu = 6.6^\circ$														
1-Jan			19.78	15.83	10.19	4.39	0.00	-.96	- 1.27	1.58	5.66			
1-Feb			18.90	15.09	9.65	4.11	0.00	- 1.70	- 0.80	2.22	6.44			
1-Mar		18.81	17.62	14.00	8.87	3.71	0.00	- 1.33	- 0.11	3.17	7.58	12.01		
20-Mar		17.79	16.66	13.19	8.30	3.42	0.00	- 1.06	0.40	3.86	8.42	12.97	16.39	
1-Apr		17.14	16.06	12.69	7.94	3.23	0.00	- 0.89	0.72	4.30	8.95	13.56	17.03	17.50
1-May			14.69	11.53	7.11	2.81	0.00	- 0.49	1.44	5.30	10.17	14.94		
1-Jun				10.72	6.53	2.52	0.00	- 0.22	1.96	6.01	11.03	15.92		
21-Jun				10.55	6.41	2.46	0.00	- 0.16	2.06	6.15	11.21	16.11		
1-Jul				10.60	6.44	2.47	0.00	- 0.18	2.03	6.11	11.16	16.06		
1-Aug			14.31	11.22	6.89	2.70	0.00	- 0.39	1.64	5.58	10.50	15.31		
1-Sep			15.62	12.32	7.67	3.10	0.00	- 0.76	0.95	4.62	9.34	14.01	17.50	
22-Sep		17.76	16.64	13.18	8.28	3.41	0.00	- 1.05	0.41	3.88	8.44	12.99	16.41	
1-Oct	17.69	18.24	17.09	13.55	8.55	3.55	0.00	- 1.18	0.17	3.55	8.04	12.54		
1-Nov			18.57	14.80	9.45	4.01	0.00	- 1.60	- 0.62	2.47	6.74			
1-Dec			19.61	15.69	10.08	4.33	0.00	- 1.91	- 1.18	1.70	5.81			
21-Dec			19.85	15.89	10.23	4.41	0.00	- 1.98	- 1.31	1.53	5.60			

H_s solar hour, Lerma's declination $\varphi = 42.03^\circ$, Wall's declination $\delta_W = -6.6^\circ$

Fig. 12 Delays in readings for Lerma's sundial. $\varphi' = 42,03^\circ$, $\mu = 0,0^\circ$

Although the graphs have been obtained for intervals of 0.1 h, for the sake of ease of analysis and neatness Table 4 includes only significant values for intervals of 1 h and for the first dates of each month plus the dates of equinoxes and solstices. Note that signs of results in Eq. 19 have been inverted for clarity due to the fact that most values for Lerma were negative. Therefore, Table 4 and Figs. 12, 13, 14 and 15 show delays instead of advances. The values of H_s correspond, as always, to true local solar times. In all cases, $\delta_w = 0$, and different hypotheses take into account different values for angles μ and φ' as defined in Fig. 10.

New Gnomon Located on O in a Vertical Plane Perpendicular to the Wall ($\mu = 0$), with $\varphi' = \varphi$

In our case, this was most probably the original gnomon, now missing. The graph for delays is that of Fig. 12.

The maximum delay, 26.25 min, corresponds to winter solstice, about 12.6 H_s . The minimum delay, 9.12 min, corresponds to summer solstice, about 11.6 H_s (again we are using decimal values for fractional hours). Daily delays increase towards noon in autumn and winter and decrease towards noon in spring and summer. During the equinoxes, values have slight variations throughout the day, remaining stable at about 18 min of delay. Note that left and right ends of the curves are cut according to the limits shown in Tables 1 and 3.

Fig. 13 Delays in readings for Lerma's sundial. $\varphi' = 42.03^\circ$, $\mu = 6.6^\circ$

New Gnomon Placed on O in a Vertical Wall Oriented South ($\mu = 6.6^\circ$), and $\varphi' = \varphi$

In this special case, $O=O'$. The delays are exactly the same along the year: all the curves are identical, except for their extreme points, as stated in Tale 3 (Fig. 13).

Minimum values for delays are 0 min every day at noon. Maximum values are for sunrise and sunset, and can reach 17.7 min. Almost negligible advances correspond to solar hours near noon between 12.1 and 12.3 (not shown in Table 4).

New Gnomon Placed on O in a Vertical Plane Perpendicular to the wall ($\mu = 0$), with φ' Different from φ

Results will depend on φ' but, in any case, no additional precision is obtained. Fig 14 shows, for example, $\varphi' = 40^\circ$.

New Gnomon Placed on O in a Vertical Wall Oriented South ($\mu = 6.6^\circ$), with φ' Different from φ

As in the previous case, a slight variation of φ' causes a great distortion in the curves. Fig 15 shows, for example, for $\varphi' = 40^\circ$.

Fig. 14 Delays in readings for Lerma's sundial. $\varphi' = 40,00^\circ$, $\mu = 0,0^\circ$

Other Combinations of μ and φ'

Different combinations have been checked and it is not worth showing all the functions in this paper. No additional precision is added, and the distributions of values seem almost random. The worst choices are those in which φ' differs from φ , which causes notables asymmetries in the functions. Other orientations can be dismissed, including intermediate positions between those analysed in “New Gnomon Located on O in a Vertical Plane Perpendicular to the Wall ($\mu = 0$), with $\varphi' = \varphi$ ” and “New Gnomon Placed on O in a Vertical Wall Oriented South ($\mu = 6.6^\circ$), and $\varphi' = \varphi$ ”, that is, values for $\varphi = \varphi'$, with $\mu \neq 0^\circ$. The most suitable possibilities are, therefore, those shown in “New Gnomon Located on O in a Vertical Plane Perpendicular to the Wall ($\mu = 0$), with $\varphi' = \varphi$ ” and “New Gnomon Placed on O in a Vertical Wall Oriented South ($\mu = 6.6^\circ$), and $\varphi' = \varphi$ ”.

Extrapolation to Sundials with Marks According to Incorrect Latitudes

Considering that in the past templates or pre-calculated tables were widely used to simplify the engraving of the marks on sundials, and that these templates were usually made for integer values of latitude, frequent errors due to faulty latitudes can

Fig. 15 Delays in readings for Lerma's sundial. $\varphi' = 42.03^\circ$, $\mu = 6.6^\circ$

Fig. 16 The best choices for the sundial's restoration: graphical comparison between gnomon perpendicular to the dial and rotated gnomon (frontal views)

be found in ancient sundials. Although Lerma's dial has no significant errors in latitude (42° vs. 42.03°), the method developed to check possible orientations and geometrical features of a new gnomon can be used for the calculation of errors in sundials with marks corresponding to incorrect latitudes. In these cases, the procedure is similar to those followed in "[New Gnomon Placed on O in a Vertical Plane Perpendicular to the wall \(\$\mu = 0\$ \), with \$\varphi'\$ Different from \$\varphi\$](#) " or "[New Gnomon Placed on O in a Vertical Wall Oriented South \(\$\mu = 6.6^\circ\$ \), with \$\varphi'\$ Different from \$\varphi\$](#) ", with φ being the correct latitude of the dial and φ' the wrong latitude corresponding to the marks. Angle μ will be different from 0 when the wall is also rotated.

For example, if we want to study a sundial whose marks correspond to a perfect South orientation⁶ for 43° and the location of our dial is at 43.42° , with a real declination of 3.5° East and with its gnomon located on a plane perpendicular to the wall (this last feature is usual in dials with marks facing South, despite their true orientations), the input datas will be

$$\varphi = 43.42^\circ \quad \varphi = 43.00^\circ \quad \mu = 0^\circ \quad \delta_w = 3.5^\circ$$

Of course, a preliminary study of the hours when the dial receives sunbeams will have to be performed to prevent obtaining nonsensical results, following the procedure explained in "[Time Intervals in which the Dial is Operational](#)".

Conclusions

For Lerma's sundial, results calculated at "[New Gnomon Located on O in a Vertical Plane Perpendicular to the Wall \(\$\mu = 0\$ \), with \$\varphi' = \varphi\$](#) " and "[New Gnomon Placed on O in a Vertical Wall Oriented South \(\$\mu = 6.6^\circ\$ \), and \$\varphi' = \varphi\$](#) " have different implications. The absence of errors at noon for the South-oriented gnomon, such as that studied in "[New Gnomon Placed on O in a Vertical Wall Oriented South \(\$\mu = 6.6^\circ\$ \), and \$\varphi' = \varphi\$](#) ", is not a negligible question in our case of study: the building which houses the dial is a church, and solar noon, was traditionally taken as a reference to ring the bell for the Angelus. Further, a constant distribution of errors throughout the year provides good grounds for choosing this option from a strictly mathematical point of view. The errors of choice shown in "[New Gnomon Located on O in a Vertical Plane Perpendicular to the Wall \(\$\mu = 0\$ \), with \$\varphi' = \varphi\$](#) " can reach more than 26 min in the days close to the winter solstice related to noon, and daily precision varies continuously. However, two facts must be taken into account. The first one is an aesthetic objection: the odd combination of a dial with symmetrical hours topped with a rotated gnomon can look bizarre, although the front view (Fig. 16) does not seem too striking.

The second objection is that purists of Lerma's history may prefer the presumably original gnomon. Let's bear in mind that an obsessive architectonic symmetry governs all the religious and civil historic buildings in Lerma; whether

⁶ Graphical programs such as Shadows (Blateyron, 2017) and Orologi Solari (Casalegno 2017) are useful again for this analysis.

this was the reason for the symmetric marks, or simply that the sundial's makers used a template neglecting the wall declination, we'll never know.

From a general point of view, the analysis of Lerma's sundial has helped us to fully elaborate an unusual procedure of calculation, since the existing treatises relative to gnomonics very often refer to the construction of new sundials, not to restoration. It has also been proved that the minimum errors for sundials with incorrect marks are associated with the better oriented gnomon, facing South. This was somewhat predictable, but we have numerically quantified these errors. And with the help of this study, we are now able to calculate reading errors in all vertical (non-inclined) sundials. We hope that this paper encourages sundial experts and enthusiasts to experiment with new proposals of restoration. In any case, and in our opinion, predictive tabulation of errors always ought to be a fact to be remarked prior to any restoration project.

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