Nexus Network Journal (2019) 21:329–345
https://doi.org/10.1007/s00004-018-00423-2

NEXUS NETWORK JOURNAL
Architecture and Mathematics

RESEARCH

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Geometric Proportions in Measured Plans of the Pantheon of Rome

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Published online: 9 January 2019

Abstract
The Pantheon in Rome has been depicted in countless paintings and measured drawings. This paper considers how the building and its subsequent representations express meaning through elementary geometric symbols, patterns, and proportions. The author observes notable discrepancies between a sampling of measured plans from Sebastiano Serlio's woodcut engravings in the Renaissance to current laser campaigns. She analyzes the different drawings for underlying geometric patterns. A pattern of rotated squares, in root-two proportion, appears consistently in the horizontal plan of each measured set and complements Mark Wilson Jones's proposed scheme of a conjoined sphere and cube. This comparative method of analysis offers students and scholars of descriptive geometry a useful tool for interpretation.

Keywords Pantheon of Rome · Serlio · Palladio · Desgodetz · Bern Digital Project · Mark Wilson Jones · Descriptive geometry · Incommensurable values · Root-two

Introduction

The Pantheon in Rome, commonly attributed to Hadrian, is one of the great iconic buildings of the western world, rivalled only by the Parthenon of Athens for its impact on subsequent architectural works in the classical tradition. One of the best preserved of ancient Roman buildings, it has remained in continuous use, claiming the largest unreinforced, solid concrete dome in the world today, with an interior diameter of 43.56 m (Albers et al. 2009: 7).

Prior to the late nineteenth century, historians credited the Pantheon to Marcus Agrippa (64/62 BCE–12 BCE), the Roman statesman and son-in-law of Rome's first

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emperor Augustus. The attribution owed largely to an inscription on the building's facade, although portions of the Pantheon were sometimes credited to the Roman Republic (Serlio 1996: 99; Palladio 1997: 285; Desgodetz 1771: 2, 4, 12; Pasquali 2015: 333). In 1891–1892, French architect Georges Chédanne (1861–1940) discovered brickstamps attached to the building, which appeared to date the aboveground structure to c. AD 118. Chédanne concluded the entire structure was constructed near the beginning of the reign of Rome's fourteenth emperor, Hadrian (AD 76–138), although recent scholarship and brickstamp analysis limit the influence of Hadrian and date the construction start to c. AD 113 during the rule of Roman emperor Trajan (AD 53–118). (See Hetland 2015: 82, 88, 94, 97; Wilson Jones 2009: 82, 84; Marder and Wilson Jones 2015: 23.)[1] Meanwhile, in the 1890s, excavations by architects Luca Beltrami (1854–1933) and Pier Olinto Armanini (–1896) revealed traces of an earlier building underneath, which they associated with Agrippa.[2]

The Pantheon has been depicted in numerous paintings, measured drawings, and other means of representation, perhaps more than any other classical work. No two studies are exactly alike. Each one "measures" and interprets the true Pantheon in its own way. This anomaly poses a challenge for anyone seeking to understand how geometric symbols, patterns, and proportions contribute to our experience of this iconic building, which lacks first-hand documentation of its original measures. How reliable are these various representations and measured drawings? Were they offered as true representations, idealizations, or opportunities to express other concepts? How did the Pantheon and its subsequent representations express meaning through geometric symbols and proportions?

To address these questions, the author analyzes a sampling of representations of the Trajanic-Hadrianic Pantheon for the presence of geometric constructions and proportions. Studies range from inexact measured plans in the Renaissance by Sebastiano Serlio and Andrea Palladio to remarkably precise laser campaigns conducted in this century. Geometric analyses reveal an elementary pattern consisting of two equal rotated squares, which is common to each horizontal plan examined, even though the measured plans themselves differ significantly. This raises the possibility that the Pantheon's creators intended this pattern to underlie the building's measures. How this knowledge would have been conveyed from one culture to another is unclear.

[1] Today, scholars believe the present Pantheon is the third building or building phase on the site. Agrippa's building, considered the first Pantheon in Rome, was built and dedicated 27–25 BCE, but suffered damage by fire in AD 80. The emperor Domitian (AD 51–96) reconstructed to some extent, c. AD 89, but lightning struck in AD 110, during the rule of Trajan, from AD 98–117. A third reconstruction concluded c. AD 125 during Hadrian's reign, hence its recognition as Hadrianic, if not Trajanic–Hadrianic. A second carving under the Agrippa inscription declares the building was refurbished in AD 202 under the emperor Septimius Severus (AD 145–211) and his successor son Caracalla (AD 188–217) (Albers et al. 2009: 7; Marder and Wilson Jones 2015: 7–8; Wilson Jones 2009: 82).

[2] Modern-day excavations in 1996–1997 by Paola Virgili and Paola Battistelli indicate that the Augustan Pantheon, attributed to Agrippa and situated directly below the present building, was north-facing, circular, and included a rectangular portico. In this interpretation, the inscription on the northern portico correctly recognizes Agrippa as the Trajanic–Hadrianic temple's true originator, with Agrippa's temple providing the original concept for the plan that survives today. This scenario could date the origins of the Pantheon's geometric scheme to an earlier period, leaving future builders to embellish the circular concept with a spectacular dome and oculus (Broucke 2009: 28; La Rocca 2015: 53–64).

Survey of Measured Drawings of the Pantheon

Measured studies selected for this comparative analysis date from the Renaissance to the present day. The woodcut engravings in the 1540 *Il terzo libro dell'architettura* of Sebastiano Serlio (1475–1554) and the 1570 *I quattro libri dell'architettura* of Andrea Palladio (1508–1580) represent the Italian Renaissance, along with the first complete English translation of Palladio, *The Architecture of A. Palladio, in Four Books* (1715), edited by Giacomo Leoni (1686–1746).[3]

Faced with contradictory representations by Palladio, Serlio, and, in 1650 France, Roland Fréart de Chambray, architect Antoine B. Desgodetz (1653–1728) produced more carefully measured studies in the seventeenth century. His twenty-three engravings of the Pantheon are profuse with detailed measurements, which he compares with Palladio and Serlio, when possible.[4] In 1682, the Académie Française financed the publication of his results in *Les édifices antiques de Rome*, considered the most accurate printed source for ancient Roman architecture in its day.

In 2005, Gerd Graßhoff initiated a building survey known as the Bern Digital Pantheon Project (BDPP), now managed in Berlin under the auspices of "Topoi."[5] State of the art laser scan technology included a Leica HDS3000 three-dimensional long-range laser scanner, which captured 620 million measuring points from twenty positions in the first campaign (December, 2005) and twenty-three new locations in the second (July, 2009). Sub-scans were aligned and merged to form the point cloud that ultimately translated to a three-dimensional computer model. The project's team of civil engineers, archaeologists, and historians of science produced the most detailed and precise representation of the Trajanic-Hadrianic Pantheon to date, with a mean maximal margin of error of 0.005 m. Such precision would have been unimaginable to ancient builders or anyone prior to the digital age. The three-dimensional computer model does not capture the original building, but rather a structure subjected to extensive renovations,

[3] Serlio assumed that ancient builders of the Pantheon employed the ancient Roman palm and he measured the Pantheon accordingly. He notes the measures of key elements in the narrative and provides a vertical scale (Serlio 1996: 100, 458).

Unlike Serlio, Palladio locates important measures on the drawings themselves, which he records in Vicentine feet, or *piedi*, the unit of measure specific to his locality. The Leoni edition employs this unit of measure also, but makes "necessary Corrections with respect to Shading, dimensions, ornaments, &c. that this Work may in some sort be rather considered as an Original, than an Improvement" (Leoni 1715: Preface to the Reader, Bk. IV, Pt. 2).

[4] Serlio provides only a few specific measures but Desgodetz gleans what he can from information offered for column diameters, which he presumes are measured deliberately (Desgodetz 1771: xiv, xvi). Desgodetz measures the Pantheon in Parisian feet. Dimensions are taken from column and pilaster centers (Desgodetz 1771: xviii).

[5] The Bern Digital Pantheon originated as a pilot project of the Karman Center for Advanced Studies in the Humanities of the University of Bern, Switzerland, before moving to Berlin in 2010. The project's web-based program, currently http://repository.edition-topoi.org/collection/BDPP, provides open access to survey data and encourages the free exchange of resource materials and research results. See http://repository.topoi.org/BDPP for an inventory of building survey studies and views.

replacements, repairs, and environmental stresses for nearly two millennia (Albers et al. 2009: 7–10).[6]

Measured Drawings Included in the Study

The drawings selected for this comparative analysis depict prominent features in each horizontal plan. The analysis compares their measures and basic features, then analyzes each plan for underlying geometric patterns and proportions. The goal is to determine if geometric patterns emerge and if any are common.

Horizontal Plan

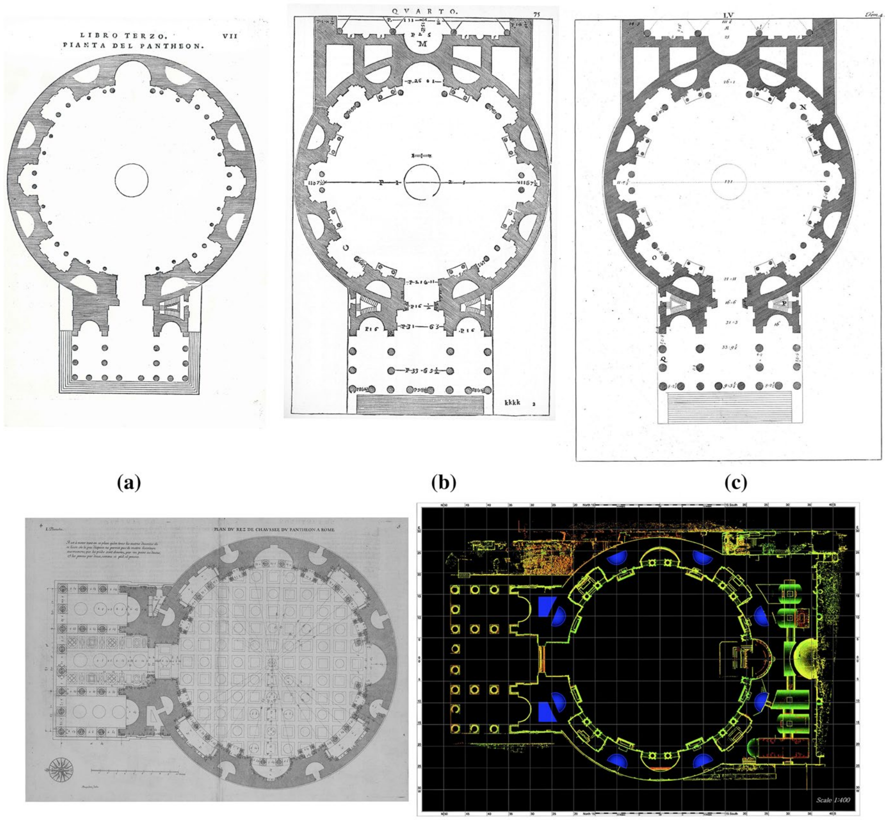
Each horizontal plan identifies an eightfold circular arrangement of recessed niches (Fig. 1a–e). The niches contain six chapels fronted by two columns each, a portal that leads from the portico, and an opposing central chapel or apse with columns on either side. Three of the seven chapels are semicircular; four are oblong. A ring of eighteen columns surrounds the rotunda pavement and central oculus. Semicircular cavities within the thick concrete rotunda wall relieve the stress of earthquakes and minimize cost and materials, according to Palladio (Palladio 1997: 285), and lighten the load, according to Desgodetz (Desgodetz 1771: 4).

The recessed niches recorded by Palladio and Serlio display subtle differences (Fig. 1a–c). The chapels, portal, and apse of Palladio, particularly the Leoni edition, appear wider than those of Serlio. The oblong chapels present more subtle detail. Differences between the two porticos are more notable (Fig. 1a–c). Fourteen steps in front lead to the portico in Palladio's plan.[7] Serlio illustrates seven steps from ground level, around the portico's three sides. On either side of the entrance, Palladio locates two hidden stairs, which lead above the chapels through a secret passage. Serlio identifies only one. On the rotunda's south side, Palladio indicates four enclosed chambers and a portion of another building interior. Serlio does not.

The narrative accompanying Desgodetz's ground plan (Fig. 1d) corrects numerous errors in Palladio, among them that "the diameter of the columns of the portico is too small by two inches." The result is that the front of Palladio's portico is "too small by four feet four inches." The inside diameter of his rotunda is "too small by two feet, and the portico much too small in proportion to the temple: nor has he even drawn the plan as he quotes it in his description" (Desgodetz 1771: 6).

[6] The Bern survey is recorded in meters. See (Marder and Wilson Jones 2015:18–47) for a survey of modifications to the Pantheon from the Middle Ages to the modern era.

[7] The Leoni edition presents sixteen steps.



(d) (e)

Fig. 1 **a**–**e** Above, left to right: **a** Sebastiano Serlio, Pantheon plan, 1540. **b** Andrea Palladio, Pantheon plan, 1570. **c** Andrea Palladio (Giacomo Leoni, ed.), Pantheon plan, 1715. Below, left to right: **d** Antoine Desgodetz, ground plan of the Pantheon at Rome, 1682. **e** Bern Digital Pantheon, Pantheon, horizontal cross-section of the point cloud with three-dimensional volumes of cavities on second level, cutting between the ground floor and the first cornice, BDPP0089, 2009

Among several horizontal cross-sections, the Bern study known as BDPP0089, which is cut between the ground floor and first cornice and locates the cavities within the cylindrical rotunda wall, best captures the horizontal plan views by Serlio, Palladio, and Desgodetz (Fig. 1e).[8]

[8] The Bern floor plan BDPP0027, measured at ground level, does not capture the cavities within the rotunda wall. Otherwise, differences between BDPP0089 and BDPP0027 appear negligible.

Mathematical Symbolism

One cannot fully appreciate the patterns and proportions of geometry without understanding their connection to basic mathematical forms, elementary symbols, and universal themes. The Pantheon expresses these connections with exceptional power and simplicity.

Sphere

Most, if not all, representations of the Pantheon describe an interior that contains a perfect or near-perfect sphere, the most ideal form that geometry can produce. Serlio reports that the interior assumes "a perfect rotundity, because its width from wall to wall is the same as the pavement to the underside of the opening." He measures the dome's hemisphere from the last "cornice to the top of the dome or vault where the opening is" (Serlio 1996: 103). Palladio says the Pantheon "is shaped like the world, that is, round, in that its height from the pavement to the opening which lets in the light is as great as its diameter across its breadth from wall to wall" (Palladio 1997: 285). In fact, the sphere that best fits Serlio's interior, when presented in section, coincides with the dome's inside surface, while the sphere that best fits Palladio's interior encompasses the dome's recessed coffers. Likewise, Desgodetz presents a rotunda interior of equal height and breadth. Like Serlio, the interior height divides approximately in half into a cylindrical wall and semicircular dome, minus the recessed coffers (Desgodetz 1771: 10, 12).

Today, laser scan technology and computer modelling capture the Pantheon's interior with remarkable precision. The Bern survey records the interior dome span, at the lowest point of the Pantheon's cupola along the north–south axis, at 43.56 m. A sphere of 43.98 m in diameter best fits the inner walls of the dome; it does not fit exactly due to irregularities and deformations in the dome and dome surface (Graßhoff et al. 2009: 168–169, BDPP0078). Whether these irregularities are intentional or due to weight, seismic stresses, or other factors is uncertain.

The Pantheon's sphericity recalls the universe and cosmos. "Pantheon" in Greek, means "for all of the gods" (*pan*-"all" + *theios* "of or for the gods"). The historian and Roman statesman Dio Cassius (AD 155–235) proposed that it "has this name" because it contains statues of several divinities, including Venus and Mars, and because the "vaulted roof…resembles the heavens" (Dio Cassius 1917: 263, 265). The Pantheon's interior setting imitates the sun's daily celestial motion. As the day progresses, the sun pierces the unglazed oculus and casts a circular sun-like image that revolves along the dome.

Sun, Moon, Planets, and Number Symbolism

Elementary numerical relationships reinforce the Pantheon's symbolic expressions of sun, moon, and planets. Sunlight is the primary source of light.

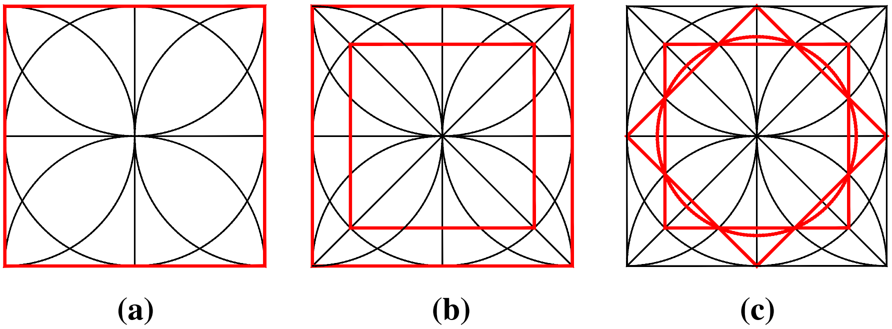


Fig. 2 **a–c** Left to right

Piercing the oculus, it illuminates five rings that encircle the hemispherical vault; each ring contains twenty-eight coffers, or the number of phases recognized in a lunar cycle.

William MacDonald and others posit that the chapels that surround the Pantheon's rotunda pavement symbolize the seven visible "planets" recognized in the ancient world—Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn. One first experiences the Pantheon's interior cosmos while standing at the entry portal. In this scenario, the portal, which completes the eightfold symmetry of the Pantheon's horizontal plan, represents the planet Earth (MacDonald 1976: 89).

Circle, Square, and *Ad Quadratum*

Specific geometric patterns, embedded in the Pantheon's measured plans, support the building's eightfold spatial symmetry and planetary symbolism. The patterns arise from the simple circle and square, which together represent complementary transcendent and finite qualities, or heaven and earth, and include *ad quadratum* and rotated squares constructions.

The geometric construction known as *ad quadratum*, which in Latin means "to the square," is a series of squares, which alternate with circles, such that the diagonal of the smaller square and the side of the next larger square are equal. The side lengths of successive squares increase in root-two ratio; the area of the larger square doubles. The *ad quadratum* construction has been observed in the plans and decoration of Roman buildings, including the tile patterns that adorn the Pantheon's pavements (Watts 1996; Williams 1997: 34–39).

Ad Quadratum, Rotated Squares, and Octagon Constructions

One method for initiating the *ad quadratum* encloses a circle in a square, then a smaller square within the circle. Draw a circle, then four semicircles of equal radius from the endpoints of the circle's vertical and horizontal diameters. The semicircles intersect at the four corners of a square (Fig. 2a). The circle intersects the square's two diagonals at the four corners of a smaller square (Fig. 2b). The side of the larger square and diagonal of the smaller square are equal. The area of the larger square is double the area of the smaller square. The sides of the smaller and larger squares are in ratio $1:\sqrt{2}$.

The *ad quadratum* construction yields octagonal symmetries. The four midpoints of the larger square locate the four corners of a square equal in size to the smaller square but rotated 45°. A new circle drawn through the eight points of intersection of the rotated squares encloses a regular octagon and inscribes a stellar octagon (Fig. 2c).

Comparative Geometric Analysis

These geometric patterns can be observed in representations of the Pantheon from Serlio to the present day. Although the measures of the representations differ, the identical geometry of rotated squares is common to each rotunda, when viewed in horizontal plan. This geometry reinforces the Pantheon's eightfold spatial plan and planetary symbolism.

Horizontal Plan and Rotated Squares

Serlio considered the Pantheon "the most beautiful, most complete and best conceived" of the ancient buildings to be seen in Rome (Serlio 1996: 99). When viewed in horizontal plan, the inside and outside walls of his massive rotunda enclosure relate in proportion to a circle that encloses two equal squares rotated 45 degrees and a smaller circle drawn through the squares's eight points of intersection, as in Fig. 2c (Fig. 3).

Like Serlio, the horizontal plans in Palladio's 1570 and 1715 Leoni editions relate the inside and outside rotunda walls according to the same geometric scheme of rotated squares (Fig. 4a, b).

Desgodetz considered the Pantheon "the most entire, and the best executed of any that have remained to our days" (Desgodetz 1771: 2). Although he detects numerous errors in the horizontal plans of Serlio and Palladio, the inside and outside cylindrical surfaces of his own rotunda follow the same geometric construction of rotated squares (Fig. 5). The inside and outside walls of the Bern rotunda follow the construction, as well (Fig. 6).

Resolution of Rotunda and North Portico

While the pattern of rotated squares relates the inside and outside surfaces of each cylindrical rotunda studied, there remains the geometric connection between the rotunda and north portico. In Palladio's horizontal plan, a square inscribes the circle of the interior rotunda wall, in *ad quadratum* fashion. An identical square, drawn adjacent to its northern side, locates the column centers along the sides and front of the portico, shown here in the 1715 Leoni edition (Fig. 7). Palladio's 1570 version

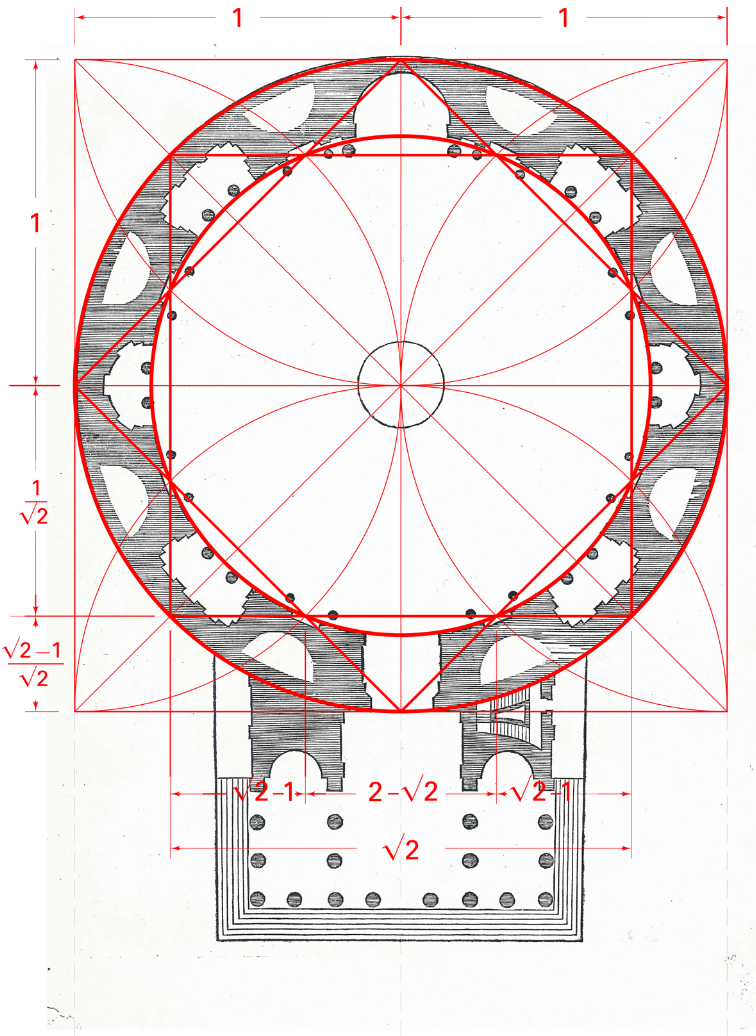


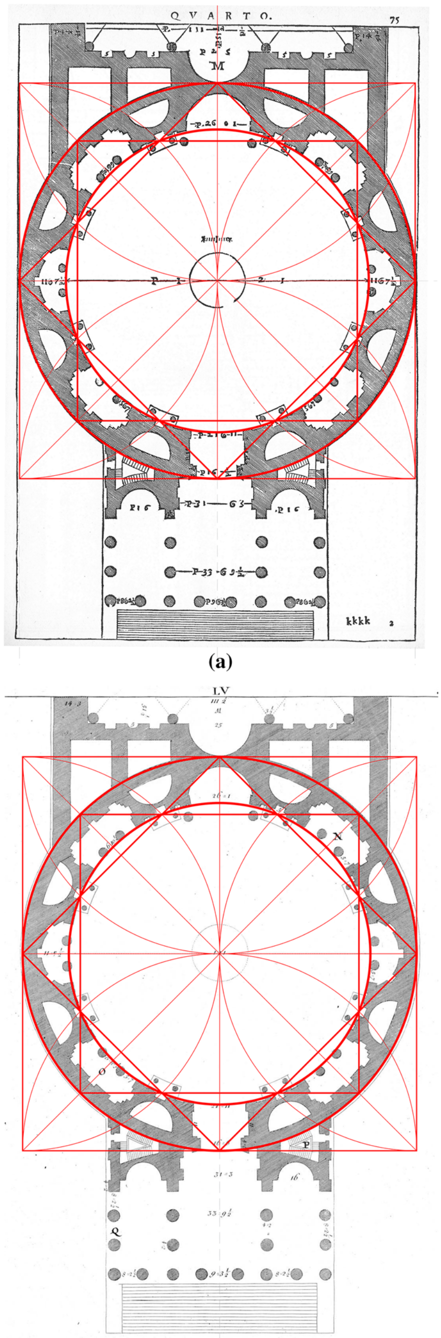
Fig. 3 Sebastiano Serlio, Pantheon plan, 1540, with overlay

reveals a noticeable axial asymmetry between the portico's right and left sides, however.

Mark Wilson Jones's Conjoined Sphere and Cube

Mark Wilson Jones proposes a similar concept, in which a square inscribes a circle that is drawn through sixteen of the eighteen column centers that surround the rotunda pavement, excluding the two columns fronting the central chapel. An identical square drawn adjacent to its northern side locates the column centers

Fig. 4 **a** Andrea Palladio, Pantheon plan, 1570, with overlay. **b** Andrea Palladio (Giacomo Leoni, ed.), Pantheon plan, 1715, with overlay



(a)

(b)

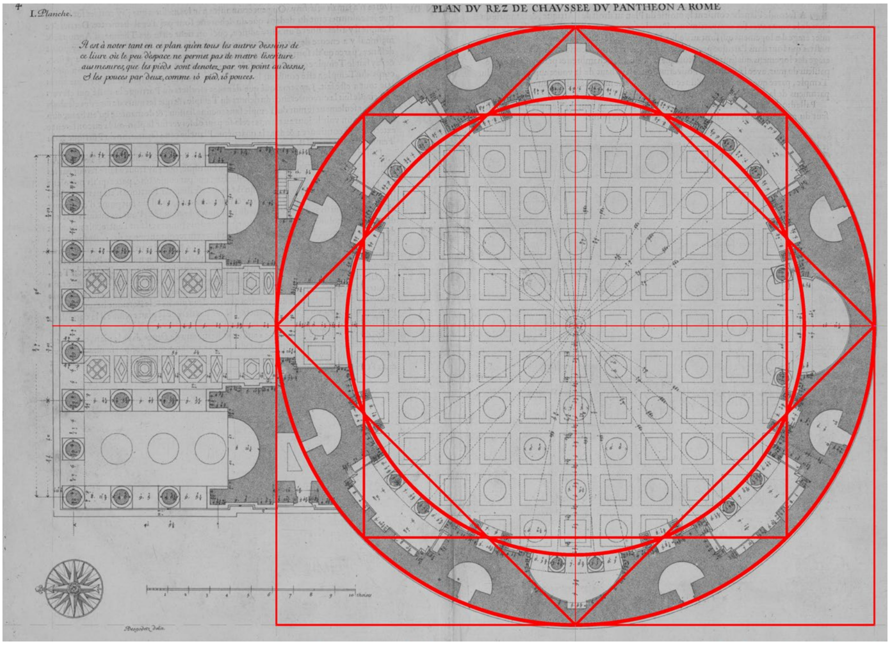


Fig. 5 Antoine Desgodetz, ground plan of the Pantheon at Rome, 1682, with overlay

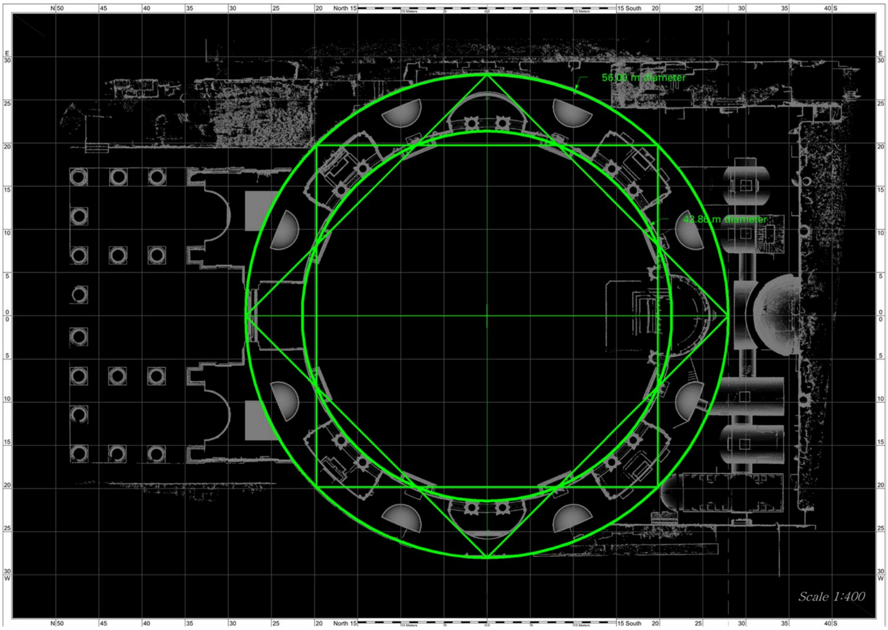


Fig. 6 Bern Digital Pantheon, Pantheon, horizontal cross-section, cutting between the ground floor and the first cornice, BDPP0089, 2009, with overlay

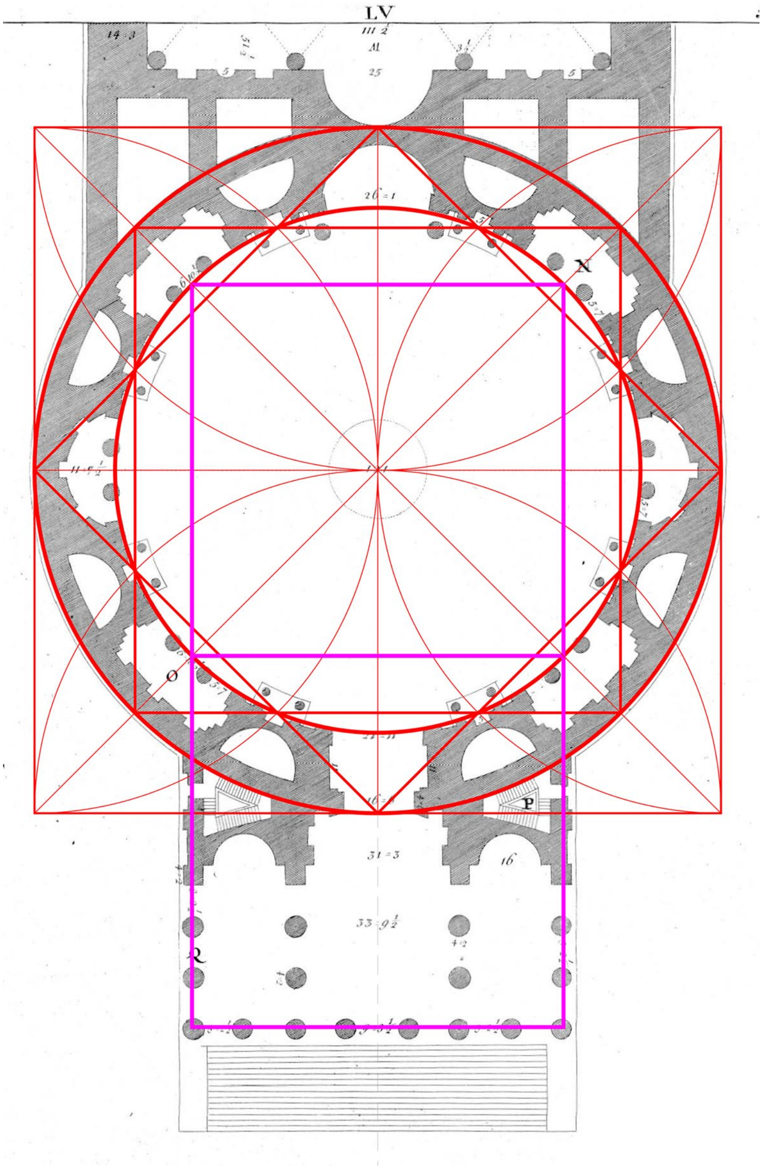


Fig. 7 Andrea Palladio (Giacomo Leoni, ed.), Pantheon plan, 1715, with overlay

along three sides of the portico. Wilson Jones's configuration supports his view that the Pantheon was conceived as a conjoined sphere and cube (Wilson Jones 2000: 184 and 208; Marder and Wilson Jones 2015: 9–10, 21–22, and PL. XII).

Wilson Jones's proposal does not align with Palladio's portico columns as precisely as the *ad quadratum* approach. However, it appears to best express

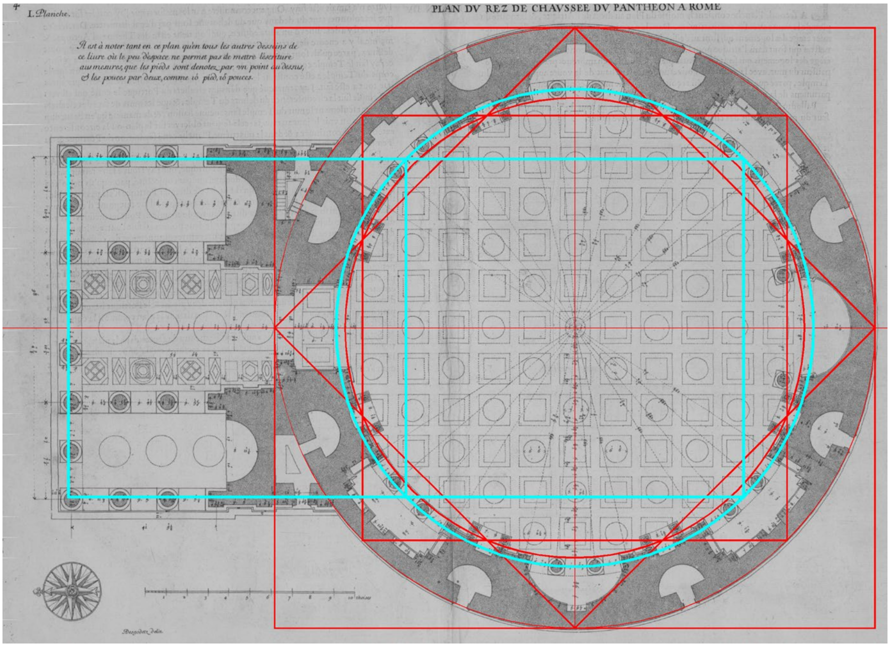


Fig. 8 Antoine Desgodetz, ground plan of the Pantheon at Rome, 1682, with overlay, after Mark Wilson Jones (in cyan)

the relationship between the portico and rotunda in Desgodetz's more accurate horizontal plan (Fig. 8).

Wilson Jones contends that the rotunda's ring of column centers is a reasonable representation of the sphere contained within the Pantheon's interior. Furthermore, a cube inscribed in such a sphere, but placed adjacent to the cube's northern side, encloses the intended portico. Wilson Jones speculates that the higher pediment in back is a vestige of an earlier design, which called for a larger portico. The height of the lower pediment in front would have matched the back pediment, had fifty-foot monolithic column shafts been available on the day (Wilson Jones 2015: 214).

The Bern survey supports Wilson Jones's conjoined sphere and cube. In horizontal plan, the square that inscribes the circle through the rotunda's ring of column centers is equal and adjacent to the square that locates the column centers along the portico's three open sides (Fig. 9). The diameter of the ring of column centers equals 44.54 m.[9] A sphere of 44.54 m exceeds the sphere that best fits the inner walls of the Pantheon dome (43.98 m) by 0.56 m, or 1.3%; it exceeds the dome's interior span at the cupola base (43.56 m) by 0.98 m, or 2.2%.

By comparison, the circle drawn through the eight points of intersection of two rotated squares, which aligns with the diameter of the rotunda wall's inside surface,

[9] Wilson Jones, incorporating Desgodetz and others, measures the ring of column centers at 44.52 m (Wilson Jones 2000: 220).

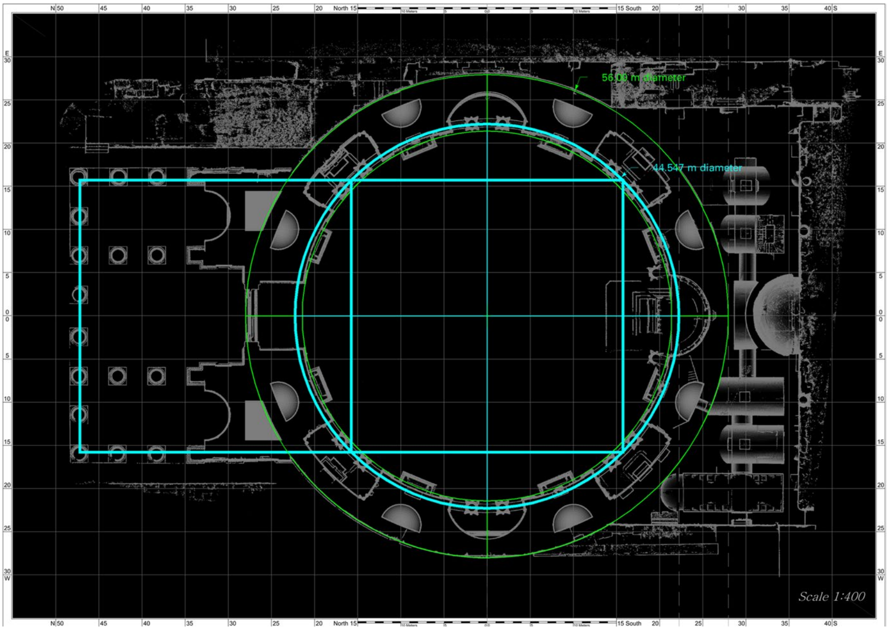


Fig. 9 Bern Digital Pantheon, Pantheon, horizontal cross-section, cutting between the ground floor and the first cornice, BDPP0089, 2009, with overlay, after Mark Wilson Jones (in cyan)

is 42.86 m (Fig. 6). A sphere of 42.86 m in diameter falls short of the sphere that best fits the interior dome by 1.12 m, or 2.5%. However, it falls short of the dome's interior span at the cupola base by only 0.7 m, or 1.6%.

Summary and Conclusion

The Pantheon has been represented in countless paintings and drawings, perhaps more than any other classical work. Various representations, within the same historical period and between one period and the next, display notable discrepancies. The purpose of analyzing a sampling of plans for underlying geometric proportions was to see if a comparative approach would yield significant results and prove to be a useful tool for analysis.

The sampling included inexact Renaissance studies by Serlio and Palladio, a "corrected" measured plan by Desgodetz, and a highly precise digital survey in the present day by the Bern Project. The analysis revealed a configuration of equal squares rotated 45° in root-two proportion, which relate the rotunda's inside and outside cylindrical walls, when viewed in horizontal plan. Remarkably, the geometric construction appeared in each plan studied, even though the measurements of the drawings themselves differ.

This geometric scheme of rotated squares compares favorably with Mark Wilson Jones's well-documented proposal, which visualizes the spatial masses of the rotunda and portico as a conjoined sphere and cube. The cube and sphere are proportioned to a square inscribed within a circle that is drawn through the rotunda's ring of column centers. Wilson Jones's proposal rests on the speculation that the higher second pediment in back is a vestige of an earlier design, which intended a larger portico. Nevertheless, there are compelling arguments in its favor. Simple geometric volumes formalize the three-dimensional spaces of the rotunda and portico. The scheme resonates the Pantheon's circle and square symbolism and its dominant decorative motif of pairing circles and squares within the tiles on the rotunda floor and in the rings of square coffers within the dome.

Wilson Jones's circle and square geometry does not align with the Renaissance plans of Serlio and Palladio. However, it accurately informs the more precise Desgodetz and Bern studies. The Bern survey shows that a sphere, which is scaled to the rotunda's ring of column centers, appears to exceed the sphere that best fits the inner walls of the dome by only 1.2%.

By comparison, the configuration of rotated squares, which relate the inside and outside rotunda walls of each horizontal plan studied, also arises from the circle and square. Moreover, its root-two proportions reappear in the tiles that adorn the rotunda and portico pavements (Williams 1997: 37–38). The rotated squares reinforce the rotunda's eightfold spatial symmetry and planetary symbolism. They do not translate to simple spatial volumes.

A comparative analysis of the Pantheon's various measured drawings raises significant questions. Why do the measured plans of Serlio and Palladio differ markedly from one another, when they were published less than 30 years apart, with access to similar measuring techniques? During the Renaissance, it was not unprecedented to alter measured plans to correct perceived irregularities or express favored mathematical concepts. For example, Serlio's interior elevation of the Pantheon's rotunda aligns the attic pilasters with columns and pilasters below, contrary to existing conditions (Serlio 1996: 109). It was not uncommon for Palladio to publish a design for a building of his own that differed from the one constructed. In the 1770s, Ottavio Bertotti Scamozzi recognized discrepancies between Palladio's published design for the Villa Emo at Fanzolo and the actual building (Bertotti Scamozzi 1976: 75–76). The opportunity to present a favored mathematical scheme may have been a factor.[10]

Why does the same geometric configuration appear in various plans of the Pantheon whose measurements differ from one another? The remarkably consistent presence of rotated squares in each horizontal plan could be interpreted to mean that the construction, perhaps intended originally, was communicated over time, from one culture to the next, independent of the building's actual measures. Thomas Jefferson's concept for the University of Virginia Rotunda, completed in 1826,

[10] Rudolf Wittkower identifies, in the published plan of Villa Emo, a mathematical system of harmony that is based on the arithmetic ratios of musical consonance (Wittkower 1971: 131). Lionel March observes still more arithmetic relationships that include the Pythagorean 3:4:5 right triangle and the Platonic *lambda* (March 2001: 96–100). However, none of these mathematical relationships align with the constructed villa's actual measures, which may express yet another proportional scheme (Fletcher 2001: 105–106).

appears to continue the tradition. Jefferson cited the Leoni edition of Palladio's Pantheon as the basis for his design. The two sets of measured plans display notable differences; yet, the geometric pattern of rotated squares, which informs Palladio's Pantheon, appears in Jefferson's Rotunda, as well (Fletcher 2003).

These and other questions warrant further study. The observations cited here, though compelling, are not conclusive on their own and would benefit from further corroboration, such as demonstrating the draftsmen's working knowledge of geometric patterns observed.[11] If utilized thoughtfully, in combination with more established analytical tools, the comparative analytical method can be a useful vehicle for exploring geometric intent.

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[11] Serlio does not provide specific instructions for producing the Pantheon plan geometrically. However, the first book of his collected works, *Tutte l'opere d'architettura*, is a treatise on geometry, published in 1545. It includes a method for deriving an octagon from a perfect square and two *ad quadratum* schemes (Serlio 1996: 9, 28).

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