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An Analytical Study of the Methods of Design and Geometric Constructions in Architectural Ornaments of the Friday Mosque of Forumad

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Abstract   The Friday Mosque of Forumad, Masjid-i Jami Forumad is a small two-iwan mosque in the northeast of Iran. Although the building has no inscription, it is believed to date back to the eleventh or twelfth century, during the late Seljuk dynasty. Nearly the whole construction is finely decorated by vegetal and geometric patterns in stucco, brickwork with glazed brick, and elaborate muqarnas. These features make this mosque a prototype for studying the history of architectural ornaments and geometric patterns in Persian mosques. This research intends to propose approaches for depicting the substructures and geometric constructions of these architectural ornaments. By doing so, the knowledge of the architects in practical geometry and their creativity in constructing exclusive geometric designs is highlighted. In addition, attempts have been made to demonstrate the influence of ornaments of this mosque as a pioneer sample of initial forms of *girih* patterns in Persian architecture.

Keywords   Persian architecture · Architectural ornaments · Geometric patterns · Friday mosque of Forumad

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Introduction

The earliest mosques were built on simple plans with limited decorations. To prevent idol worship in the religious contexts, the application of accurate representations of living beings was particularly avoided in Islamic architecture. Therefore, vegetal motifs and geometric patterns were gradually applied in Islamic art and architecture under the influence of ancient cultures. Among those, geometric patterns became the most remarkable ornamental system in Islamic art and architecture. These patterns are based on the mathematical division of a surface and associate woven patterns with the reticular system (Clivenot 2000). The meetings and collaboration between mathematicians and architects which were mentioned in some historical documents,[1] provide insight into how such collaborations in the early and medieval Islamic world aided the creation of complex geometric design (Ozdural 2000). The progress of mathematical knowledge was due to the elementary geometrical knowledge of the ancient Greeks, Persians, and Syrians which reached Damascus and Baghdad, the capitals of the Umayyads[2] and the Abbasids.[3] This led to the flourishing of mathematics in the early Islamic culture (Sezgin 2010). There exist very few written records to explain the various methods that architects employed for the constructions of design of geometric patterns in Islamic art and architecture (Berggern John 1986). However, it is obvious that the traditional tools of medieval times (a compass, straightedge, and set square) were used for creating ornamental design (Bonder 2012). These factors had a great influence on geometric design and led to the formation of exclusive geometric patterns in Islamic art and architecture.

The richest medium of architectural decoration in the early Islamic period was stucco which followed Sasanian[4] and Central Asia styles (Grabar 1975). Previous studies have made it evident that the application of geometry was common among architects at the same time as Sasanian stucco ornaments were being constructed (Kharazmi et al. 2012). Henceforth, it could have certainly influenced the stucco ornaments and geometric constructions of the early Islamic architecture. Besides stucco, architects also used brick to adorn the buildings in Islamic architecture. The techniques of decorative brickwork gradually developed and found a decorative role in addition to its structural role. The brickwork of the Seljuk[5] period in Iran and

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[1] Examples include *A book on those geometric constructions which are necessary for a craftsman* and the descriptions in *Interlocks of figures* written by Abu'l-Wafa al'Buzjani (940–998 CE), one of the well-known Islamic mathematicians and astronomers.

[2] The Umayyad dynasty, also spelled Omayyad, was the first great Muslim dynasty to rule the Empire of the Caliphate (661–750 CE).

[3] The Abbasid Caliphate (750–1258 CE) was second of the two great dynasties of the Muslim Empire of the Caliphate. It was ruled by the Abbasid dynasty of caliphs, who built their capital in Baghdad, Iraq, after overthrowing the Umayyad caliphate.

[4] The last Persian Empire before the rise of Islam, ruled by the Sasanian dynasty from 224 to 651 CE. The Sasanian Empire was recognized as one of the main powers in western and Central Asia, alongside the Roman-Byzantine Empire, for a period of more than 400 years.

[5] The Seljuk Empire, also spelled Seljuq, (1040-1157), ruling military family of the Ghuzz Turkic tribes that invaded southwestern Asia in the 11th century and eventually founded an Empire that included Mesopotamia, Syria, Palestine, and most of Iran.

Central Asia was particularly elaborative. In the earliest monuments brickwork was undecorated, but in the later buildings it was introduced as a decorative tool and reached its culmination with complex geometric patterns and inscriptions under the Seljuk dynasty. In this epoch, two techniques of brickwork decoration were used. The first technique was based on employing brick in standard size which was followed by arranging in simple patterns. In the second one, shaped brick that were especially manufactured for the purpose of brickwork decoration were utilized. The latter technique was more suitable for inscriptions and creating complex motifs. In addition, during the Seljuk period, buildings began to be decorated with glazed brick and colored ceramic tile inlays (Peterson 2002). Glazed tiles initially appear in the twelfth century in light blue. In the early thirteenth century, white and dark blue tiles were also used (Wilber 1939). The homeland of Seljuk architecture was Iran, but the Mongol invasions destroyed most of the brickwork ornaments of the buildings and only a few remain (Peterson 2002). The Friday Mosque of Forumad is one of which back to the last Seljuk period in Iran. This mosque was introduced by French archaeologist and art historian, Andre Godard, who excavated the mosque in the 1920s. He classified this two-iwan[6] mosque as Khurasan architectural style of Persian architecture (Godard et al. 1992).

The whole structure is decorated on inside and outside regions. Furthermore, the mosque has a unique mihrab[7] in the southern iwan that is decorated with stucco and geometric patterns. There are outstanding muqarnas[8] ornaments above the mihrab which could be considered as one of the earliest examples of muqarnas ornament in Persian mosques. These features made this mosque a pioneering example in the application of complicated ornaments in the Persian architecture of the late Seljuk era.

There have been some studies done on this mosque like the archaeological research by (Godard et al. 1992), and the historical and archaeological description of the mosque by Molavi (1974). They mainly concentrate on the history of the mosque and an introduction into its architecture. However, there are no detailed studies about the overall architectural ornaments, geometric constructions, and design methods of its ornaments. Moreover, since the location of this mosque is isolated, its unique decorations have been extremely damaged and methods to preserve what are left need to be studied further. Hence, in this paper we attempt to extract the geometric constructions and methods of design for each ornamental

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[6] An *iwan* is a vaulted hall, walled on three sides, with one end entirely open. The four-iwan mosques that became one of the classical features of the religious architecture of Iran are in fact a continuation of two-iwan mosque style.

[7] A *mihrab* is usually a niche set into the middle of a wall in a mosque to indicate the direction of Mecca. Certainly the mihrab became a focus for architectural decoration and was often embellished with the latest artistic techniques.

[8] *Muqarnas* is one of the most characteristic features in Islamic architecture. It is the Arabic word for stalactite vault. It is a type of wall or ceiling decoration, which is used to make a smooth transition from the rectangular basis of the building to the vaulted ceiling. It is usually associated with domes, doorways and niches, although it is often applied to other architectural features and is sometimes used as an ornamental band on a flat surface. By providing a diffused method of transition from flat to curved, muqarnas zones of transition were able to break down the distinction between vertical and curved, domed and horizontal.

pattern of the mosque, analytically. By doing so, the role of knowledge of practical geometry on their construction is demonstrated. Although various studies discussed the knowledge of geometry in geometric patterns of Persian architecture (Abdullahi and Embi 2013; Bonner 2003; Cromwell and Beltrami 2011; Cromwell 2009; Lu and Steinhardt 2007; Sarhangi 2012; Tennant 2003), there are a few specific studies about the historical monuments of Seljuk period in Iran. Therefore, the Friday Mosque of Forumad could be a worthy sample for studying the earliest history of geometric patterns in Persian architecture of Islamic period.

Method

In the analysis section of the present paper, the geometric methods and their varieties for designing the ornaments are discussed. Following the underlying designs, the effect of the practical geometry and discern design methods which could be created by only using the compass and the straightedge is illustrated. Besides the intact ornaments, there are some heavily damaged ornaments of which only small fragments have survived. The analysis of these ornaments includes envisioning the whole decorative pattern and offering the geometrical substructure and the methods of design. The ordering of figures and their analysis has been done based on the similarity of patterns and is explained from the simplest toward the most complex substructures. The similar patterns with different substructures and methods of design are analyzed in tandem to demonstrate the advanced ability of the architects and their creativity in applying various geometrical methods for drawing the substructure and making various unique ornaments.

The repetitive units, axes of symmetry, vectors which transfer motifs and design methods used in construction of ornaments has been demonstrated through analyzing the underlying designs. Moreover, in order to provide a more comprehensive perception of applied symmetries in the generation of patterns, the symmetry aspects of motifs will be discussed by showing the repetitive units. These units will be classified based on the symmetry groups:[9] the 17 wallpaper groups,[10] and the seven frieze groups.[11] After analyzing the ornaments located in

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[9] Symmetry groups in a design which decorates a surface by the regular repetition of a unit by translational symmetry fall into one of two categories: (1) a mono translational design (otherwise known as a one-dimensional design, a one-sided band, a strip or frieze group, a border, or a periodic border design) or (2) a distranslational design (otherwise known as a two-dimensional design, a wallpaper group or wallpaper design, a crystallographic group, a periodic group, a plane, a network or an all-over pattern, a periodic planar design, a plane group, or an n-dimensional space group ($n = 2$) (Horne 2000).

[10] A periodic pattern extended in two linearly independent directions to cover the 2D plane is known as a wallpaper pattern. The two smallest linearly independent translation vectors T1 and T2 in the pattern's symmetry group are generators for the underlying lattice structure of the pattern. The lattice divides the plane into identical parallelogram shaped sub-images, called lattice units or tiles. The symmetry group of a wallpaper pattern has to be one of the 17 distinct wallpaper groups. A lattice unit is typically chosen with centers of highest order of rotation at the vertices (Liu et al. 2004).

[11] A frieze group is a mathematical concept to classify designs on two-dimensional surfaces which are repetitive in one direction, based on the symmetries in the pattern. Such patterns occur frequently in architecture and decorative art. The mathematical study of such patterns reveals that exactly 7 different types of patterns can occur.

the outside and inside area of the mosque, the ornaments of the mihrab and the muqarnas will be analyzed, accordingly. Finally, patterns, their geometric methods for drawing, the repetitive units, and the geometric features of their substructures will be put in a table separately and compared with each other. All geometric constructions and drawings have been created by the authors.

The Friday Mosque of Forumad

The Friday Mosque of Forumad, *Masjid-i Jami Forumad,* is made up of two iwans facing each other on either side of a courtyard (Fig. 1). Although this mosque has no inscription, it is believed by most archaeologists that the architectural style is in the style of either the eleventh or twelfth century. This is resulted from comparing this structure with other securely dated ones which have the two-iwan plan, similar applications of brick and glazed brick ornaments, and were built in the same region, being the northeast of Iran (Godard et al. 1992). However, the complex tiling and the geometric design of ornaments raise the assumption that some parts of the ornaments might be repaired or constructed during the later centuries, e.g. in the Ilkhanid period.

This mosque has a total area of 820 m^2 which is covered by stuccowork on the inside region and by stucco and brickwork on the outside part. The portal entrance of the mosque (Fig. 2) is located on the north side of the building and is decorated by rich and elegant brickwork, stucco, and inscription. This portal is similar to a broken-arch iwan in which two brick facade pillars are used. Interior walls and vault of the portal are decorated with the stucco in the form of small squares. The mosque has two iwans in the northern and southern area; each one has about 6.6 m width. The northern iwan is 12 m height and the height of the southern iwan is 14 m. There are two vaults on both sides of each iwan which are connected to the naves with arches of 22 m wide. There are two naves in the east and west sides of the mosque
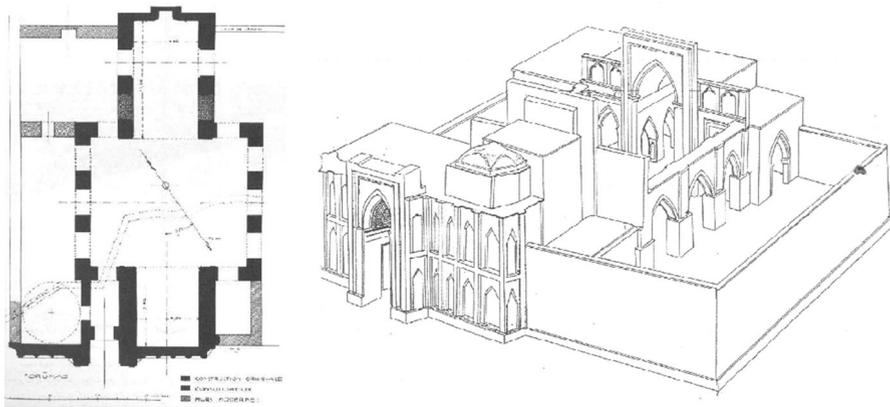
Fig. 1 *Left* the plan of the Friday Mosque of Forumad (Godard et al. 1992). *Right* schematic view of the mosque (Roohparvar 1994)

Fig. 2 *Left* the outside view of the mosque. *Right* the entrance view of the mosque
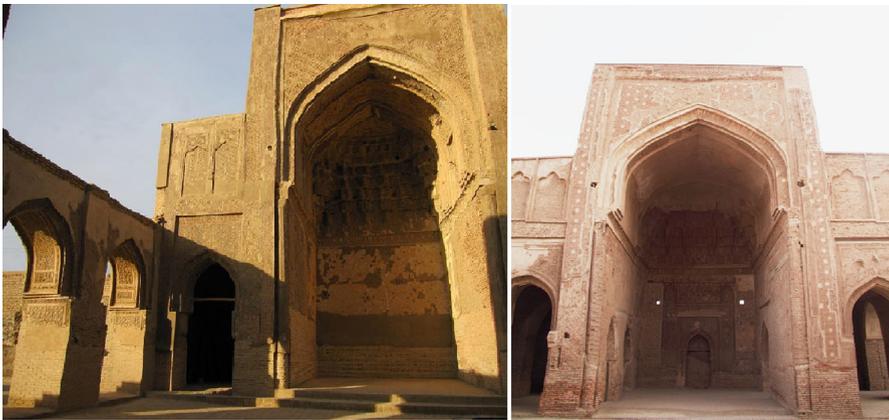
Fig. 3 *Left* the northern iwan of the mosque. *Right* the southern iwan of the mosque

that their roofs and walls were destroyed and just three of their arches with 24 m width have survived. There are exquisite patterns and elegant stuccoworks on these arches. Each pedestal has also a particular inscription in stucco by calligraphy (Molavi 1974) (Figs. 3, 4).

The two iwans are nearly symmetric with similar parts and elements. In order to give more attention to the southern iwan, it was built taller than the northern one. Moreover, the exclusive mihrab and moqarnas were constructed with elaborate ornaments inside the southern iwan.

Analysis of the Ornaments

Figure 5 shows a part of an ornament which was constructed on the curved surface of the wall in the southern iwan; however, the main pattern might be drawn in a planer surface.

In order to prepare the substructure of design for this specific study, we first inscribe an octagon with a diagonal d_1 inside of a square. Then draw a smaller

Fig. 4 Detailed view of the architectural ornaments located at the interior parts of the mosque
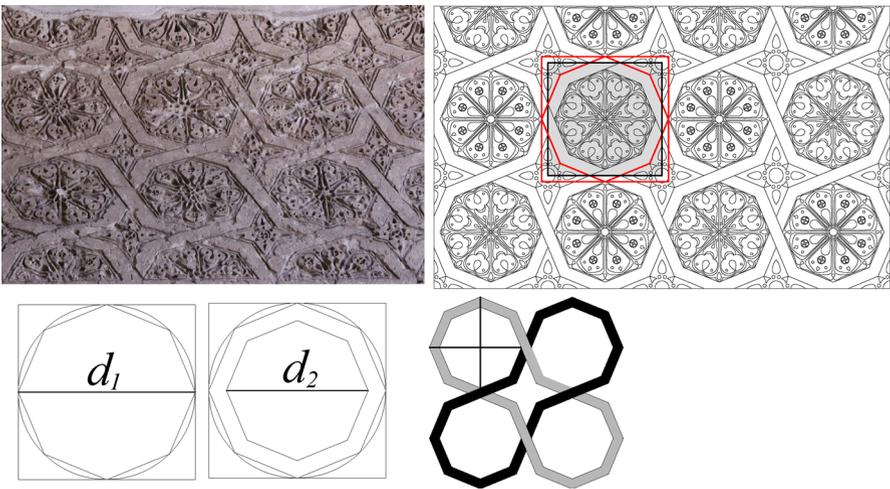
Fig. 5 The stucco design in the southern iwan and the analysis of its layout
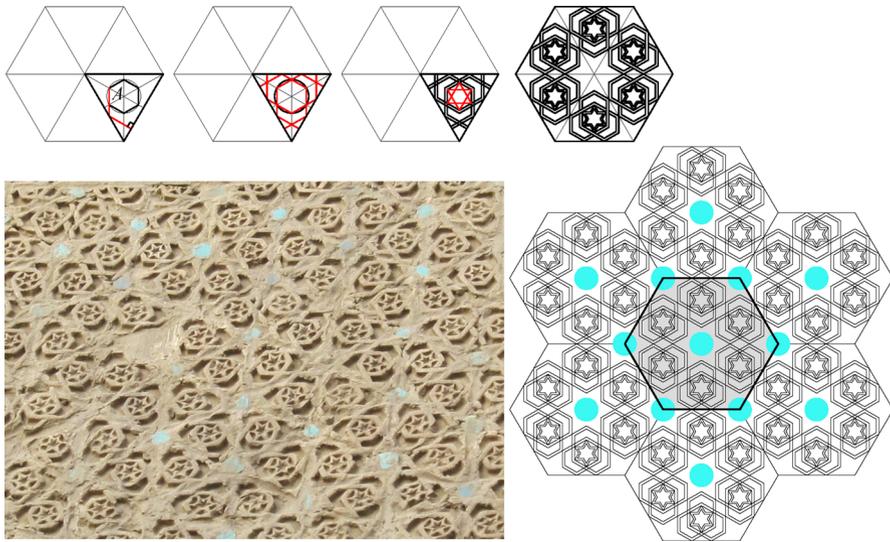
Fig. 6 The stucco design in the northern iwan and the analysis of its layout

octagon inside of the first one with a diagonal d_2, to create the necessary interlocking strips,[12] which we show them in two colors. The entire substructure in this figure is created by transferring the octagon vertically and horizontally using a vector with a magnitude of $\frac{d_1+d_2}{2}$. The star forms are created among this kind of transformation to complete the layout. In the figure, the two different motifs are alternately placed inside the octagons to complete the design. All motifs in each frame are centrally symmetric. If we ignore the up and down interlocking strips and consider the octagons and their motifs separately, this design can be classified as the symmetry type 'p4m'.

The ornamental pattern in Fig. 6 shows one of the earliest examples of glazed brick application in Persian mosques. The substructure of the pattern here is based on designing one hexagon. The entire decorative pattern is generated by translating the hexagon in three directions based on its three pairs of parallel sides. The method for the construction of the internal patterns is described in four steps in the figure as follows: divide a hexagon into six triangles and draw the angle bisectors of one of the triangles. Let A be the meeting point of the angle bisectors, make a circle with center A and radius equal to one half of the bisector. Then, as it is shown in the figure, inscribe a hexagon inside of it. Next, extend the sides of the hexagon to meet the sides of the triangle and then from these meeting points draw orthogonal lines to the adjacent sides of the triangle. Complete this pattern by adding interlocking strips and transferring the pattern to the other triangles using rotation. One (6, 2) star

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[12] Interlocking strips are interwoven going up and down from other strips based on a rotational symmetry about the center of each motif. They originated in ancient ornamental design and found specific feature in geometric design in Islamic art.
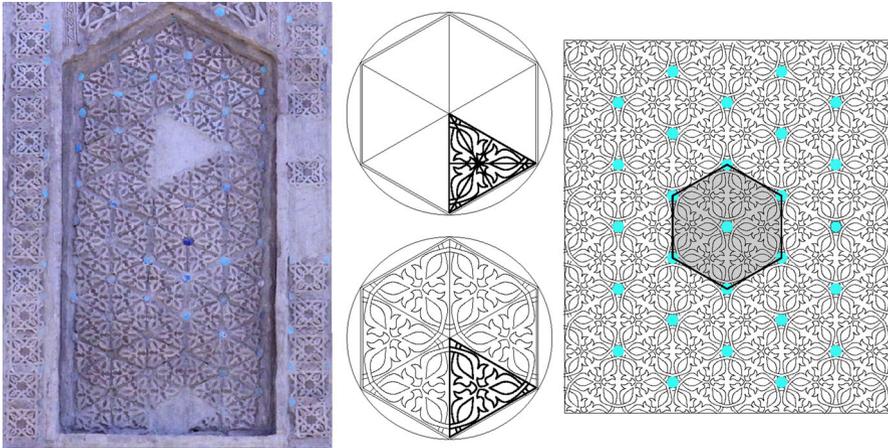
Fig. 7 The stucco and glazed brick in the northern iwan and the analysis of its substructure

polygon[13] is constructed inside of each hexagon. The entire pattern is generated when this decorated hexagon, as a repetitive unit, is translated in three directions. This design has the symmetry type denoted '*p6*'.

Similar to the designs demonstrated in the past figures, the pattern in Fig. 7 is drawn based on one hexagon. Draw a hexagon and divide it into six triangles. Draw the main motif, which has threefold symmetry, in one triangle. Rotate the motif of the triangle about the center of the hexagon to cover the entire shape. Finally, add interlacing strips to complete the design. This pattern has the symmetry type denoted '*p6*' in the wallpaper symmetry groups.

The pattern in Fig. 8 is drawn by translation of one hexagon that can be created as follows: first, construct the hexagon *A* in a circle. Then, draw the hexagon *B* inside the first one in such a way that its vertices stand at the midpoints of sides of the hexagon *A*. For interlocking strips draw another hexagon inside the hexagon *B*. In the next step, translate the hexagon *B* in six directions using vectors that are originated from the center of hexagon *A* and terminated to its vertices. The entire tessellation can be made by the translation of decorated hexagon *A*. As it is shown in the figure, the designer located small hexagons in the vertex points of the hexagons which were used for putting glazed brick inside. This design possesses the symmetry type denoted '*p6*' in the wallpaper symmetry groups.

The layout of the stuccowork in Fig. 9 is generated based on a hexagonal repetitive unit which can be extended in every direction. The method for creating the motif for this design which is borrowed from a geometric construction by (Said Issam 2001), is as follows: draw a circle and inscribe a dodecagon inside of it. Then, connect every second vertex of the dodecagon in one direction. This will generate two concentric congruent hexagons, (12, 2) star polygon, as is illustrated in the first circle in the middle six images from top-left to the bottom-right. Next, consider the

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[13] A figure created from connecting every second vertex in a set of six equally spaced points on a circle in one direction using two strokes.
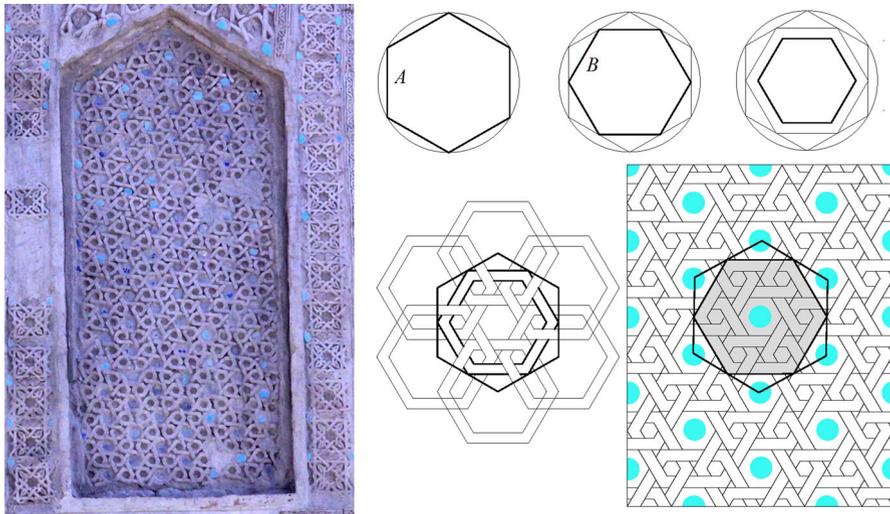
Fig. 8 The stucco and glazed brick in the northern iwan and the analysis of its layout
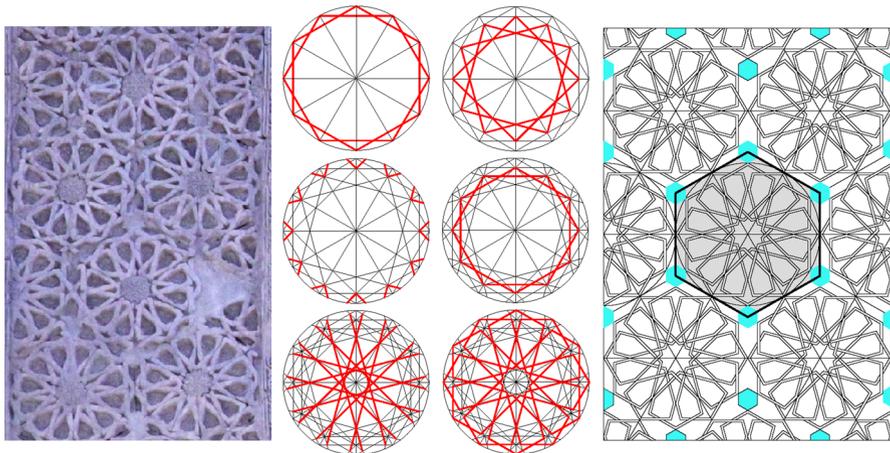
Fig. 9 The stucco and glazed brick in the northern iwan and the analysis of its layout

midpoints of the sides of the two inscribed hexagons. Connect every third vertex to make three concentric, congruent squares. Use the segments created from the intersections of the hexagons and squares to make more segments and continue completing the pattern according to the illustrated images in the figure. The entire pattern, which is shown in the right image, is made from copies of the star presented in the last step of the middle construction. By extending the sides of the stars, the architects made appropriate places for decorated glazed brick which most of them
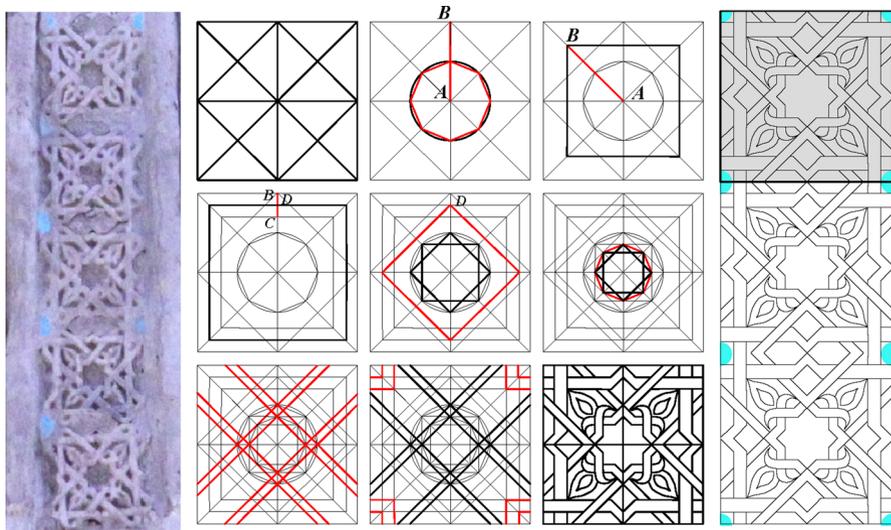
Fig. 10 The stucco and glazed brick in the northern iwan and the analysis of its layout

have been destroyed so far. This pattern has the symmetry type denoted '*p6*' in the wallpaper symmetry groups.

The decorative stucco stripe in Fig. 10 is created by the repetition of a one square shape motif. The geometric construction of this unit is demonstrated in nine steps using the middle images from top-left to the bottom-right. Draw a square and its reflection axes; then, using the midpoints of its sides, construct another square inside of it. Rotate this new square 45° to make another square. Add stripes properly and draw a circle centered at the center of the original square with radius equal to 1/4 of its sides. Make an (8, 2) star polygon with vertices on this circle. Make some new stripes by adding another (8, 2) star polygon inside the previous one. Use the sides of the star polygons to make off-diagonal stripes as those illustrated in the last row of the middle images and then complete the motif. This design has the symmetry type denoted as frieze pattern, '*p112*' in the symmetry groups.

The stuccowork appearing in the eastern porch of the mosque in Fig. 11 is an example of the application of geometric constructions for drawing vegetal patterns. Since the stuccowork was built in a curved surface, the architects had to change the accurate scale of the pattern in some parts. It seems that the pattern might be done based on an exact substructure in this way: draw the equilateral triangle *ABC*, then make three circles with centers at the vertices of the triangle and by a radius equal to one half of a side of the triangle. The three circles meet each other at the midpoints of the triangle's sides. With a similar approach, two more circles can be added, as is illustrated in the second image from the left in the figure. The thick line shows the boundary of the motif that creates the existing substructure for the stuccowork. Another approach in constructing this motif, as is shown in the third image, is to select the 60° arc of *AB* (which is equal to 1/6 of the circumference of a circle) and then apply a 180° rotation which creates the curve *ABC*. Two reflections of this arc
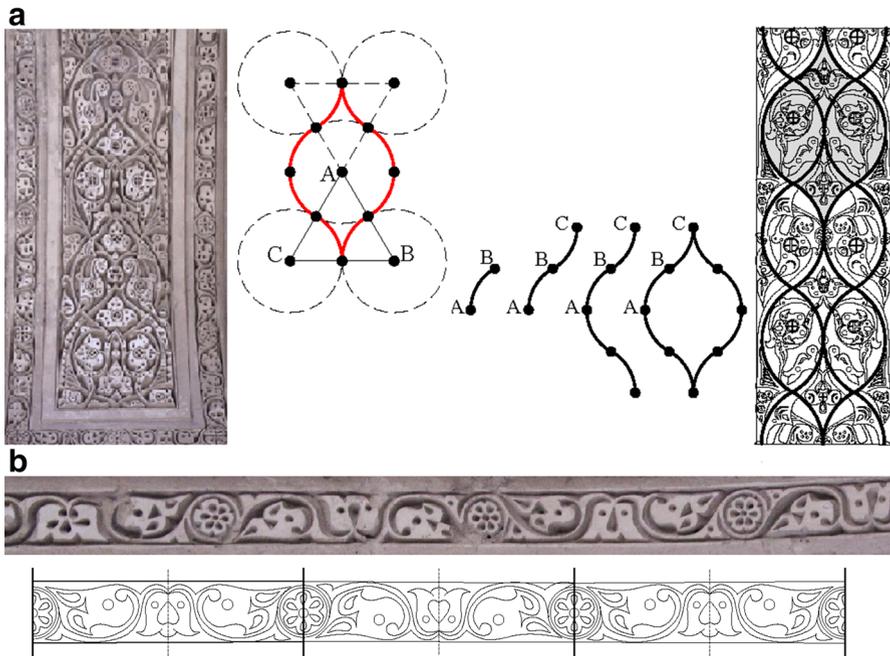
Fig. 11 a The stuccowork in the eastern porch and the analysis of its layout. **b** Analysis of the decorative frame around the main stuccowork

complete the first ornamental frame over a horizontal line from *A* and then over a vertical line from *C*. The fourth image is created by the repetition of the hatched form within the ornamental frame. Moreover, secondary frames are made through these movements in which the internal vegetal motifs are placed. This pattern has the symmetry type denoted as frieze pattern, '*p1m1*'.

The frieze on the frame of the stuccowork in Fig. 11 exhibits a combination of two symmetry transformations that are used alternately and a reflection followed by a half turn rotation. As it is shown in Fig. 11a, dotted lines are the lines of reflection and each motif is separated by the bold lines. This frieze is classified within the seven frieze groups as '*pma2*'.

The vegetal patterns in Fig. 12 are surrounded by a stem-like frame. At a glance, the stuccowork inside the frame may be considered as a free-hand drawing of abstracted flowers, leaves, and stems, exhibiting a snake-like movement. Nevertheless, it will be shown that the architect applied his knowledge of geometry for creating an accurate geometric layout for this pattern. The substructure of this ornamental pattern might be done in the following way by the architects: draw the ornamental frame as is shown in the middle image of the figure like the method mentioned in Fig. 11. Then, as it is shown in the bottom of the middle image, the ornamental frame is transferred based on the vertical lines tangent to its sides. These movements create secondary frames for vegetal motifs. In order to complete the whole decorative stucco ornament, each frame should be transferred based on a
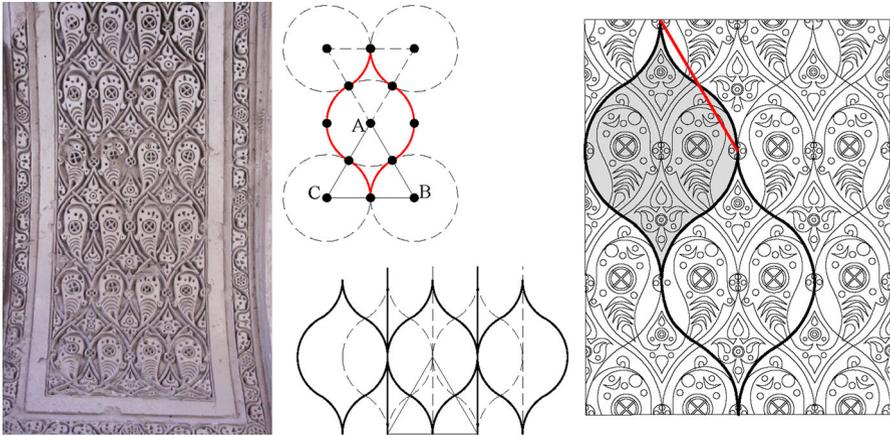
**Fig. 12** The stuccowork in the western porch and the analysis of its layout

a
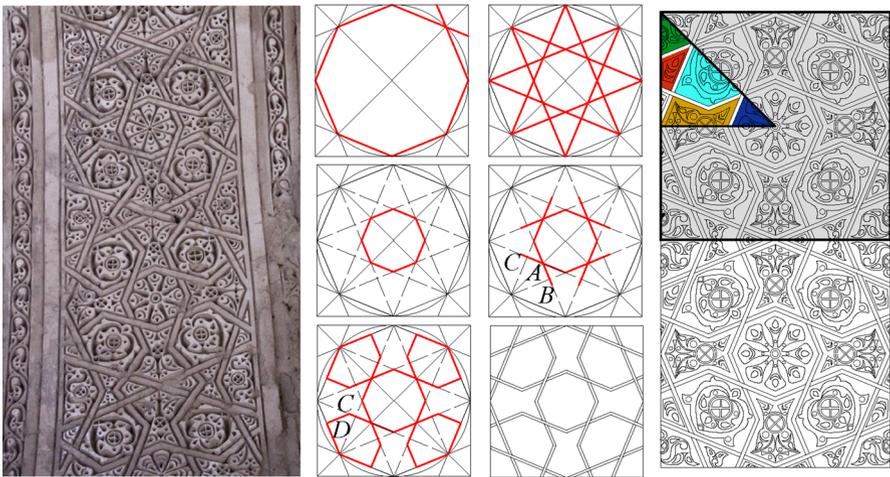
b
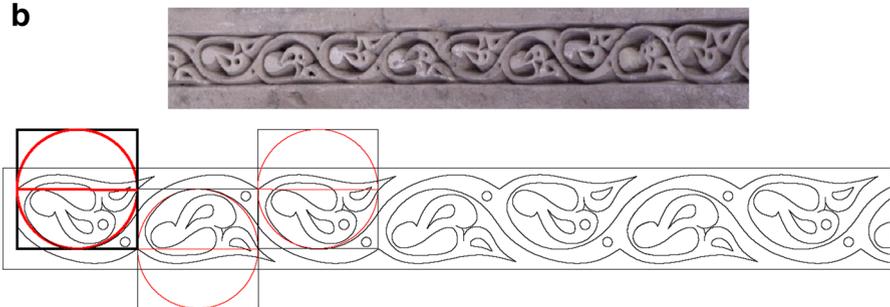
Fig. 13 a The stuccowork in the eastern porch and the analysis of its layout. **b** Analysis of the frieze decorated the frame around the main stuccowork

vector which is shown in the third image. This pattern has the symmetry type denoted 'cm'. The decorative frieze patterns around the stuccowork are exactly the same as Fig. 11b.

For creating the layout of the stucco pattern in Fig. 13, an octagon is inscribed inside of a square and its sides are extended. Then, a (8, 3) star polygon is constructed inside of it. Now, concentrate on the inner octagon and extend its sides to a length equal to each side (here we extend segment *AB* from its *A* side to find *C*). Next, from the free endpoints of these segments (here in our example *C*) make perpendicular segments to the sides of the big octagon (to find point *D*). Repeat this process for other directions to complete the design. For adding interlocking strips to the pattern, one should consider the way which the strips should be interwoven to preserve the rotational symmetry of interlocking strips. This stuccowork has the symmetry type which is classified as '*p112*' in frieze groups.

The stuccowork in Fig. 13a presents another frieze group. The design is based on a glide reflection (a reflection based on a horizontal line that passes through the
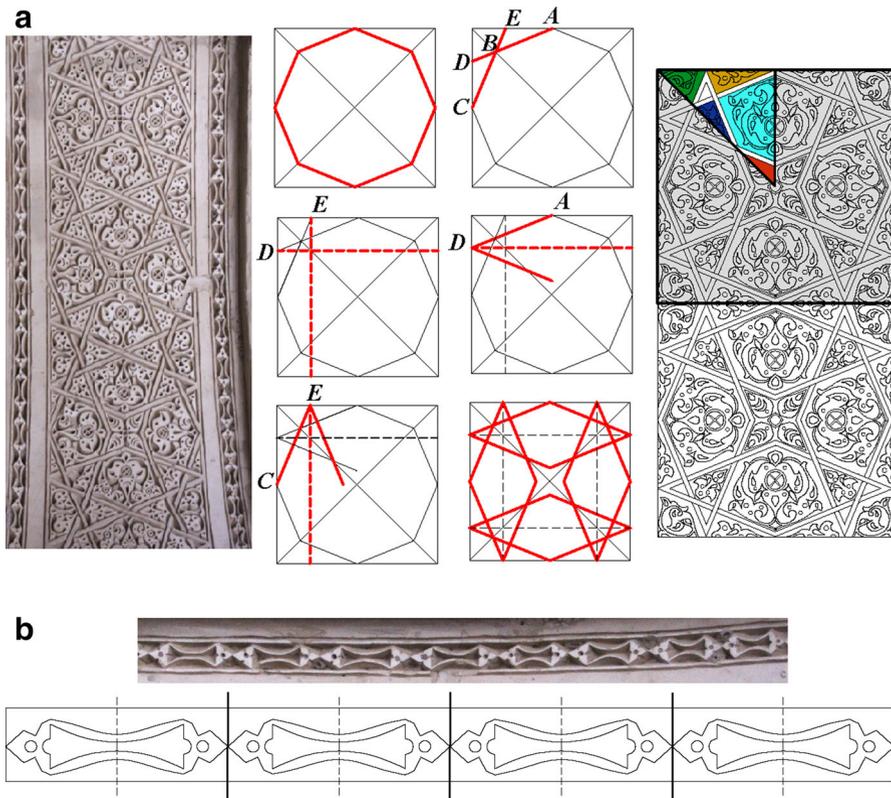
Fig. 14 a The stuccowork in the western porch and the analysis of its layout. **b** Analysis of the frieze decorated the frame around the main stuccowork

middle of the frieze and a translation). The frieze pattern of this stucco follows '*p1a1*'.

In Fig. 14, the underlying design of the stuccowork of the porch is studied. The six drawings in the middle of the figure illustrate how one can complete the motif by starting with an octagon which is inscribed in a square. The fundamental region for this design is one-fourth of the square (or the octagon) that produces the pattern using a horizontal and a vertical reflection. From top-left to bottom-right for the middle six images, construct the octagon and extend sides *AB* and *BC* to meet the square's sides at *D* and *E*. Then, draw a horizontal line from *D* and use this line to reflect *AD*. With a similar approach, make a vertical line from *E* and then use this line to reflect *EC*. Finally, the underlying design is completed through horizontal and vertical reflections of the one-fourth part. This design has the symmetry type denoted '*p112*'.

The stuccowork in Fig. 14 presents a different frieze group. The design is based on a horizontal and vertical reflection as well as a translation. The frieze pattern of this stucco follows '*pmm2*' in the frieze groups.

The underlying design of the existing stuccowork on the left in Fig. 15 is based on a square and an inscribed octagon inside of the square. The middle images from
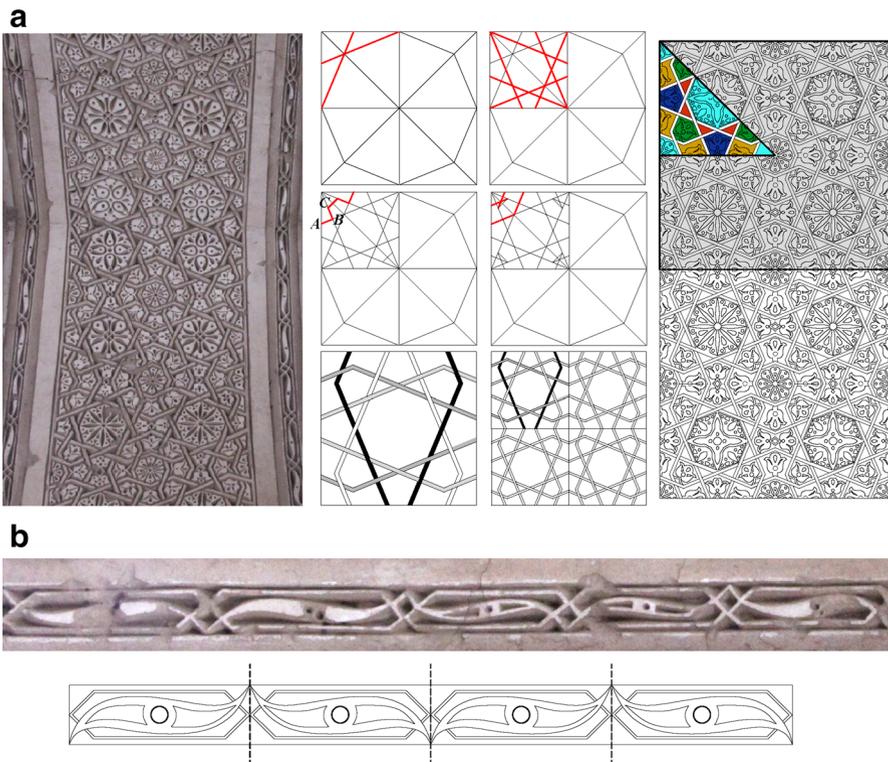
Fig. 15 a The stuccowork in the western porch and the analysis of its layout. **b** Analysis of the frieze decorated the frame around the main stuccowork

top-left to the bottom-right exhibit the process of geometric construction for this pattern: extend the sides of the octagon to meet the square's sides. Then, use reflections to construct the second image. In the third step, draw segment *BC* equal to *AB* and complete the corners as in the fourth image. The fifth image is the enlarged version of the small square in the previous image. In this step, interwoven strips are added to the design. The sixth image illustrates the complete pattern that covers the main square. The large image on the right is a tessellation of the square pattern that is constructed in the middle images. This design has the symmetry type denoted '*p4*'.

The frieze on the frame of the stuccowork in Fig. 15 exhibits a combination of two symmetry transformations, that are used alternately; a reflection followed by a half turn rotation. This frieze is classified as '*pma2*'.

The stuccowork in the left image in Fig. 16, which is located on the entrance of the southern iwan, is created based on the repetition of a regular dodecagon, a 12-gon, about the vertices of a hexagon. As it is illustrated at the top part of the figure, the underlying geometric construction can be made by drawing a square and
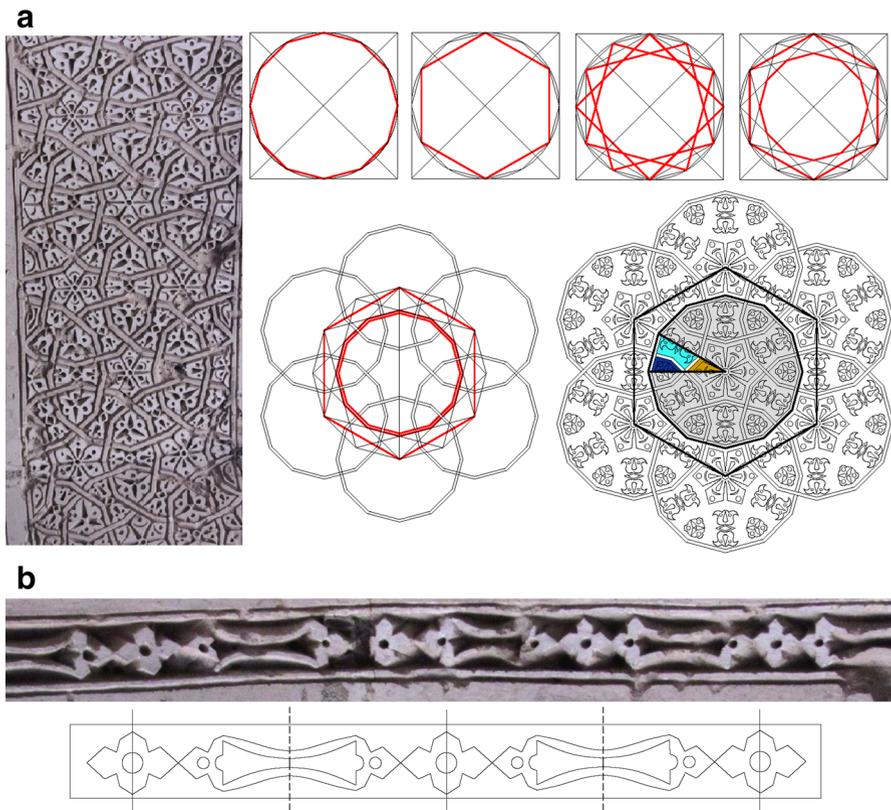
Fig. 16 **a** The stuccowork on the entrance of the southern iwan and the analysis of its layout. **b** Analysis of the frieze decorated the frame around the main stuccowork

inscribing a dodecagon inside of it. Then, we draw a hexagon inside of the dodecagon by connecting every other vertex. The next step is to connect every third vertex of the dodecagon by segments in one direction to create three inscribed squares. The intersections of these three squares are the vertices of another dodecagon. This dodecagon is a repetitive unit for the construction and should be copied on the vertices of the hexagon in repetitive format, as is shown in the middle image. The interlocking, interwoven strips should be added to the design. The details in this stuccowork highlight the creative mind of its designers who used simple tools such as a straightedge and compass to demonstrate proportional properties of regular hexagons and dodecagons and their relationships many years ago. This design has the symmetry type denoted '*p6*'.

The frieze of the stucco in Fig. 16 contains two different motifs. The motifs are symmetric about themselves and the pattern follows '*pmm2*' type of frieze groups.

The stuccowork in Fig. 17 is similar to the work in Fig. 16 but there are some differences in details. Draw a dodecagon in a hexagon as in the previous figure. Then, draw a circle at center *O* with radius congruent to segment *AB* and inscribe a dodecagon inside of it. After that, make three concentric, congruent squares by connecting every third vertex of the dodecagon. This will be resulted in a six-petal star as in the top-left corner of the above images. Make another circle with the same radius as *AB,* tangent to the previous circle, and inscribe three petals of the same star as in the top-right image. Extend the two segments inside of the circle with the same length but in the opposite direction as is shown in the following image. In this stage we can complete the design by choosing an angle smaller than 90° (here in the figure we chose 75°) and by adding segments inside of the circle proper to save its threefold rotational symmetry. Then, extend some segments to complete the design as is shown in the last hexagon of the middle image. One should rotate the generated design about *O* with an angle of 60° five times more to complete this pattern. This pattern has the symmetry type denoted '*p6*'.
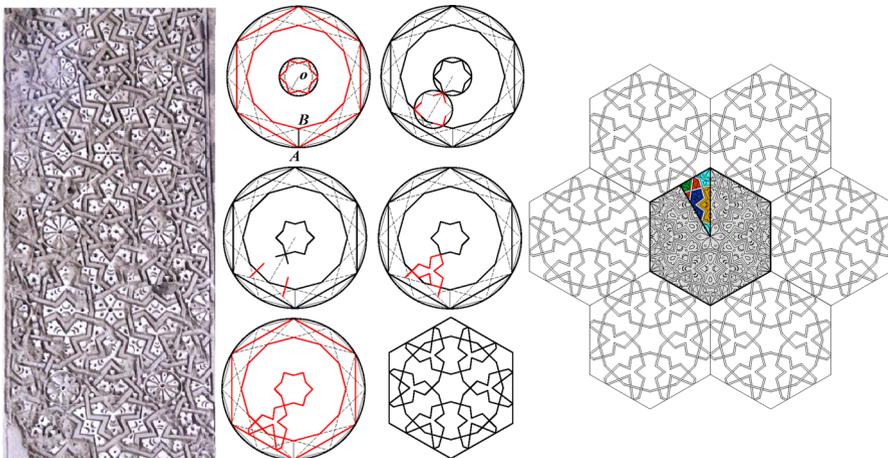
Fig. 17 The stuccowork on the entrance of the northern iwan and the analysis of its layout
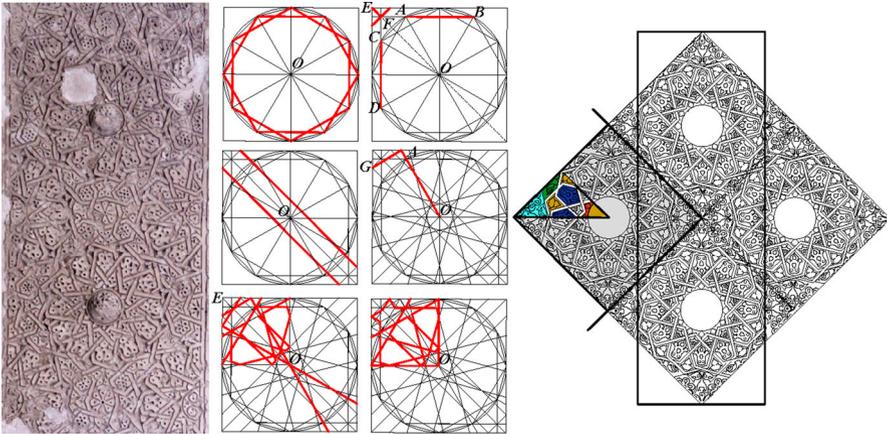
Fig. 18 The stuccowork located at the northern iwan and the analysis of its layout

The design of the stucco in Fig. 18 is similar to the illustration design of Mamluk Quran, which has been previously analyzed (Broug 2006). Inscribe a dodecagon inside circle *O*. Connect every other vertex to make two concentric, congruent regular hexagons. Extend segment *AB* and *CD* to the square's sides. Find segment *EF* that is constructed based on the extensions of segments *AB* and *CD*. In the next step, draw two parallel segments as in the figure. Then, follow the images in Fig. 18 to find the main points for drawing the final pattern. As it is depicted in the figure, we draw one-fourth of the square as a repetitive unit and transfer it to the rest of the square to complete the design. The main pattern has strips which are revolved up and down. As it is shown in the right image in Fig. 18, by interactions of the lines of symmetry, some decorative forms are shaped which are exhibited here in color. This design has the symmetry type denoted '*p4*'.

Stucco in Fig. 19 is a pattern which decorated a plinth in the southern iwan of the mosque. It is one of the samples which have been destroyed and only part of it has survived. This issue makes it difficult to visualize the main pattern. We will draw the substructure of this stucco pattern based on an imagination of the whole design. Then, we will analyze the substructure of this ornament by two different methods. The first method is shown in Fig. 19b and the second method is provided in Fig. 19c. The first method for design of this pattern is based on the traditional way of constructing *girih* patterns in Persian architecture. For drawing the frame of the repetitive unit, divide the right angle *O* into three congruent angles. Then, place an arbitrary point *A* on the first radial. Make perpendiculars from *A* to the sides of the right angle to find points *B* and *C*. The rectangle *OBAC* is the frame of the *girih*. Draw the midpoint of *OC* and call it *D*. Also find *E*, which is the midpoint of *OD*. After that, construct two arcs as in the figure. Point *F* is located where the small arc and the second radial intersect. From *F* make a parallel line to *OC* to find *G*. Connect *G* to *H*, which is the intersection of the small arc with *OB*. Find *I* on *OC* in such a way that *FG* = *FI*. Now, make a segment from *D* to the other end of the big arc on *OB*. This segment will intersect the extension of *GF* on *J*. Now find *K*, *L*,
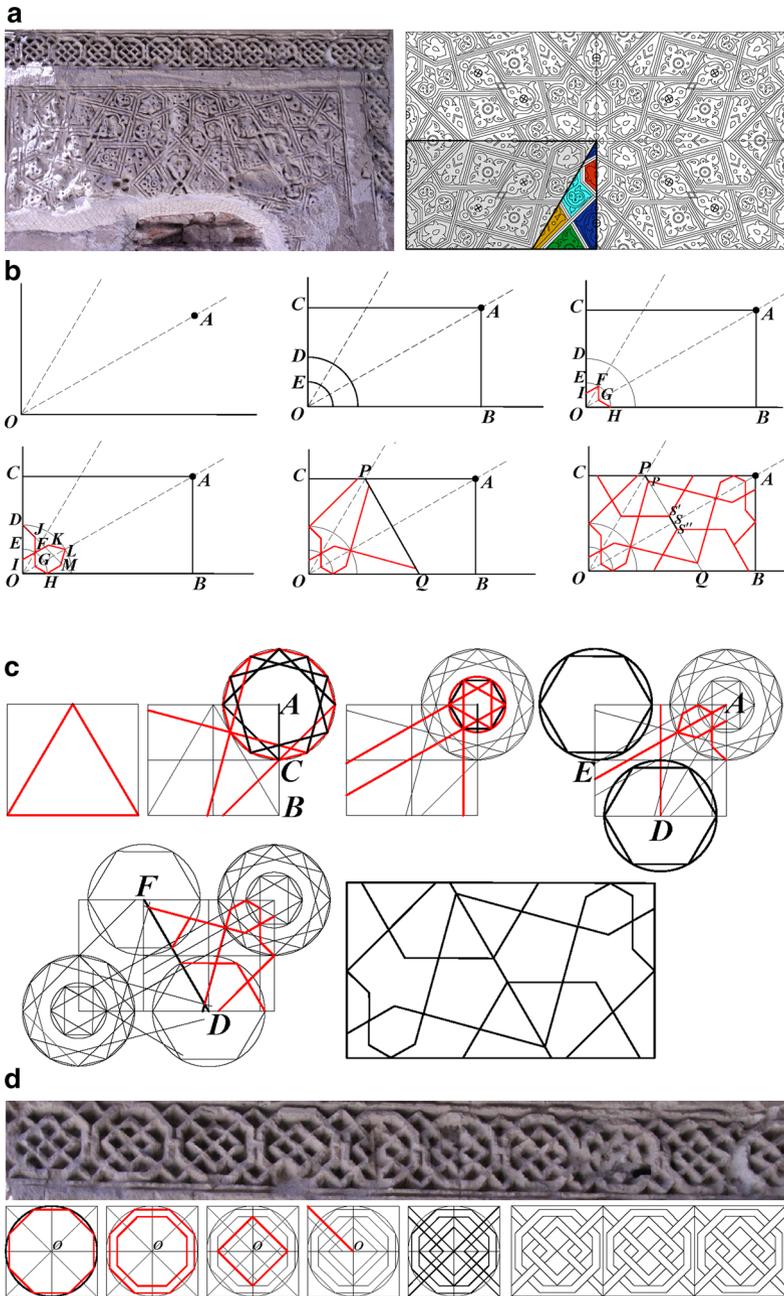
**Fig. 19** **a** The stucco of a plinth in the southern iwan and the analysis of the substructure. **b** Analysis steps of the substructure of the stuccowork. **c** Analysis steps of the substructure of the stuccowork. **d** Frieze pattern of the stucco and the analysis of its substructure

and M using facts such as $FK = FJ$ and K is on line of IF. There is also $KL = JD$. HM is parallel to OA and $HM = FK$. P is the intersection of the second radial with AC. Find Q on OB in such a way that $CP = BQ$. Extend ML and KL to the intersect PQ. From D make the reflection of the extension of ML based on the reflection line of OP. Now use the compass to find S' and S'' in such a way that $PP' = SS'' = SS'$. From S' make a segment parallel to OB to cut OP and transfer the segment based on the reflection line of PQ and then on the line OP. Like the previous step, make a point from S'' and use S as the center for a 180° rotation, the pattern is then completed. In this way, we create one-fourth of the whole decorative pattern which could be extended. It is completed by proliferation of this part based on the lines of symmetry. This design has the symmetry type denoted '*p4*'.

The second method which we consider for constructing the geometric substructure of the stucco ornament of Fig. 19 is based on the extension of an equilateral triangle which is shown in five steps in Fig. 19c. Draw an equilateral triangle and then draw two vertical lines from its angles to make a rectangular. Then, draw a circle with a center at A and radius $AC = 1/2\ AB$. Draw a dodecagon inside the circle and make three concentric, congruent squares by connecting every third vertex of it. Extend three segments of the squares as in the figure. In the next step, draw another circle inside the first one, with a center at A and radius $1/2\ AC$ and inscribe a hexagon inside of it. Make a (6, 2) star polygon by connecting every other vertex of the hexagon and extend three segments like in the figure. After that, draw a circle with center at D, a perpendicular line of the vertex of the first triangle, and with radius AC and then inscribe a hexagon inside of it. In the next step, divide angle A into three parts and transfer the last circle and hexagon based on AE, one of the lines which divides angle A. As it is shown in the figure, the main lines for drawing the substructure are generated which are shown in bold format. Now we transfer motifs based on FD, the segment which connects the centers of circles F and D. In this way, we create one-fourth of a whole decorative pattern which is completed by proliferation of this part based on the lines of symmetry.

Stucco in Fig. 19d shows a frieze of the stucco of Fig. 19. For drawing the substructure, one should draw a circle inside a square and inscribe an octagon inside of it. For interlocking strips, draw a smaller octagon inside the first one. Now, draw segments from the midpoint of four sides of the octagon as illustrated in the figure. After that, draw a line from the center O to one angle of the square and add interlocking strips to it. Transfer these segments based on the lines of symmetry. This pattern denoted '*p4*' in the frieze groups.

Stucco pattern in Fig. 20 is another example of the complex design with an accurate geometric substructure. Like the previous figure, we will analyze the substructure of the stucco ornament of Fig. 20 by two different methods. The first method which is offered for creating this decorative pattern is as follows: divide the right angle O into six congruent angles. Then, place an arbitrary point A on the fourth radial. Make perpendiculars from A to the sides of the right angle to find points B and C. Then transfer the rectangle $OBAC$ based on the reflection line AB in order to create a rectangle for the frame of the *girih*. After that, draw a circle with center B and tangent to the fourth radial. Then, inscribe a nine-sided polygon inside of the circle. Draw another circle with center B and tangent to the second radial and
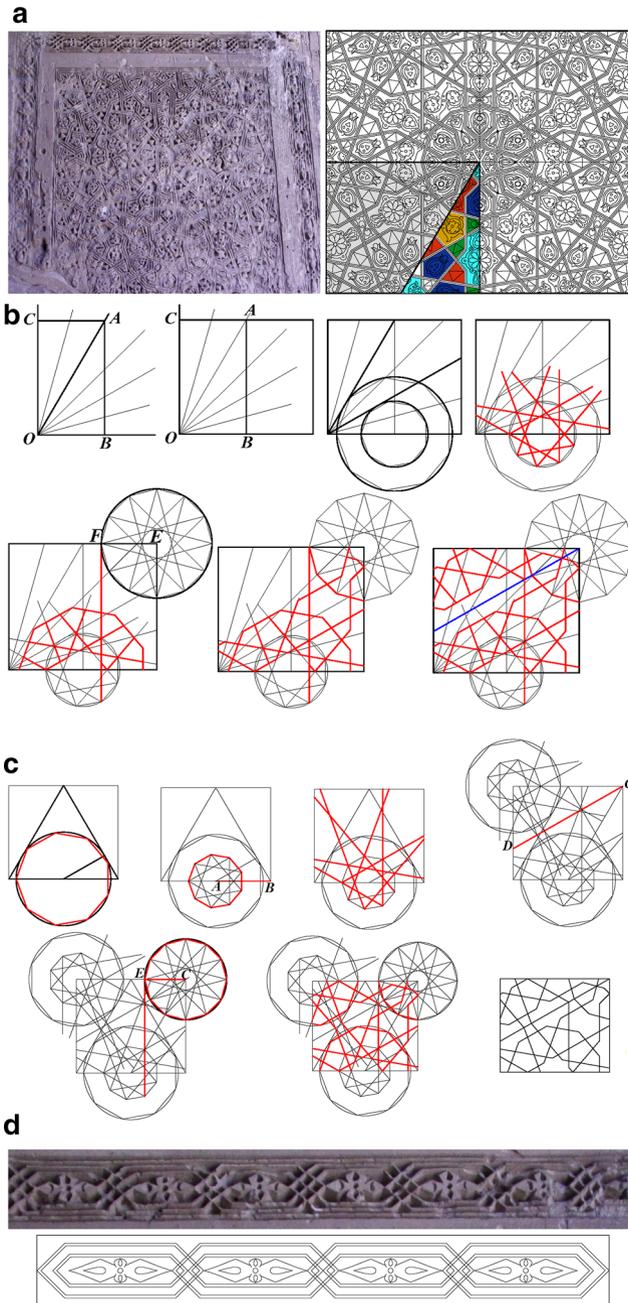
Fig. 20 a The stucco of a plinth located at the southern iwan and the analysis of its substructure. b Analysis steps of the substructure of the stuccowork. c Analysis steps of the substructure of the stuccowork. d Frieze pattern of the stucco and the analysis of its substructure

inscribe a nine-sided polygon inside of it. Make three concentric, congruent triangles by connecting every third vertex of the small nine-sided polygon and extend the sides of these triangles like in the figure. Draw a circle with a radius of *EF* and inscribe a dodecagon inside of it. Then, connect every fifth vertex to each other. The next step is to divide angle *E* into three congruent angles and transfer the motifs in the last step based on the second radius as in the figure. The last step shows the underlying design for this pattern. Now we can find the lines for creating the main motifs as there are shown in bold. The whole design could be completed by transferring this part based on the symmetry lines. This design has the symmetry type denoted '*p4*'.

The second method which we setup for the geometric substructure of the stucco ornament of Fig. 20 is like Fig. 19c, based on the extension of an equilateral triangle. For constructing the substructure, draw an equilateral triangle and then draw two vertical lines from its angles to make a rectangle. Inscribe a circle in the triangle and then inscribe a nine-sided polygon inside the circle. Draw another nine-sided polygon with radius equal to *1/2 AB*. Make three concentric, congruent triangles by connecting every third vertex of the nine-sided polygon. Extend these triangles' sides like in the figure. The next step is to divide angle *C* into three equal parts and transfer the motifs in the last step based on the segment *CD*. As it is shown in the figure, draw a circle with radius *CE* and inscribe a dodecagon inside of it and then connect every fifth vertex of the dodecagon. The last step shows the underlying design for this drawing which is shown in bold format. By doing so, we are able to find one-fourth of the whole design. Transferring this part based on the symmetry lines completes the pattern.

The frieze pattern which is shows in Fig. 20d is also designed based on the repetition of one motif. This motif is symmetric and the whole pattern follows the '*pmm2*' type of frieze groups.

The stuccowork in Fig. 21 is another ornament of the Friday Mosque of Forumad that has been damaged severely. Like the two previous examples, we prepare the substructure of design by two different methods. The first method which is shown in Fig. 21b is drawn based on a similar method mentioned in the book *Arabic Geometrical Pattern and Design* (Bourgoin 1973). The book does not provide step-by-step instructions for the geometric constructions but there are underlying circles and segments using thin dashed lines that are instrumental for performing such instructions. As it is shown in the figure, the substructure for this design is based on a rectangular frame. Make five radials to divide the given 90° into six equal angles. On the fourth radial, select an arbitrary point *P* and make the rectangle frame for the *girih*. Make the bold circle which is tangent to the two sides of the rectangle and the second radial. The center of this circle is the intersection point of the three angle bisectors of the triangle *ABC*. Other small circles are congruent with the bold circle. The larger circle with center at *A* is tangent to the bold circle. The radius of the smaller circle with center at *A* is created by dropping a perpendicular from the intersection of the third radial and the second circle. As it is shown in the third step, the substructure for drawing this pattern is made and the motifs are shown in bold. In this way, the repetitive unit is created and the whole design can be completed by transferring this rectangle based on the lines of symmetry. This stucco pattern has
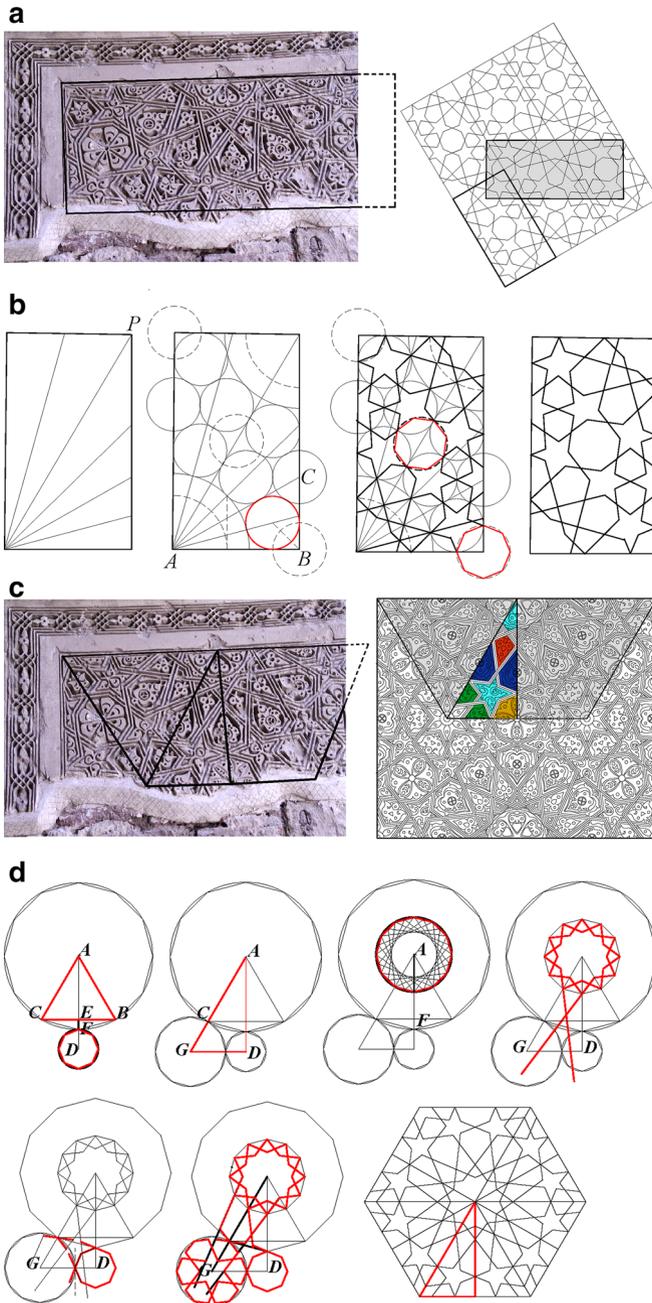
Fig. 21 **a** The stucco of plinth of the southern iwan and the analysis of its substructure (Method 1). **b** Analysis steps of the substructure of stuccowork. **c** The stucco of a plinth of the southern iwan and the analysis of its substructure (Method 2). **d** Analysis steps of the substructure in the stuccowork

the symmetry type denoted '*p6*'. The frieze of this stucco is also similar to the frieze of the stucco in Fig. 20.

The second method which we offer for constructing the substructure of the stucco ornament in Fig. 21d is based on the extension of an equilateral triangle like the two previous examples. First, draw an equilateral triangle *ABC*. Draw a circle with center at *A* and with the radius *AB* and inscribe a dodecagon inside of it. The next step is to extend the perpendicular segment of *AE* and mark the segment *FD = EF* on it. Then, draw a circle at the center *D* with the radius *FD* and inscribe an octagon inside of it. After that, draw a horizontal line from the point *D* and extend *AC* to meet the last line at *G*. With center at *G* and radius *CG,* draw a circle and inscribe a dodecagon inside of it. In the next step, draw a circle with center *A* and radius *1/2 AF* and inscribe another dodecagon inside of it. Connect its vertices in such a way that each segment is from one vertex to the middle point of its fourth vertex. Finally, we have a star form and like in the figure, extend two segments from this form. Then extend the sides of octagon *D* into the circle *G* and transfer them on the base of the inscribed dodecagon in *G*. As it is shown in the figure, the substructure for drawing this pattern is made by this way and the motifs are shown in bold format. The repetitive unit is a right triangle and the whole design is completed by transferring this triangle based on the central symmetry.

The Ornaments of the Mihrab and the Muqarnas

Figure 22 shows the detail of the mihrab and the muqarnas ornaments in the southern iwan of the Friday Mosque of Forumad. The stucco patterns which are applied to decorate muqarnas ornaments might be designed based on a two-dimensional view, later it is exerted on a concave surface. Although the northern iwan is also decorated by muqarnas ornaments, most of them have been destroyed so far. Therefore, we will analyze the substructure of design taken from the vegetal patterns of muqarnas in the southern iwan in the following figures. Since these ornaments are made on a concave surface, the stuccoworks are a little different from the patterns which are drawn on plane surface.

The stucco pattern in Fig. 23 is a part of the decorative stucco on the mihrab of the southern iwan. The analysis for drawing the substructure is shown step by step in the middle figure. Draw a circle and inscribe a dodecagon inside of it. Make three concentric, congruent squares by connecting every third vertex. Draw another circle at the center *O* equal to the half of the first circle and inscribe another dodecagon inside of it. This time make four concentric, congruent triangles by connecting every fourth vertex. In the next step, extend the segment *AB* to point *C* where *AB = BC*. Draw a vertical line from point *C* and transfer segments *DE, DA,* and *AB* based on the line which divides angle *B* into two parts. Now at center *F* with a radius *FG,* draw a circle and inscribe an octagon inside of it. As it is shown in the last step, one repetitive unit is completed by transferring a quarter of the pattern. One repetitive unit is a square which can be transferred based on the lines of symmetry, the squares' sides. This design has the symmetry type denoted '*p4*'.
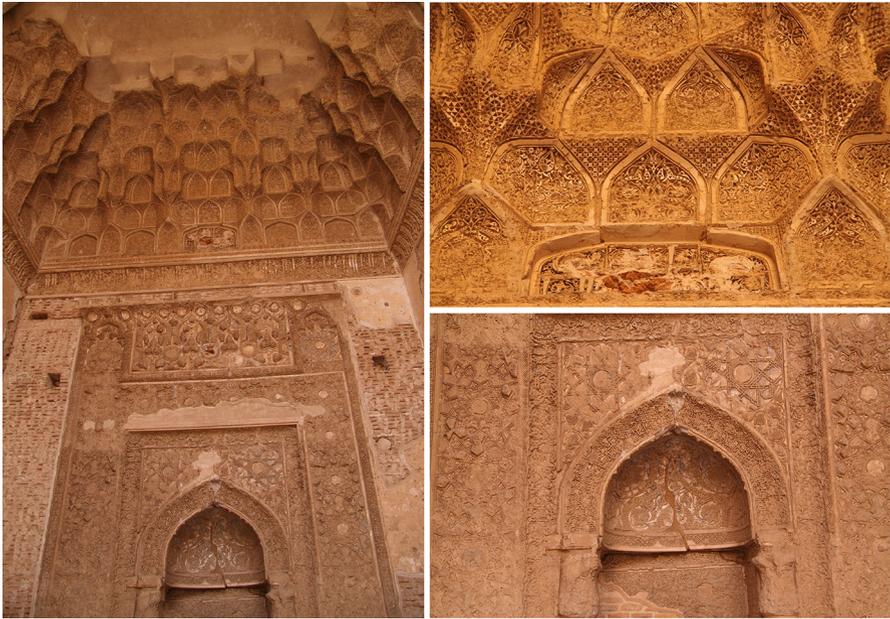
Fig. 22 The mihrab and the muqarnas of the Friday Mosque of Forumad
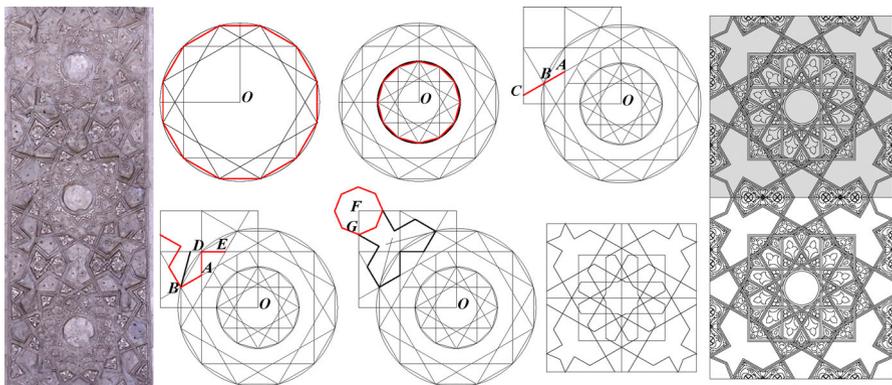
Fig. 23 The stucco on the mihrab and the analysis of its substructure

By comparing Figs. 17 and 24 one can conclude that the architects applied the same motifs for decorating the stucco on a side of the mihrab as the stucco on the entrance of northern iwan. However, in the mihrab these motifs by a different substructure method were used. This pattern is made by proliferation of a hexagon as a repetitive unit. For drawing the substructure, draw a circle with a center at O and radius OA and inscribe a hexagon inside of it. Then, draw another circle with a center at O and radius equal to *1/2 OA* and inscribe another hexagon inside of it. In the next step, extend the inner hexagon's sides to the first hexagon's side so it makes
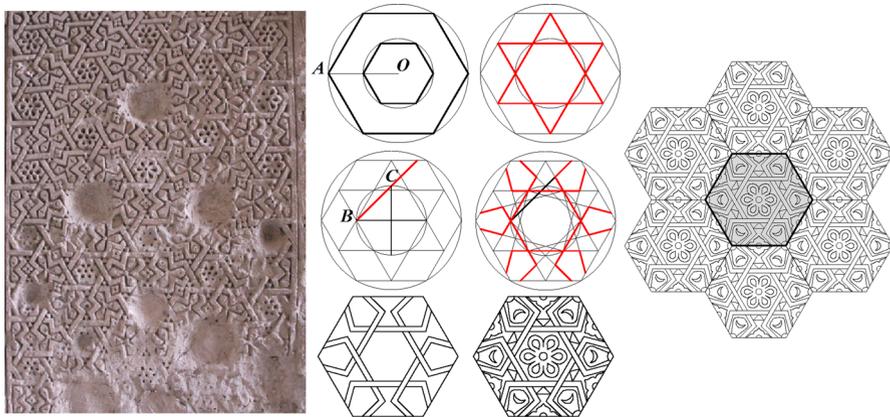
Fig. 24 The stucco on the mihrab and the analysis of its substructure

two concentric, congruent triangles. After that, draw segment *BC* by connecting a quarter of the inner circle's points. Extend segment *BC* to the first hexagon's side. In the last step, transfer the segment *BC* around the center *O* to gain the underlying substructure. Motifs for this drawing are created from the aforementioned steps and they are shown in bold format. As it is shown in the figure, the whole decorative pattern is completed by transferring the repetitive unit around its sides. This stucco has the symmetry type denoted '*p6*'.

For drawing the substructure of design of the stucco in Fig. 25, the first step is to make the frame. Draw square *ABCD*, then draw an arc at center *A* and radius *AB = BC = 1*. After that, draw another arc at center *B* and radius *AB*. These two arcs meet each other at a point which makes the vertex of the ornamental frame. For drawing vegetal motifs as is shown in the figure, a geometric division is offered for making the substructures. The main frame is divided and six circles are drawn which determines the best places for vegetal motifs. The analysis demonstrates that the application of geometry for drawing the substructure of vegetal motifs was applied in this pattern.

The stucco in Fig. 26 illustrates another pattern that is used for decorating parts of the muqarnas ornament. Draw a frame like in the previous figure. Now, for creating the substructure, divide the frame and make four congruent circles inside the square *ABCD*. Then, extend *AD* and *BC* to provide another division for placing circles in design. Although this vegetal decorative pattern does not exactly follow this geometric substructure, the main division for selecting the ideal places for the motifs might follow a kind of geometric division as in this analysis.

The frame in Fig. 27 is the same as the previous examples. The suggested underlying method for drawing this pattern is to divide the frame and make congruent circles like in the previous figures. This kind of similar frame and geometric division for drawing various forms of vegetal patterns demonstrate the advanced creativity of the architects; and that's why there is no monotony in the ornaments.
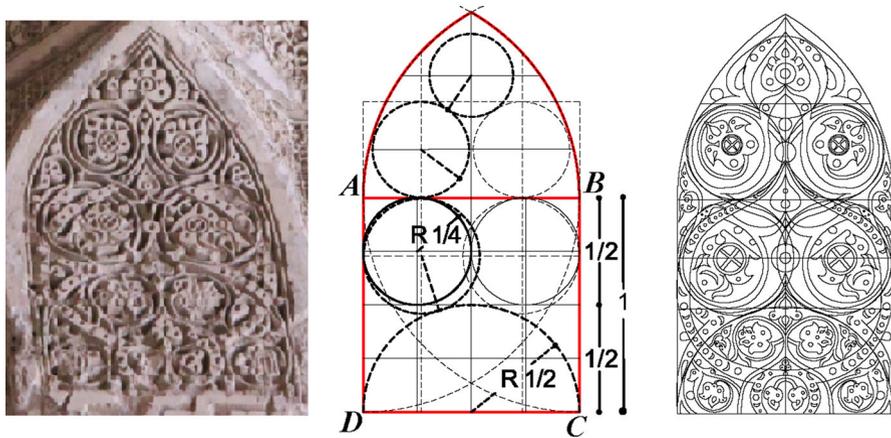
Fig. 25 Detail of the muqarnas stucco ornament of the southern iwan and the analysis of its substructure
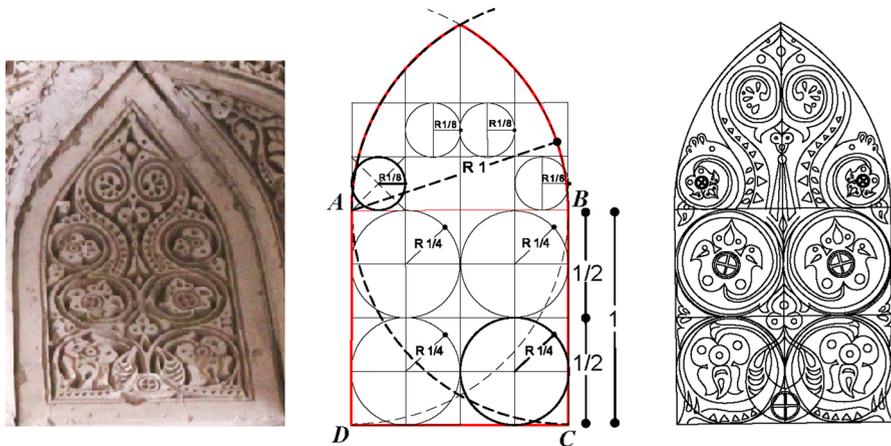
Fig. 26 Detail of the muqarnas stucco ornament of the southern iwan and the analysis of its substructure

For drawing the substructure of the stucco which is shown in Fig. 28, we make a frame like in the previous figures. The suggested substructure for this stucco is a little different from the previous examples. In the upper part of the square ABCD, draw a circle inside the divided surface. Then make two other congruent circles above and below the main circle for creating ideal places for vegetal motifs.

For drawing the frame of the stuccowork in Fig. 29, first draw rectangle ABCD. Then make a circle at the center in the middle of the segment AB and radius equal to half of AB. An arc can then be drawn based on this division. This frame is filled with a single motif which is symmetric; thus, we separate this motif into two parts. Then, in order to create an accurate pattern inside the main frame like in the figure, one
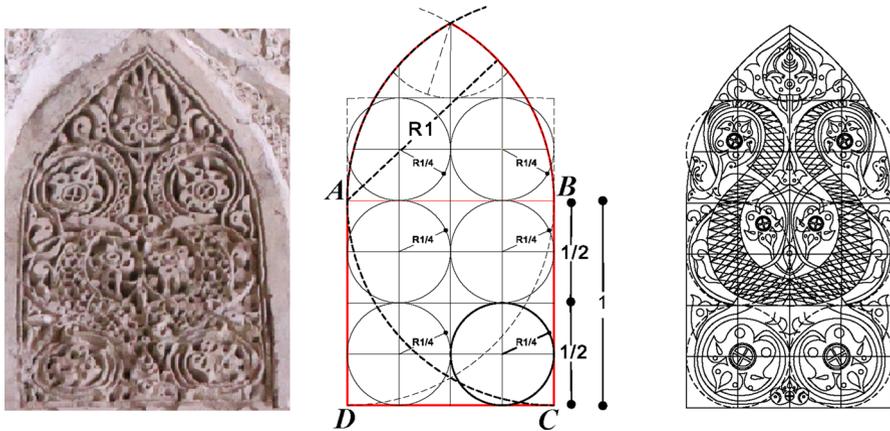
Fig. 27 Detail of the muqarnas stucco ornament of the southern iwan and the analysis of its substructure
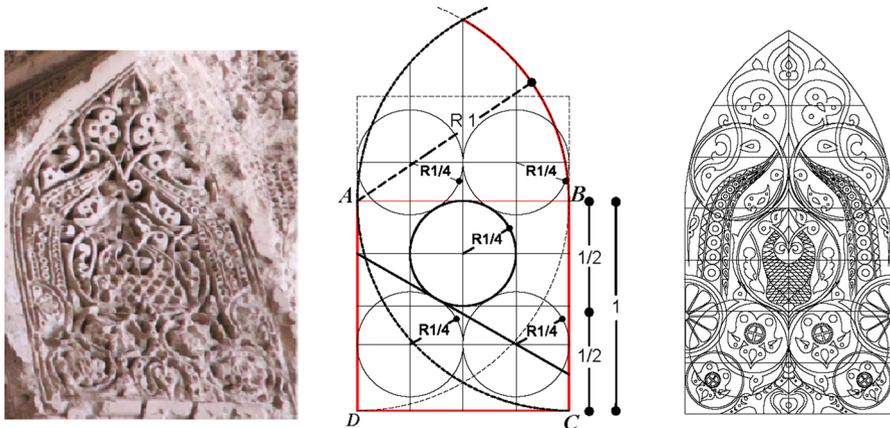
Fig. 28 Detail of the muqarnas stucco ornament of the southern iwan and the analysis of its substructure

row is started with the motif shown by the vector *AB* whereas its above and below rows should start with the motif shown by the vector *BA*.

The stucco pattern in Fig. 30 is created by the application of one motif. The frame is the same as the previous ones and it is filled with the repetition of one motif. Moreover, the motif utilizes the same frame as the stucco ornaments in Figs. 11 and 12.

All the analyses of the substructures, design methods, and geometric constructions which were discussed in this part were based on the imaginations of the whole decorative pattern and by considering the simple instruments for constructing these designs. We cannot assure that the methods which were shown here are the ones used by the original architects. However, these analyses indicate the knowledge of the architects in practical geometry and support the idea that the architects would
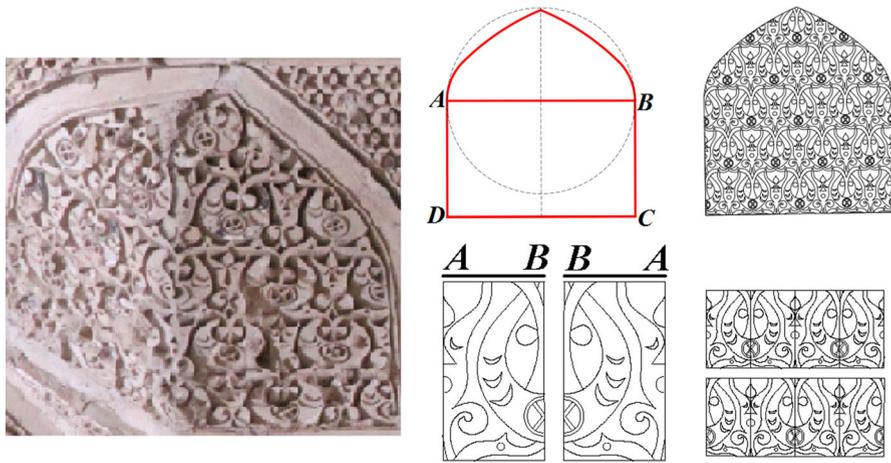
Fig. 29 Detail of the muqarnas stucco ornament of the southern iwan and the analysis of its substructure
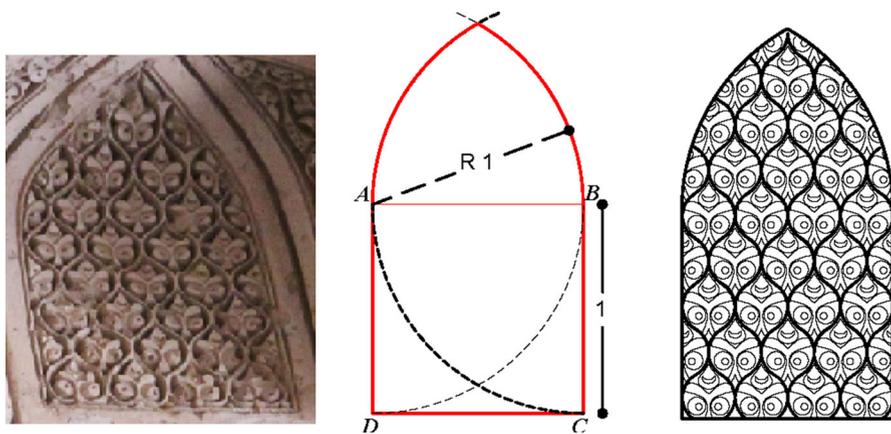
Fig. 30 Detail of the muqarnas stucco ornament of the southern iwan and the analysis of its substructure

have been aware of the methods of geometric divisions, symmetrical design, and the ways of proliferations of the patterns to create geometric substructures for the exclusive architectural ornaments.

Discussion

In order to draw a more substantial conclusion and follow the substructures and geometric methods of designs of the ornaments, we present the analyses in a table. Therefore, herein, the design methods, substructures, geometric constructions,

Table 1 Analysis of the ornaments

Architectural ornament	Substructure	Repetitive unit	Motifs	Geometric features
				Application of two different symmetrical motifs alternatively. Proliferation of the repetitive unit for making substructure by a vector in vertical and horizontal directions. The symmetry type is 'p4m'
				Translation of the hexagon in three directions. The whole pattern has rotational symmetry. The symmetry type of the substructure is 'p6'.
				Proliferation of one unit based on the lines of symmetry. The motifs in one triangle have 3-fold symmetry. The whole pattern has rotational symmetry. The symmetry type is 'p6'.
				Proliferation of one unit around one hexagon's vertex by a vector. Translate the repetitive unit in six directions. Decorative pattern has rotational symmetry. The symmetry type is 'p6'.
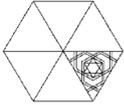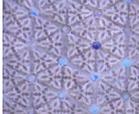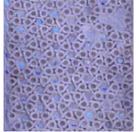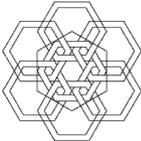
Table 1 continued

				Proliferation of one unit based on the lines of symmetry. Extend the motifs in every direction. The whole pattern has rotational symmetry. The symmetry type is 'p6'.
				Proliferation of one unit in one direction. Repetitive unit is symmetric. The symmetry type of the substructure is 'p112'
				Proliferation of one unit in one direction by a vector. Repetitive unit has reflection symmetry. Secondary frames are made through the movements. The symmetry type is 'p1m1'.
				Alternatively contrariwise application of one symmetrical motif. The symmetry type is 'pma2'.
				Proliferation of one unit in one direction by a vector. Repetitive unit has reflection symmetry. Secondary frames are shaped through the movements. The symmetry type is denoted 'cm'.
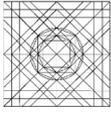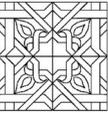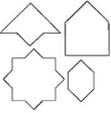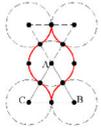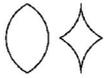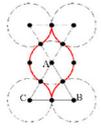
Table 1 continued

				Proliferation of one unit in one direction by a vector. The repetitive unit is symmetric. The symmetry type of the ornament is '*p112*'.
				Contrariwise application of one motif in one direction. The symmetry type is '*p1a1*'
				Proliferation of one repetitive unit. The repetitive unit is symmetric. The symmetry type of the ornament is '*p112*'.
				Application of one motif in one direction. The symmetry type is '*pmm2*'
				Proliferation of one unit in one direction by a vector. Repetitive unit is symmetric. The symmetry type of the ornament is '*p4*'.
				Application of one motif. The symmetry type is 'pma2'.
				Proliferation of one repetitive unit, a dodecagon, around the hexagon's vertex. The pattern has rotational symmetry. The symmetry type is 'p6'.
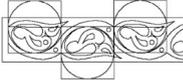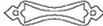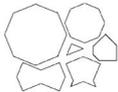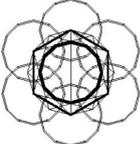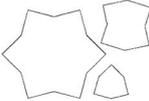
Table 1 continued

				Two different symmetrical motifs. The symmetry type is 'pmm2'
				Proliferation of one unit based on the lines of symmetry. Decorative pattern has three-fold rotational symmetry. The symmetry type is 'p6'.
				Proliferation of one unit in one direction by a vector. Secondary frame are shaped from the proliferation of repetitive unit. The symmetry type is 'p4'.
				Proliferation of one-fourth of the whole pattern. The symmetry type is '_p4_'.
				One repetitive square unit. The symmetry type is 'p4'.
				Proliferation of one-fourth of the whole pattern. The symmetry type is 'p4'.
				Proliferation of one symmetric motif. The symmetry type is '_pmm2_'
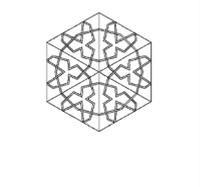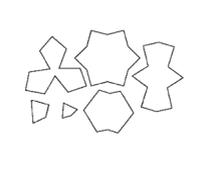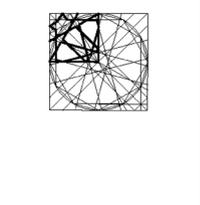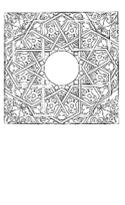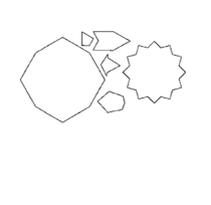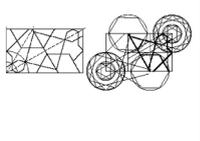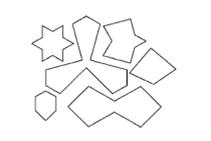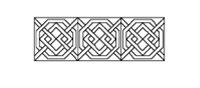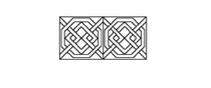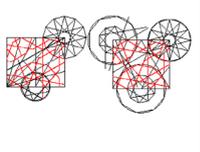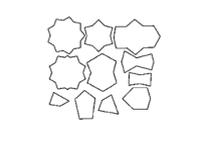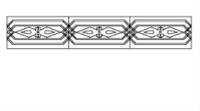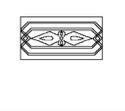
Table 1 continued

				Proliferation of one-fourth of the whole pattern. The symmetry type is '*p6*'.
				Proliferation of one unit in one direction by a vector. One repetitive unit is a square which can be transferred based on lines of symmetry. The symmetry type is 'p4'.
				Proliferation of one unit based on the lines of symmetry. Decorative pattern has rotational symmetry. The symmetry group in substructure is '*p6*'.

repetitive units, and proliferation methods in design of the ornaments are being studied and compared.

By reviewing the analyses of the ornaments in Table 1, it becomes more obvious that multiple methods and geometric constructions were used to draw and construct the ornaments of the Friday Mosque of Forumad. Based on the analysis, we can categorize the patterns according to similarities as follows:

Similar Substructure with Different Patterns

There are some substructures in design of the ornaments which have similar repetitive units and the same form of proliferation but they are made with different patterns. For instance, in Figs. 6, 7, 9, 17 and 24 the repetitive unit is based on the symmetrical transition of one hexagon alongside of its sides. The other method is based on the transition of a repetitive unit from the center of a hexagon around its vertices by a vector equal to half of the hexagon's diagonal like the ones shown in Figs. 8 and 16a. Moreover, the translation of a square as the repetitive unit of the substructures is used in Figs. 5, 10, 13a, 14a, 15a, 18 and 23 to create the ornamental design.

In some other substructures it is observed that they are designed beyond the main frame, and the architects just selected a part of the substructures to create the ornamental patterns, e.g. in Figs. 6, 7, 8, 9, 16a, 17, 18, and 24.

Similar Patterns in Different Substructures

In addition to the first category mentioned, there are some patterns which might be seen similar at a glance but they have different substructures like Figs. 16a and 17; Figs. 24 and 17; Figs. 9, 18 and 23.

Patterns with Similar Symmetry

As it was shown in the analysis part, architects used one repetitive unit to complete the whole pattern through its symmetric translations. We classified each pattern based on the type of symmetry groups. The symmetry group '*p6*' is used for a large number of patterns such as the ones in Figs. 6, 7, 8, 9, 16a, 17, 21, and 24. In Figs. 15a, 18, 19, 19d and 20a, the symmetry group '*p4*' is utilized. The symmetry group '*p112*' is applied for creating patterns in Figs. 10, 13a and 14a. The symmetry group '*pmm2*' is used for the ornaments of Figs. 14b, 16b, and 20d. Moreover, in Figs. 11b and 15b, the symmetry group '*pma2*' is exerted. There are also some symmetry groups which are applied for only one particular figure like: '*p4m*' in Fig. 5, '*plml*' in Fig. 11a, '*cm*' in Fig. 12, and '*plal*' in Fig. 13b. These features demonstrate the advanced ability of the architects to apply various symmetrical approaches for constructing the substructures of ornaments. It is mentioned that in most cases the interlocking strips were added to the patterns. These strips should be made in such a way to create the upward and downward format at every other intersection. Therefore, one repetitive unit is symmetric apart from the interlocking strips and the architects had to make the whole patterns in separate parts when interlocking strips were added to them. We can see that this condition exists for the repetitive units of all the figures with interlocking strips. Thus, when finding symmetry groups matters we ignored the upward and downward format of interlocking strips and considered the substructure of design, separately.

Conclusion

The construction of geometric substructures in the ornaments of the Friday Mosque of Forumad has been analyzed in this paper. The analysis shows that the architects applied different approaches for drawing the substructure of similar patterns. Moreover, diverse methods of proliferation with many different motifs in the construction of the ornaments were used. These features illustrate the knowledge of practical geometry as well as the creativity of the architects in design. In addition to the geometric patterns, the analysis demonstrates that there are some vegetal patterns with the curved stems in the ornaments of this mosque which are made based on the geometrical substructures. So that, in the results, there is no heterogeneity or clutter in designs. These characteristics in design of the ornaments make the Friday Mosque of Forumad as an exclusive pioneering mosque in applying complicated geometric ornaments in the history of Persian architecture.

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