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Proposal of a New Tool for 3D Pattern Generation

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Abstract The behavior of sound in an enclosed space is very complex. This behavior is closely related to boundaries, material properties and with the form of the space they are in. In this paper, the image source method as a method for modeling the sound field in enclosed spaces is re-visited and patterns of image sources and their relationships with the geometry of enclosing space is studied by using polygons/polyhedrons in 2D and 3D. Symmetries of these image source points, identification of possible polygons enabling simple geometrical shape formations and knowing the resulting geometrical pattern, is used as an input to an acoustic model, reducing the complexity and computational time. Visualization of such complex patterns provides a solid tool to grasp complex relationships. Therefore transcoding the invisible relationships of acoustics into geometric patterns of mathematics provides a valuable means to improve the cognition of sound phenomena and to extract information as design input.

Introduction

The behavior of sound in an enclosed space compared with free field conditions is very complex. This behavior is closely related with the boundaries, their material properties and with the form of the space they are in. Several methods have been used to model this behavior in closed spaces such as image source method (ISM), ray tracing method, and hybrid models governing multiple methods (Egan 1988). In

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these methods, the bounding surfaces are treated as the planes of reflection where the rules of symmetry have been directly applied and the resulting patterns have been employed to model the sound fields. As in any computational model, the precision, reliability, and validity of these acoustic models and their computational costs are the major criteria to evaluate their efficiency in modeling the sound fields. When the ISM is inspected in particular, it is seen that ISM is simple and easy to apply since any order image source is obtained by mirror symmetries in any form of space. In addition, ISM is able to model all possible reflections occurring in an enclosed space while statistical methods such as ray tracing method only traces finite numbers of rays to model reflections. However, when the computational times of those methods are compared, despite all the advantages of the image source method, ISM is used for the calculation of the first few reflections where the sound pressure levels are high and the rest of the sound pressure level decay is calculated via other methods such as the ray tracing method (Chen 2007). Also, ISM is supported with a scattering algorithm as scattering caused by surfaces is disregarded in ISM. An example of sound energy decay in an enclosure and methods used as an example are shown in Fig. 1.

In this paper, the ISM is re-visited and the patterns of image sources and their relationship with the geometry of the enclosing space have been studied by the use of regular and irregular polygons (polyhedrons) in 2D and 3D. Influenced by the symmetries of image source points in space, the identification of possible polygons (polyhedrons) permits us to model the reflection patterns as just simple geometrical shape formations. Knowing the resulting geometrical pattern, data is used as an input to the acoustic model, reducing the complexity and thus the computational time required by ISM.

The paper also aims to employ these geometrical patterns to visualize the behavior of the sound source in an enclosed space and the relationship between the source position and the form of space. The authors believe that the visualization of such complex patterns provides a solid tool to grasp complex relations. Therefore transcoding the invisible relations of acoustics into the geometric patterns of mathematics provides a valuable means to improve the cognition of sound phenomena.

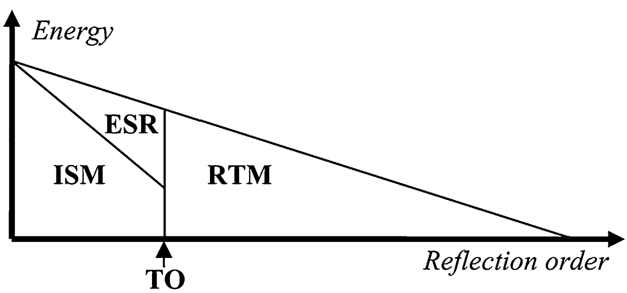


Fig. 1 Hybrid calculation method of ODEON Software (*ISM* image source method, *ESR* early scattering rays, *RTM* ray tracing method) (Christensen and Rindel 2005)

Image Sources Method

Image source method is a commonly used method for analysis of sound fields in enclosed spaces. ISM employs an optics analogy to construct images of a sound source to set up a sound field involving all possible reflections (Rossing 2007). In the course of achieving this goal several assumptions are made; the sound source is treated as an omnidirectional point source, thus the sound emitted from the sound source is homogenously distributed from the sources without having any directionality, and the source is treated as a dimensionless entity—as a point—so that the volume and area of the source is ignored in order to obtain omni-directionality to attain a simplistic approach (Kuttruff 2006). In the course of generating image sources, all the boundary surfaces of the enclosed space are treated as planar surfaces. Hence, it is possible to generate images of the sound source as a plane mirror. Firstly, first order image sources are generated with respect to the original sound source, and then first order image sources are treated as primary sound sources to obtain second order image sources. This generation process is continued until a termination criteria (order dependent or energy dependent) is reached. When the termination criteria is reached, boundary surfaces are neglected and the sound field is treated as a free field (Allen and Berkeley 1979). In the process of generating image sources several tests can be employed to increase the precision of the method, such as validity and visibility tests. In the scope of this

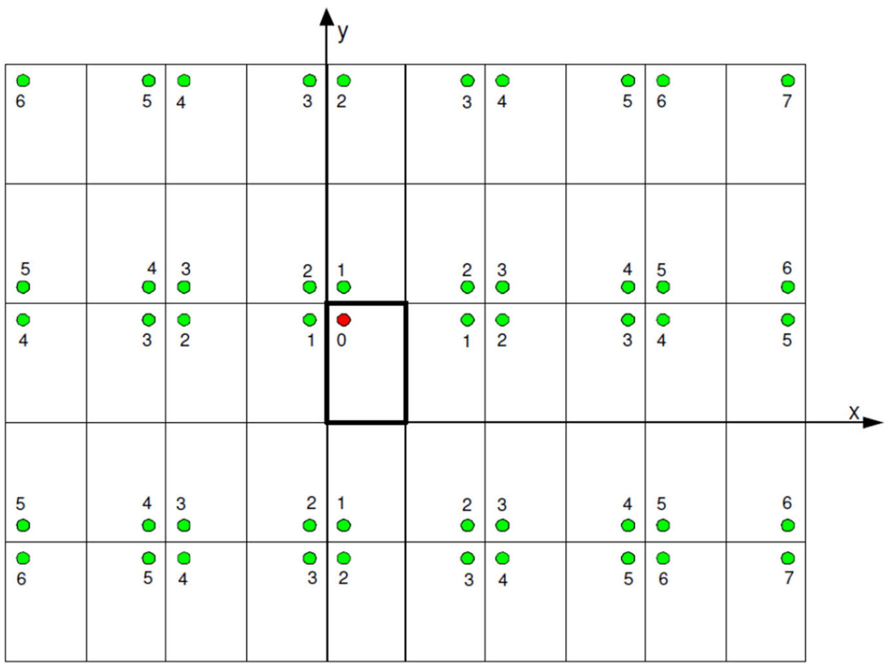


Fig. 2 Image generation in a rectangle (Collins 2004)

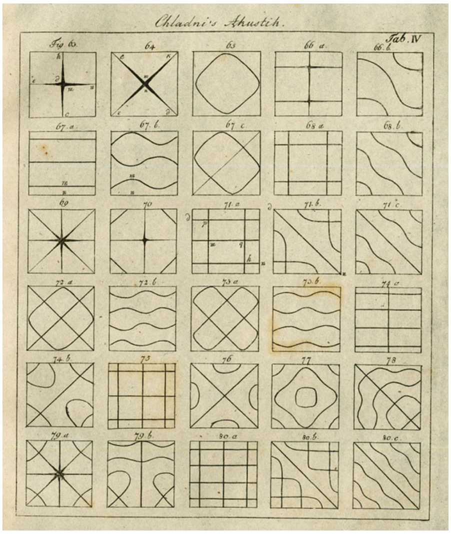


Fig. 3 Chladni plate (Chladni 1802)

research only the validity test is employed, which is dealing with the omission of reflections from the surfaces which the source has primarily generated in a previous iteration. The generation of images in a rectangular room can be seen in Fig. 2.

Visualization of Sound Fields in Enclosed Spaces

It is hard to visualize the form of the enclosure simply by listening to the sound field, as the gap between each reflection reaching our ear is shorter than the interval we can perceive as distinct. If two sound waves incident to the ear are received in a shorter time than 40 ms, we cannot distinguish two separate sound waves, which is explained as the Haas effect or precedence effect. Yet it is possible to visualize sound by mapping the relationships governing sound to another medium. The Chladni plate can be given as an example of such efforts to visualise sound. In the application of the Chladni plate, a metal plate is excited with a sound source and small particles such as sand or salt are poured on the metal to observe the modes of vibration of the speaker. An example of the application is shown in Fig. 3.

However, efforts such as the Chladni plate, which can be grouped under the term cymatics studies, try to visualize the sound source not the sound field. In order to visualize the sound field within an enclosure, one may be influenced by the similarity between optics and acoustics.

As mentioned before, ISM treats surfaces as mirrors and works with an optics analogy. Two projects are shown below showing the potential of ISM for the visualization of a sound field (Fig. 4).

As a result, the following study and the results shall also be perceived as an attempt to visualize the sound field by using an optics analogy and the ISM for more complex geometries than rectangular rooms.

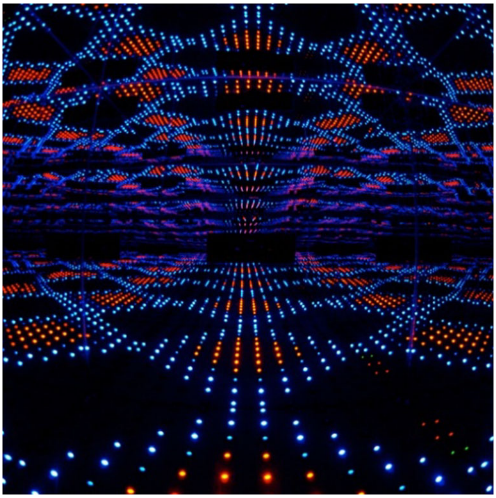


Fig. 4 Infinity Mirrored Room—LEDs Forever by Corey Menscher (image retrieved from http://portfolio.menscher.com/Infinity-Mirrored-Room-LEDs-Forever)

Identification of Patterns Generated by the Image Sources

ISM is based on reflection with respect to the axis defined by the boundary surfaces. Hence, it is expected to achieve highly symmetrical patterns while working with symmetrical boundaries. Yet, although the rules employed by ISM are simplistic and solid, these symmetrical patterns can be used to reduce computational time, and even to predict new case results without generating all image sources but employing necessary transformation functions to the patterns achieved.

For the sake of simplicity, the first test cases are constructed in 2D with regular polygons. The source, which the image sources will be generated from, is located at the centroid of the polygons and symmetrical relations are inspected to construct a base line. The image sources are treated as separate groups with respect to their order of reflection and are displayed with color codes of red, green, blue, yellow and black as the order of reflection increases, and the image sources of a particular image order are connected in a radial order to increase traceability of the

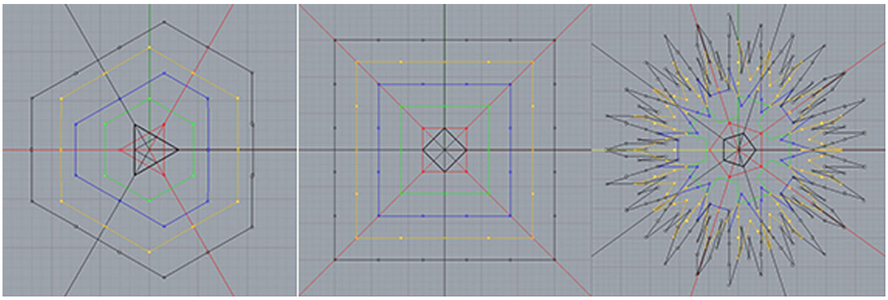


Fig. 5 Symmetrical patterns generated from *triangle*, *square* and *pentagon*

symmetrical patterns. After the construction of the baseline, two relationships of pattern and boundary surfaces and original source are inspected as the relationship between patterns and the radius of the polygons and the relationship between the location of the original source and patterns.

The symmetrical patterns, which are achieved by generating image sources from a source located at the centroid of regular polygons with the number of edges from 3 to 9, are shown below.

As can be seen from the Figs. 5, 6 and 7, two types of lines (black and red) are dividing the space in which the patterns are generated. Patterns can be decoded with two types of symmetrical relations as rotation and reflection. Each of the patterns has rotational symmetry with the order of n, which is also the number of edges of the polygons they are generated from. These symmetrical parts are the polylines residing between two black lines dividing the space. In addition, each of these parts has a reflection relation with respect to the red line dividing the part in half. Hence, once we obtain 1/2n part of the pattern, we are able to obtain the full pattern by following the symmetrical operations as: $R_n \times \sigma_1$. This function reduces the computational cost of ISM by an order of 1/2n where n is the number of edges of the polygon which is the boundary of the space concerned.

Secondly, the relationship between the radius of the polygon and the pattern is inspected. It is observed that exactly the same pattern is obtained as expected as the radius of the polygon is changed. The radius of the polygon is linearly proportional with the distance of the image sources to the origin even though the generation of image sources has more complex relations. As the patterns obtained by scaling the polygon are exactly the same, the resultant patterns are not shown.

The third aspect within the scope of this research is the relationship between the pattern and the position of the original source. In this operation, the source location is changed firstly in x directly and then in both the x and y directions. Dislocation in only the y direction is excluded as the patterns have rotational symmetry and the resultant pattern will have the same behavior with the dislocation of the source in only the x direction. The resultant patterns obtained by dislocating the source are shown in Figs. 8, 9, 10, 11, 12 and 13.

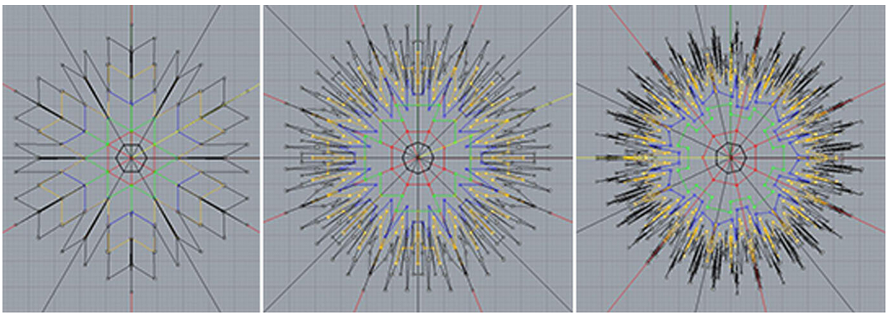
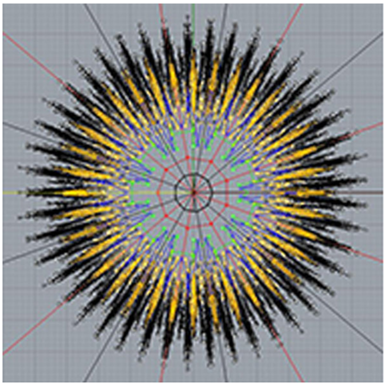


Fig. 6 Symmetrical patterns generated by *hexagon*, *heptagon* and *octagon*

Fig. 7 Symmetrical patterns generated by *nonagon*



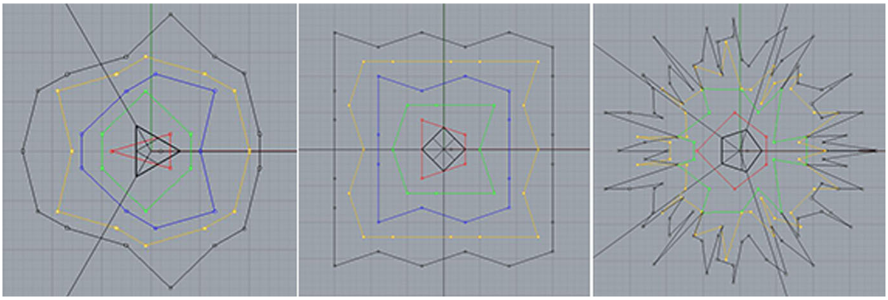


Fig. 8 Patterns obtained by dislocating the source in the x direction for *triangle*, *square* and *pentagon*

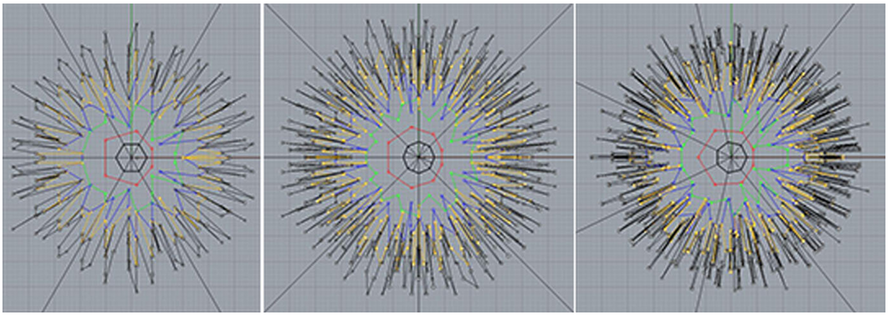


Fig. 9 Patterns obtained by dislocating the source in the x direction for *hexagon*, *heptagon* and *octagon*

It is observed that the pattern achieved in the case of the source being at the center of the concerned polygons forms the base of patterns achieved by dislocating the source. Patterns for dislocated sources have an additional relation resulting in the stretching and scaling of the patterns of centered sources. As the transformation function cannot be explained with symmetrical relations only, research of this function is left for future studies.

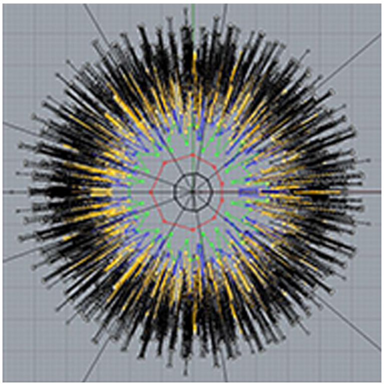


Fig. 10 Patterns obtained by dislocating the source in the x direction for *nonagon*

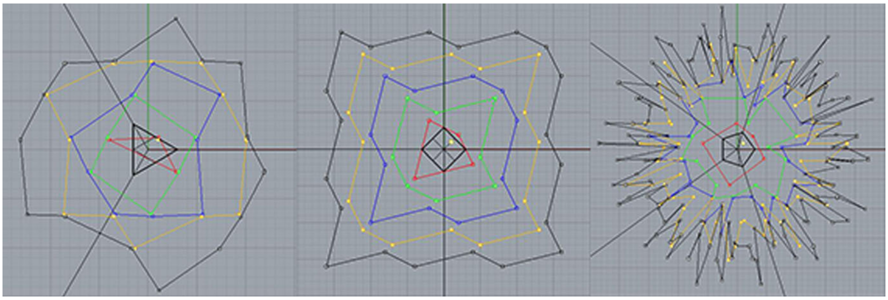


Fig. 11 Patterns obtained by dislocating the source in x and y directions for *triangle*, *square* and *pentagon*

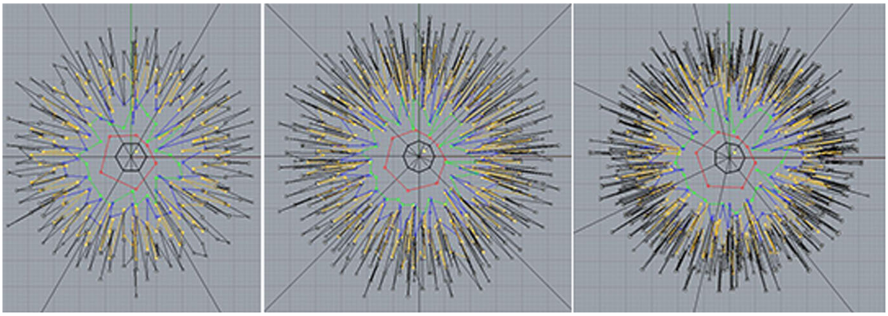


Fig. 12 Patterns obtained by dislocating the source in x and y directions for *hexagon*, *heptagon* and *octagon*

Projection of the Proposal to 3-Dimensional Geometries

The proposal, in which the results on 2D enclosures are inspected above, is also applied to 3D spaces to investigate the correspondence of a potential room acoustics case and the proposed model.

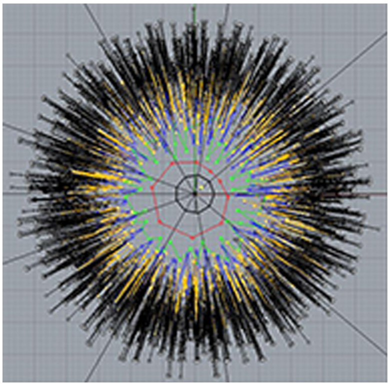


Fig. 13 Patterns obtained by dislocating the source in x and y directions for *nonagon*

Similar symmetrical patterns are also observed in 3D regular polyhedrons. Regular polyhedrons selected are the tetrahedron, cube, octahedron and dodecahedron. Polyhedrons with more faces are not included in the scope of this study, as the number of image sources would exponentially increase and it is almost impossible to visualize the symmetrical patterns. The results of the proposed method for polyhedrons are shown in Fig. 14.

In 2D applications, we divided the space by connecting the original source and vertices of the polygons, which results in triangles, and extending this triangle to encapsulate image sources in this region. In addition, we divided this region in two as half of this region is reflection symmetry with respect to an axis determined with the connection of source and midpoint of the edge of the concerned polygon. In 3D geometries we apply the same rules. As we connect the original source and the vertices of each face of the polyhedron, we obtain pyramids dividing the solution space which the image sources are residing in. In this way, we reduce the computational cost by 1/n where n is the number of faces of the regular polyhedron. The resulting patterns are shown in Fig. 15.

In addition, we divide the pyramids we obtain with respect to plane defined by original sound, midpoints of the edges of the faces and area centroid of the pyramid base. In this way, we reduce the computational cost of 1/n with 1/k where n is the number of faces and k is the number of edges of the faces and obtain a reduction value of 1/nk. Resulting patterns are shown in Fig. 16.

Computational Cost

In order to assess the computational cost differences between the proposed method and conventional ISM, case studies are both simulated in acoustical software and calculated by means of the proposed method. Illustration of the abstraction made while switching between a real case simulation and mathematical model is shown in Fig. 17.

The number of image sources generated at each reflection can be calculated with the formula:

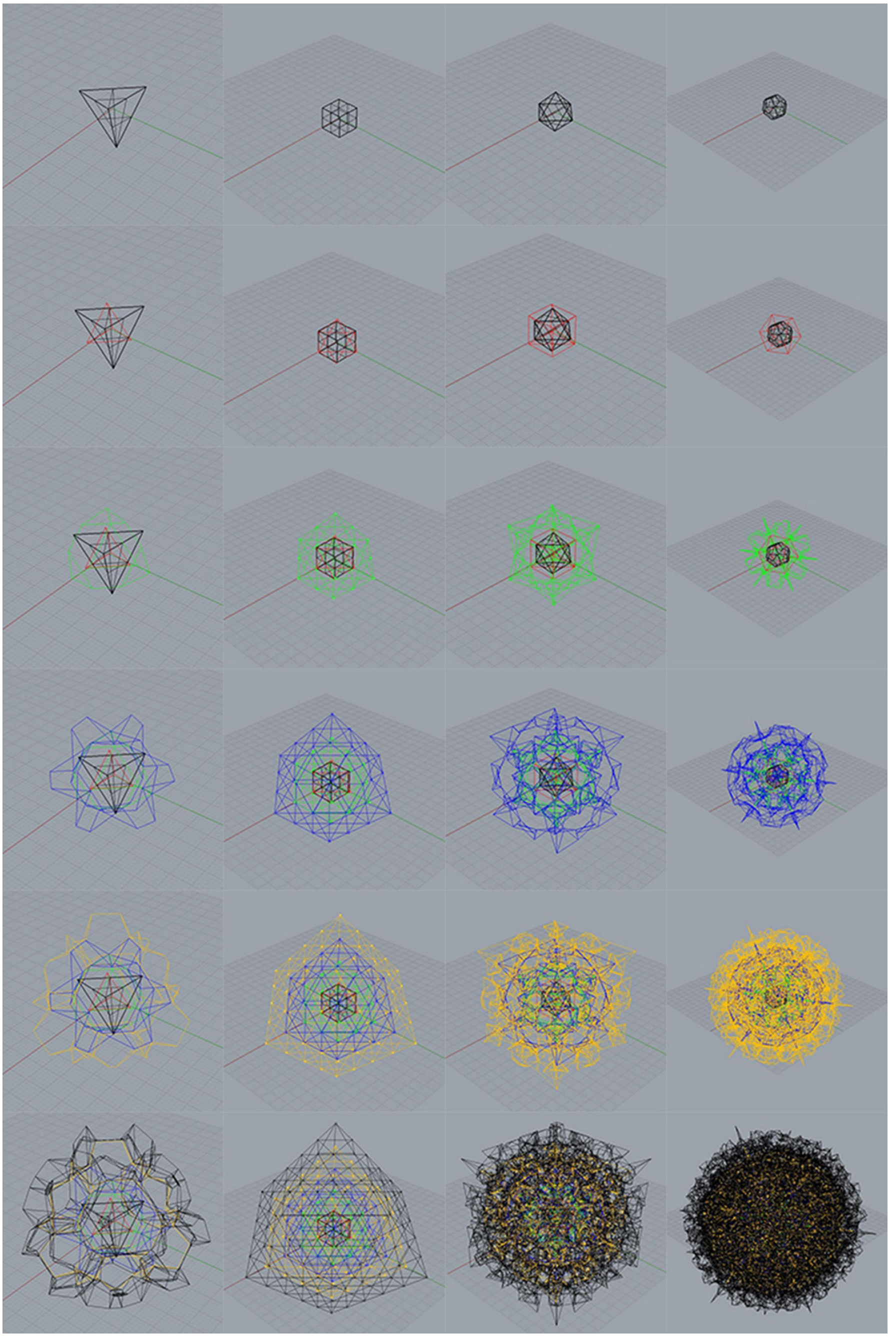


Fig. 14 Symmetrical patterns in 5 order reflections obtained from *tetrahedron*, *cube*, *octahedron* and *dodecahedron*

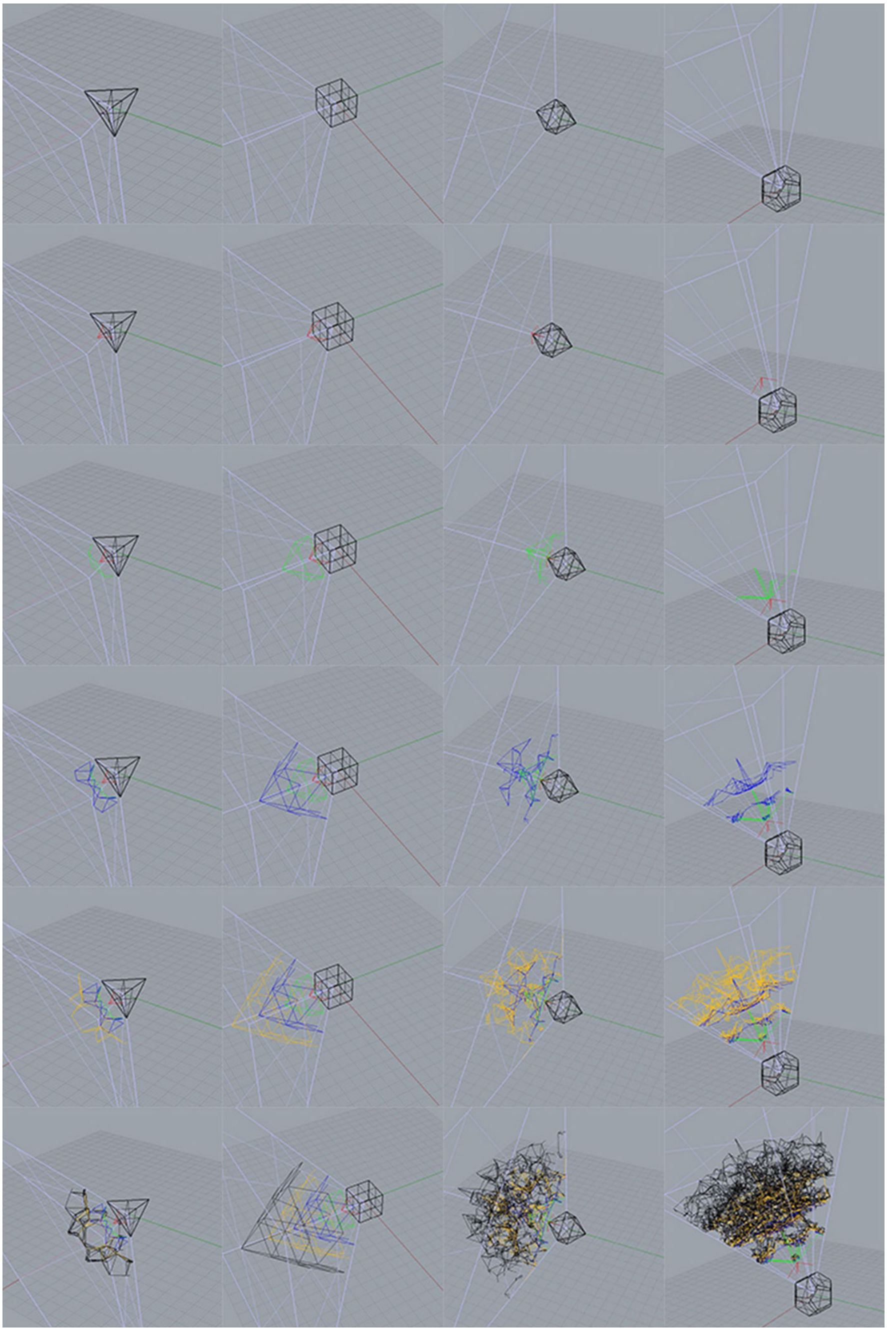


Fig. 15 5 order reflections residing in 1/n part of the symmetrical pattern for *tetrahedron*, *cube*, *octahedron* and *dodecahedron*

Fig. 16 5 order reflections residing in 1/nk part of the symmetrical pattern for *tetrahedron*, *cube*, *octahedron* and *dodecahedron*

$$N(i) = N\frac{(N-1)^i - 1}{N-2}$$

where N(i) is the number of reflections, N is the number of planes and i is the order of reflections to be calculated (Kuttruff 2001). So, the computational cost of ISM is an exponential function and is expected to increase dramatically as the number of reflections increase.

As mentioned in the previous section, in 2D applications two different methods of dividing the space were employed to result in a 1/2n reduction in the computational cost. When we switch to 3D geometries, the factor of 2 is replaced by the number of edges of a face of a regular polyhedron. In this way, it is observed that the computational cost reduction is even more efficient in 3D geometries. The comparison of computational times required by both the conventional ISM and the proposed method are compared below for each order of reflection. As the data obtained within the scope of the study is limited, curve fitting is done to get an equation for the expected computational time for both methods (Table 1; Figs. 18, 19).

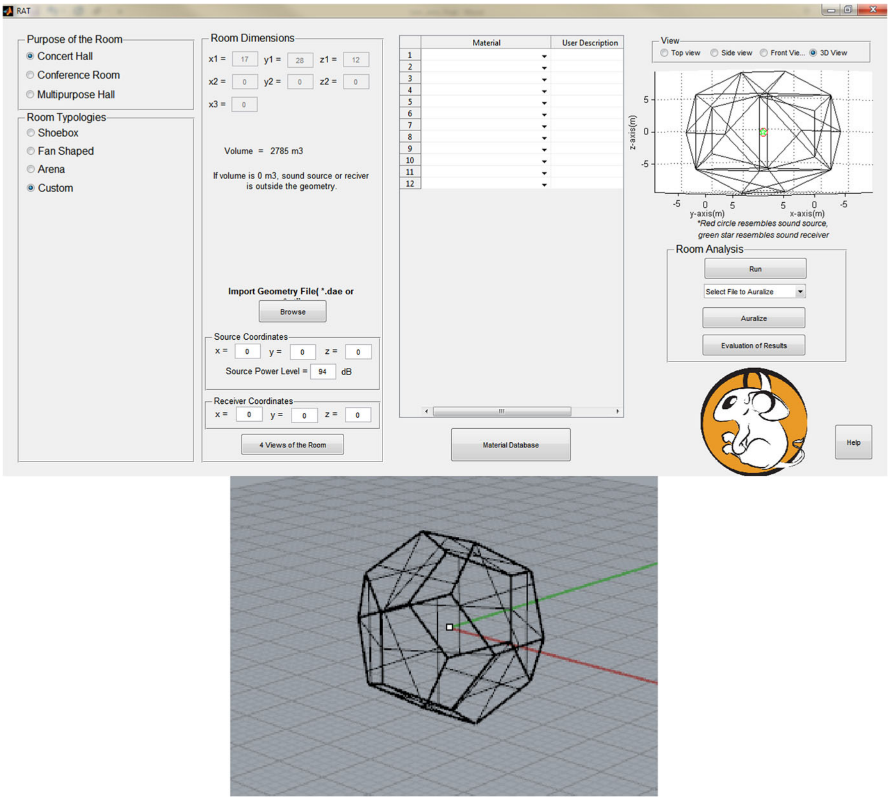


Fig. 17 Analysis of a *dodecahedron* in a room acoustics simulation software and visualization of the mathematical model

Table 1 Comparison of conventional ISM and the proposed method in terms of computational cost with respect to the number of faces and number of edges

Name of platonic solid	Number of faces	Number of edges of a face	Number of points generated (up to 5 order reflections)	Computational time (conventional ISM) (s)	Computational time (proposed method) (s)
Tetrahedron	4	3	388	0.4212	0.0131
Cube	6	4	2190	0.7807	0.0139
Octahedron	8	3	9448	1.5131	0.0146
Dodecahedron	12	5	81872	15.9954	0.0167

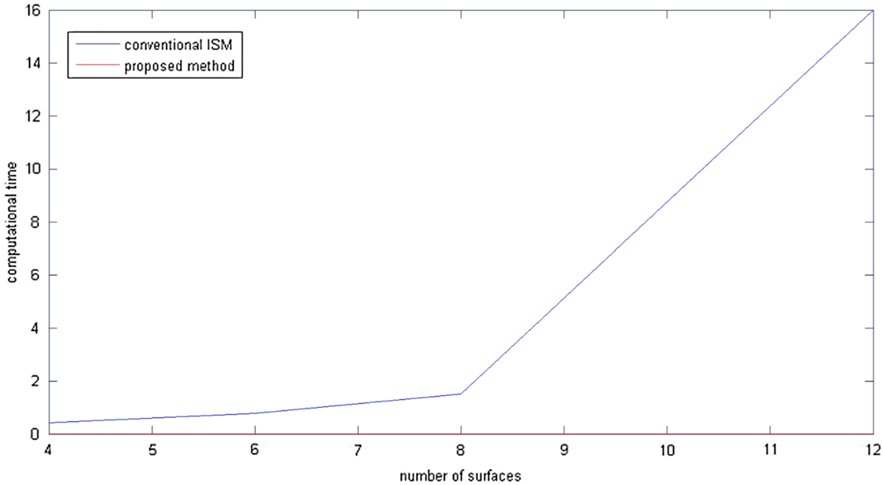


Fig. 18 Computational times of ISM and proposed method for computing 5 order reflections from 4 different regular *polyhedrons*

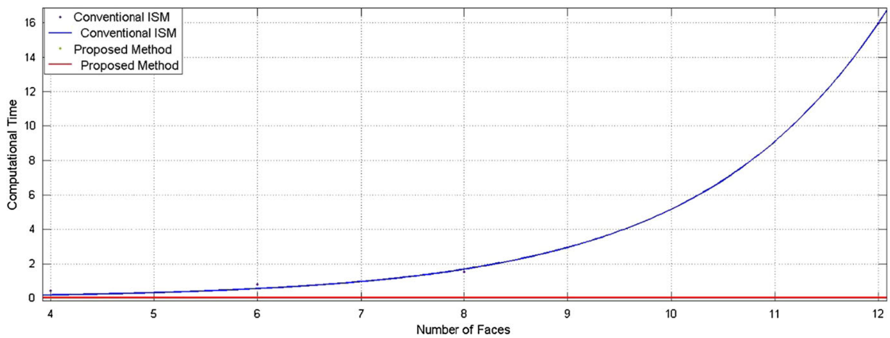


Fig. 19 Curve fitting results on computational time data

Table 2 Curve fitting results and error values

Name of the method	Equation	SSE	R-square	Adjusted R-square	RMSE
Conventional ISM	$0.01853 * e^{0.5633*n}$	0.1453	0.9992	0.9987	0.2679
Proposed method	$0.00045 * n + 0.0122$	6×10^{-8}	0.9916	0.9874	0.0001732

After fitting curves to the attained data, the divergence between the computational performances of the two methods becomes striking. It is possible to predict the computational cost of analysis and simulation to be done with polyhedrons with higher faces by means of the equations obtained. These equations and error values are shown in Table 2.

Conclusion

The tendencies of the human brain to search for patterns in nature as a way of simplifying complex problems inspired the authors of the present study. In the quest for these patterns basic geometrical transformations and uniform and semi uniform polygons and polyhedrons are employed to transcode acoustics to mathematics. Congealing the relationship between the sound source and the space that the source is in, as the geometric patterns in 2D and 3D can be considered as a demystification of the sound phenomena.

The attempt at mapping what is aural to visual provided significant data to map what is visual to reduce the computational cost of the ISM algorithm. As a result, two objectives of this research are accomplished: using an optics analogy for visualization as a tool for understanding sound phenomena in enclosed spaces, and reduction of the computational cost of ISM by using symmetrical relations among image sources.

In this study, the physical realm of acoustics and the abstract world of mathematics are brought together with the use of simple symmetry relationships inherited in polygons (polyhedrons) to reduce computational cost and complexity of the model. It is shown that instead of calculating all the reflections, calculation of the image sources in the major symmetry space reduces the computational time and overcomes the major drawback of the ISM for symmetrical spaces as shown in the examples. Further studies of these patterns and relationships, enabling modeling of the relationship between the space and the sound source, promises exploration in more intricate spatial forms as well.

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