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The Design of The Great Pyramid of Khufu

Christopher Bartlett

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Abstract This paper examines the geometrical design of the Great Pyramid of Khufu. It discusses theories derived from the golden ratio and pi which seek to explain the Great Pyramid's plan and the most recent surveys of its dimensions. The paper concludes by offering a theory which suggests that the Great Pyramid's design was intentionally based on the 4th Dynasty Egyptian canons of the proportion of the human figure.

Keywords Geometry · Golden section · Fibonacci numbers · Incommensurable ratios · Pyramid · Metrology

Introduction

The Great Pyramid stands on the Giza Plateau, outside Cairo, Egypt. Accuracy in orientation, shape, measurements and precise tolerances on this immense structure obviously seemed of paramount importance to its architects. It covers 13 acres but its orientation to north deviates only 3′ 6″ (within the N–S margins that NASA uses). The maximum difference in the side measurements (230 m each) is just 4.4 cm, and the space between blocks is 1/50th of an inch. It was built for Pharaoh Khufu in the 4th dynasty, 4,707 years ago,[1] taking perhaps 20 years from 2694 BC to complete. The 2,300,000 blocks each weigh from 2.5 to 80 tons. It required a build rate of an average size block every 2–3 min in a 10-h day. As the last remaining of the Seven Wonders of the Ancient World it is still the world's largest

[1] Based on recent carbon dating (Smith: 280).

C. Bartlett (✉)
1316 Ivy Hill Road, Hunt Valley, MD 21030, USA
e-mail: cbartlett@towson.edu

building, if now not the tallest. It is an enigmatic feat of architecture that has captured the imagination of people for centuries.

There are two primary opinions in contention for a theory of the geometrical design of the Great Pyramid. One is that the Great Pyramid of Khufu was designed based on the proportions of the golden ratio, specifically that the cross-section triangle forms a golden ratio triangle. There is also a strong following for a pi (π) theory that argues that the height of the pyramid is equal to the radius of the circle that has a circumference equal to the perimeter of the base of the pyramids.

Herodotus and the Golden Ratio Theory

The very popular belief that the Great Pyramid was built according to a design based on the golden ratio owes much to the credence given to one statement by the famed Greek historian, Herodotus (485–425 BC.), who traveled in Egypt two millennia after the construction of the Giza pyramids. This quote is constantly repeated today in math texts and even by scholarly authors. The statement, as such, doesn't exist. The notoriety of his purported geometric description of the Great Pyramid can be attributed to the claims of the pyramidologist John Taylor (1859). Taylor imaginatively misinterpreted Herodotus as saying that the Great Pyramid's dimensions were such that the area of a square with a side the height of the pyramid equaled the area of its face triangle. If true, this would have proved a definite intent to design a shape that would correspond to the golden ratio, where the ratio of the height and half side is the root golden ratio ($\sqrt{\Phi}$) that would give a triangle with sides in a geometric progression of $1{:}\Phi{:}\sqrt{\Phi}$ (also known as the 'Kepler' triangle).

What Herodotus actually wrote is:

"τῇ δὲ πυραμί δι αὐ τῇ χρό νον γενέ σθαι εἴ κοσι ἔ τεα ποιευμέ νῃ · τῆ ς ἐ στὶ πανταχῇ μέ τωπον ἕ καστον ὀ κτὼ. πλέ θρα ἐ ού σης τετραγώ νου καὶ ἴ σον, λί θου δὲ ξεστοῦ τε καὶ ἁ ρμοσμέ νου τὰ μά λιστα· οὐ δεὶ ς τῶν λί θων τριή κοντα ποδῶν ἐ λά σσων"

…which translates as: *it is square*, *eight hundred feet each way*, *and the height the same* (Rawlinson 1897, 208). One hundred Greek feet (a plethron) is now calculated as 29.608 m, and 8 plethra therefore is 236.864 m.; that deviates from later surveys of the sides by a very small amount. But the vertical height is only about 146 m not 236 m. Since Herodotus had no way of determining the vertical height, he would have meant the measurable corner (arris) at about 219 m and if he was estimating this visually he was only off by around 11 m from the actual arris (Verrall 1898, 197) On the mathematical authority of Euclid limiting its usage, the word

"ὕ ψος"

has been translated as "vertical" (i.e. vertical to the base). Herodotus was not a scientific writer and in popular description the word has ambiguity. It seems what Herodotus actually said adds nothing to a contemporary debate concerning the intentional geometrical design of the Great Pyramid and certainly should not be the reason for dismissing the golden ratio triangle out of hand. But based on this erroneous interpretation many mathematicians now do. If one examines the actual dimensions, however, there is still support for a golden ratio design even without the disproven textual evidence of Herodotus.

Dimensions

The cross-section through the Great Pyramid produces a right-angled triangle, with the vertical from the apex, a half base and the slant side (Fig. 1). Then, if we take the measures of the Great Pyramid, we can find if this triangle exhibits golden ratio proportions. Theories about its proportions rely heavily on knowing the exact dimensions of the Great Pyramid. This is why there have been so many expeditions through the centuries, from as far back as John Greaves in 1638, in search of the true measures, and Napoleon in 1798, to try to find some definitive answer. Even now, because the casing stones and the courses at the top are missing, there are still no unimpeachable perfect measures of the Great Pyramid. What has been done is to project the original dimensions from the fragmentary data that remain using the best modern techniques in metrology. With those measures, we can calculate the ratio of the height of the bisector of the slant side (apothem) of the pyramid to half the length of the base and see if it indeed is the golden ratio, or 1:1.618 (Φ).

I began by taking the averages of the most reliable published data from three highly respected surveyors/researchers, Cole (1925), Dorner (1981) and Lehner (1997). The height of the Great Pyramid averaged at 146.515 m and the base at 230.363 m. Half of the base is 115.182 m and so, by the Pythagorean theorem $146.515^2 + 115.182^2 = 34{,}733^2 = 186.369$ m (apothem). Dividing the apothem by half the base gives $186.368 \div 115.182 = \mathbf{1.61803}\ldots$ which is exactly Φ (1.61803…) to five decimal places and the ratio of height to half side is 1.27203… and similarly an excellent approximation of $\sqrt{\Phi}$ (1.27202…). This is very compelling evidence to conclude the Great Pyramid's cross section triangle's sides, whether by accident or design, are in a harmonic geometric progression of 1, $\sqrt{\Phi}$, Φ (Fig. 1).

Nevertheless, to counter any argument that may suggest the data was conveniently chosen to prove the appearance of Φ, I examined other published measures. (I should also emphasize I recognize that finding an appearance of the golden ratio in the measurements does not prove that it was the intentional design). I took over 20 sets of published empirical data from a range of authors from 1840 to 2012:

Year	Source	Side	Height
1840	Howard Vyse	232.8	148.2
1883	Flinders Petrie	230.348	146.71
1925	J. H. Cole	230.364	146.731
1971	Peter Tompkins	230.365	146.729

continued

Year	Source	Side	Height
1971	Livio Stecchini	230.363	146.512
1981	Josef Dorner	230.360	–
1992	George Markowsky	230.365	146.731
1997	Mark Lehner	230.33	146.59
1998	Miroslav Verner	230.38	146.5
2000	Lawton–Herald	230.330	146.59
2000	Roger Hersch-Fischler	230.4	146.6
2001	John F. Pile	230.356	146.649
2002	Mario Livio	230.365	146.731
2004	Craig B. Smith	230.38	146.649
2006	Stephen Skinner	230.27	146.53
2007	John Romer	230.35	146.71
2008	Paul Calter	230.5	146.8
2009	Farid Atiya	230.356	146.649
2010	Stephen Brabin	230.348	146.59
2007	Zahi Hawass	230.37	146.59
2012	Glen Dash	230.329	–
Average		230.478	146.726

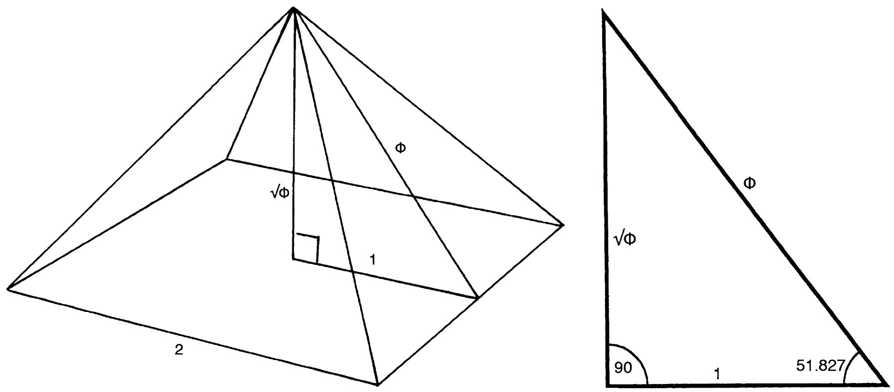

Fig. 1 Cross section of Khufu's pyramid

Using these averages, the ratio of apothem to half base is within three decimal places of Φ at **1.61893**….

Over 2 years, Sir Flinders Petrie (1883), made a very careful survey of the pyramid. He indicated that the original base of the pyramid was 230.3475 m. (his probable 'error margin' was ±0.03 or 1/260,000 of the whole). Petrie's estimation of the vertical height was 146.7104 m. Thus $146.7104^2 + 115.17375^2 = 34{,}788.93414^2 = 186.5179$ m and dividing the apothem by half base gives

1.61944… Dr. Mark Lehner is, perhaps, the foremost contemporary archeologist on the Giza Plateau with Zahi Hawass. His major current preoccupation, though, is not finding more accurate measures of the pyramids but mapping the plateau and identifying the archeological remains of the town housing the work force that built the Giza Pyramids. He had, nevertheless, all of the surveys that went before to guide him in that task. Lehner (1997, 108) estimates a base side of 230.33 m (with base error margins of just 1.27 cm), a height of 146.59 m and a gradient of 51°50′40 (51.844°). Thus, $146.59^2 + 115.165^2 = 34{,}751.6053^2 = 186.4178$ m and $186.4178 \div 115.165 =$ **1.61870**… Yet, Glen Dash (2012, 2) presents the most recent and most accurate measures of the base sides of the Great Pyramid. He analyzed Mark Lehner and David Goodman's meticulous and comprehensive 1984 survey data from The Giza Plateau Mapping Project using a linear regression analysis. It yielded mean lengths of each side at 230.329 m, he did not, though, publish an estimate of a new height or apothem. The Lehner–Goodman measurement differs in mean of all four sides from Petrie by just 1.8 cm. If we use Lehner's height of 146.59 m and Glen Dash's average side measure of 230.329 m, by those figures we would get **1.61819**… an excellent approximation of the golden ratio.

Metrology

Most Egyptologists believe that the pyramids were built using the Egyptian royal cubit measurement, where a royal cubit = 7 palms and 1 palm = 4 digits (fingers) and which may measure 523.55 mm. Using cubits, the Great Pyramid theoretical dimensions would be height: side = 280:440 (round figures that would be very valuable to the practical concerns of the builders in coordinating the work-force). This would also imply the side length is 230.384 m and not differing much from Dash's latest estimate of 230.329 m. However, since there is some disagreement on the cubit's equivalency in modern measures, in reverse, it could also very well imply that if 230.329 m is the side length, then the cubit actually measures 523.475 mm. The *seked* (the horizontal run to a rise of one cubit) of the Great Pyramid is calculated at 5 palms, two digits (5.5 palms). It is identical to a rise and run of 14–11 (Fig. 4) and corresponds to the ratio of the height and half base of the Great Pyramid (i.e. 280:220). Also with a small approximation the 10/9 arris (corner edge) corresponds to an apothem of 14/11 and gives excellent agreement (51°50′39) to the theoretical angle of 51°50′34. This would have given the builders a means to verify the accuracy of the construction sighting the straight line of the arris. It could also be a means of identifying the intent of the builders post construction by a simple measurement of the corner slant. Petrie (1883, 93) also concluded: *The profile used for the work being thus a 14 rise on 11 base*.

Sequential numbers in the architecture of the Great Pyramid appear to be 7, 14, 28 and 5.5, 11, 22, together with multipliers of 4 and 10. These numbers reflect an angle of slope of the Great Pyramid as 51°50′35 (51.843°) differing fractionally from the observed 51.844° and the height: half base = 1.27272′ ($\approx\sqrt{\Phi}$). The Pyramid's apothem then would be 356 cubits and if divided by the half base of 220 the result is 89/55, two successive terms in the Fibonacci sequence, or **1.61818**…[2] and interestingly within almost five decimal places of the Lehner–Dash number.

The pi (π) Theory

The other popular alternative geometrical design theory in contention has a reasonable fit to the empirical measurements. The pi (π) theory makes the height of the pyramid equal to the radius of the circle that has a circumference equal to the perimeter of the base of the pyramids, $4 \times base \div 2\pi$ ($\pi = 3.14159\ldots$), so using the most recent estimate of the base as 230.329 m, then the perimeter is 921.316 m which if divided by 6.2857 gives a height of 146.5733 m (Using cubit dimensions $4 \times 440 = 1{,}760 = 2 \times 22/7 \times 28$). This certainly is very close to Lehner's estimate of height at 146.59 m except there is an inherent issue. If the Great Pyramid exhibits the dimensions of the golden ratio, then mathematics automatically implies the pi ratio, so that $0.618\ldots = 1/\Phi \approx (\pi/4)^2 = (3.1416/4)^2 = 0.617\ldots$. The Φ (Kepler) theory gives a theoretical angle of 51°49′38 (51.827°), while the π theory results in an angle of 51°51′14 (51.854°), very close to each other. The Φ and π theories are in such great contention as the primary design theory because of this close similarity. The dimensions and particularly cubit measures described above could therefore reasonably fuel either viewpoint. It also provides an argument that the Great Pyramid may have been designed to exhibit both ratios. In particular, the π theory is very appealing if one subscribes to the generally accepted idea that the height and base of the Great Pyramid is 280–440 cubits and thus the implied 22/7 ratio (the approximation of π). Even the consummate surveyor Petrie subscribed to the π theory since it had corollary appeal in the British scientific milieu of the late 19th century. This ratio is also particularly interesting to those who would relate the design of the pyramid to the circumference of the earth and embody it with other cosmological ideas. On the other hand, it is hard to find any use of π as any organizing principle in the art or architecture of the ancient Egyptians, let alone in more contemporary art. In fact, Giedion (1957, 473) asserts: *the circle and the curve, apart from a few special exceptions, are banished from Egyptian art.* It is clear that the circle, as a solar disc was the symbol reserved for the images of the gods.[3]

[2] Obviously, the ancient Egyptians were not aware of the Fibonacci sequence since Leonardo of Pisa was a 13th century monk, but they were aware of and used summation series that were close in concept. (Rossi and Tout 2002: 104).

[3] Amongst a number of other possible theories, one in particular has been proposed by Fischler (1979: 92), who asserts that*: the correct theory*, *verified by both extant Egyptian manuscripts (the Rhind Papyrus) (Gillings* 1972*: 185*; *Petrie* 1883*: 42, 162) and by archeological evidence (Petrie* 1892*: 2, 37, Plate VIII) has been available since the end of the nineteenth century*. Unfortunately, he does not elaborate and it requires considerable peregrinations to try to identify this correct theory. His paper, *On Applications of the Golden Ratio in the Visual Arts*, (1981), mentions that of eight proposed theories on the form of the Great Pyramid: *two of these involve the Golden Ratio irrational number. One of the two has an excellent agreement with the actual measurements, but it is based on a quotation that doesn't exist [7]. There is, however, a simple hypothesis based on rational numbers that gives as good an agreement and, furthermore, this hypothesis is supported by archeological and textual evidence [6,7].* With this

Proportions in Egyptian Art and Architecture

Since there is no direct textual evidence to point to the design of Khufu's Pyramid, I will examine the compositional relationships incorporated into the art and architecture of the Old Kingdom. We may then infer certain motivations by means of the ancient Egyptian religion, aesthetics and use of proportion. In general, in an overview for assessments of intention for architectural authorship in ancient Egypt one can conclude that golden ratio related geometrical figures and mathematical relationships could be found (Rossi 2004, 32–56, 86]. Without doubt, there are difficulties encountered with any singular theory applied to the architecture of ancient Egypt but one in particular has gained some credibility. Of course, any a posteriori attempts to deduce a plan is always speculative, but in measuring 55 Egyptian temples there was consistent evidence of a simple golden ratio proportion

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Footnote 3 continued
contention, it is puzzling he doesn't name his hypothesis. The two theories he indicates that involve the golden ratio are outlined in his 1978 article, *Theories Mathematiques de la Grande Pyramide*. One is the nombre d'or (golden number) where base to height = 1:Φ. He compares this to the theory, base to height = 8:5, because 8/5 = 1.6 and is close to 1.618… The second is the Kepler's triangle. His 1979 text, *What Did Herodotus Really Say Or How to Build (A Theory) Of The Great Pyramid*, is a discussion of the equal area theory (area of square on height = area of face) and the misrepresentation of the original statement by Herodotus (see below). He goes on to make a connection between the area theory and the infamous 'golden number' (his quotation marks) and its appearance in Kepler's triangle, since each gives the same result. In doing so, he rejects the possibility of either appearing in the construction of the Great Pyramid because of the Herodotus misinterpretation. The reference to Petrie (1883: 42) provides only a mention of measuring angles of the pyramid faces and on page 162, Petrie remarks that the design of various Egyptian pyramids: *appears to be always a simple relation of the vertical and horizontal distance*. In Petrie's book, *Medum* (1892: 2) page two was about copying sculptures. This must have been a typo, since on page 12, Petrie mentions red and black markings on the mastaba (tomb) at Meidum that indicated a rise and run of 1 to 4 for its construction. On page 37, Plate VIII, Griffith notes*: the short inscriptions against the architect's lines read meh 5 (and 8) kher nefru or kher n nefru and must mean '5 (and 8) cubits respectively beneath the ground level'*. We are also referred to the *Rhind Mathematical Papyrus* (Ahmes c.1650 B·C.) and Gillings, who translated the RMP problem # 56 that calculates a 5.25 *seked* of a pyramid. (The *seked* can be considered the co-tangent of the angle of slope of the face of the pyramid and is the run to a rise of seven, a cubit). It seems the *seked* is then what he believes is the correct theory of the intentional design. Some Egyptologists, though, believe these calculations are only theoretical problems. In Fischler's 1978 article, of the eight theories he presents, referenced in his 1981 paper, the first is the *seked* theory. He notes that this hypothesis is ratified by Petrie's (1892) discovery of 'regulating lines' (tracés régulateurs) at the Meidum mastaba (and cites the same sources mentioned above). Otherwise, he does not distinguish the seked theory over the seven other theories as the correct theory. Fifteen historical theories as claims for the shape of the Pyramid are detailed by Herz-Fischler (2000). He names separately some theories that produce the same result to attribute specific design intent. Although here he just treats rise and run as a subset of the seked theory and doesn't elaborate. (He doesn't comment either on the parenthetical *and 8* in the Griffith translation above). In researching Petrie (1883, 1892) we find he eschews any use of the word seked and relies entirely on a rise and run of simple integers. In the book, Herz-Fischler's initial enthusiasm for a proposed correct theory seems to have waned, since he also finds that this theory does not quite fit other pyramid shapes in that the seked would have to involve small fractions of digits. He comments that the design of the Great Pyramid was: *most likely driven by aesthetic and constructability considerations, rather than any theoretical method*, (2000: 114) and finally concluding: *with the present state of our knowledge it is not possible to arrive at a definite conclusion as to the shape of the Great Pyramid* (2000: 168).

(1:1.6) by applying a 5:8 isosceles triangle to the plans (Badawy 1965, 19–40).[4] Furthermore, an investigation over 12 years at Luxor Temple demonstrated that its architectural plan was rigorously based upon Egyptian aesthetic proportions of the figure and corresponded to vital anatomical parts of the body, especially the navel. The use of golden ratios was also quite evident (Schwaller de Lubicz 1998, 66).

Undoubtedly, artists and architects through the ages, especially Greek sculptors and particularly the architects Vitruvius and Le Corbusier, have presented us with their versions of the ideal canons of human proportions. It is well documented that Le Corbusier explicitly used the golden ratio in his Modulor system for the scale of architectural design. He saw his system as a continuation of the long tradition of Vitruvius.

Modern Egyptologists agree that the pharaonic rule of proportion for depicting the standing human figure used a modular system of 18 squares from the soles of the feet to the hairline, (thus allowing for various heights of crowning head-dresses). The navel (the important symbol of maternal attachment, birth, and continuity) was placed just above square 11 (or about 11.1) and clearly seen in Fig. 2. This proportion, 18:11.1, is an excellent approximation of Φ. The navel divides the height of 18 squares by a proportion of about 11:7. By combining the septenary unit (based on the royal cubit of seven palms) and the factor of 11 the ancient Egyptians were easily able to solve a number of practical geometric calculations. Incidentally, the numbers 7, 11 and 18 are also in a Fibonacci-like (Lucas numbers) summation sequence of 1, 3, 4, 7, 11, 18… (Fig. 3). It is also interesting that the vertical grid line through the eye and navel bisects the triangle of the Pharaoh's apron, perhaps a significant reflection of the shape of the pyramid as well as an obvious center of procreative continuity.[5]

Well known for his 1992 *Upuaut* robot investigations of the 'airshafts' and the discovery of a 'door', Gantenbrink (1997), also found 14:11 and 11:7 proportions evident in interior measures and in the horizontal exit points of the 'airshafts'. Actually, these are not airshafts, but believed to be symbolic exits for the Pharaoh's *ka* (spirit) to reach the after-life and return to the mummified body at will. Many consider these shafts astronomically or cosmologically important, or perhaps, simply as sightings to guide the construction. Whatever their function, they must have had very special significance for the builders to incorporate diagonal tunnels through successive layers of tons of horizontally laid rock. They are placed at a 14:11 division of the vertical height. This proportion would give a pyramid angle of 51.843°, virtually the same as the observed angle of 51.844° (Fig. 4).

The square grid proportion of the Egyptian sitting figure is 14 squares high with the navel at 7 squares. If we compare this to the side elevation of the pyramid, we

[4] This might provide strong support for the appearance of golden ratio related proportions in the architecture of the time. Unfortunately, if one applies Badawy's 5:8 isosceles triangle proportions to the right angled triangle of Khufu's Pyramid it would give a ratio of 5:4 (height: base) and a theoretical angle of the slope of 51.340°. The observed angle is estimated at 51.844, and so the 5: 8 proportion is not a very good fit for the design intent of the Great Pyramid.

[5] Schwaller de Lubicz measured scores of pharaonic aprons and found their base angles were $\sqrt{\Phi}$ and Φ (1957. Vol. 1, ch. 6, Figs. 145B and D), although Robins (1994: 222) argues that the slope angle of the apron may be a determinant of the grid on which the figure is drawn.

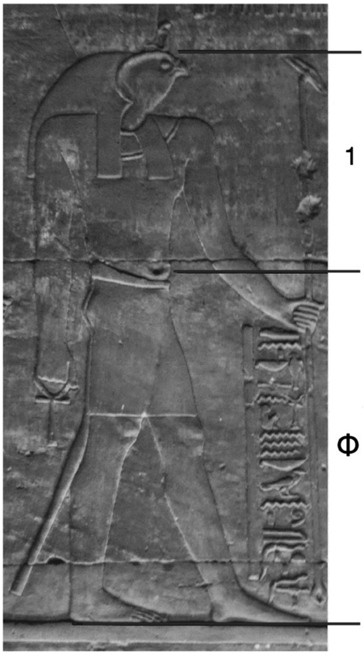


Fig. 2 Horus; navel:height Φ proportion (author's photo)

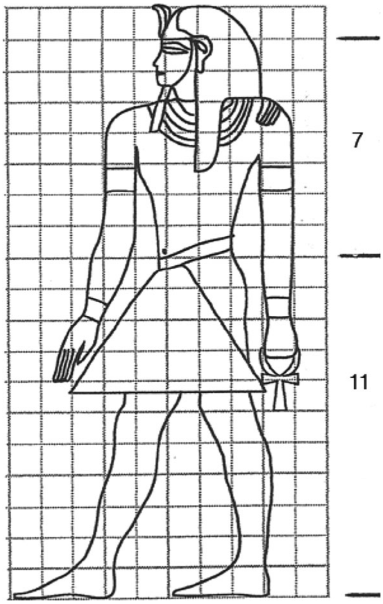
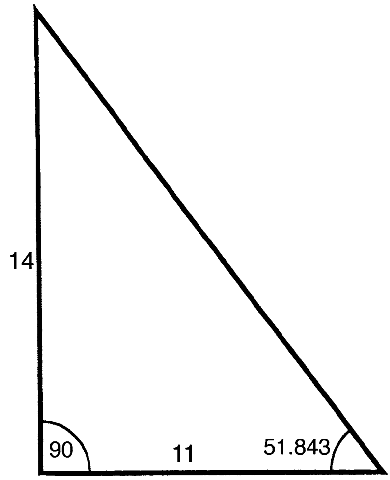


Fig. 3 Standing figure on the original grid in KV22 of Amenhotep 11, after Robins (1994, 46)

can see that it perfectly matches the 11:14 proportion of the exit heights of the shafts. The crown of the figure at square 14 matches the exit heights of the shafts. Significantly, the navel at square 7 aligns with the position of the Royal burial

Fig. 4 Slope angle



chamber. Similarly, the airshafts meet at a point at the base of the Royal burial chamber that is 11/18 of the horizontal distance between the outer openings of the two shafts. If we position the standing figure on its side to correlate with the prone position of the Pharaoh's mummy, then the offset Royal burial chamber is exactly in vertical alignment with the navel of the standing figure (Fig. 5). This offset from center of the Pharaoh's chamber has been puzzling to many, especially since the so-called Queen's chamber below is at a perfect center. The significance of the modules of 7:11 rectangles is hereby explained. The alignment of the navels of the sitting and prone figures perfectly determine the location of the burial chamber, the raison d'etre of Khufu's Great Pyramid.

Conclusion

It is hard to believe that with the precision with which the Egyptians built this massive pyramid that there was not an intention to build it based on some important specific design. After all, the perfection of the exterior proportions is precisely matched in the interior chambers and corridors and work in exact accord. Moreover, a collaborative piece of evidence that the design is a decisive choice is that the earlier pyramid of Sneferu (Khufu's father) at Meidum and the later 5th dynasty Pyramid of Niuserre exhibit the same proportions as the Great Pyramid. The empirical evidence of the surveyed dimensions presented here repeatedly reveals the presence of the golden ratio and by implication (but with slightly less correlation) the pi ratio. It would also suggest that the ancient Egyptians intentionally built the Great Pyramid to exhibit the mathematical concepts of either Φ or π, or perhaps to reflect both. Yet, there doesn't seem to be any good reason to conclude that it was, unless to demonstrate to the future an advanced mathematical knowledge. A notion

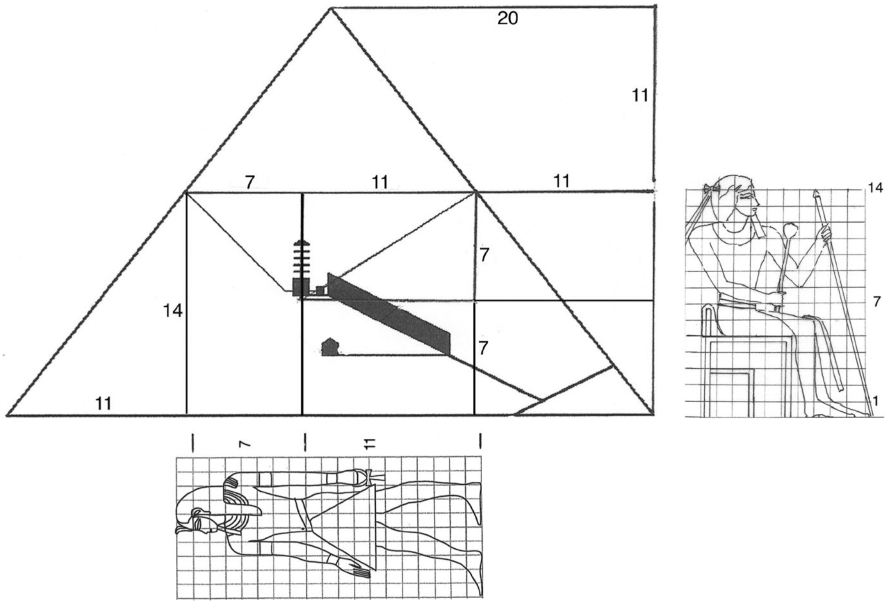


Fig. 5 **a** Pyramid diagram extrapolated and revised from Gantenbrink (1997); **b** Sitting figure: King Thutmose 111 with an original grid drawn on a wooden board with a plaster coating, British Museum, EA 5601, sketch by author from Russmann (2001),154 [see also (Robins 1994), 92 pl. 5.1]; **c** Proportions of standing figure (see figure 3)

that would seem, in my opinion, rather oblique to their other very real spiritual and funereal concerns.[6]

Consequently, in my view the ancient Egyptians had no implicit intention of incorporating the geometrical and mathematical theories of either Φ or π, but the shape of the Great Pyramid was based on the same religiously significant idealism of human proportions used to depict the gods and the pharaohs in their reliefs, paintings and sculpture. The studies mentioned above corroborate the view that the Egyptians had a predilection for simple golden ratio proportions in their art and architecture. Since the figural formula they employed gave a primary golden ratio division of 7:11, then it follows that if the same human proportion was used for the design of the Great Pyramid the golden ratio would appear as its consequence rather than its cause.

[6] Unlike the golden ratio, a visual proportion which is mentioned as an ideal relationship in virtually every text on art and design, it would be hard to argue that pi is an aesthetic proportion for the visual arts. There is no pi school of artistic thought that I am aware of that believes pi produces an aesthetically pleasing visual result, while on the other hand, the golden ratio has a strong following in the arts. Actually, that even the mathematically interesting golden ratio proportion has any implicit aesthetic beauty is debatable and psychologically hard to prove. Nevertheless, as a basis for compositional design the golden ratio has, unarguably, the important quality of bringing harmony and unity through self-similarity and continuity of proportional divisions. My 2013 paper with Dirk Huylebrouck examined the math and art of my discovery of what I call the 'chi ratio' rectangle (1:1.356…) and its applications in the geometrical composition of painting. It reveals a proportion that has the same generative properties as the golden ratio rectangle, dividing into similar 1.356 rectangles and golden ratio rectangles *ad infinitum.*

The architects of Khufu's vast Pyramid in choosing its slope would not want to encounter the structural problems of the Bent Pyramid and they would surely want to follow the architectural and artistic traditions of the time. The choice of one angle over another would seem to be of paramount importance for many reasons, and it would need to be harmonized in the interior dimensions. The builders also needed to coordinate thousands of workers on four faces, and so simple whole integers for the rise and run would be desirable. Designing the great Pyramid to correlate with the Egyptian canon of proportion used in their art for millennia had to carry the same religious and cultic importance as the depiction of the figures themselves.

For the ancient Egyptians the concept of renewal, the idea of generative continuity and a permanency of corporeal life after death was the all-important religious concern and the pyramids exemplified this. Undeniably, the pyramids were built to provide this continuity from life to after-life, and so the Φ concept of self-similarity and generative continuity as it appeared in their depictions of the human figure, I believe, seems a natural choice for the design of Khufu's tomb.

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Christopher Bartlett is Director of the University Art Galleries at Towson University, MD, USA. He took a First Class degree from Bristol University, England and received his M.F.A. from Syracuse University. He has published on the geometry of composition in painting, notably, "Decoding Fairfield Porter's July Interior", in Smithsonian's *American Art Journal* where he detailed Porter's compositional use of $\sqrt{\Phi}$. His most recent paper, co-authored with Dirk Huylebrouck, "Art and Math of the 1.35 Ratio Rectangle", was published in *Symmetry: Culture and Science*, 2013, Vol. 24, 451–462.