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Letter from the Guest Editor

Persian Architecture and Mathematics: An Overview

Abstract. *NNJ* Guest Editor Reza Sarhangi introduces the Editorial Committee for this issue: Carol Bier, B. Lynn Bodner, Douglas Dunham, Mohammad Gharipour, and Hooman Koliji, and the papers dedicated to Persian Architecture and Mathematics in *NNJ* vol. 14, no. 2 (Autumn 2012).

1 Persian architecture and mathematics: an overview

Persian architecture has long been known as the embodiment of mathematical and geometrical premises. From remote history to recent times, edifices and landscapes were designed based on rules of mathematics; their implementation required skill in practical geometry. One could consider the mutual interaction between the disciplines of mathematics and architecture as divided into three major periods: ancient Mesopotamia, pre-Islamic Persian Empire, and the Islamic Era.

Extant buildings of pre-Achaemenid architecture are few, but the ruins that remain provide evidence for the dominance of geometry in conceiving architectural space. Informed by the Babylonian culture, ancient Persians used geometric shapes as ordering tools for their monumental buildings. The Chogha Zanbil ziggurat in southwestern Iran, a stepped pyramid temple, used concentric ascending square forms in its design. The use of mathematical computation was not, however, limited to architecture. In drainage and sewer systems, Persians employed advanced knowledge of mathematics. Sewer systems in Shahr-i Sukhteh (“Burnt City”) in southeastern Iran (around 3200 B.C.), and Nari Qanat (about 3500 B.C.) demonstrate their builders’ knowledge of geometry. Arts and crafts of this period represent ornamental motifs such as animals and flora in highly abstract geometric forms.

The second period, marked by the founding of the Achaemenid Empire (550-330 B.C.) by Cyrus the Great, left numerous marks on the civilizations and cultures in a large part of the world from North Africa to Europe, India, and China. Achaemenid Persia was a large empire that encompassed modern day Iran, Iraq, Armenia, Azerbaijan, Afghanistan, Tajikistan, large parts of Pakistan, Central Asia and India. Unifying diverse ancient cultures, the Achaemenids brought together scholars and scientists of all fields, including mathematicians and astronomers, from different parts of their own empire as well as their neighbors and rivals. As significant testimonies of the knowledge of mathematics of this period, one could refer to buildings, gardens and irrigation systems, bridge construction, and arts and crafts. Applied geometry was used as an ordering tool in conceiving building plans and façades. Roof structures of this period were built of both wood timbers (long spans) and stone (small chambers). Achaemenid building ornament, often related to political and cultural rituals, represented abstract portrayal of vegetal forms.

The Achaemenid Empire was succeeded by the Seleucid Empire (the Hellenistic conquerors who were influenced by Buddhism), Parthians (from the Eastern part of the Iranian plateau who were influenced by Hellenistic culture), and Sassanids (who

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established Zoroastrianism as a state religion). The Sassanid dynasty ended with the Muslim Arab conquests (651 C.E.). The interchange of cultures and combinations of arts among nations living in a vast area conquered by Arabs created a type of art known as Islamic. Sassanids introduced domed structures into their buildings. The catenary arch of Ctesiphon (today in Iraq) demonstrates use of curvilinear geometric forms in space. The squinches of the dome in the palace of Sarvistan, Fars Providence, Iran, are late pre-Islamic examples of complex curved surfaces.

The flourishing of geometry and mathematics in the Islamic periods of Persia is found roughly between the tenth and eighteenth centuries. The Persians contributed to the flowering of knowledge in Abbasid Baghdad. Because of their background in art and architecture, Persians became very influential in lands governed by Islamic rulers. The Seljuk period exemplifies material exploration through geometry; brick structures demonstrate the dominance of geometric knowledge applied to construction at various scales, from ornaments to domes. Seljuk architecture showcased some of the purest and most sophisticated forms of geometric design. The Friday mosque of Isfahan is one of the most elegant monuments of this period. The monochromatic brick that was the dominant building material prior to the thirteenth century urged architects-engineers of the time to conceive highly elaborate and sophisticated geometric forms to enliven their architecture. These designs were later widely used in Iran and Central Asia. The Timurids' use of geometric patterns has been handed down to us through rare extant scroll drawings. Geometric interlocking patterns, such as *girih* (Persian, 'knot', geometric lines that form an interlaced strapwork), were widely used as a geometric grammar to order forms of two- and three-dimensional ornaments in architectural revetments and domed spaces.

Knowledge of mathematics resulted in the erection and embellishment of a variety of buildings and landscapes in Persia. This tradition culminated in the Safavid dynasty (1501-1736 C.E.), when the application of the knowledge of geometry can be found from the scale of urban design, garden design, architecture and building ornaments, to forms of arts and crafts. The Safavids redesigned their new capital, Isfahan, around a new urban core, today called Naghsh-i Jahan Square. As in other new developments, the new urban development of the capital embodied geometric forms on massive scales, articulated by small tectonic modules. The significant scale of landscape architecture not only required the extensive use of geometry in design and layout of gardens but also in the design of systems for their irrigation. This period also marks a physical and intellectual change in the use of geometry. The emergence of vegetal ornament, ordered by underlying geometric forms, became increasingly popular among architect-engineers, introducing a new era of Islamic architectural ornament. The realistic vegetal and abstract geometric forms alluded to the spiritual attributes of architecture.

2 Mathematics: an intellectual and practical vehicle

Since early times, Persians regarded mathematics as an area of knowledge essential for thinking as well as for very pragmatic reasons. The use of symbolic Mandala forms in the architecture of the time alludes to metaphysical and intellectual attributes of geometry. The Islamic era, as evidenced by Abbasid Baghdad (the cultural and scientific capital of the Islamic world at the time) embraced mathematics, along with the philosophies of the Alexandrian and Persian schools. These two key realms of thought highly influenced Islamic art and architecture. Gundeshapur, the Sassanid intellectual center for the study of philosophy, sciences, theology, and medicine, along with the School of Alexandria, served as intellectual resources for the Islamic court. With the establishment of the House of Wisdom (Bayt al-Hikma) in Abbasid Baghdad, a scientific institution, and the movement to translate Greek and Persian texts, scholars were able to paraphrase and develop earlier knowledge and disseminate it throughout all Islamic lands. The early Abbasid court, benefiting from Sabians' knowledge of mathematics and astronomy, developed the knowledge of mathematics and its applications on many fronts. The Sabian mathematician Thabit ibn Qurra's contribution with Banu Musa in translating Euclid's *Elements* is an example in this regard. Euclid's *Elements* were long used by Islamic and Persian scholars in early Islamic centuries, until the fourteenth century when prominent Persian polymath Nasir al-Din Tusi's edition of Euclid's *Elements* became widely accepted as a major mathematical resource.

Mathematicians became responsible for disseminating mathematical knowledge to the artisans and craftsmen. Treatises on practical geometry assisted architect-engineers in their conceiving of buildings and structures. Mainly addressing issues concerning *ilm al-hiyal* (a subcategory of practical geometry), practical geometry provided craftsmen and architects with essential "know-how" for working with geometric shapes and figures. The treatises were simplified forms of theoretical geometry explained in a practical manner to be used in the real world. One surviving example is *Kitāb fīmā yahtā ju ilayhi al-sāni' min a'māl al-handasa* (*A book on those geometric constructions which are necessary for a craftsman*) by Abū'l-Wafā Muhammad al-Būzjānī (940-998 C.E.), a Persian polymath, mathematician and astronomer of the tenth century, who lived most of his life in Abbasid Baghdad.

An anonymously-authored attachment to one of the copies of al-Būzjānī's treatise provides a glimpse into the depth of investigations of geometric patterns in the eleventh to thirteenth centuries. Most likely added to al-Būzjānī's treatise in the twelfth century, the *Fi tadakhul al-ashkal al-mutashabiha aw al-mutawafiqa* (On Interlocking Similar or Congruent Figures) introduces about 110 various *girih* patterns. Al-Bayhaqi (1100-1169 C.E.), a Persian historiographer and biographer, cited the astronomer and mathematician al-Isfizari (1123 C.E.), who regarded the science of geometry as the foundation that "architects and bricklayers had to follow." Another entry about the geometer al-Hakim Abu Muhammad al-'Adli al-Qajini "establishes a hierarchy based on the differing levels of geometric knowledge required by the designing architect and the mason executing his designs; the architect with his practical knowledge of geometry follows after the theoretical geometrician, and the bricklaying mason comes last." In this sense geometry and mathematics provided a connection between the very practical stages of building construction and the associated conceptual and transcendental ideas. Mathematics was regarded as an intellectual tool capable not only of providing answers to abstract mathematical problems, but also of penetrating the spiritual realm. Concurrently, mathematics was a tool available to engineers and artisans for everyday practice in making edifices, tools, or objects.

3 The scope of this issue

The present issue of the *Nexus Network Journal* is an attempt to offer a variety of approaches and interpretations of the presence and use of mathematics and geometry in Persian architecture. The papers are conceptually arranged. A chronological order provides the reader with a historical understanding of the subject matter. Papers with similar themes are ordered in a way that physical and tectonic descriptions come first and more interpretive and conceptual themes follow.

This issue begins with Alain Juhel's "Touring Persia with a Guide Named … Hermann Weyl," an overview of the presence of mathematics in Persian architecture using an approach similar to that taken by Herman Weyl's book *Symmetry*. Next, in "A Study of Practical Geometry in Sassanid Stucco Ornament in Ancient Persia," Mahsa Kharazmi, Reza Afhami and Mahmood Tavoosic examine pre-Islamic ornaments of the Sassanid period in regards to geometric frieze -patterns. This article contains drawn analyses of various types of patterns that in repetition create groups of associated forms.

"Along the Lines of Mathematical Thought: The Decagonal Tomb Tower at Maragha" by Carol Bier is a critical study of overlapping polygons and radial symmetries, which includes analyses of the geometric patterns that appear on the tympanum at Gonbad-e Surkh in Maragha and on the western tomb tower at Kharraqan. In "Significance of Conical and Polyhedral Domes in Persia and Surrounding Areas: Morphology, Typologies and Geometric Characteristics" Maryam Ashkan and Yahaya Ahmad introduce typologies of dome structures and their tectonics by examining a variety of dome structures across history. This is followed by a semantic discussion of the role of geometry in the construction of domical structures: Hooman Koliji's "Revisiting the Squinch: From Squaring the Circle to Circling the Square", which takes an interpretive approach to the intellectual role of geometry in vault structures, and, in particular, the case of the Friday Mosque in Isfahan. B. Lynn Bodner's "From Soltaniyeh to *Tashkent Scrolls*: Euclidean Constructions of Two Nine- and Twelve-Pointed Interlocking Star Polygon Designs" is an analysis of two star polygon *girih* patterns used from the fourteenth to the seventeenth centuries, both of which are considered to be of Persian origin. Carl Bovill looks into the geometric patterns of Mirza Akbar, an architect of late-eighteenth-century Qajar, Iran. "Using Christopher Alexander's Fifteen Properties of Art and Nature to Visually Compare and Contrast the Tessellations of Mirza Akbar" is a discussion of the use of tessellations in pre-modern Iranian ornament. Finally, in "Interlocking Star Polygons in Persian Architecture: The Special Case of the Decagram in Mosaic Designs," Reza Sarhangi studies a series of Persian mosaic designs that have been illustrated in scrolls or decorated the surfaces of old structures.

As the reader will notice, this issue, while including a wide historical, tectonic, and conceptual spectrum of Persian architecture, is by no means an exhaustive representation of the rich tradition of Persian architecture as it pertains to mathematics. We have attempted to include a variety of papers that represent diverse attributes of Persian architecture and mathematics. However, the review process was rigorous and did not allow an ideal diversity of papers to be included in this issue. The Call for Papers announced in September 2010 for this special issue was widely answered by scholars from different fields. In addition to a thorough review by the editor, each paper underwent blind peer-review by two reviewers. Finalists received comments from the reviewers and revised their papers accordingly. At this stage, a group of scholars comprising academic mathematicians, architectural historians, and architects was invited by the editor to serve on an editorial committee. This group, which included Carol Bier, B. Lynn Bodner, Douglas Dunham, Mohammad Gharipour, and Hooman Koliji, was responsible for making final reviews of the papers and cross-reviews among all papers. In this second stage, further comments were shared with authors and final revised papers were resubmitted to the editorial committee. The committee oversaw overall integration and connections among final papers in terms of content and form.

I thank the authors for their contributions, Kim Williams for giving us this unique opportunity, and the editorial committee for their hard work.

4 Glossary

Because it was also found that most papers used terminologies specific to Persian architecture, which may be foreign to Western readers, the board decided to provide the following brief glossary.

Azaj: Cradle Vault, Barrel Vault

Girih: Literally meaning knot in Persian. In architecture, it refers to the interlocking geometric patterns found in two-dimensional revetments or three-dimensional vaults or structures.

Gunbad: Dome

Handasa: Geometry

Iwan: Vaulted hall or space, walled on three sides, with one end entirely open.

Kar-Bandi, Rasmi-Bandi: Technical terms used by masons and architects as an act of making interlocking patterns in construction.

Madrasa: Islamic academy, where not only theology, but also literature, poetry and sciences were taught.

Mashrabiyah: A type of window enclosed with carved wood latticework.

Mihrab: A niche space in the wall of a mosque indicating the direction towards *Qibla* (direction to Mecca). In plan, *mihrab* is often found in semicircular or half-octagonal forms.

Minar: Minaret, a cylindrical structure of the mosque used as a visual landmark for the mosque and a means to deliver *adhan* to the public as a call for prayer.

Minbar: Pulpit; often a wooden structure similar to a pulpit for the *imam* or the clergy to deliver sermons.

Muqrnas: Stalactite-like structures built and hung under vaults or half-vaults. These structures were built out of plaster of paris in horizontal layers and vertical faces and were often covered with glazed tiles, colored glasses, and mirrors.

Pishtaq: Portal projecting from the facade of a building, usually decorated with glazed tile-work, mixing calligraphy and geometric designs.

Shabistan: A vaulted space adjacent to the mosque's courtyard. The plan of *shabistan* is a checkered-grid of columns.

Taq: Vault

About the guest editor

Reza Sarhangi is a professor of mathematics at Towson University, Maryland, USA. He teaches graduate courses in the study of patterns and mathematical designs, and supervises student research projects in this field. He is the founder and president of the Bridges Organization, which oversees the annual international conference series "Bridges: Mathematical Connections in Art, Music, and Science" (www.BridgesMathArt.Org). Sarhangi was a mathematics educator, graphic art designer, drama teacher, playwright, theater director, and scene designer in Iran before moving to the US in 1986. After completing a doctoral degree in mathematics he taught at Southwestern College in Winfield, Kansas (1994-2000), before moving to Towson University. In addition to writing many articles in mathematics and design, Sarhangi is the editor/coeditor of fourteen Bridges peer-reviewed proceedings books. He is an associate editor of the *Journal of Mathematics and the Arts*, published by Taylor & Francis in London.