A Geometrical Analysis of the Layout of Acaya, Italy

Abstract. The analysis of the urban fabric contained within the city walls of the town of Acaya, made possible by a new integrated survey involving manual, topographical, photogrammetric and 3D laserscan techniques, has cast doubts on the conventional attribution of the city layout to Gian Giacomo dell'Acaya. A rectangular layout consisting of six blocks divided by six longitudinal streets and three lateral streets is indicative of a medieval date. The geometrical analysis shows how the site of the ancient town of Salappya was transformed by Charles I d'Anjou in 1273, renaming it Segine, and how, in about 1500, Alfonso dell'Acaya enlarged the city and its walls according to the same proportional criteria. In 1536 Gian Giacomo dell'Acaya succeeded his father as Baron, redesigning the city walls in order to make them suitable lines of defense against firearms, renaming the city Acaya.

And he that talked with me had a golden reed to measure the city, and the gates thereof, and the wall thereof. And the city lieth foursquare, and the length is as large as the breadth: and he measured the city with the reed, twelve thousand furlongs. The length and the breadth and the height of it are equal. And he measured the wall thereof, a hundred and forty and four cubits, according to the measure of a man, that is, of the angel. And the building of the wall of it was of jasper: and the city was pure gold, like unto clear glass [Revelations 21: 15-18].

Preliminary remarks

The construction of a town creates a perimeter, a shape, attributing to these actions the religious mystic value of separating the land. The internal and the external part of a town are defined by the functional meaning of the urban enclosure, which creates an order: a conceptual work of the architect which skillfully contains a great quantity of meanings and theoretical and functional codes. The surrounding walls, having always been part of the development of towns, constitute the main point of reference for comprehending the consistency of the value of what they delimit. From the time of picket fences, people have viewed the enclosures as both a protection and the mark of having achieved political and cultural order. In the region of Puglia such meanings have been preserved since the fortified towns of the Messapi people (as early as the eighth century B.C.) and of later Roman and medieval towns. In particular, medieval Acaya was part of a network of villages and castles that resulted from a political and administrative reorganization of the territory. In many cases the walls show recognizable signs of the changes occurred over the centuries, caused not only by administrative arrangements but also by their adaptations to be a proper defense against firearms. The legacy of knowledge contained in this perimeter has to be emphasized. The walls of a town are not only of monumental and historical value, but cultural as well, perceived as their own by the inhabitants who lived and live within that territory and enclosure.

The richness of Italy's cultural heritage is remarkable. Many towns have a notable architectural, artistic, and archaeological heritage, a great quantity of cultural assets that deserve to be known, exploited and preserved in the best way, starting with a careful documentation and ending with a range of conservation and restoration interventions where necessary. The essential requirement for the valorization of our cultural heritage is to know it. Survey has always been the privileged means of documenting the natural and manmade reality, but, both in theory and in practice, but only in the last few decades has the survey process turned into a scientific discipline, officially becoming the first step and essential operation for any new research on architectural heritage. In recent years the science of surveying has been deeply renewed. Now, although still strongly rooted in already consolidated scientific knowledge, surveying offers new perspectives of analysis in an increasingly wide range of situations and applications. The scientific survey is the instrument and means for documenting, investigating and understanding shape and historical events directly from the object surveyed, considered the principal "self-document". With a renewed confidence in the undeniable results obtained from an accurate survey, studies on historical architecture based on the analysis of the built environment are emerging, deserving the attention of the scientific community. Experts in this field have been recognized as capable of critical investigations in the meaning of what has been surveyed, since they have the possibility to compare drawings with parallel studies, checking and validating any hypothesis.

The considerations that follow arise from the precise measurement of the walls of the town of Acaya, a small fortified village near Lecce. The analysis of the survey has unveiled the signs of the different authors involved in the building. This research clarifies, by means of hypotheses and confirmations, the meaning of the shape, and shows how arithmetic, geometry and astronomy (three of the four disciplines of the *quadrivium*, which, together with those of the *trivium*, constituted the basis of liberal arts education in antiquity, propaedeutical to the knowledge of theology and philosophy) act as the generating instruments for the design of the walls and the urban grid of the city of Acaya.

The integrated survey of the town walls of the village of Acaya

In 2009 the city of Vernole launched a plan to collect funds from the Region of Apulia (Puglia) for a project aiming at the recovery and restoration of the town walls of the village of Acaya, one of the five outlying villages of Vernole.¹ That was the first step of a long process of recovery to be done in the immediate future, an important step that led this small town to use all its resources to organize a workshop for the knowledge of the town walls. At the city's invitation, the present author organized a survey class for the students of the Universities of Ferrara Bari, Milan and the Faculty of Arts of the Telematic University Ecampus. This experience has had different outcomes: training in knowledge and critical analysis, knowledge of the built environment as a prerequisite for its optimal preservation and use by both the administration and the inhabitants. The inhabitants of the town have gained awareness of the presence of the town walls as part of their heritage and have also rediscovered the symbolic value of a civic memory.

The student teams who conducted the survey operations directly in situ throughout the different steps, analyzing the collected data, came to understand the value of scientific integration between operators and the need to bring to a good end a process subtended by a scientifically and technically correct methodology.

In a way similar to what happened with drawing (where digital drawing having almost completely substituted hand drawing) computer and information sciences have

moved on. Thus new, technologically advanced instruments have substituted the traditional ones. These changes have altered the meaning of the survey intended as a model and have even prompted the development of new methodologies to acquire and manage data,² which represents an improvement with respect to the traditional methods. Therefore, the new survey is the result of a combination of different methods aiming at a scientific description and knowledge of the architecture and producing an image embedding its value as an image.³

The survey of the town walls of Acaya is a significant case study of an integrated survey, oriented towards management, conservation and restoration. The administrative requirement to have a complete digital survey of the entire complex of the town walls arose from the necessity of having scientific documentation in order to begin an iter for future restoration interventions characterized by respect for those unwritten rules that every architect should know. The city technical office realized that a survey on digital support is not a static database, but can be implemented and provides valuable support for the one who maintains it, who will be able to integrate it with all future information pertaining to this object. The reading of a document of this kind, accurate and detailed, could provide answers and raise questions both for the restorers and for all the scholars willing to investigate and increase the knowledge of the architecture.⁴ The remarkable dimensions of the building to be surveyed and its morphological features required a great effort of organization and a notable quantity of working force.⁵

The integration of the different survey systems (manual, topographical, photogrammetric and 3D laserscan) has been fundamental for setting up a database that had would encompass characteristics just described. The measurements were conducted following two methods. The topographic survey was used to fix points of first kind and to identify the shape of the plan enclosed by the entire circuit of town walls; this grid was implemented with pieces of information obtained by measurements made with laser meters and traditional instruments. The principal elevations and the cross sections have been obtained by integrating topographic, traditional (where possible), photogrammetric surveys with a 3D laserscan survey.

The result is a survey model that guarantees an elevated degree of metric precision, with a series of detailed cross sections along the town walls. The wall stretches are more than sixteen. The drawing of the elevations of each stretch has been obtained using the photogrammetric technique. The photos mapped in the vectorial model (obtained by integrating different survey systems and methods) make it possible to have a functional document for specific estimations: in particular, to check the state of deterioration of the masonry and to draft some thematic maps useful for the evaluation of the extent of the intervention. The survey with a 3D laser scanner has been realized in those stretches where there are partial collapses, in particular in some areas on the east side where the walls are connected to the rock mountain. The survey operations have been extended to the dyke. The survey of the thickness of the walls and that of the internal elevations has been partial, as the main part of the walls are included in private houses or in private properties. The topographer created a closed polygon that included all of the external perimeter, putting it into relation to some fixed station points, determined by the present author, to create a valid support for the measurements of important points for the plan drawing of the urban blocks. The survey of the town walls of Acaya is the result of the integration of different methods and instruments. The computerized drawing in a database of vectorial drawings has produced a document useful for the knowledge of the town walls and for a future restoration, both in terms of management and of analysis.

Fig. 1. Integrated survey of some of the stretches of the city walls of Acaya

Fig. 2. Integrated survey of the stretch of the city walls of Acaya with the entrance portal

The metric analysis of the survey: from Segine to Acaya

Through the metric analysis of the survey (which is in itself an instrument of knowledge) we have the chance to reveal, on the basis of hypotheses and confirmations, the meaning (theoretical or not) of the shape and stratification of the artifact. The actual state of a historical compound often reveals morphological elements as well as those technical and scientific aspects of the original design which even highly esteemed historiographical criticism would neglect. The most important works of architecture have been mainly studied from the points of view of aesthetical/artistic criteria, and thus finding the practical 'reason' for an architectural shape, through the generating elements and the geometric constructions which are revealed by this kind of analysis, allows a researcher to complete the historical and documentary reading of the project with the detection of the original pattern. This procedure, which seems to be very simple in theory, in practice proves to be very complex, especially, when the work to be analysed is the result of a complex project and belongs to a remote past. Analysing a survey with the aim of unveiling the original project of the designer, means starting from the metric measures and converting them into the ancient metrological system used for the construction of the building. From this conjecture, we obtain some numerical quantities about which, however, we need to guess⁶ the geometrical meaning to be explained according to a logical scientific reasoning starting from an hypothesis (going through some major postulates) and arriving at defining and explaining the detailed calculation of the quantities involved, through a series of theorems and corollaries. To guess the geometrical meaning means to formulate a hypothesis of work depending on the specific knowledge of the researcher.

The village of Segine

In the case of Acaya, the careful observation of the walls' shape led me to detect the urban structure enclosed within them. A first measurement of the block showed some

incongruities and a discontinuity of the scheme commonly attributed to Gian Giacomo dell'Acaya, a military architect, who, as Jacopo Antonio Ferrari wrote in his *Apologia* [1977],

...in his time, not only planned the fortification of Acaya, but also the Castle of Lecce, providing the walls of this City with many bulwarks and walls you can still see today; he was also involved in the edification of the Neapolitan Castle of S.Eremo, the Castles of Capua and Cosenza and many fortresses in the Kingdom; for the edification of these fortresses he was employed by the Emperor Carlo V, as he was considered a gifted architect.

The hypothesis that this discontinuity had some precise historical reasons takes shape. The metric analysis of the plan leads to logical results if we consider as a unit of measure the Neapolitan palm (26.3670 cm). A particular use of the historical metric structure emerges with multiples and submultiples and in particular the unit of length called *canna per le stoffe* (equal to 8 palms or 2.109360 m), linked to the square measure called *canna quadra* (equal to 64 square palms = 4.4494 m²). Each block measures 16.89 m x 84.44 m,⁷ which in Neapolitan palms⁸ would be 64 x 320.⁹

Fig. 3. Historic units of measurement, with multiples and submultiples, in use in Acaya up to the second half of the nineteenth century

If you consider that a *canna quadra* is given by 64 x 1 palms, it becomes quite easy to calculate the block surface, which proves to be 32 *canne quadre*. The street is 16 palms wide and 320 long; consequently, the (square) surface is 80 *canne quadre*.¹⁰ The number of the lots for each block is 40 (16 x 32 palms = 8 *canne quadre*¹¹ each). Given this basic model of 8 *canne quadre*, it becomes easy to calculate accordingly the lots and the surface occupied by the solids and the voids.

Counting the number of the city blocks and streets that make the city, one can see that the pattern proves unstable and the city so far described is actually older. A rectangular scheme composed of six blocks ($1/2 + 5 + 1/2 = 6$), broken up respectively by six streets lengthwise and three streets vertically, seems to be a medieval scheme.

From a careful reading of the historical written sources, which prove hard to interpret without this kind of background information, it turns out that the ancient village of Salappya was transformed by Charles I of Anjou¹² into a new land renamed Segine around 1268. Only later, in 1294, Charles II of Anjou¹³ gave Gervasio of Acaya (Gian Giacomo's ancestor) the village of Segine. It was probably already a city surrounded by walls with a central church and a central square, measuring 480 x 752 Neapolitan palms or 5640 *canne quadre*.

Segine, according to the archaeologist Giacomo Arditì,¹⁴ was given in 1269 to the monastery of San Giovanni Evangelista di Lecce, in 1271 moved to Filippo de Tunzico Ammirato del Regno, in 1283 to Raimondo Gubaldo and in 1285, the year of Charles I's death, to Gervasio dell'Acaya, "valiant captain, as a reward for his service to Charles I of Anjou, having already been given by him Galugnano and part of Cesareo". It is difficult to establish the exact year of the foundation of Segine.

No doubt that, if the hypothesis of the foundation of a new land is justified, then the beginning of its construction can be traced back to between 1288, when Charles became king of Naples, and 1269, the year in which Segine was made over to the monastery of San Giovanni. The city would have had the shape of some French bastides and in particular, those founded by Alphonse of Poitier (between 1250 and 1271)¹⁵ and by King Louis IX of France.¹⁶ The reasons for the foundation of the bastides were political,¹⁷ economic,¹⁸ demographic¹⁹ or of security,²⁰ whereas Segine was certainly founded for political reasons. Charles I's organisation of the State was meant to consolidate his monarchy. He distributed among his knights what he had taken away from his feudatories (lieges). That is why Segine is given to Gervasio dell'Acaya.

Once we have made clear why a city was founded in that place, in that precise historical moment and with those dimensions, we have to clarify all the arithmetical and geometrical steps which led to its final shape. The starting module/unit is a square whose side is 160 palms producing an area of 400 canne quadre. Six square units form a rectangle of proportions 2:3 (320 x 480 palms = 2400 square canne), the value of the diagonal line of this rectangle is $160\sqrt{13}$ palms; if we invert this value, we obtain a new rectangle of proportions $3:\sqrt{13}$ (480: $160\sqrt{13}$ palms).

The length of this new diagonal is equivalent to $160\sqrt{22}$. If we repeat the operation, we get a new rectangle of proportions equivalent to $3:\sqrt{22}$ (480: $160\sqrt{22}$ palms). The general dimensions of the city, then, are 480 palms widthwise and $160 \times 4.6904157 = 750.4665$, or approximately 752 palms.²¹

The calculation of the city area gives an natural integer of canne quadre (5640) which, if transformed into metric units of superior surfaces, does not produce an integer value. In fact, given that a *quarta* is equivalent to 4840 square palms the surface of the rectangle that originates the city is 74.578512 *quarte*, equivalent to 7.4578512 *moggi*; if we consider that Segine had to have a moat all around, then the total starting surface may be 8 *moggi*.

The project of Segine shows a strong arithmetic-geometrical coherence. The area dimensions are defined in order to divide its precise geometrical shape into a given number of blocks. In this way the calculation of both the area and the number of lots becomes easy. Indeed, if we consider that each lot measures 16 x 32 palms = 8 canne and that each block contains 40 lots, the computation of the total number of lots can be given by the following count: 10 blocks (5+5) for 40 lots make 400 lots to which we must add those lots contained in the four half blocks (20 lots x 4). We finally have 480 lots. We still must add the 60 lots distributed along the short sides of the triangle (480 palms : 16 palms = 30 lots which will be multiplied by two) giving a total of 540 lots.

This computation still lacks the 4 lots that occupy the space of the length of the two vertical peripheral streets, thus we have a total of 544 lots. From this number we must subtract the twelve lots attributed to the Piazza and the twelve lots occupied by the church. The number of taxable lots is 520. This kind of calculation is very useful for managing the local taxes paid by property owners.

Let us compute the street area, a very useful datum. When paving the streets, the City needs to know how much material is required. Segine's rational layout allows us to easily calculate this surface. Knowing the width of the streets (16 palms) and the length of the blocks (320 palms) the area of the longitudinal streets can be easily computed by multiplying the number of streets by the area inbetween each block.

The orientation

A first hypothesis about the orientation based on the measurement of the main axis (from Google Earth) shows that the eastwest axis has an azimuth equivalent to 72.94° . This value reported in the hypothetical year of the foundation can be identified with 15 August, the day on which according to the Neapolitan Marmoreo calendar (ninth century)²³ the feast of the Assumption is celebrated. In Acaya, the cult of the Assumption is still alive and well. During restoration works in the castle, a fresco was recovered in a wall interstice: the *Dormitio Virginis*, or sleeping Virgin, measuring 4×3 meters, traceable to the second half of 1300. The picture is perfectly preserved, and represents the Apostles witnessing the death of the Virgin and Jesus picking her soul and presenting it to the Father, according to the iconographic tradition referring to the apocryphal Gospels. What is certain is that it is difficult to establish whether the city was really devoted to the sleeping Virgin.

Fig. 5. Orientation of the city plan

Fig. 6. Orientation of the city of Acaya

The value of 72.94° is relatively close to 75° . If we consider Vitruvius's indications on the orientation of the city in relation to the wind – that the wind should not be allowed into alleys²⁴ – another conjecture can be made: the orientation of 72.5° does not allow the wind named Carbas (see [Vitruvius 2009: I.vi.10, p. 30]) to enter Segine's alleys. If we consider Vitruvius's wind rose, it can be seen that Carbas blows in direction of 75° .

Carbas is a wind that blows in a direction that is between the Grecale and the East wind. In winter it is a cold wind associated with freezing temperatures, blowing with a moderate to strong intensity. To protect oneself from cold implies to protect oneself from both the Grecale and the Eastern wind. Astronomy, the fourth liberal art, turns out to be here very useful to find out a precise orientation whose meaning can be both symbolic and practical (to avoid cold winds).

Fig. 7. left) Wind rose, Codex Vigilanus, Spain, tenth century; above after Vitruvius

Enlarging the city

Let us now return to Acaya's plan. The question is: how did the present drawing come to be and when was the city expanded? The actual situation is certainly the result of an enlargement of Segine. In the light of what is described about Segine, a rereading of the historical documents clarifies that which was, in Arditi's words,²⁵ "the third transformation". Alfonso dell'Acaya, father of Gian Giacomo, in 1506 "restored (the town) and gave it, as legend has it, walls of marble" [Arditi 1879: 4]. Arditi here uses the expression "restored it", but what does this mean? Again Foscari in an article on Gian Giacomo writes about his father Alfonso "in the following year [1510] he was charged with the safety of Otranto as, in those seas, the Venetian navy had appeared, and restored the family castle, the keeps and all that remained of the defense of 1506". Again the verb "restored" is used. Therefore, a fortified city with a castle already existed and the discovered fresco of a sleeping Virgin, dated 1300, is a testimonial. Proof of our hypothesis is another inscription on the south face of the bastion. It reads as follows: "This fortress, begun by his forefathers and enlarged by his father, to hold it faithfully for Charles V, invincible emperor, was completed by Gian Giacomo dell'Acaya with great efforts in the year of grace 1536". This epigraph demonstrates that an old fortification existed in that place, was enlarged by Alfonso, and was completed with further modifications by Gian Giacomo dell'Acaya.

Hereafter we will attempt to explain, starting from measurements, Alfonso's "enlarged" city. The restorations were necessary since Segine, less than four kilometers

from the sea, was exposed to attacks by Turks who raided along the Salento coast. Alfonso's *caput magister operis* has the same criteria regarding proportions. He added two blocks of the same size as those already extant on the long side to the eEast and to the west, bringing the general dimensions of the city to (731 x 794.52 palms, exactly 12 *moggi* (12 x 48.400 = 580.800 square palms; $580.080 : 731 = 794.5280$ palms). The enlarged city was recentered and maintains its original orientation. The cross axis remains the same and the longitudinal axis is moved one block. From the drawing we can see that the enlargement of the center is done in only one direction. This allows the city to economize resources, reinforcing the extant walls on the short side, extending them and using the demolished materials of the long side of the first *enceinte* to build the second. The southwest corner of the city has always had a fortification, certainly reorganized on this occasion. From the documents it is evident that the first circular tower, the one with the smallest diameter, is from 1500 and the other, with a larger diameter, is from 1506. The castle is located exactly on the diagonal of the new city walls and one of the two towers, the one with the hexagonal hall, has its center of basic circumference on the same oblique line. In 1521, Gian Giacomo inherited the barony of Segine from his father and decided to improve the manor by surrounding it with strong bastions, suitable as defense against firearms. Three lanceolated bastions were used here for the first time. The construction of the new walls was concluded in 1536. The three bastions show involuted walls that were less vulnerable to the cannons used at the time. The defenses ensured flanking for the curtain walls and the defense of the facades, guaranteeing a perfect defense of the town. The medieval city was giving way to the Renaissance city.

The plan of the walls begins with a rectangle built by Alfonso (8731 x 794.52 palms), which are given a double rotation to define the most useful directions for setting the lanceolated bastions.

To locate these directions it is necessary to draw a circumference which has its center at the crossing of the axes of the Alfonsine city and a radius equal to 625 palms (55.5 pertiche), dividing the circumference into sixty equal parts ($360^\circ : 60 = 6^\circ$) with the principal diameters oriented to the cardinal points. The direction of the first side rotated by the new rectangle can be obtained by joining two points of the circumference, A and B, the first extending the diagonal to the point of touching the circle (point A is from north to 24° northeast = 6×3) and the second, chosen conveniently in function of the length that the bastion will require (point B is south to 39° southeast = 8.5×3). The median point of the distance between A and B is thus found, using the half-distance of the long side ($794.52/2$) of the rectangle, and a rectangle is then drawn of the same dimensions. The second rotational direction is obtained by joining two more points of the circumference. The first is always A and the second is chosen conveniently on the circumference (point C is at north to 63° northwest = 10.5×6).

The median point of the distance AC is determined, referred to the half-distance of the short side of the original rectangle ($731/2$) and a rectangle is drawn of the same dimensions. Extending the sides of the two rectangles thus determined, we obtain the directions to contain the bastions. The iconography of the lanceolated bastions varies from bastion to bastion, the dimensions of the sides go from 280 palms to 330 palms, increasing in a counterclockwise cycle. In summary, the proportioning of the medieval city is arithmetic-geometric, that of the Renaissance city enlarged by Alfonso is arithmetic and the redrawing of the walls of Gian Giacomo is geometric. The subsequent layering of these three phases creates the design of the city, symbol of the Salentine Renaissance.

Fig. 8. Geometric schemes for identifying the shape of the city plans laid out by Alfonso and Gian Giacomo dell'Acaya

Conclusions

In the thirteenth century there was a vigorous philosophical debate about institutions that had the responsibility of educating citizens, and organizing and building cities. This debate is documented in the entire western area. During the Classical period the philosophical debate on cities and citizenship gained new vigour. Knowledge of Greek literature was revived and considered useful for the purpose of reorganizing the local territory, creating new ideologies and consequently new formal interpretations of cities. Religious philosophers wrote about the differences between celestial cities and earthly cities. As with the Greek *polis*, the medieval city was supposed to be the educational instrument for strengthening citizenship, and therefore had to show harmony, geometric rigor, symmetry, numerical order, measure as the symbolic reference. All these qualities were present in Segine. The founders of the new cities borrowed from classical Greek literature the concept of city, both in its physical dimensions, easily manageable from an administrative point of view, and in its educational task towards the citizens. The example of Segine (Acaya) immediately shows affinities with Plato's ideal city: its size is that of a typical agricultural village, the street plan is an octagonal network creating rectangular blocks. No part of the city is far from view. From one end of the city, the other can always be seen. The image of the city that fascinated Plato was absolutely geometric. The principal characteristics were smallness, isolation, autonomy, enclosure within a well-protected valley, living in Puritan rigor on the fruits of one's own soil. He declared:

...if men must have a wall of sorts, they should construct their own dwellings from the outset in such a fashion that the whole town forms one unbroken wall, every dwelling house being reentered readily defensible by the uniformity and regularity with which all face the streets. Such a town, with its resemblance to one great house, would be no unpleasing spectacle, and the ease with which it could be guarded would give it an unqualified advantage over any other in point of security [Plato 1961: Laws VI.778e, p. 1355].

Nonetheless, in this description Plato stands aside from the concrete image of the traditional Hellenic city. In the Laws, the ideal number of citizens was limited to 5040: the maximum number that allowed for direct participation and democratic elections. The number 5040 indicated by Plato is obtained multiplying the first seven natural numbers, that is, it is equal to 7! (7!=1x2x3x4x5x6x7=5.040). For the founders of the Ideal City, the number 5040 was supposed to be divisible by the first ten numbers, allowing for the redistribution, at least theoretically, of lots to each newcomer.

1	12	35	72	168	504
2	14	36	80	180	560
3	15	40	84	210	630
4	16	42	90	240	720
5	18	45	105	252	840
6	20	48	112	280	1000
7	21	56	120	315	1200
8	24	60	126	336	1260
9	28	63	140	360	2520
10	30	70	144	420	5040

Fig. 9. List of the sixty divisors of 5040, the number of Plato's ideal city

The table in fig. 9 shows the 60 divisors of 5.040. Some of these numbers can also be found in general architectural dimensions, both civil and religious, of the medieval period. Suffice it to mention the example of 144 as the dimensions of the Celestial

Jerusalem. From the metric analysis of certain information about churches of the medieval period, one notes that many of them are modulated on numbers which are divisors of 5.040. The reasoning of the medieval architects is, probably, to a reference to an ideal network. Choosing as size a divisor of 5.040, one is aware of operating within a certain number of integer divisors.

The other interesting number indicated by Plato in the *Republic* is 12,960,000, also called the nuptial number, equal to $60^4 = 3^4 + 4^4 + 5^4$. 12,960,000 also has a large number of integer divisors (225), from which one may choose to select notable numbers which themselves contain a series of divisors that help in the redistribution of whole quantities. In the table of its 225 divisors, many numbers can be found which were used in Segine. 16, 32, 40, 64, 80, 320, 480, 540, are all numbers that were used to define shape and assign dimensions.

$$3^4 \times 4^3 \times 5^4 = 12\,960\,000$$

[1]

2	100	540	2 000	6 912	27 000	144 000
3	108	576	2 025	7 200	28 800	160 000
4	120	600	2 160	7 500	30 000	162 000
5	125	625	2 250	8 000	32 000	172 800
6	128	640	2 304	8 100	32 400	180 000
8	135	648	2 400	8 640	33 750	202 500
9	144	675	2 500	9 000	34 560	216 000
10	150	720	2 592	9 600	36 000	240 000
12	160	750	2 700	10 000	40 000	259 200
15	162	768	2 880	10 125	40 500	270 000
16	180	800	3 000	10 368	43 200	288 000
18	192	810	3 200	10 800	45 000	324 000
20	200	864	3 240	11 250	48 000	360 000
24	216	900	3 375	11 520	50 625	405 000
25	225	960	3 456	12 000	51 840	432 000
27	240	1 000	3 600	12 960	54 000	480 000
30	250	1 080	3 750	13 500	57 600	518 000
32	256	1 125	3 840	14 400	60 000	540 000
36	270	1 152	4 000	15 000	64 800	648 000
40	288	1 200	4 050	16 000	67 500	720 000
45	300	1 250	4 320	16 200	72 000	810 000
48	320	1 280	4 500	16 875	80 000	864 000
50	324	1 296	4 800	17 280	81 000	1 080 000
54	360	1 350	5 000	18 000	86 400	1 296 000
60	375	1 440	5 184	19 200	90 000	1 440 000
64	384	1 500	5 400	20 000	96 000	1 620 000
72	400	1 600	5 625	20 250	101 250	2 160 000
75	405	1 620	5 760	20 736	103 680	2 592 000
80	432	1 728	6 000	21 600	108 000	3 240 000
81	450	1 800	6 400	22 500	120 000	4 320 000
90	480	1 875	6 480	24 000	129 600	6 480 000
96	500	1 920	6 750	25 920	135 000	12 960 000

Fig. 10. List of the divisors of Plato's nuptial number

Plato states that,

...for divine begettings there is a period comprehended by a perfect number, and for mortal by the first in which augmentations dominating and dominated when they have attained to three distances and four limits of the assimilating and the dissimilating, the waxing and the waning, render all things conversable and commensurable with one another, whereof a basal four thirds wedded to the pempad yields two harmonies at the third augmentation, the one the product of equal factors taken one hundred times, the other of equal length one way but oblong – one dimension of a hundred numbers determined by the rational diameters of the pempad lacking one in each case, or of the irrational lacking two; the other dimension of a hundred cubes of the triad. And this entire geometric number is determinative of this thing, of better and inferior births [Plato 1961: Republic VIII.546b-c].

The geometric number, indicated by Plato, expresses a law of growth which was useful to medieval planners, a sort of mental abacus that rationalized arithmetic. The medieval city of Acaya (Segine) is its emblem.

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Notes

1. The more recent population data of Vernole and its outlying villages (the census of 20 October 2001) attribute to Vernole 3,031 inhabitants; Strudà 1,704; Pisignano 1,058; Acquarica 1,047; Acaya 494; Vanze 306. In total Vernole and its fractions reach 7,640 inhabitants.
2. Viewed as the complex of the investigations and operations aimed at identifying the significant qualities under the morphological, dimensional, figurative and technological aspect. See [Cundari and Carnevali 2005].
3. For Le Goff "image" or "representation" includes every possible mental translation of an external reality perceived: "Representation is related to the process of abstraction. The representation of a cathedral is our idea of it" [1992: 1].
4. The perimeter of the walls measures approximately 950 meters and encloses an area of about 55,300 square meters.
5. More than thirty people have worked at the survey: Dr Giampiero Mele was coordinator of the project; the Longo and Spongano architectural firm, with the contribution of three collaborators, provided a detailed topographical survey; Federico Cortese offered his valuable contribution as an expert in the field of GPS survey; Prof. Vincenzo De Simone of the Politecnico di Bari contributed as an expert in the field of photogrammetry; students from more than thirty faculties in the 2009-10 academic year were involved in the project. Special thanks go to the students of the Faculty of Architecture of the University of Ferrara.
6. The intuition depends on the researcher's idea and capacity to formulate a hypothesis to demonstrate.
7. The measure is obtained to work out an average of all the lengths obtained from the measures that vary for the short side from 16.85 to 16.92 and for the long side from 84.33 to 84.52.
8. $64 \times 0.263670 = 16.87488$ m.; $320 \times 0.263670 = 84.3744$ m.
9. $16 \times 0.263670 = 4.21872$ m, the measure relieved from the lengths of the streets varies from 4.18 to 4.23.
10. $16 \times 320 = 5120$; $5120:64 = 80$ square canne.
11. 320 square canne : 8 square canne = 40 lots.
12. Charles I of Anjou, brother of Louis IX of France (Saint Louis), conquered the kingdom of Naples in 1266, defeating in Benevento the last Swabian king, Manfredi of Sicily, illegitimate son of Frederick II. Charles died in Foggia on 7 January 1285.
13. Son of Charles I and King of Naples from 1285 to 1309.
14. In *La corografia fisica e storica della provincia di Terra d'Otranto*, Stabilimento tipografico Scipione Ammirato, Lecce 1879.
15. Alfonso, brother both of Charles I of Anjou and of Louis IX king of France, founded about fifty cities during these years. They were: Verfeil sur Seye (1250); Calmont (1252); Monflanquin (1252); Montclar (1252); Villefranche de Rouergue (1252); Villeneuve sur Lot (1253); Montpezat (1255); Villefranche de Lauragais (1254); Ste Foy la Grande (1255); St Sulpice sur Lèze (1255); Montréal du Gers (1255); Palaminy (1255); Ste Gemme (1256); Carbonne (1256); Tournon d'Agenais (1257); Montjoie en Couserans (1256); St Pastour (1259); Castillonnes (1259); Labastide du Temple (1260); Villefranche du Périgord (1261); Labastide Castel Amouroux (1264); Donzac (1265); Gimont (1266); Cordes Tolosannes (1267); Labastide St Pierre (1267); Lavelanet de Comminges (1267); Laparade (1267); Mirabel (1267); Monclar de Quercy (1267); Montalzat (1267); Lavelanet de Comminges (1267); Villebrumier (1267); Villenouvelle (1267); Villeréal (1267); Montjoie du Quercy (1268); Gaillac Toulza (1268); Lavardac (1268); Angeville (1269); Castelsagrat (1269); Dunes (1269); Damazan (1269); Septfonds (1270); Alan (1270); Salles sur Garonne (1270); Eymet (1270);

- Castelnau de Gratecombe (1270); Castelsarrazin(1270); Caumont (1270); Molières du Quercy (1270); Najac (1270); Le Fousseret (1270); Puylagarde (1270).
16. Aigues Mortes
 17. The settlement of the French royal power on the annexed county is also the reason for a foundation, as well as the need for autonomy of some lords.
 18. The rent and value of the uncultivated land or woods not exploited is one of the economic reasons.
 19. The grouping of scattered habitat motivated the foundation, as well as the displacement of the population following the destruction of a castelnau.
 20. Providing protection of the population and the liege lord from brigands and wars was a complementary cause of some constructions.
 21. Integer value multiple of 8 which approximates the one individuated multiplying 160 for $\sqrt{22}$.
 22. The value of the square canna.
 23. The Marble Calendar of Naples, an important historical/hagiographic/liturgical monument and among the most ancient ones recovered so far (the visigotic monument of Carmona, near Siviglia is antecedent (sixth century), was probably carved between 847 and 877, on the base of a Latin and Greek calendar, as not only the problematic and crowded santorale shows, but also the mention of the four feasts of the Virgin Mary deriving from the Byzantine religion: the Purification of Mary (February 2nd); the Assumption (August 15); the birth of Mary (September 8); and, above all, the Immaculate Conception (December 8).
 24. Vitruvius, *On Architecture*: “When the walls have been built around the city, the lots for housing inside them must be allocated and the main avenues and narrow cross-streets oriented so that they take account of climactic conditions: if the winds are cold they damage the health, if hot, they are infectious, and if humid, they are noxious” [Vitruvius 2009: I. iv.1, p. 27].
 25. “... qui però debbo soffermarmi sulle persone di Alfonso e di Gian Giacomo, padre e figlio Acaja, perché furono essi i rigeneratori ed i punti di partenza della terza trasformazione di questa Terricciuola” (“... here however I would like to examine in greater depth the figures of Alfonso and Gian Giacomo, Acaja father and son, because they were the regenerators and point of departure for the third transformation of this land”); see [Arditi 1879: 4].

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