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Keywords: ovals, vaults, arches, history of construction, the Escorial, Juan de Herrera

Research

# *Ovals for Any Given Proportion in Architecture: A Layout Possibly Known in the Sixteenth Century*

**Abstract.** Oval forms have been used in architecture since antiquity as arch elevations, cross and horizontal sections of vaults, profiles of arches and building plan layouts. The present paper aims to approach the knowledge and application of oval layouts for any given proportion, that is, those which fit a particular place, either as the span and height of an arch or vault, or as the length and width of a plan, by comparing written sources and built heritage. This is, on one hand, research on architectural treatises since the sixteenth century in order to find where the geometrical construction for this kind of layouts appears for the first time; and on the other, a study of the application of ovals in the vaults of the Escorial (1563-1584). Although 1712 could be considered the date of the first published geometrical construction for an oval to fit a given place, this work hypothesizes the possible application of a layout of this type in the construction of the Escorial.

## *Introduction*

An oval is a closed convex planar curve, with double orthogonal symmetry, made of at least four tangent circular arcs. The figure can be drawn using a compass and a ruler. In the words of Spanish architect Fray Lorenzo de San Nicolás, "the oval is an elongated circular figure" (my translation) [1639: 149]. Ovals and ellipses show a clear graphical similarity, but are quite different in terms of their generation, layout and drawing processes, as well as the implications concerning their architectural construction. In fact, the use of ovals in building is much more common [Huerta 2007: 211; López Mozo 2009: 508]. The word oval has been used, however, to refer to both oval and elliptical layouts [Gentil 1996: 83-84]. Juan Caramuel de Lobkowitz was the first among the authors of architectural treatises to apply the names of oval and ellipse correctly [1678 Treatise IV: 28, Fernández Gómez 1994: 349]; Caramuel described the oval as an imperfect ellipse. Although the use of oval forms in architecture comes from antiquity, starting in the Middle Ages oval domes were frequently used to cover rectangular rooms and, in particular, surbased arches and vaults were constructed in height-limited spaces [Palacios Gonzalo 1990, 2003; López Mozo 2000, 2003, 2005; Huerta 2007; Martín Talaverano 2007, 2009].

A comparison of written sources and built heritage shows that the two sides of reality, theory and practice, do not always concur. The study of the oval layouts found in the treatises confirms the state of the knowledge at a certain moment; on the other hand, the analysis of the ovals actually built at the Escorial gives us data on architectural building practice. As we will see, oval layouts in the Escorial were constructed to fit any measure by trial-and-error adjustment, as had been usual since the late Gothic, when the use of surbased vaults spread [Huerta 2007: 222].

Nexus Network Journal 13 (2011) 569–597 
DOI 10.1007/s00004-011-0083-3; *published online* 22 November 2011

The present paper contains the following parts: a short discussion of the geometrical properties of ovals and ellipses and the surveying criteria to determine the nature of an oval form actually built; a research on oval layouts for any given proportion in architectural treatises since the sixteenth century; an analysis of the application of ovals in the stone vaults of the Escorial and a description of the oval layout for any given proportion that could have been used there.

## *Ovals and ellipses: Geometry and surveying criteria*

Although the discussion *oval versus ellipse* is not the main focus of the present paper, it is necessary to begin by defining geometrical differences and their consequences in architectural application. As noted, an oval is a figure made of tangent circular arcs. The condition of tangency only demands an alignment of the two centres of the consecutive arcs with the point of change of curvature. The layout of a complete symmetrical oval requires four circular arcs and their corresponding centres, but this number may be increased. Drawing parallel ovals in order to generate walls or arches with a constant thickness or surrounding aisles of a given width is simple as all of them are concentric circular arcs. It may be noticed that proportion between the two axes of these parallel ovals is not a constant number. The layout of orthogonal lines on an oval in order to locate, for instance, the bed joints of the voussoirs of an arch is easy, as they are aligned with the centres of the arcs. As we will see, the drawing of ovals generated from a previous geometrical construction historically began with the layout of oval figures of some standard proportions, which do not fit any pair of axes: the first published proposals come from the sixteenth century.

The ellipse was known from antiquity, and its generation was explained geometrically as one of the three kinds of planar sections obtained from a cone; metrically, it was defined as the locus of the points of the plane whose sum of distances to two fixed points gives a constant result. Drawing an ellipse by the so-called 'gardener's method' applies the last definition and is useful in large drawings; at a small scale, however, the manual procedure is only an approximation, placing isolated points and requiring the final line to be drawn freehand.[1] In the mentioned *gardener's method* – also called the *string method* [San Nicolás 1639, Part I: 67r; Huerta 2007: 215] – the free length of the string, once it is tied, coincides with the dimensions of the major axis. In addition, half the value of this length is the distance from the endpoint of the minor axis to any of the foci, which can be easily located in this way. Bachot [1598] clearly explains the method by three figures [Huerta 2007: 234]. The curvature of the ellipse varies at all points, which complicates the division into equal parts and the layout of orthogonal lines to distribute the voussoirs of an arch and the implementation of instruments for dressing and checking. On the other hand, an equidistant line of an ellipse is not another ellipse but a fourth-degree curve called a *toroid* [Gentil 1996: 81; Huerta 2007: 217], which can only be drawn by approximation. It is thus complicated to lay out the parallel curves that usually form the extrados of walls and arches and width of surrounding areas.

Determining the nature of an oval form that is actually built means discarding the elliptical layout on one hand, and identifying the original oval layout on the other. If we are studying a set of parallel ovals we should identify the primary one, where geometrical constructions were applied. As noted, equidistant ovals do not maintain proportional properties: we will not find an equal division of the axis to place the centres or a certain ratio between maximum and minimum diameter if we are not working on the first designed oval of the set. Given the slight differences among all oval and elliptical options, these decisions are difficult to make, even if the measurements of the data are exhaustive and rigorous [Huerta 2007: 217]. The surveying process demands a specific control of both measuring and data analysis, which will now be described for a barrel vault.

There is a relationship between the deformation of a vault and the inclination of its walls – by means of the thrusts – which is larger in a wall with doors or windows and smaller if the vertical load coming from above is larger. Thus, if we want to approach the original layout of the vault, the profile should be measured in an area with sufficient counter-thrust; if walls are supporting the vault in the direction of the cross-section, this is the right place, as the deformation beside them will be almost zero. If we are measuring a barrel vault around a cloister, we should take points near the corners, where the inclination of the walls and the deformation of the vaults will be smaller.

The analysis of data from a surbased profile could be done according to the following steps. We should first draw the ellipse whose axes coincide with the main dimensions of the arch and compare the curve to the measured points. If the match is not satisfactory, then the layout corresponds to an oval; the next step will be to check the oval layouts. Even if the coincidence is fully satisfactory, we still cannot conclude that there is a layout of an elliptical nature, because it is possible to draw an oval fitting any measure by means of trial-and-error adjustments, even before the geometrical construction for that problem has occurred. So the next step will always be to check the oval layouts in order to either determine which kind of oval it is or to conclude that it is an ellipse. We should first compare the measured points to the existing layouts of fixed proportions, all along the arc. The coincidence of proportions between major and minor axes is not enough evidence of the application of an oval layout: as we have said, it is easy to fit different ovals into the same place. If the existing oval models do not coincide with the data of the vault, we should adjust an oval layout by the modern method: we first try applying a circular arc into the area over the springing line and drop the length of its radius from the key of the arch; we draw a straight line from that point to the centre of the circular arc and then its median line, which will meet the vertical axis in the centre of the large arc of the oval. By drawing a straight line from one centre to the other and extending it towards the lateral arc, we define the radius of the central arc, which can then be drawn. After this first approach to the layout, we should adjust the position of the centres and the size of the radius by comparison with the original measuring unit and by checking common construction procedures, such as the division of the span into equal parts, or placing the centres to form an angle of 30°, 45° or 60° to the horizontal line.

Despite all the precautions taken, it is usually necessary to fall back on original written or graphical sources or arguments of logical constructive practice to be able to determine the nature and original layout of an oval form.

## *Oval layouts in the treatises since the sixteenth century*

It is generally accepted that the geometrical construction for drawing an oval for any given proportion was not known in the sixteenth century. Although, mainly since the Middle Ages, masons used to build oval forms fitting any measure by a trial-and-error adjustment, it would be important to know where and when the geometrical solution to the problem was reached, and when the architects had more design tools at their disposal. Research of this kind of oval layouts in the treatises on architecture is shown below. The sixteenth century was considered to be a good starting point, since there we find the first published oval layouts. These Renaissance drawings have been carefully studied by scholars [Fernández Gómez 1994; Gentil 1996; Huerta 2007]. They do not fit any given proportion, but they are included here because they were copied by almost all subsequent authors, establishing a starting point and fundamental reference.

Serlio was the first to solve the problem of laying out a surbased arch of a certain height and span by an arc that is now called elliptical and explained by an affine transformation of the circumscribed and inscribed circumferences (radii equal to the half-span and height of the arch [1545: fol. 13-14]). The layout is easy to draw, as what is necessary is not to divide the circumferences into equal parts, but rather to draw any set of straight radial lines. As a result, the problem of laying out arches of any height was solved; the same cannot be said of the ovals of fixed proportions that Serlio shows afterwards [1545: fol. 17-20]. The author draws four different ovals that are in fact three layouts, as the first one is only a general description of the drawing of parallel ovals with their centres aligned so as to form a 60° angle to the horizontal. Each of them has a fixed proportion between its axes: $\sqrt{2}$, 1.3203 and 1.3227 [Gentil 1996: 84-90]. Therefore, none of Serlio's ovals can be adjusted to fit any given place, but only those which have the same proportion (fig. 1).

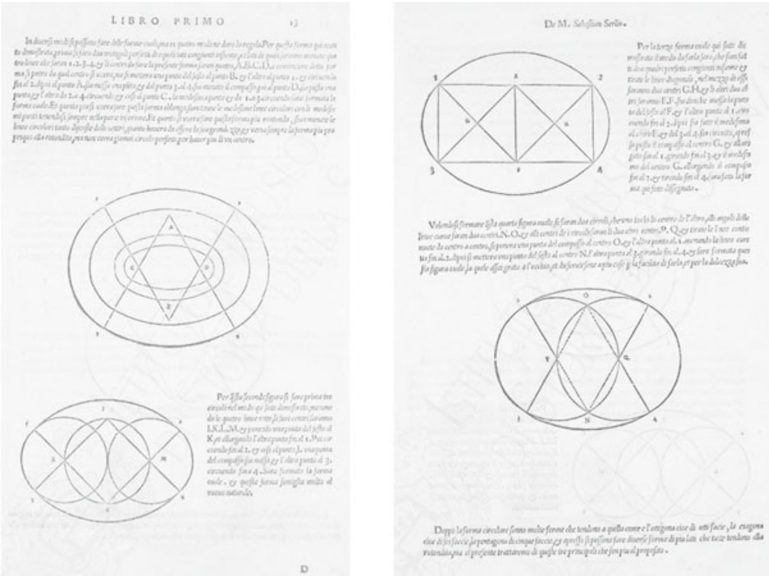



Fig.1 Four methods of drawing ovals from *Il Primo libro d'Architettura di Sebastiano Serlio* [1545]

The manuscript of the Spanish architect Hernán Ruiz shows a copy of the ovals by Serlio and includes a very interesting and unpublished layout, as it starts from the height of the arch, and not from the span [c. 1560: fol. 24v]. The drawing process is as follows: the centres of the large arcs are placed on the key of the arch and its symmetric point; two straight lines forming 30° to the vertical axis of the arch are drawn from those centres, determining in the horizontal axis the centres of the two minor arcs; the meeting point of the extended straight lines with the central arc determines the change of curvature, the radii of the minor arcs and, finally, the length of the major axis or span of the arch (fig. 2). The proportion between major and minor axes is 1.4226 [Gentil 1996: 90]. Hernán Ruiz also copies the surbased "elliptical" layout by Serlio and proposes two new ways to lay out surbased or stilted arches by stretching a circumference [c. 1560: 23, 37, 41v and 47v; Gentil 1996: 97; Huerta 2007: 225] (fig. 3).

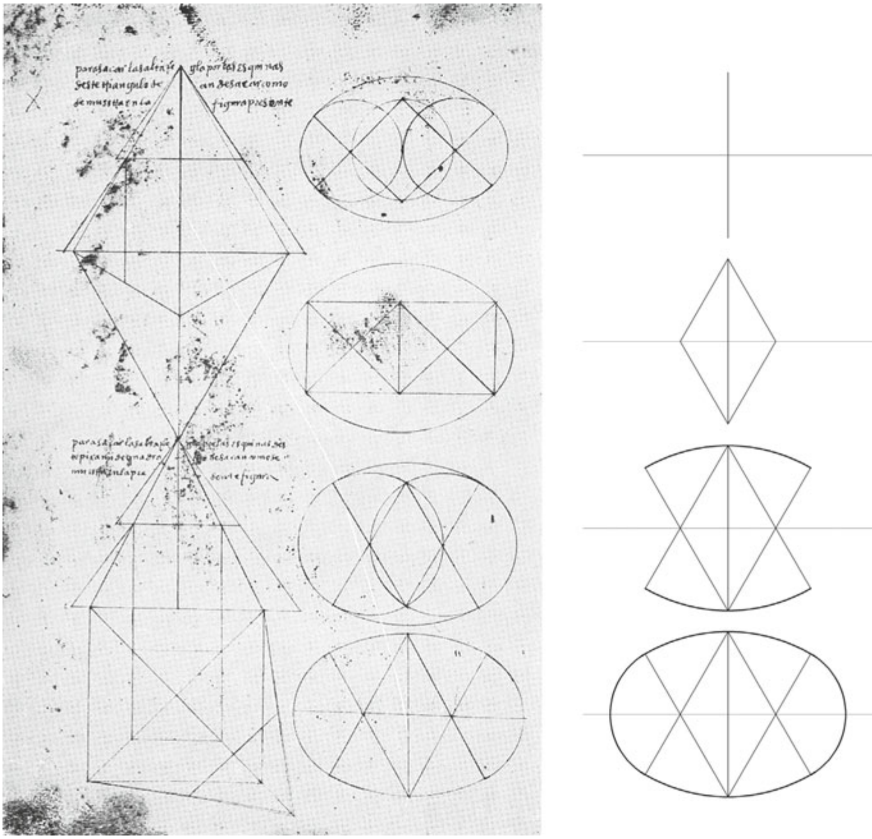


Fig. 2. Left, Hernán Ruiz, *Libro de Arquitectura*, c. 1560 [Navascués 1974]. On the right, possible drawing process of the oval below

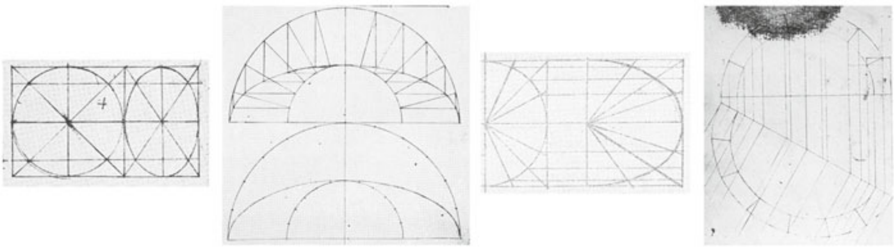


Fig. 3. Hernán Ruiz, *Libro de Arquitectura*, c. 1560 [Navascués 1974]

Philibert De l'Orme [1567: 118] draws a surprising oval cross-section for a surbased dome with a double proportion between its axes [Gentil 1996: 97-98]. It does not seem to follow a controlled geometrical construction: the span is not divided into equal parts to locate the centres of the lateral arcs, and the straight line connecting the change of curvature point with the centres of the consecutive arcs does not form a fixed angle 30°, 45°… to the horizontal springing line (fig. 4).

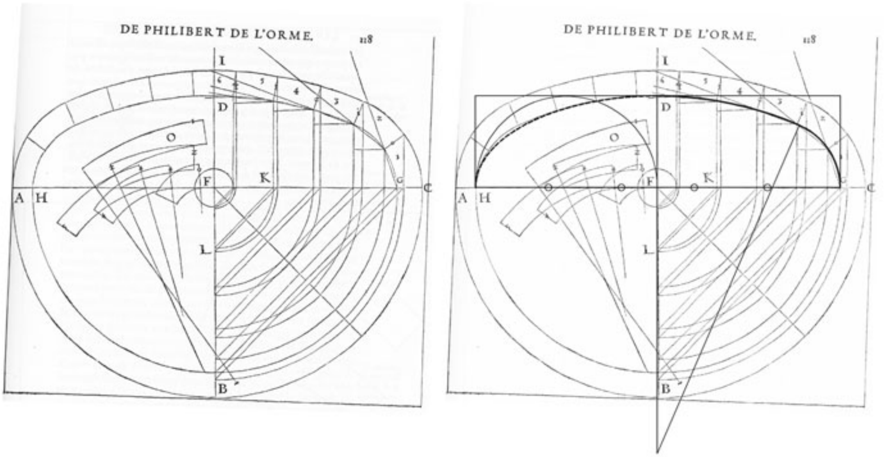



Fig. 4. De l'Orme, *Le premier tome de l'Architecture*, 1567. On the right, overlapped, an elliptical layout (dotted line) and an oval one (continuous line)

The stonecutting manuscript by Vandelvira [c. 1580] follows the storyline posed by Serlio: he describes only one oval layout with a fixed proportion between its axes and, then, in the next sheet of paper, copies the "elliptical" layout of Serlio, citing the author, to solve the problem of adapting an arch to a given proportion and "lay out a vault according to the demands of the place in question" (my translation). The oval arch or *carpanel*[2] by Vandelvira is lower than those by Serlio, with a proportion between axes of 1.5773. The geometrical layout of the arc is similar to the fourth oval by the Italian architect, with the alignment of centres forming 60° to the horizontal axis, but starting with a division of the span into four rather than three equal parts [Vandelvira c.1580: 18r] (fig. 5).

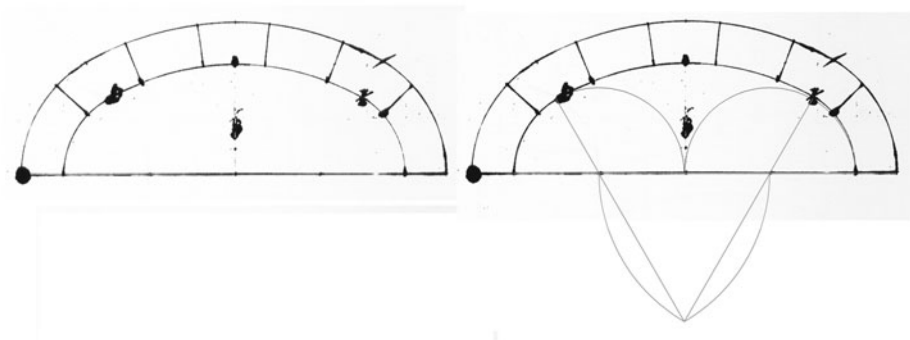


Fig. 5. Vandelvira, *Libro de Traças de cortes de piedras*, c. 1580.
On the right, overlapped, explicit geometrical layout

This is the only kind of oval layout used by Vandelvira in the entire treatise – which comprises 141 different stonecutting layouts – even though he knew the ovals by Serlio, which certainly give less surbased arches. Vandelvira's manuscript is a theoretical text, that is, he is free to design the measures and proportions of all the architectural models explained. His oval layout is simple and useful, as it can be drawn using only a compass. In the last page of the manuscript the author sets out an unpublished construction to draw an oval figure – we would now say "elliptical" – into a non-rectangular parallelogram. The procedure can now be explained as an affine transformation of a circumference [Vandelvira c.1580: 126r] (fig 6).

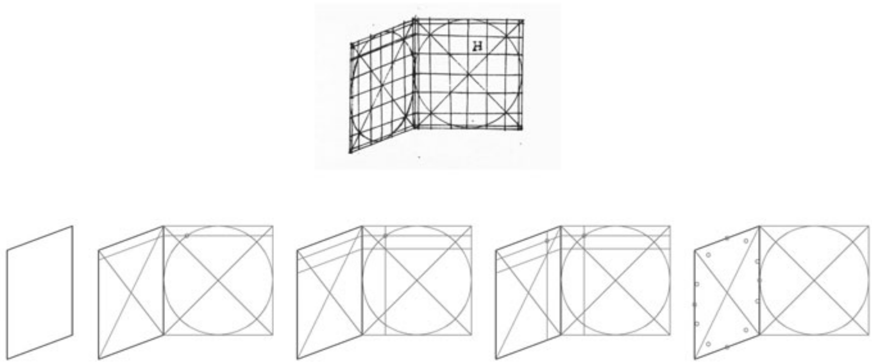



Fig. 6. Vandelvira, *Libro de Traças de cortes de piedras*, c. 1580, method of inscribing an "oval figure" within a non-rectangular parallelogram and graphical interpretation by the author

Cristóbal de Rojas [1598: 98] follows the idea of the layouts for arches by Vandelvira [c.1580: Tít. 21], but his drawing for the oval arch corresponds to the fourth layout by Serlio, with the alignment of the centres forming a 60° angle to the horizontal springing line, perhaps because it was clearly described graphically (fig. 7).

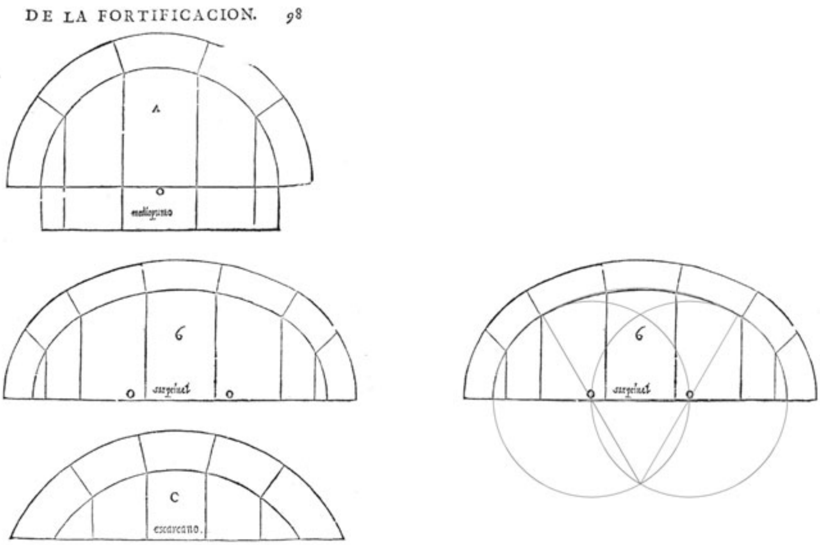



Fig. 7. Cristóbal de Rojas, *Teoría y Práctica de Fortificación*, 1598.
On the right, overlapped, explicit geometrical layout

The carpentry manual by Mathurin Jousse contains a drawing for an oval roof structure, with a layout that could follow the fourth oval by Serlio [(1627) 1702: 156] (fig. 8).

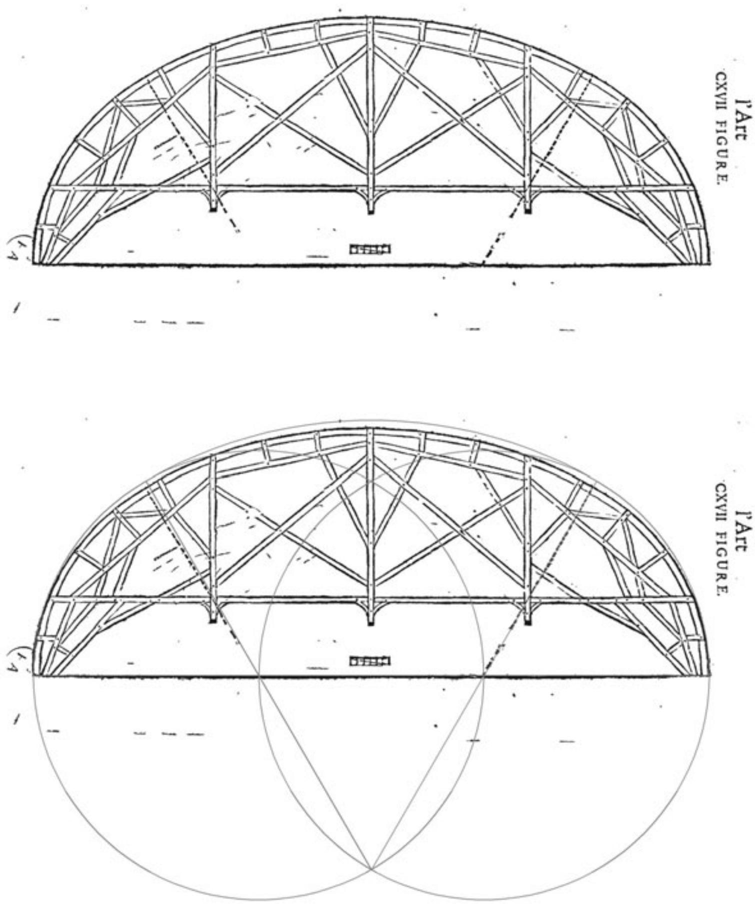



Fig. 8. Mathurin Jousse, *L'Art de Charpenterie*, (1627) 1702.
Below, overlapped, explicit geometrical layout

In the seventeenth century the Spanish Fray Lorenzo de San Nicolás [1639] follows, as did Vandelvira, the method set out by Serlio for drawing oval figures: when they have to be built into certain given measures he uses the gardener's method, an elliptical layout; when there are no constraints on proportions he proposes four oval layouts. In the chapter entitled *On Arches and the Way and Form in Which They Must Be Made* (my translation), San Nicolás draws the fourth oval by Serlio – without citing Serlio – obtained by dividing the span into three equal parts and having the centres form a 60° angle to the horizontal springing line. The author then explains the possibility of surbasing this arc by dividing the span into a larger number of equal parts, although he points out that a better way is the string method/gardener's method, with the advantage that it "can be surbased as it is desired" (my translation) [1639: fol. 66v-67v]. The drawing more closely approaches an oval than it does an ellipse, and the author, in fact, points out that it can be dressed with three *cintrels* (fig. 9).

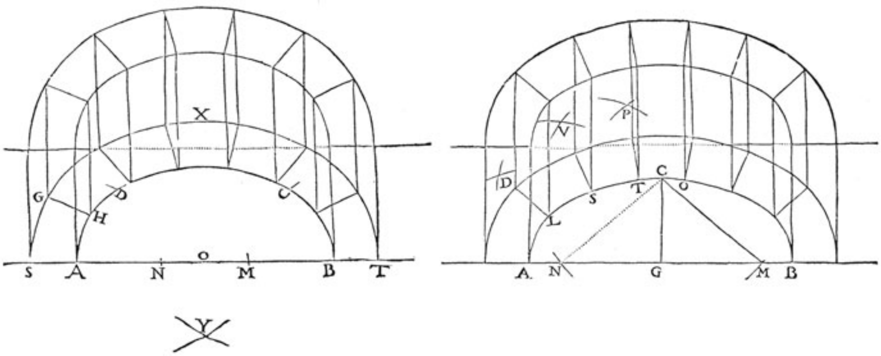


Fig. 9. Fray Lorenzo de San Nicolás, 1639

Later on, in the Chapter titled *On the Construction of Ovals and Their Measures and Other Advices* (my translation), San Nicolás describes three unpublished layouts and repeats the fourth oval by Serlio [1639: 150,151]. Following the order of his drawings, the first one is similar to the second oval by Serlio with a free placement of the centres, only keeping an equal distance to the centre of the oval. The process of drawing starts with the major axis: extending the straight line which connects the centres, he starts by drawing the small arcs, which define the radii of the large ones in the intersection with the mentioned line. The measure of the minor axis of the oval is obtained as a result of the drawing process. It could have also been done in reverse order, starting out from the minor axis and defining the major one. The second oval proposed by San Nicolás does not comply with the tangency conditions: the centres are not aligned with the change of curvature point. The third layout starts from a square that does not contain the final points of the axes, so the main measures of the oval are not previously controlled, but the construction is geometrically correct. Two of the centres are located at the midpoint of the square sides, and the remaining two can be found in the centres of the two rectangles obtained by the division of the square into two equal parts (fig 10).

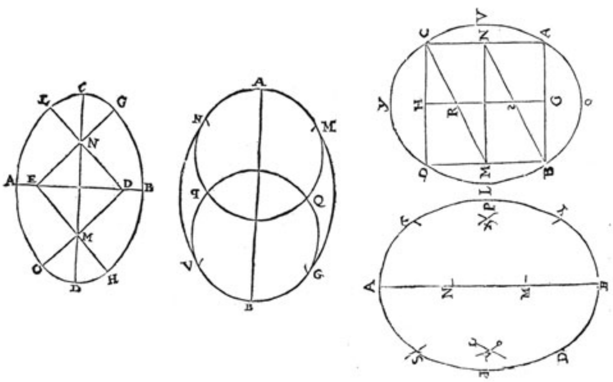


Fig. 10. Fray Lorenzo de San Nicolás, 1639

The stonecutting manuscript by Mathurin Jousse shows an odd oval layout [1642: 8]. Although the drawing process seems to be explicit both graphically and in written form, it is difficult to understand. The text proposes a division of the major axis into three equal parts, where the centres of the lateral arcs will be placed. Then the author suggests taking points 1 and 4 of these arcs, without explaining how to determine them, '*pour faire D, H, égal*', placing the compass on H and opening it to 1 or 4 to lay out the central arc. The drawing is quite inaccurate: the initial division into equal parts is only an illusion and the supposed change of curvature point is not aligned with the two centres. The steps of the process could be: to divide the span into three equal parts; with the centre placed on those points, to draw two circumferences to a certain point which, connected to the centre of the arc and extended to the vertical axis, determines the centre of the large arc (fig. 11).

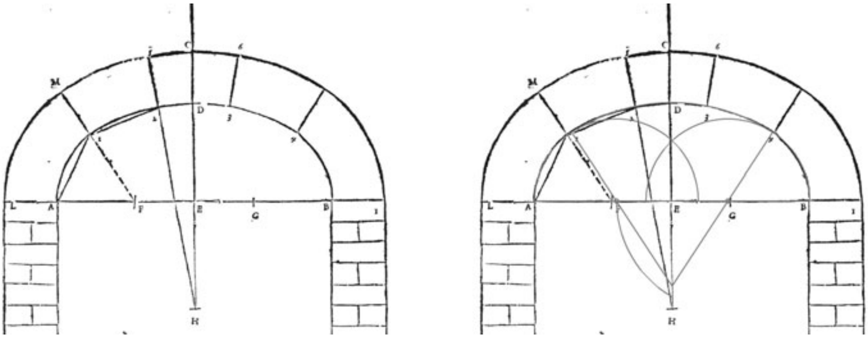


Fig. 11. Mathurin Jousse, *Le Secret d'Architecture*, 1642.
On the right, overlapped, explicit geometrical layout

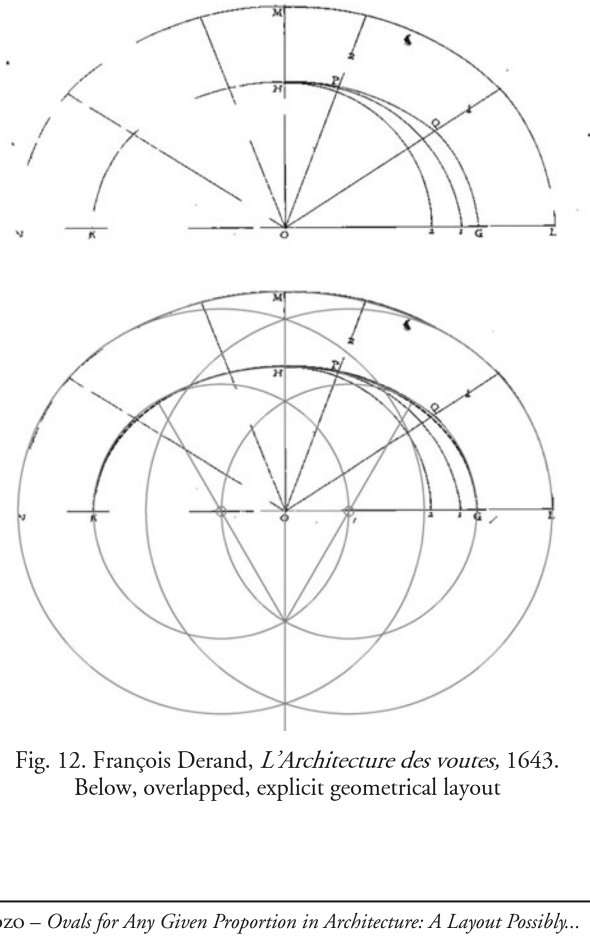


Fig. 12. François Derand, *L'Architecture des voutes,* 1643.
Below, overlapped, explicit geometrical layout

The stonecutting treatise of the French Jesuit François Derand [1643: 302] does not include explicit oval layouts but he does draw some. For instance, in the layout for a *Trompe en niche en demy-ovale, ou surbaisée, ayant mesme cintre que son plan*, the text describes an elliptical centring, but the drawing follows the fourth oval by Serlio (fig. 12).

The manuscript by the Majorcan architect Joseph Gelabert [1653: 21r] proposes a layout for an oval arch. Departing from the division of the span into five equal parts, the centre of the lateral arc is placed on the first point, and a straight line forming a 60° angle to the horizontal line is drawn, whose intersection with the vertical axis will determine the centre of the large arc [Rabasa 2011] (fig. 13).

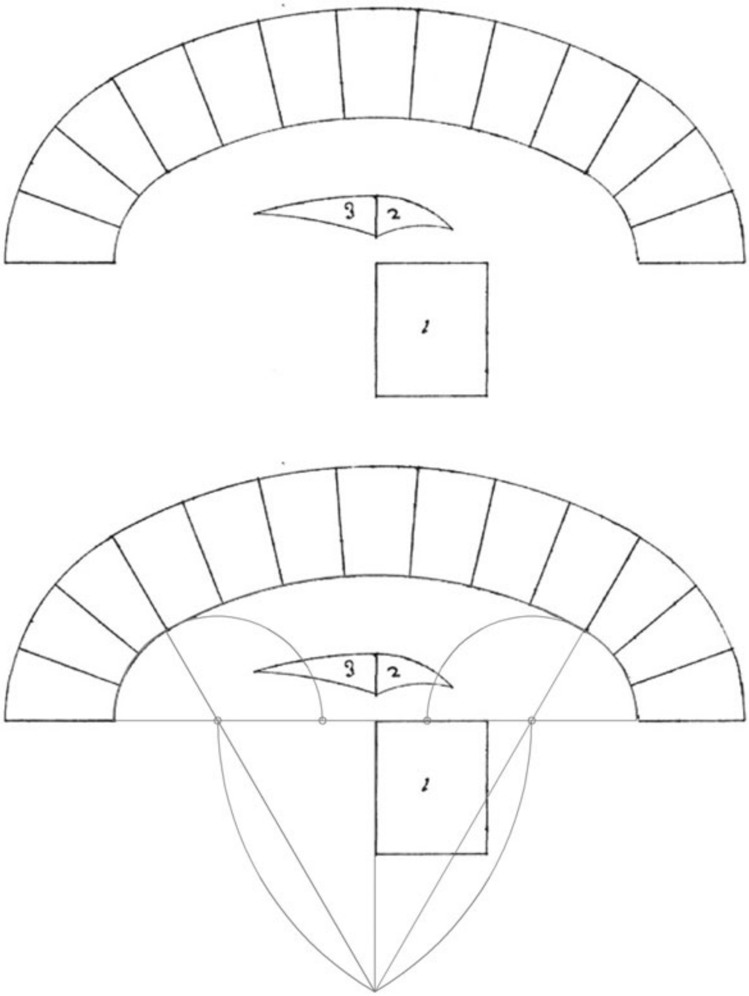



Fig. 13. Above, Joseph Gelabert, *De l'art de Picapedrer*, 1653;
below, an oval layout after Rabasa [2011]

The French Jesuit Claude François Milliet-Dechales includes the second and fourth ovals by Serlio in his encyclopedia on mathematics [1674: Vol.2, 8] (fig. 14).

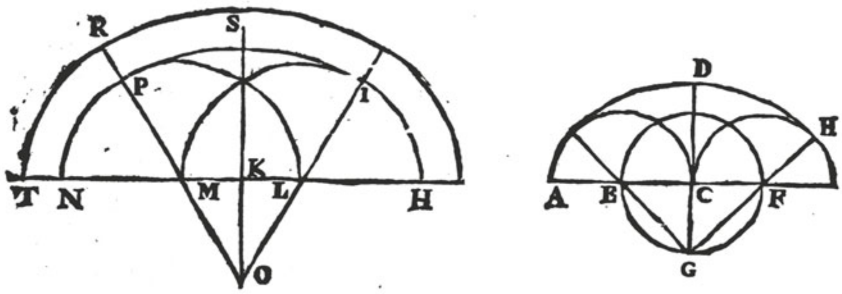



Fig. 14. Claude François Milliet-Dechales, *Cursus seu mundus mathematicus*, 1674

The treatise by Juan Caramuel de Lobkowitz describes the oval line as an "imperfect ellipse that is made of portions of circles" (my translation) and repeats the second oval by Serlio [1678: Treatise IV; 28, Planche VII] (fig. 15).

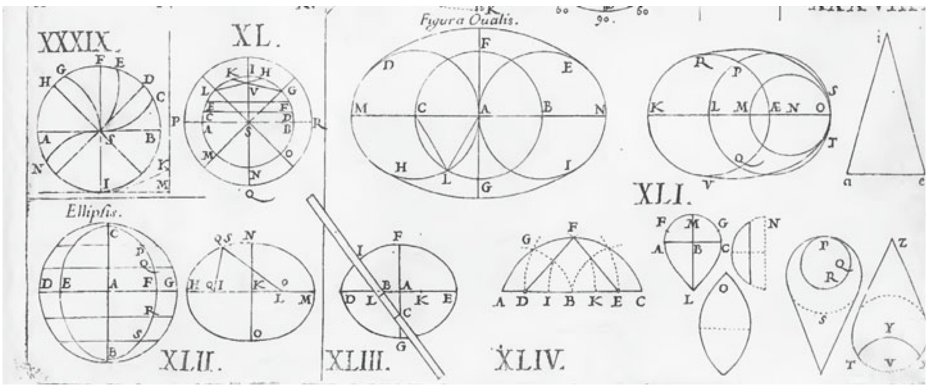



Fig. 15. Juan Caramuel de Lobkowitz, *Arquitectura civil recta y oblicua*, 1678

Simón García also tackles the problem of laying out ovals [1681: 65, 77v]. As the first part of the manuscript is considered a copy of the one by Rodrigo Gil de Hontañón,[3] in this topic we find the influence of Serlio and, mainly of Fray Lorenzo de San Nicolás (fig. 16). His original contribution is a layout for a semi-ovoid shape which is not geometrically correct: the points K, L and B are not aligned.

All the layouts analysed up until now draw ovals of a certain fixed proportion, that is, ovals that cannot be adjusted to fit to any given measures. The Spanish Tomás Vicente Tosca [1712] might possibly have given the first, general construction for drawing an oval for any given proportion. His *Compendio Mathematico* was first published between 1707 and 1715; the fifth volume, containing the *Tratado XV sobre Montea y Cantería,* was published in Valencia in 1712, and includes the outstanding contribution we are describing. A second edition of the complete *Compendio* was published in Madrid in 1727 and another one in Valencia in 1757 [Fernández Gómez, 1994: 349; 2000: 21]. Tosca's *Compendio* is believed to have been influenced by the work of Milliet-Dechales [Rosselló 2004: 160] and the text on stonecutting is said to be a copy of the French treatise [Rabasa 2000: 234, 306]. Tosca describes five ways to layout a surbased arch: two oval and three elliptical ones [1712, Treatise XV: 99-104 and 108]. The first oval method is the fourth proposed by Serlio and the second, which looks like a segmental arch, is the most interesting for the present research (fig. 17).

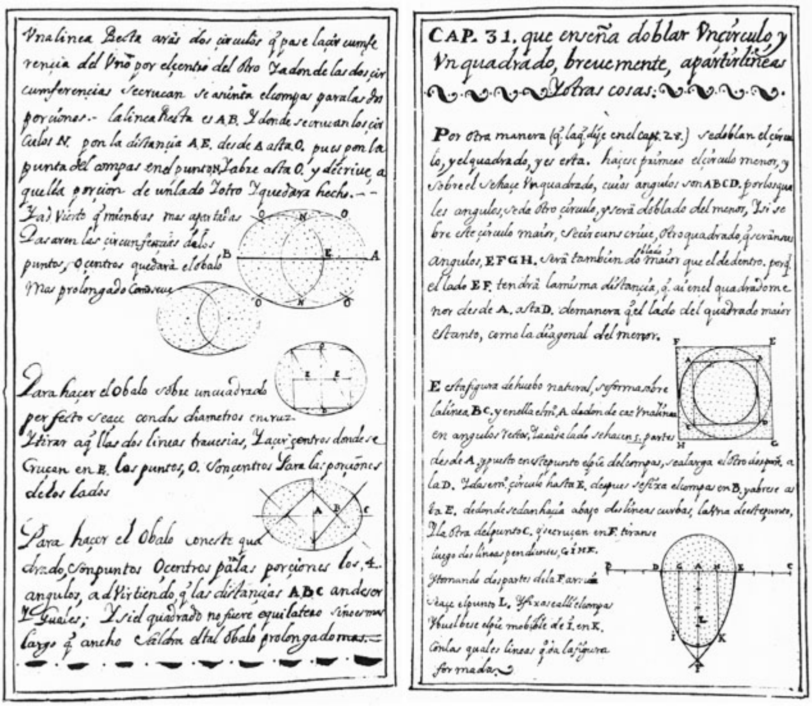



Fig. 16. Simón García, *Compendio de arquitectura y simetria de los templos*, 1681 [1990]

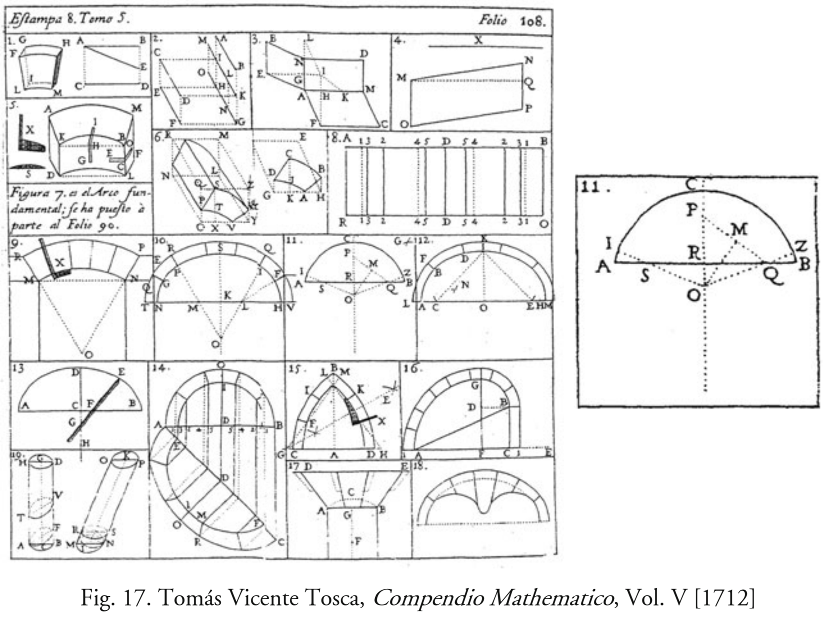



Fig. 17. Tomás Vicente Tosca, *Compendio Mathematico*, Vol. V [1712]

The translation of the text describing the way of drawing an oval fitting any given proportion is the following:

> Given the horizontal diameter AB of an arch (fig. 11) and CR the height that it must have: cut arbitrarily, but equal, the distances AS, BQ, CP. Draw the line PQ and its median line MO, which will meet to the CR lengthened to O. Draw a line from O to Q and Z; and another one from O to S and I; and from O with the distance OC draw the arc ICZ; and from Q with the distance QZ draw the arc ZB: and from S the arc IA, and the intrados will be formed: with the same centres the extrados will be drawn (my translation).

Indeed, choosing the radii of the lateral arcs of the oval (AS and BQ in the text of Tosca) fixes one of the infinite solutions to the problem. The importance of Tosca's contribution should be studied and analyzed taking into consideration the sources which he could have employed for this topic.[4]

Jean-Baptiste De La Rue ignored the contribution by Tosca, as he proposes a layout to draw a semi-oval given the diameter and the height, which, however, is not geometrically exact [1728: 5]. A graphical translation of the method by De La Rue is shown in fig. 18.

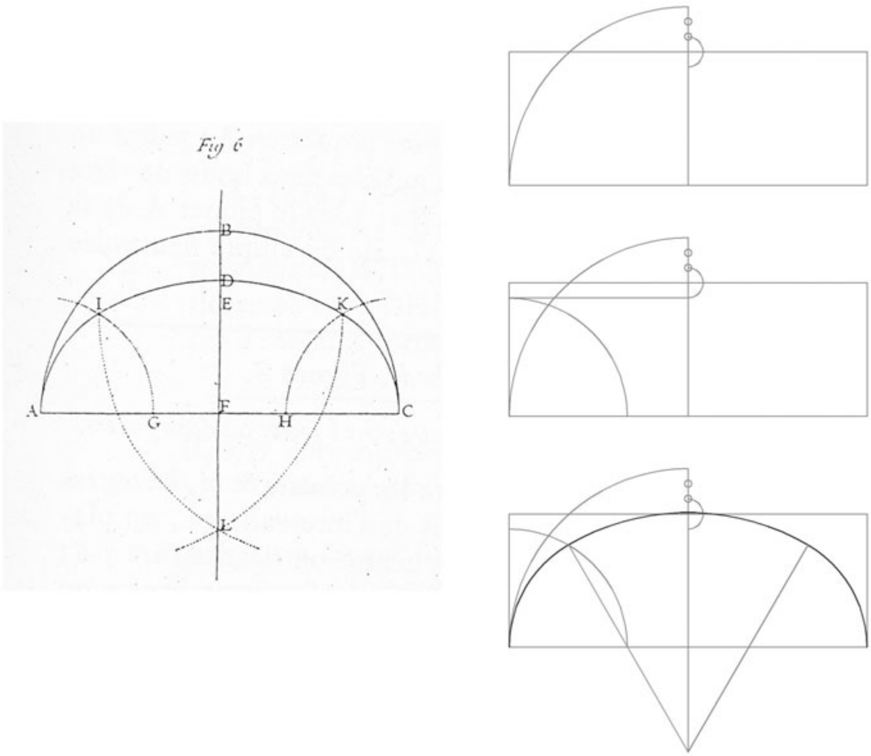



Fig. 18. Jean-Baptiste De La Rue, *Traité de la Coupe des Pierres* [1728].
On the right, drawing process

The mathematical proof of the erroneous construction by De La Rue is given below. As the centres are aligned forming 30° to the vertical axis of the oval, if the layout is correct, dropping the radius of the lateral arcs from the key and drawing a straight line to one of their centres should give an angle of 15 ° to the horizontal line. Letting *a* and *b* represent the lengths of the semi-axes, the proof would be:

$$tga = \frac{\frac{1}{3}(a-b)}{a-b+\frac{1}{3}(a-b)} = \frac{\frac{1}{3}(a-b)}{\frac{4}{3}(a-b)} = \frac{1}{4} = 0.25; \quad \alpha = 14.03° \neq 15°$$

The contribution by De La Rue is quite ingenious, as the drawing is very close to the exact one, especially in arches that are not excessively surbased.

Frézier, critical of the use of ovals [Huerta 2007: 234], poses the problem, "given two axes, imitate an ellipse by the union of four arcs of circumference" (my translation) and finds an interesting solution. Frézier describes an exact oval layout for any given proportion, with the alignment of the centres forming a 60° angle to the horizontal line, of which the mathematical proof is given but not completed [1737: 183, Pl. 14]. The drawing begins by taking the length of the minor semi-axis on *By*; the rest of the process is clearly explained in the figure (fig. 19).

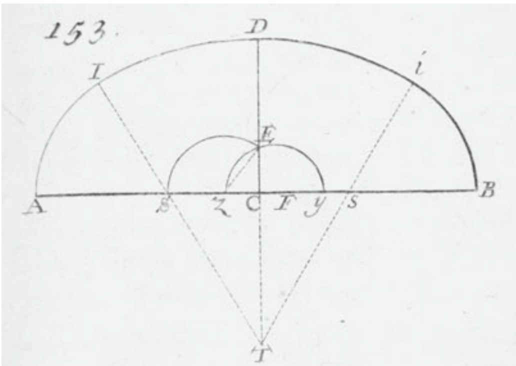



Fig. 19. Amédeé François Frézier, *La théorie et la pratique de la Coupe des Pierres*, 1737

The mathematical proof, for the oval starting from A and B and exactly reaching D, would be the next one:

$$AS + ST = DC + CT \, .$$

With *a* and *b* as the lengths of the semi-axes, we have

$$Cy = (a-b) \text{ and } CZ = \frac{1}{2}(a-b).$$

The first step should deduce the values of CE and SZ:

$$tga = \frac{CE}{a-b}$$

$$tga = \frac{\frac{1}{2}(a-b)}{CE}$$

$$\frac{CE}{a-b} = \frac{\frac{1}{2}(a-b)}{CE};\; CE^2 = \frac{1}{2}(a-b)^2;\; CE = \frac{a-b}{\sqrt{2}}$$

$$SZ = ZE = \sqrt{CZ^2 + CE^2} = \sqrt{\frac{(a-b)^2}{2^2} + \frac{(a-b)^2}{2}} = (a-b)\sqrt{\frac{3}{4}} = (a-b)\sqrt{3}\,\frac{1}{2}$$

Going back to the equation we have to prove, we will work separately with each of the members of the equation. Considering that the triangle *SST* is equilateral, it must be hold true that $ST = 2SC$.

$$AS + ST = [a - (SZ + CZ)] + 2SC = a - (SZ + CZ) + 2(SZ + CZ) = a + SZ + CZ$$

Replacing *SZ* and *CZ* by the mathematical expression found above:

$$AS + ST = a + (a-b)\sqrt{3}\,\frac{1}{2} + \frac{(a-b)}{2}$$

$$AS + ST = a + \frac{(a-b)(\sqrt{3}+1)}{2}$$

$$= \frac{2a}{2} + \frac{\sqrt{3}a - \sqrt{3}b + a - b}{2} = \frac{3a + \sqrt{3}a - \sqrt{3}b - b}{2} = \frac{a(3+\sqrt{3}) - b(1+\sqrt{3})}{2}$$

If we operate in the second member of the equation:

$$DC + CT = b + ST\sqrt{3}\,\frac{1}{2} = b + 2SC\sqrt{3}\,\frac{1}{2} = b + 2(SC + CZ)\sqrt{3}\,\frac{1}{2}$$

$$= b + 2\left[\frac{(a-b)(\sqrt{3}+1)}{2}\right]\sqrt{3}\,\frac{1}{2} = b + \left[\frac{(a-b)(\sqrt{3}+1)}{2}\right]\sqrt{3}$$

$$= \frac{2b}{2} + \frac{(\sqrt{3}a - \sqrt{3}b + a - b)\sqrt{3}}{2} = \frac{3a + \sqrt{3}a - \sqrt{3}b - b}{2} = \frac{a(3+\sqrt{3}) - b(1+\sqrt{3})}{2}$$

Therefore, it has been proven that $AS + ST = DC + CT$.

The oval by Frézier is based on an exact geometrical construction, offering a unique solution to the problem: an oval that fits the given measures of the axes, with its centres aligned forming a 60° angle to the horizontal line. Like De La Rue, Frézier ignores the contribution by the Spaniard Tosca, who had found the most complete solution twenty-five years earlier.

The book *Escuela de Arquitectura Civil* by the Spanish architect Brizguz y Bru [1738: 16-17], published in Valencia like Tosca's *Compendio*, shows four oval layouts. The first three start out from the major diameter and use a similar method; two equal circumferences are drawn, passing through the end points of the major axis, and may be secant, tangent or exterior one to each other depending on the relative sizes of their radii and the length of the major axis. If they are secant (fig. 25 of the original text) and the centres of the large arcs are located on the points of intersection of the small ones, then this corresponds to the case of the second oval of Fray Lorenzo de San Nicolás [1639: Chapter LXXVIII], but now complying with the conditions of tangency. If they are tangent (fig. 26 of the original text) and the centres are aligned to form a 60° angle to the horizontal line, as Brizguz proposes, then the oval is the one by Vandelvira [c. 1580: fol. 18r]. If the two equal arcs are exterior to one other (fig. 27 of the original text) the centre of the large arcs can be placed anywhere, as long as they respect the tangency conditions. Nevertheless the drawing by Brizguz may cause confusion, as it seems that the oval passes through those centres (E and its symmetric) and that is not the case. The last layout proposed by the author (fig. 28 of the original text), in spite of its intention of "describing an oval given the major and minor diameters", copies the proposal of De La Rue, which does not follow an exact geometrical construction (fig. 20).

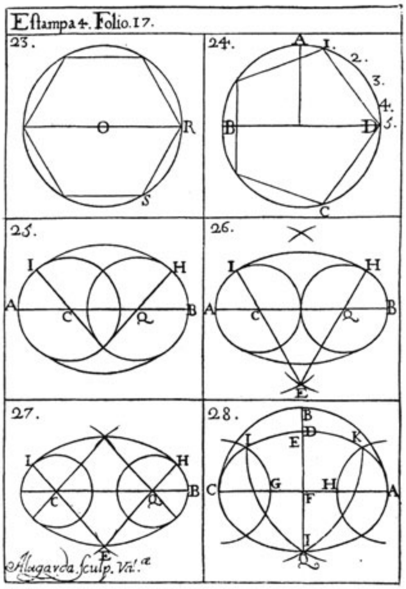



Fig. 20. Atanasio Genaro Brizguz y Bru, *Escuela de Arquitectura Civil* [1738: 17]

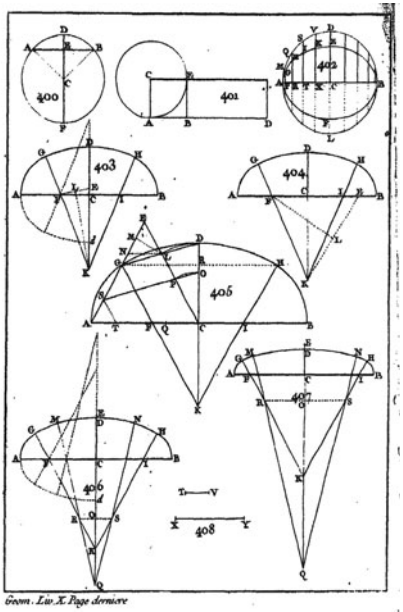


Fig. 21. M. Camus, *Cours de Mathématiques*, [1750]

The contribution by M. Camus [1750: vol. 2, 504-538] is a complete study of oval layouts for any given proportion for arcs of three and five centres, comprising thirty pages of the book [Huerta 2007: 234] (fig. 21).

As we have seen, the solution to the geometrical problem of laying out an oval fitting any given proportion would only be achieved at the beginning of the eighteenth century, in the *Compendio Mathematico* of Spanish author Tosca. We can hypothesize that perhaps the problem was not solved earlier because it was not necessary: builders had been constructing ovals by the trial-and-error method of adjustment. An analysis of the vaults at El Escorial will lend support to this argument, but it will also raise other doubts.

## *Ovals for any given proportion in the Escorial*

The study of the stone vaults at the Escorial shown in the present paper is part of the Ph.D. dissertation of the author [López Mozo 2009]. The work proved that one of the most renowned vaults in the building, the famous flat vault in the forechurch, is not the only outstanding contribution out of the problems tackled during its construction, and is perhaps not even the most interesting. A whole body of knowledge was developed: oval arches of all kinds were constructed; new solutions for pointed lunette and sail vaults were put forward, and, in particular, extradosed stone domes on drums, unprecedented in Spain. The general context of all these contributions is the aim of translating the Renaissance repertoire of vaults, which were mainly masonry brick built in Italy, to the language of stonecutting, carried out in the sixteenth century in Spain and France. This effort is reflected in the texts by De l'Orme [1567] and Vandelvira [c. 1580]. Moreover, original documents from the seventeenth century prove that a short treatise – now lost – was written at the Escorial worksite [Marías 1991], something easy to consider if we analyze the vaults and domes and stonecutting layouts that were designed without the support of a specific practical constructive tradition.

The Escorial, built by Philip II in the town of San Lorenzo de El Escorial about 50 km away from Madrid between 1563 and 1584, is one of the first Renaissance buildings with a new plan constructed in Spain. The king chose the Spanish Juan Bautista de Toledo to be "his architect for life" (my translation). Toledo was working in Naples at that time, and had been Michelangelo's second architect for two years during the construction of Saint Peter's Basilica [Rivera 1984: 47, 125]. After his death in 1567, the works continued, mainly under the supervision of stonecutting master builders. Juan de Herrera, who had been working at Toledo's office in Madrid since 1563, gradually began to assume more responsibility. In 1576, during the beginning of the construction of the church, Herrera was already the head of the chain of command: he reorganized the stonecutting works, removing most of the stone dressing process to the quarry, and dismissed both of the master builders who had been at the Escorial since the beginning of the works. Juan de Minjares, confidant of Herrera, was appointed sole stonecutting master builder and he remained in that position until the end of the building works [Bustamante 1994].

When the study of the vaults of the Escorial began, it was necessary to analyze the profile of the surbased vaults present throughout the building. The cylindrical vault is the most frequently used type: in the main level of the building there are 132 stone vaults; 106 of them are cylindrical and 69 surbased (fig. 22). All the significant vaults have been measured with a laser station to provide accurate data.

The existing groin and cloister vaults in the building prove that our masters knew how to reduce the height of an arch without changing the span, in order to correctly solve the intersection of the barrel vaults. We cannot determine if they used the layout by Dürer [1525], Serlio [1545], De l'Orme [1561] or Vandelvira [c. 1580], but the one by Serlio can be found in an original drawing – perhaps by Toledo – kept in the Library of the Royal Palace in Madrid [Bustamante et. al. 2001]. The method to elongate a semicircular arc was also known, based on the procedures just mentioned, but these were not specifically explained until the manuscript of the Spaniard Ginés Martínez de Aranda [c. 1600]. This case is actually built at El Escorial in the intersection of two small oblique barrel vaults with different, pre-fixed widths, behind the church cornice [Calvo 2002: 419-435].

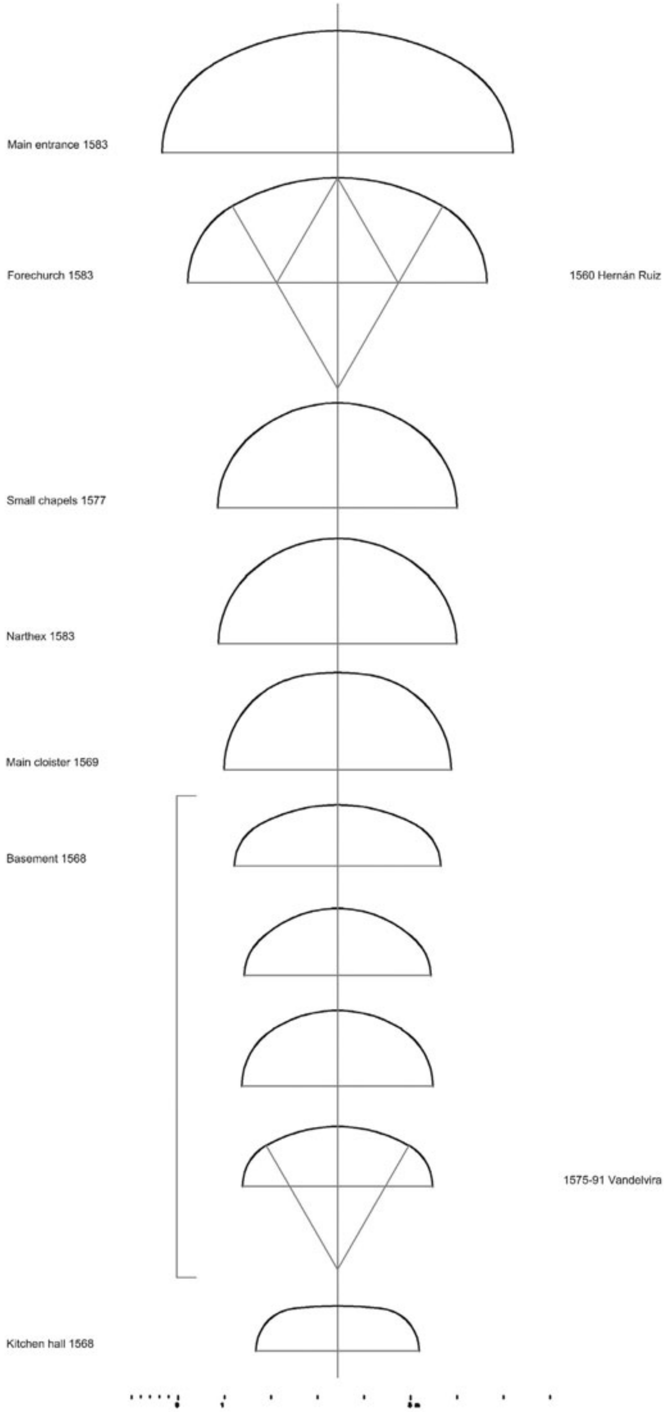



Fig. 23. The most significant oval layouts in vaults of the Escorial, drawn at the same scale [López Mozo 2009: 509]

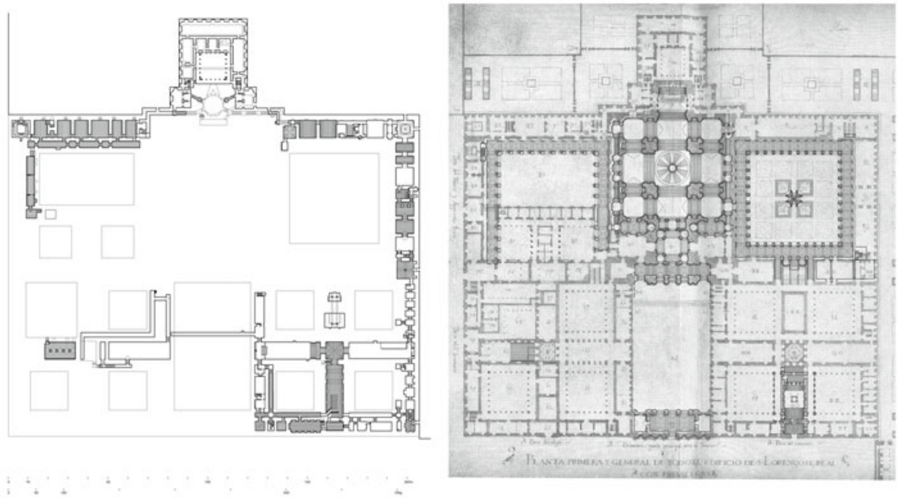


Fig. 22. Barrel vaults in the Escorial [López Mozo 2009: 263]

It is generally accepted that there was no known geometrical construction for drawing an oval for any given proportion in the sixteenth century. The layouts of the treatises at that time only present standard proportions that cannot be changed. But the oval figures at the Escorial only fit those layouts in two cases (fig. 23), so our builders were able to construct them fitting any measure by a trial-and-error adjustment, overcoming the limitations expressed by Serlio, and especially by Vandelvira, who proposes a single oval layout of a standard fixed proportion, and when it has to be built "according to the demands of the place in question" (my translation) he proposes the use of the "elliptical" layout by Serlio (fig. 24).

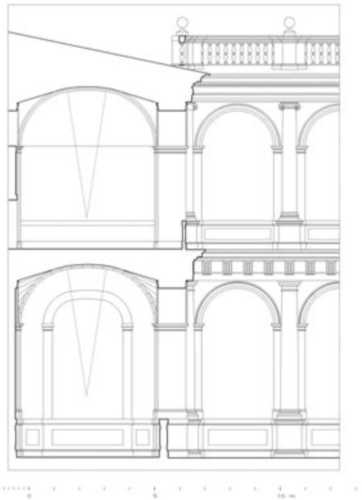


Fig. 24. Oval layouts in the vaults of the main cloister in the Escorial [López Mozo 2009: 283]

The analysis of the oval layouts of the vaults of the church would give unexpected conclusions. The surbased profile of the narthex vault fully fits an elliptical layout (fig. 25). However, for the sake of simplicity of the construction process alone, it would be more sensible to discard this option: on one hand, the mouldings of constant width in the arches and in the front walls cannot be laid out with ellipses; on the other hand, all the templates needed to dress the voussoirs of half the arch would be different; and, finally it is not possible to divide an ellipse into equal parts in order to distribute the voussoirs. These disadvantages of the elliptical layout versus the oval one would explain the fact that we only find built elliptical forms at the Escorial in small elements such as niches, lantern pilasters profiles (main dome, Fountain of the Evangelists) and in places where they were unavoidable (oblique barrel vaults behind the cornice of the church and surface intersections in groin and cloister vaults) [López Mozo 2009: 273-278].

Fig. 25. The Escorial, vault in the narthex

Once an elliptical layout for the narthex vault was discarded, the comparison of the measured points to the oval layouts of the treatises of the sixteenth century proved to be unsuccessful. Applying the modern method, described by Tosca in 1712, the oval layout that best fitted the vault cross-section showed an alignment of the centres forming a 45° angle to the horizontal line, a condition also satisfied in the determination of oval layouts in other church vaults (fig. 26). The recurrence of this situation made it necessary to consider the possibility that an oval layout for any given proportion with alignment of centres forming a 45° angle could have been known at the Escorial.

A simple mathematical deduction, set out in fig. 27, gave an easy graphical translation. The process was as follows: given an oval of semi-axes $a$ and $b$, it is possible to find the value $h$ which determines the distance of the centres to the vertical axis and to the horizontal springing line, resulting in an oval that passes through the final points of the axes.

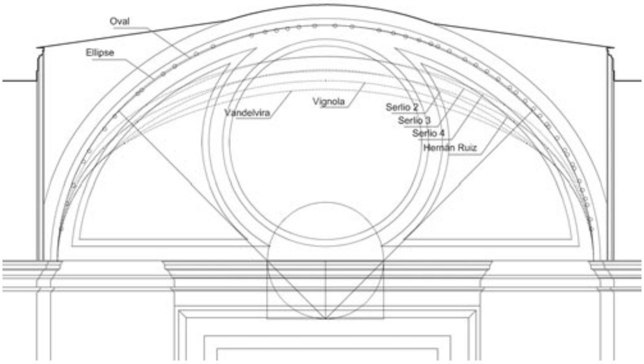



Fig. 26. Cross-section of the vault in the narthex,
comparing measured points and both elliptical and oval layouts

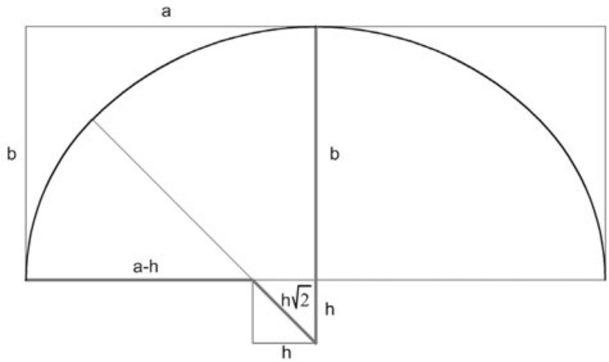



Fig. 27. Oval layout for any proportion with centres forming an angle of 45° to the vertical axis: setting out [López Mozo 2009: 288]

$$a - h + h\sqrt{2} = h + b$$

$$a - b = 2h - h\sqrt{2}$$

$$a - b = h(2 - \sqrt{2})$$

$$h = \frac{(a-b)}{2 - \sqrt{2}}$$

$$h = (a-b)\frac{2+\sqrt{2}}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)} = (a-b)\frac{2+\sqrt{2}}{2} = (a-b)\left(1 + \frac{1}{\sqrt{2}}\right)$$

$$h = (a-b) + \frac{(a-b)}{\sqrt{2}}.$$

Fig. 28 shows a graphical translation of the terms of the last mathematical expression. The corresponding process to lay out the oval is described by three steps in fig. 29.

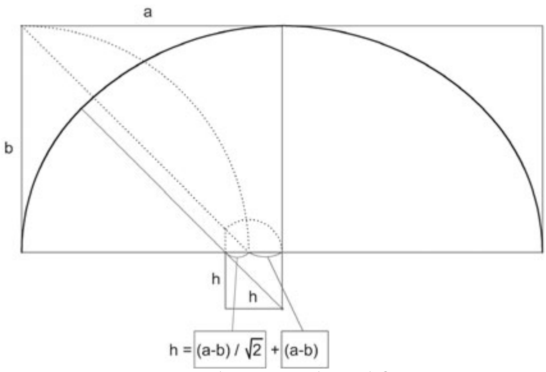



Fig. 28. Oval for any given proportion with centres aligned forming a 45° angle to the horizontal line. Graphical translation

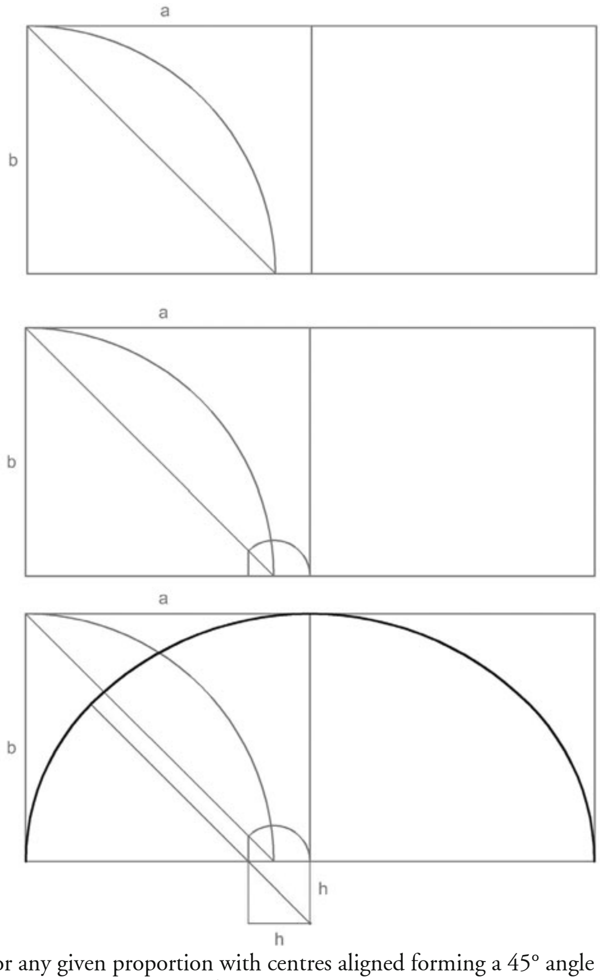



Fig. 29. Oval for any given proportion with centres aligned forming a 45° angle to the horizontal line. Layout process

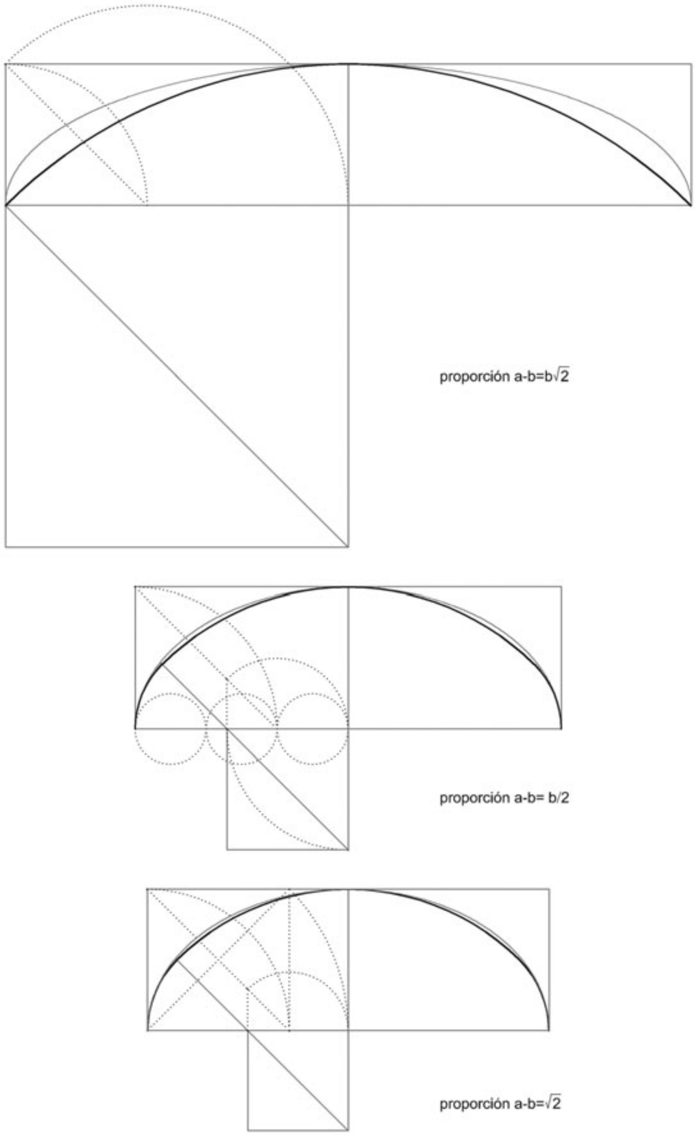



Fig. 30. Oval for any given proportion with centres at 45°. Specific cases.
The elliptical layout is also drawn

The layout just explained contains certain restrictions: it is not practical in cases with low proportions, since the first centre exceeds the segment for the span of the oval; the condition only holds for $a - b < b\sqrt{2}$, in which case the starting arch is at the extreme, its radius is zero, and the result is a segmental arch. In very low, but possible, proportions, the layout differs greatly from an ellipse. It would be more logical to make the arch starting from the proportion: $a - b = b/2$ (fig. 30).

Fernández Gómez [1994: 351, 378] describes this oval when listing current layout methods, without pointing out the condition satisfied by the centres, aligned at a 45° angle with respect to the axes. Neither the graphical drawing process nor the written explanation coincide with that offered in the present paper (fig. 31).

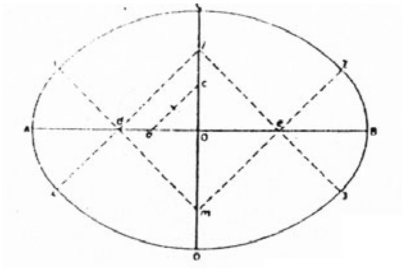
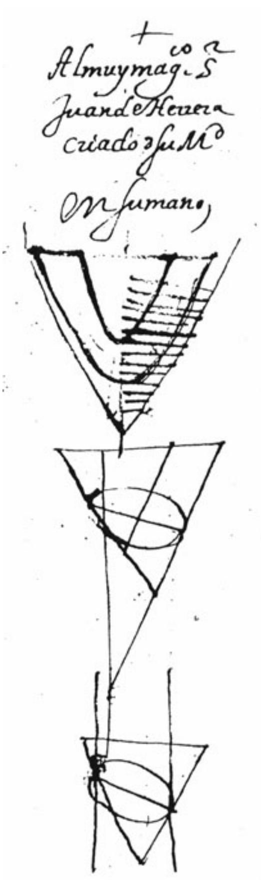


Fig. 31. Margarita Fernández Gómez [1994: 378]

## *Conclusion*



Judging by the number of times Serlio's oval layouts appear in subsequent treatises examined in this study, his enormously influential treatise enjoyed a very wide-spread dissemination. We find his proposals in the works of Hernán Ruiz [c. 1560], Jousse [1627], Fray Lorenzo de San Nicolás [1639], Simón García [1681], Derand [1643], Caramuel [1678], Milliet Dechales [1674] and Tosca [1712].

Vandelvira poses the problem of oval layouts in a manner similar to that of Serlio, but with a single possible oval, of fixed proportions; in the cases where the oval does not fit the given place, he proposes the elliptical layout posed by the Italian architect. It is strange that Vandelvira did not include the other three ovals by Serlio, especially since one of them has a $\sqrt{2}$ proportion and could fit the intersections of the cylinders forming the groin vaults. On the other hand, it is surprising that Vandelvira, who possessed abundant experience from his father's works, did not mention the possibility of drawing oval arches by approximating the centres. The work carried out in the construction of the vaults at El Escorial does offer evidence in this direction, since the majority of the ovals that were used do not fit either the fixed proportions mentioned in the treatises, nor an elliptical shape: the practical mastery evident in their execution – the skilful adjustment of the oval arches to the measures imposed by the worksite – seems to be present throughout the entire building.

Fig. 32. Drawings of conical sections attributed to Juan de Herrera (Archivo General de Simancas)

The precise geometrical construction necessary for drawing an oval adjusted to a given set of dimensions of the axes probably did not appear until 1712, in the *Compendio Mathematico* by Spanish author Tomás Vicente Tosca. It is the oval layout most frequently used nowadays. Two important later French authors, De La Rue [1728] and Frézier [1737], ignore this contribution by Tosca. The study of this topic with regards to his sources, dissemination and later influence has yet to be carried out. The oval layout put forth by Frézier is less interesting than the one by Tosca, since it is a specific case. However, it solves the problem for a given inclination of 60° of the line segment that joins the centres.

This work puts forward the possibility that a correct geometrical method for laying out ovals for any given proportion might have been known at the Escorial at around 1576. Nevertheless, no evidence exists of this fact; it is hypothesized based on observation of the second stage of the works, carried out by Juan de Herrera and his master builder Juan de Minjares. If this hypothesis is correct, it seems strange that the method would not have spread. However, it might be that the lack of a subsequent echo was due to the fact that builders did not need such a precise tool: they laid out oval arches for any given place by means of trial and error. On the other hand, Herrera's solid mathematical and graphical knowledge lends support to this hypothesis. Herrera, who founded the Academy of Mathematics in Madrid in 1582 at the behest of Philip II, possessed a library containing dozens of books related to mathematics, including several editions of works by Euclid, Archimedes and Apollonius [Aramburu-Zabala 2003]. A set of drawings containing conical sections found within a document sent to Herrera by the Secretary of Philip II are attributed to Herrera himself [García Tapia 1990] (fig. 32).

A new graphical description of a layout that is not widely known today is shown in this paper: it is the one that draws an oval beginning from its axes, satisfying the condition that the line segment that connects the two centres form a 45° angle with the minor axis. This is a particular case, like that by Frézier, but for a different angle.

## *Acknowledgments*

This paper is part of the research project "Ashlar construction in the Mediterranean basin and Atlantic area. Analysis of built heritage", which is funded by the Ministry of Science and Innovation of Spain (BIA2009-14350-C02-01). I wish to express my gratitude to Enrique Rabasa for his kind permission to use his work on Gelabert's manuscript and to José Calvo Lopez for making available an electronic facsimile of the 1515 edition of Serlio's Books I and II.

## *Notes*

1. Vandelvira proposes drawing the "elliptical" line of the *arco painel* by taking points in sets of three and drawing the circular arc that passes through them [c. 1580: Section 22; Barbé-Coquelin de Lisle 1977: Vol. I, 56].
2. The term "carpanel" might have originated in the French *anse de panier*, which refers to a basket handle, in the same manner as the term *ansapaner* used by the Majorcan Gelabert [Rabasa 2011]. Juan Bautista de Toledo, the first architect for the Escorial, used the term *ançarpanel* in 1565 [Portabales 1945: XXXIV-XXXVII; López Mozo 2009].
3. According to Antonio Bonet Correa, the first four chapters are either a word-for-word copy or at least very faithful extracts of the text and ideas found in the manuscript by Rodrigo Gil de Hontañón. Other authors attribute to him chapters 1 through 6 at the most [1991: 14]. The topic on ovals is found in Chapter 20.
4. Margarita Fernández Gómez describes the layout, indicating that it starts out from the axes of the oval [1994: 351].

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