António Nunes Pereira

Rua Antero de Quental, 35, 4º Dtº
1150-041 Lisbon PORTUGAL
anunespereira@gmail.com

Keywords: proportional systems, Goa, ad quadratum, Renaissance architecture, sacred architecture, harmonic proportions, commensurable dimensions,

Research

Renaissance in Goa: Proportional Systems in Two Churches of the Sixteenth Century

Presented at Nexus 2010: Relationships Between Architecture and Mathematics, Porto, 13-15 June 2010

Abstract. The sixteenth-century Cathedral of Goa and the Jesuit church Bom Jesus reflect mainly European architectural concepts, before local influences appeared in the Portuguese Christian architecture of the following centuries. The research presented here investigated the use of proportional systems. The results show that both the Cathedral and the Bom Jesus have proportions that are usually found in Renaissance architecture of their time, namely, the "ad quadratum" progression and the use of a 4:3 rectangle.

Introduction

Two churches in the former capital of the Portuguese State of India, the Cathedral (1564-1652) and the Jesuit church Bom Jesus (begun 1594, consecrated in 1605), were begun during the sixteenth century and still reflect mainly European architectural concepts, before local influences appeared in the Portuguese Christian architecture of the following centuries. But even if the churches' typology reflected the contemporary religious architecture in Portugal, a unique formal language was developed in Goa. This language showed Portuguese and of course Italian, but also French and Flemish influence through the circulation of etchings, treatises and artists between Europe and the Orient. It became a specific Goan synthesis of the European Renaissance, unknown elsewhere in its purest form [Pereira 2003: 235-239; Pereira 2005: 315-321].

The central question of the following research project was to inquire about the use of proportional systems in these churches' designs and consequently to find out how much they owned to the European culture of the Renaissance. The project's outcome shows that both the Cathedral and the Bom Jesus have proportions that are usually found in Renaissance architecture of their time. In both buildings the proportional systems coincide in ground plan, elevation and sections. This demonstrates that proportions were consciously used to design a whole building, thus reflecting an attempt to achieve harmony through similar dimensions of different parts of a building. But whereas the Cathedral was designed with the *ad quadratum* progression, the proportions of Bom Jesus indicate the use of a 4:3 rectangle. Neither of these churches presents a highly complex proportional system based in musical theory such as Paul von Naredi-Rainer found in Alberti's Tempio Malatestiano and Palazzo Rucellai [Naredi-Rainer 1995: 163-172) or Stefan Fellner in the three Paraguay churches of the Swiss Jesuit Pater Martin Schmid (1694-1772) [Fellner 1993]. Rather, the churches in Goa show a pragmatic use of simple ratios between numbers such as 2:1, 3:2 or 4:3. If music was at all considered in the design process of these churches, their designers chose to privilege the intervals of octave, perfect fifth and perfect fourth and thus the most harmonic of all. The results of this research also confirm Wittkower's assertion, that Renaissance architects thought and

DOI 10.1007/s00004-011-0071-7; *published online* 8 June 2011

created in commensurable dimensions and seldom used irrational proportions. The proportions and geometric patterns of the Cathedral of Goa and of the Jesuit church Bom Jesus show that during the sixteenth century even in the most remote areas of the Portuguese colonial empire the means to achieve architectural harmony remained very close to those of Europe. This fact enlightens us about the processes of transmission of architecture to the new discovered world in the context of the emerging colonial system.

State of the art

Since Rudolph Wittkower's seminal book *Architectural Principles in the Age of Humanism* of 1949 launched the study of architectural proportions in the Renaissance, there has much research in the field of architectural proportion, a great part of which dealing with Renaissance architecture. This and some of the subsequent, more recent publications, were of determining importance for the present research. In his book *Architektur & Harmonie* [Naredi-Rainer 1995], – which has become a standard work in the German speaking world – Paul von Naredi-Rainer deals with number, measure and proportion in the western architecture in addition to period and style. He shows in various buildings the *ad quadratum* and the *ad triangulum* progressions, as well as incommensurable, musical proportions. Karl Freckmann's *Proportion in der Architektur* [Freckmann 1965], Hans Junecke's *Die Wohlbemessene Ordnung* [Junecke 1982], Barbara Böckmann's *Zahl, Mass und Massbeziehung* [Böckmann 2004] also give very informative examples and clues for the present research. Roman architecture was also considered in the present research, even though it was also known to the majority of non-Italian architects and master-builders of the sixteenth century through Renaissance treatises. Mark Wilson Jones' *Principles of Roman Architecture* [Wilson Jones 2000] is one fundamental publication on this topic, even more so as the author also deals with proportion issues. Paul A. Calter's handbook comprehensive display of proportional systems and geometric constructions in *Squaring the Circle* [Calter 1999] was very useful as a guide book as of their appearance throughout History.

Proportional systems and master builders in Goa

Proportional systems of buildings certainly did not depend only on its owner or function, but also very much on the architect. They reflected the designer's design options as well as his cultural context of origin or training. Therefore, master builders – as little as we might know about them – are key figures to unveil the proportional systems issue. During the Renaissance we find on one hand great innovators like Leon Battista Alberti, who developed new architectural concepts, including proportional systems. According to Naredi-Rainer's research, Alberti designed his buildings using sophisticated musical proportions [Naredi-Rainer 1995: 163-172]. Whereas his Tempio Malatestiano shows simple proportions based on the octave (2:1) and the fifth (3:2), the proportions of the Palazzo Rucellai includes many other intervals' ratios, like the fourth (4:3), major third (5:4), minor and major sixth (8:5 and 5:3) and minor and major seventh (9:5 and 15:8). It is likely that we could find such creators in the intellectual humanist circles mainly in Italy or central Europe. This is the case of the Swiss Pater Martin Schmid (1694-1772), a Jesuit musician who went to Paraguay in 1728. His work is known through Stefan Fellner's doctoral dissertation at the Technische Universität Berlin, who studied the musical proportions in the architecture of Schmid's three churches of wood and clay [Fellner 1993].

But on the other hand there were a great number of architects and master-builders who relied on Vitruvius, but certainly also on the medieval Vitruvian tradition. Their concern was to revive Antiquity through building design, approaching what they thought to be Roman architectural concepts. This is implied in the sixteenth-century Portuguese expression to designate Renaissance architecture: *ao (modo) romano*, i.e., "in the Roman (way)". The goal of these architects and master-builders was not to imitate the creations or to adhere to architectural concepts of Italian architects, but to relate to the Roman Empire. In many building tasks the Roman reference was politically important and expressly demanded from the consigner. This was the case in Goa, where the Portuguese were eager to legitimise their news conquests before the other European nations. But antique Rome was not assessable to all sixteenth-century architects and master builders. Those living and working outside Italy had to look at the work of Renaissance architects (through drawings and etchings) but also to authors of Renaissance treatises, as many of them did not have the possibility to travel to Rome and observe the ruins of Classical Antique. In particular, the books of Sebastiano Serlio were a source of graphic information, easily applicable to the design and building tasks these architects had to respond [Kruft 1995: 80]. The adoption of proportional systems remained close to the pragmatic Roman design approach.

Such a pragmatic approach must have been typical of the most important architects in the Portuguese colonies during the sixteenth century, the so-called king's master builders. The office of the king's master builder enjoyed a position of vital importance in the colonies. It was given to an expert trained above all in military architecture, as he had to design, restore and modernize the fortresses that guaranteed Portugal's military dominance in the Orient. These officers were also able to supervise civil and religious construction, as their education comprehended exercises in civil architecture and the knowledge of the most important architectural treatises, as well as of mathematics [Moreira 1993]. Because the Portuguese humanist Francisco de Holanda travelled to Italy and was offered either the *Libro IV* or the *Libro III* by Serlio himself in 1540 [Deswarte 1981, 252-254] his works became very popular in Portugal. Several features in the very churches which proportions we will analyze later testify that this popularity extended to Goa [Pereira 2005: 175-180, 197-201, 203-207, 305-311]. Vitruvius's *Ten Books* were never completely forgotten and circulated now in translations with prints, of which still exist copies in Portuguese libraries of Jean Martin's French translation and Daniele Barbaro's translation and commentary. The Spanish Daniel de Sagredo also contributed with a Vitruvian treatise in 1526, which was printed in Lisbon in 1541 and 1542. And, of course, every new accomplishment in military architecture was immediately introduced in the architectural training. Thus, during the second half of the sixteenth century the leading master-builders and architects, especially those sent to the Orient, were well versed in the latest developments coming from Italy and Central Europe. Following governmental orders in the name of the royal patron (see below), the master builders would draw plans, give advice, or supervise a building site for a religious order. It is highly probable that this was the case of the Goa Cathedral and that its designer was Inofre de Carvalho, the king's master builder at that time [Moreira 1988].

Goa as the capital of the Portuguese State of India

The Portuguese conquered Goa in November of 1510 through the military action of Afonso de Albuquerque. Even though at the time the town was already a very important commercial place, it remained in the first two decades after the conquest similar to the

many town-fortresses the Portuguese held along the African and the Asian Atlantic and Indic coasts. But the Christianization of people and territory began immediately after 1510. Before the first parish church was begun in 1514 and dedicated to Saint Catherine of Alexandria, Albuquerque built several votive chapels, thanking God for his help in this important military victory for the Portuguese Crown. In 1543 two other parish churches were founded: Our Lady of Light and Our Lady of Rosary. Two convent churches were also begun in the middle of the century: St. Francis (later Holy Spirit) in 1521, and St. Dominique, in 1550. Of all these churches, only Our Lady of Rosary still stands in the outskirts of Old Goa. All the others were either destroyed or rebuilt. We know therefore very little about the architecture of these early times. The main information comes from written sources, mostly brief building descriptions included in letters, reports, petitions, complaints, and contracts. From these we conclude that with the possible exception of St. Dominique, all these chapels and churches, as Our Lady of Rosary, were still medieval, either Gothic or Manueline Style.

But things were changing both politically and religiously. In 1530 the capital of the State of India (founded in 1505) was transferred from Cochim in Kerala to Goa. In 1533/1534 Goa became a diocese, in 1557 an archdiocese. Goa was now the centre of the small, but numerous Portuguese possessions spread all over the Orient. Goa was also the residence of both the governor or viceroy of India and of the archbishop. It was the only town in the whole Portuguese Empire to have a status very similar to that of Lisbon. State and religion were entangled, as all the territories in the Orient under Portuguese rule were also under the king's religious patronage. The king's obligations extended to the material support for the missionary work of the Catholic Church. This included not only the financial support of parish churches and convents, but also their construction. In this sense, the large majority of churches and convents were royal buildings, just like fortresses or government facilities were.

It is precisely at this point that our two churches diverge. The Jesuit church Bom Jesus is not part of this group. Whereas the Cathedral is the best example of a royal-religious building in Goa, the Bom Jesus Professed House and church was the result of a solely Jesuit initiative and was carried out without royal support – the crown had had already financed the earlier Saint Paul College (built between 1541 and 1576) and with it the king saw his obligations towards the Society in his capital Goa fulfilled. The different proportional systems we are going to find in each church are certainly a consequence of their different status.

In spite of these issues, there is a formal language common to all churches of the sixteenth century. This can be explained not only by of the smallness of Goa's territory, but also by the very architectural activity of master builders. Religious orders seldom had trained architects in their service. Even the Jesuits, who certainly were the best organized of all religious orders, often complained that they did not have learned architects in Goa for all their needs. In most cases, the only solution was to appeal to the king's master builder. It was most certainly this centralized system that gave rise to Goa's homogeneous architectural language, independent of the tradition of any particular order. The resultant visual homogeneity corresponded to the need for identification of an image of this new religion for the local people. It is not surprising then, that these churches, especially their architectonic "face", tended to display a relatively homogeneous configuration. The codification of the architectural rules of the Renaissance style supported the creation of repeatable models, which was furthered by the dissemination of treatises, in particular

those of Serlio with its easily adaptable drawings. In Goa, one very particular type became a typical solution for buildings of this time. The Cathedral and the Bom Jesus church represent different stages on the evolution towards this type, which I considered to have been best concretized in the ruined Augustinian church Nossa Senhora da Graça, of 1597 [Pereira 2005: 320-321]. Nevertheless, the proportional systems of both churches differ according to their function and representativeness. We are going to focus on their analysis after giving a short history of these buildings.

The Cathedral

The Cathedral of Goa was built after a royal order to substitute the old church of Saint Catherine, which had been begun in 1514 as the first parish church in Goa. But in 1533/1534 Goa became a bishopric and in 1557 an arch-bishopric, following the restructuring of the Portuguese bishoprics in the 1550s. In 1562 King Sebastião – or rather the regent in his name, as he was just eight years old – ordered a new cathedral to be built. Although the cathedral was begun shortly after 1564, the work was delayed because of lack of funding. It was during the second decade of the seventeenth century that the building progressed swiftly, and was finished between 1651 and 1652. In 1776 the two upper storeys of the north tower collapsed. There is no record of what they looked like. However, as the cathedral is entirely symmetrical and symmetry (in modern, not in a Vitruvian sense) was a very important feature in Renaissance buildings, the façade's drawn reconstruction shows a north tower very similar to the south one. Similarly to most medieval and sixteenth-century cathedrals in Portugal, the Goan Cathedral adopted a Latin cross plan. The church is a false basilica (no clerestory windows between the nave and aisles) with nave, two side aisles and side chapels, transept and choir. The façade shows three bays with portals in the first storey and rectangular windows in the second. The additional central bay linked to its sides through curved walls resolves the height difference between nave and aisles and hides the roof behind it. An entablature between the storeys marks the gallery floor level over the entrance. Portals, windows and niches have detailed, erudite architectural frames, very close to the figures shown in the treatises, especially those of Serlio.

Apart from its scale, the Cathedral is an inconspicuous building, conceived in a somehow discrete Renaissance style (figs. 1-3). Nevertheless, its design is very coherent and homogeneous, which means that in spite of the long building time the original project must have been respected. Although its designer is unknown, there are strong reasons to believe that Inofre de Carvalho was involved in it, as the Cathedral was in fact ordered by the king [Moreira 1988]. Besides some other master builders who worked there, Júlio Simão is known to have had a prominent role in the building during the first decades of the seventeenth century. It is uncertain though, whether he had much influence on the final result or did little more than carry out the initial design.

Architectural proportions in the Cathedral of Goa

As the Cathedral of Goa is a Latin plan church, the starting point of the proportion analysis was its crossing. Firstly, I considered the crossing, including the built elements, i.e., the piers supporting the groin vault (fig. 4). The side of this square was denominated with the letter A.

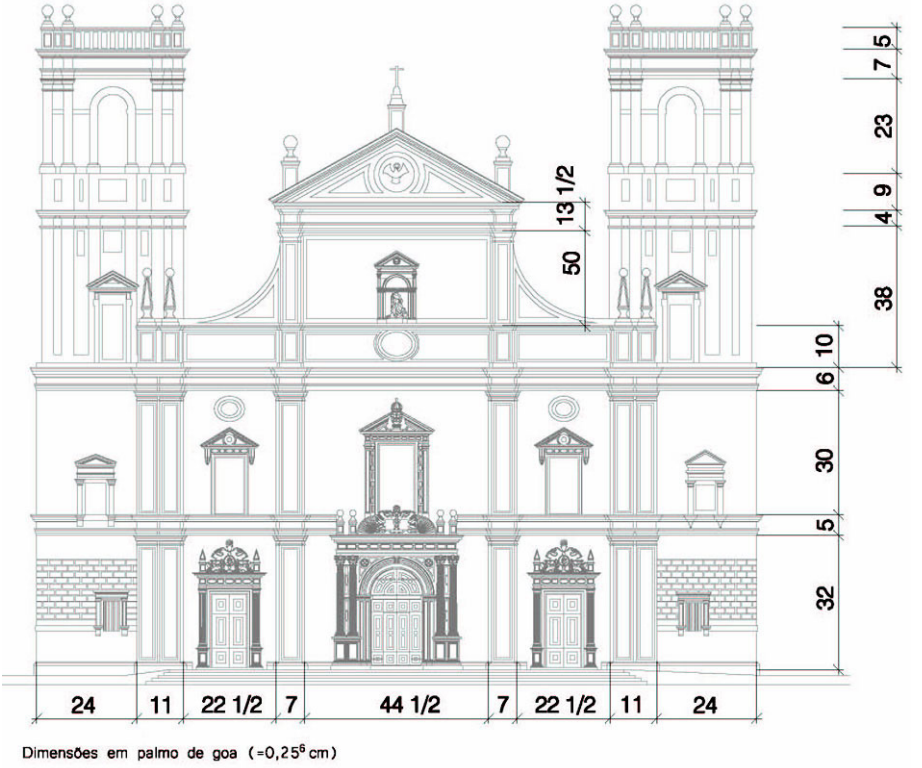

Fig. 1. Elevation, Cathedral of Goa. Dimensions in goa palms

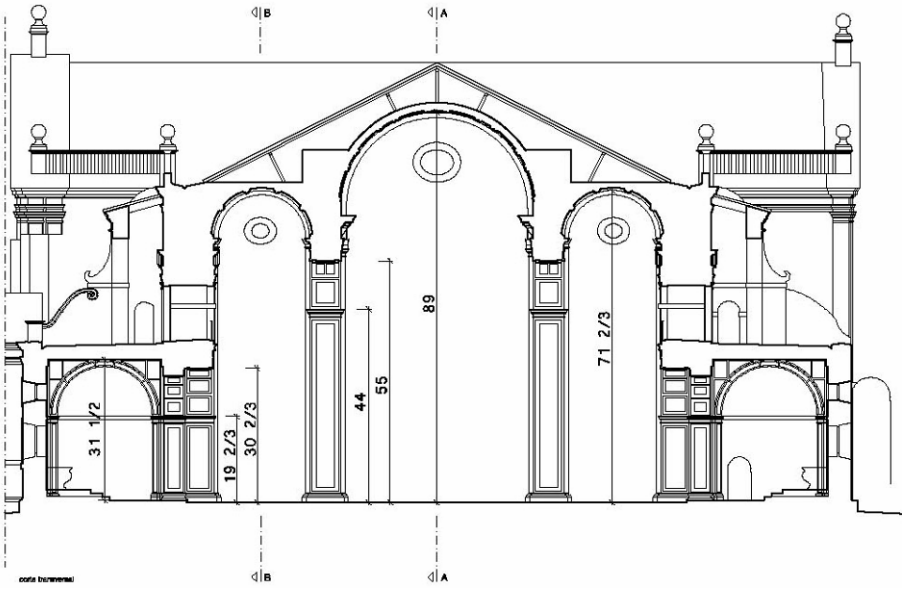

Fig. 2a. Transversal section, Cathedral of Goa. Dimensions in goa palms

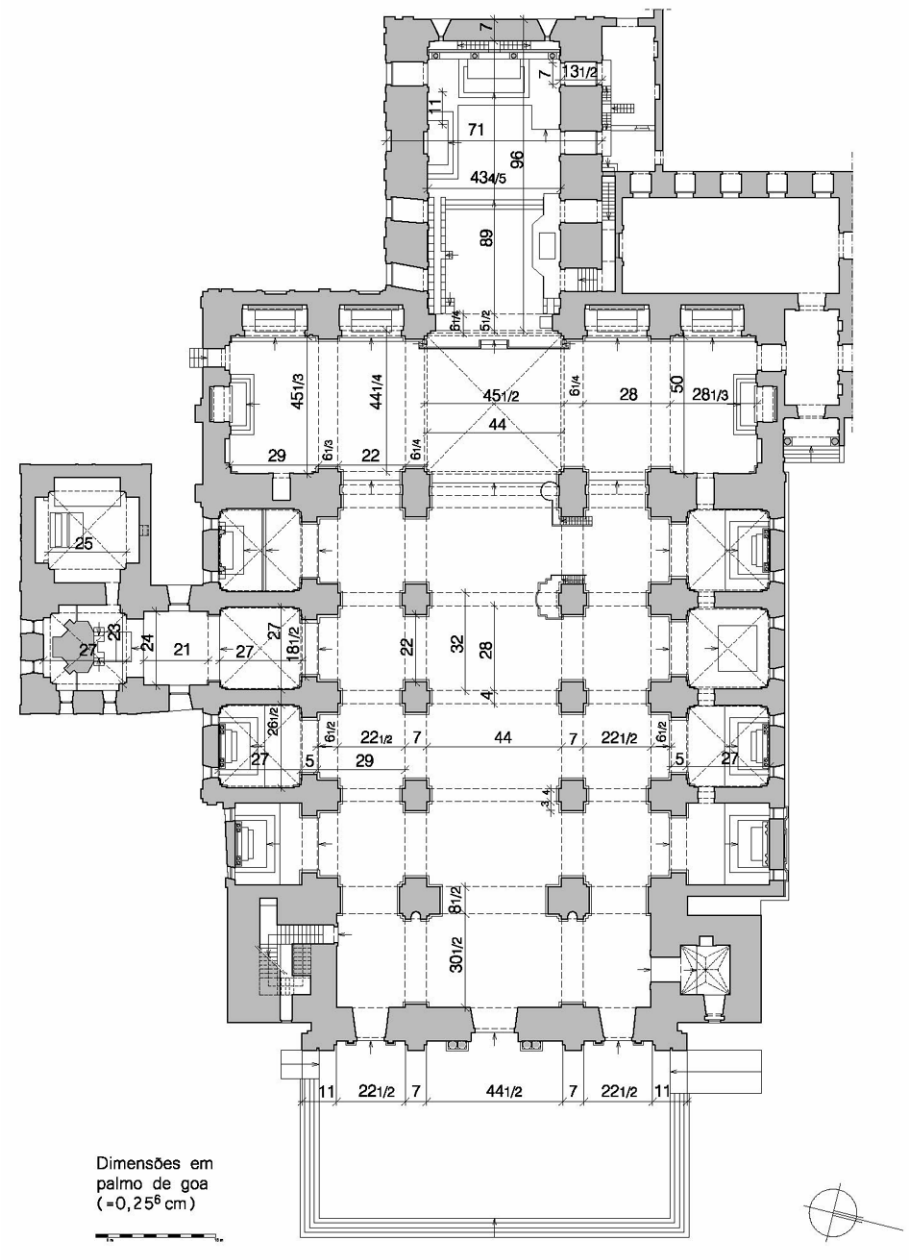

Fig. 3. Plan, Cathedral of Goa. Dimensions in goa palms

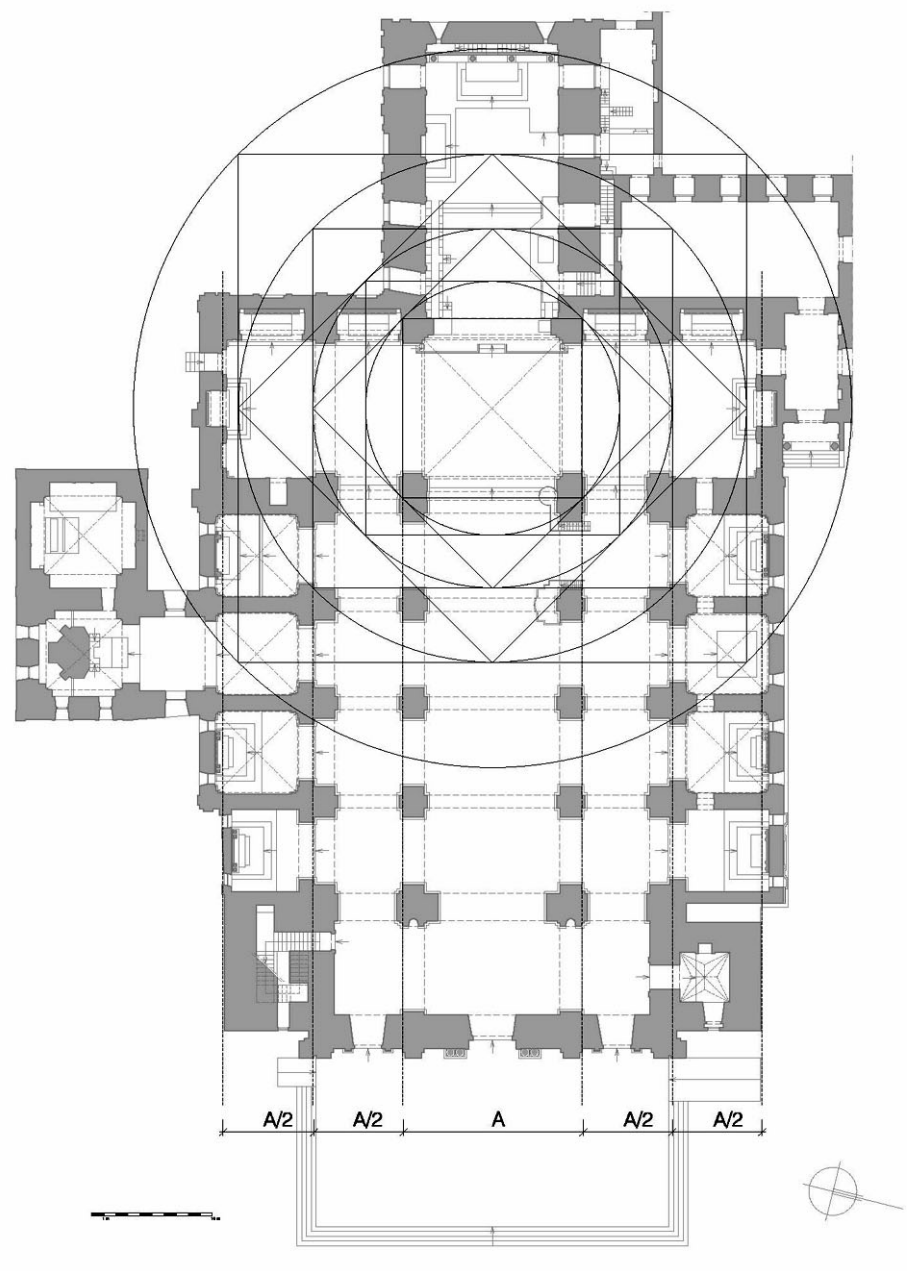

Fig. 4. Proportional analysis based on module A, Cathedral of Goa

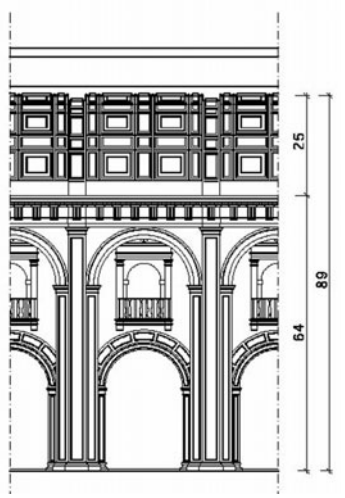

Fig. 2b. Segment of the longitudinal section, Cathedral of Goa. Dimensions in goa palms

Taken from the crossing side A measures ca. 14.86 m (1.61 m for each of the piers and 11.65 m for the span of the crossing),[1] which corresponds roughly to 58 goa palms[2] (14.87 m : 0.256 m = 58.08). It also corresponds to the nave and transept width including the pillars. Setting here the square/circle construction to double the initial square, we get a third square as wide as the side aisles including the pillars forming the outer arches of the side chapels. This proportion corresponds to the architectural morphology, as these pillars have a very similar configuration as the pillars between the nave and aisles.[3] The same construction may be applied to the façade (fig. 5). Even considering only the ground plan, the correspondence between nave and aisles width respectively with the façade central and side bays, as well as between the pillars in the inside and the pilasters – including the angulation – in the outside is evident.

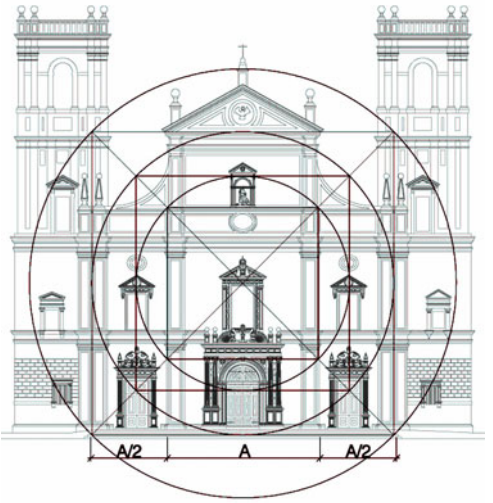

Fig. 5. Proportional analysis of the façade based on module A, Cathedral of Goa

The square A encloses the central bay and the adjoining pilasters, whereas the top square side matches the attic cornice. The third square (side dimension 2A) encloses the inner row of the double corner pilasters, which correspond in the interior to the side chapels' pillars. The outer rows, which do not have any correspondence in the interior, are left out of this construction.[4] The top square line runs along the cornice below the main pediment, where the dove representing the Holy Ghost is. The middle square (side dimension equal to $\sqrt{2}A$) divides the lateral portals and windows roughly through their middle axis.

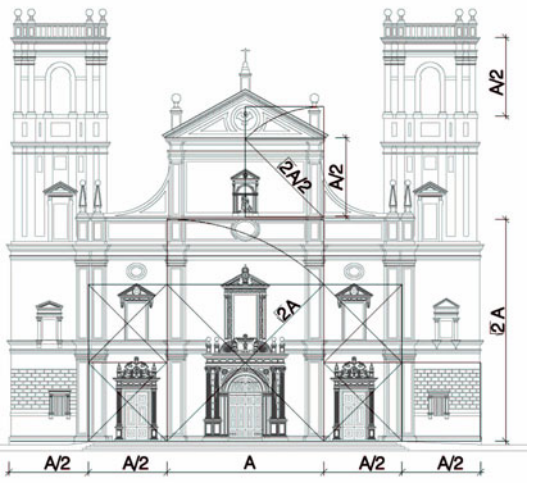

Fig. 6. Proportional analysis of the façade based on module A, Cathedral of Goa

A second drawing (fig. 6) shows further geometrical relations relating to the incommensurable dimension √2A: The height of two storeys of the façade plus attic corresponds roughly to √2A. The height of the additional bay with the sculpture of Saint Catherine triumphing over the Adihl Shah is A/2, whereas the spheres' base over the side acroteria on the pediment corresponds to half of √2A. Further commensurable proportions are the towers width on the façade's basis, which corresponds to half of A, measuring from the inner row of the double corner pilasters; and the height of the towers' last storey, excluding both plinth and balustrade.

The sections (fig. 7) show similar proportions based on the square A. In the transversal section the nave's width equals the height of its side pilasters including the capitals. The entablature above them and the vaulting correspond in height to half of A. From the same point to the roof's top the distance is √2A/2. The aisles' height also matches A, but here the capitals that are at the same level as the walls' cornices are excluded. The longitudinal sections show that the bays between pilasters are A/2 wide.

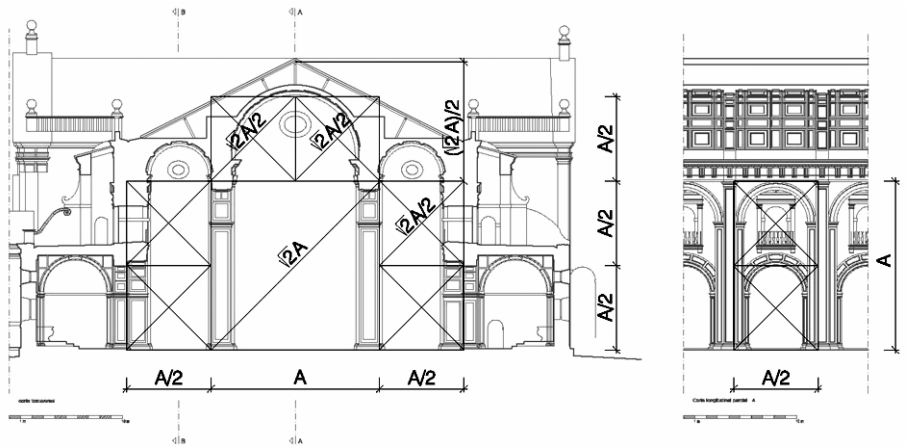

Fig. 7. Proportional analysis of the interior section and elevation based on module A, Cathedral of Goa

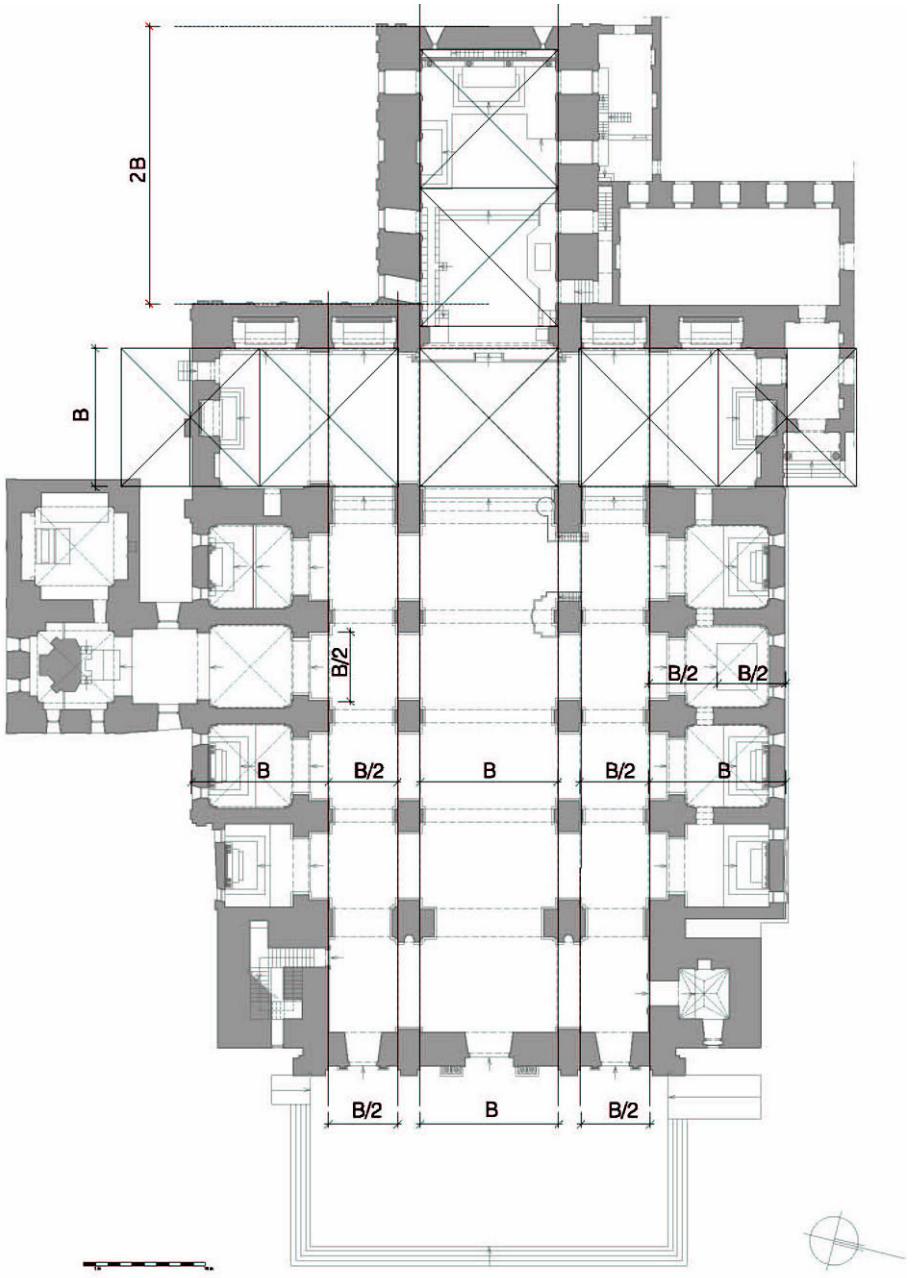

Fig. 8. Proportional analysis based on module B, Cathedral of Goa

So much for proportions built on square A. If we consider only the crossing's free space without angulation, we get another square, which is nominated B (fig. 8). B is ca. 11.23 m, which corresponds to little less than 44 goa palms. The transept is slightly wider (varying between 11.38 m and 11.59 m ≈ 44½ to 45¼ goa palms), but both the choir (11.21m ≈ 43 4/5 goa palms) and the nave (11.29m ≈ 44 goa palms) vary little.

Instead, the transept's arms are $\frac{3}{2}$ B deep, having a *sesquialtera* proportion, the choir is $\frac{2}{1}$ B deep, showing an even more "harmonic" proportion, the *diapason*, as a sacred space. The aisles are half as wide as the nave and the side-chapels including walls and arches are as wide as the nave, corresponding to B.

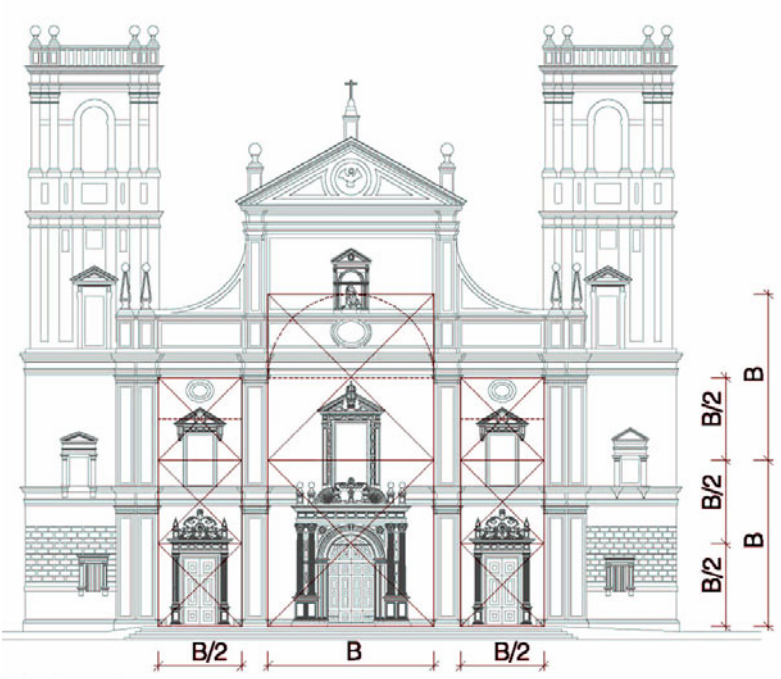

Fig. 9. Proportional analysis of the façade based on module B, Cathedral of Goa

As with the square A, so the façade shows proportions after the square B, although they only concern the bays corresponding to nave and aisles (fig. 9). There is a slight dimensional deviation, as the central bay is 11.39 m and therefore ca. 44½ goa palms wide, in contrast with the crossing's 11.23 m or the nave's 11.29 m (both ≈ 44 goa palms). These deviations were not so evident when we compared the plan with the façade concerning the proportions after square A. Their origin lies mainly in the width variation between pillars in the interior (1.81 m) and pilasters on the façade (1.77 m). This may be due to two reasons: the fact that the Cathedral is plastered and the successive renovations certainly altered the initial shape; but above all that the façade was built almost a century after the foundations and main walls in the interior had been laid, so that minor variations may have occurred. As the crossing is our starting point, we will consider B as corresponding to 11.23 m, even though the nave's width with its 11.29 m is even closer to the considered 44 goa palms. Back to the façade, the central bays – including the

entablature that divides them but excluding the upper entablature – have a proportion of *sesquialtera* (3:2), whereas the lateral bays – again excluding the upper entablature – are three times higher than wide, a proportion of *diapason diapente* (3:1). The transversal section (fig. 10) shows that the nave is two times higher (2B) than wide, whereas the longitudinal one evidences that the space between the main pillars also corresponds to B/2.

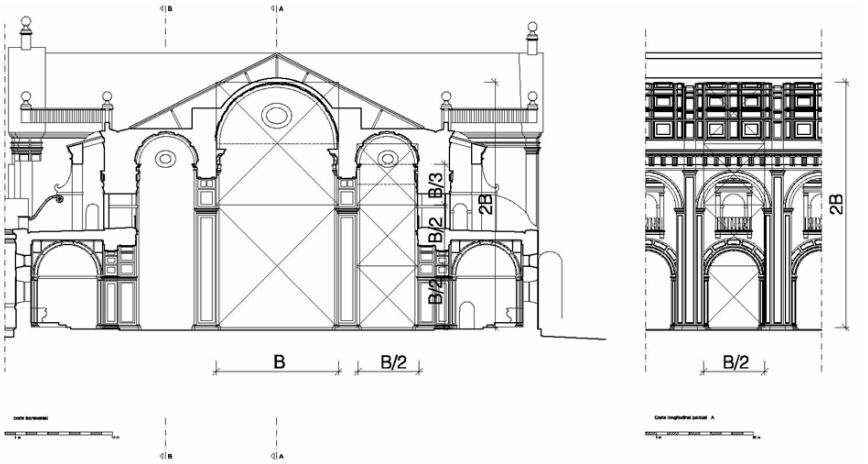

Fig. 10. Proportional analysis of the interior section and elevations based on module A, Cathedral of Goa

Church of Bom Jesus

Bom Jesus was conceived as the church of the Jesuit Professed House in Old Goa. The Professed House itself was begun eight years before the church, in 1586, as the Jesuits had not the means to finance both building sites simultaneously. The visitor Alessandro Valignano claimed to be the designer of the Professed House. For the church a large rectangular plot by the façade was kept free, as Valignano's drawings sent to Rome show [Wicki and Gomes 1948-1988: XIV, 274-281, 293-295]. Interestingly, the legend in the rectangle representing the church in the drawings sent to Rome by Valignano informed the General of the Society of Jesus that it had not yet been decided, whether the church should have a nave and two aisles or just a nave. Although the architects in Rome had some remarks about the whole Professed House complex – it was considered too much monumental for the Jesuit way, "*modo nostro*" – no comments were made about the vagueness concerning the church's building type [Wicki and Gomes 1948-1988: XIV, 702-706]. Only in 1594 would a legacy of the late captain of Hormuz make it possible to begin it. Bom Jesus was designed with a single nave and two chapel-like spaces to the right and left of the main chapel (figs. 11-13). These spaces and main chapel are narrower and lower than the nave, which means that the architectural space of the Bom Jesus is not cruciform like the Jesuit mother-church Il Gesù in Rome. Rather, the Goan Jesuit church follows the space type of the Portuguese Jesuit churches as Espírito Santo in Évora (1565) and São Roque in Lisbon (1567), as the late art historian Mário Chicó very wisely noticed [Chicó 1956: 267-268]. A cloister-like courtyard was built adjacent to the south side of the church, with cells lining the south side. Two later changes in the church and its surrounding buildings are determining for the proportions analysis: in 1652 a new sacristy was begun to replace an older one; and a few years later the chapel-like spaces by the main chapel were amplified, so that in the south one could receive the tomb of Francis Xavier[5] and the north one the Blessed Sacrament.

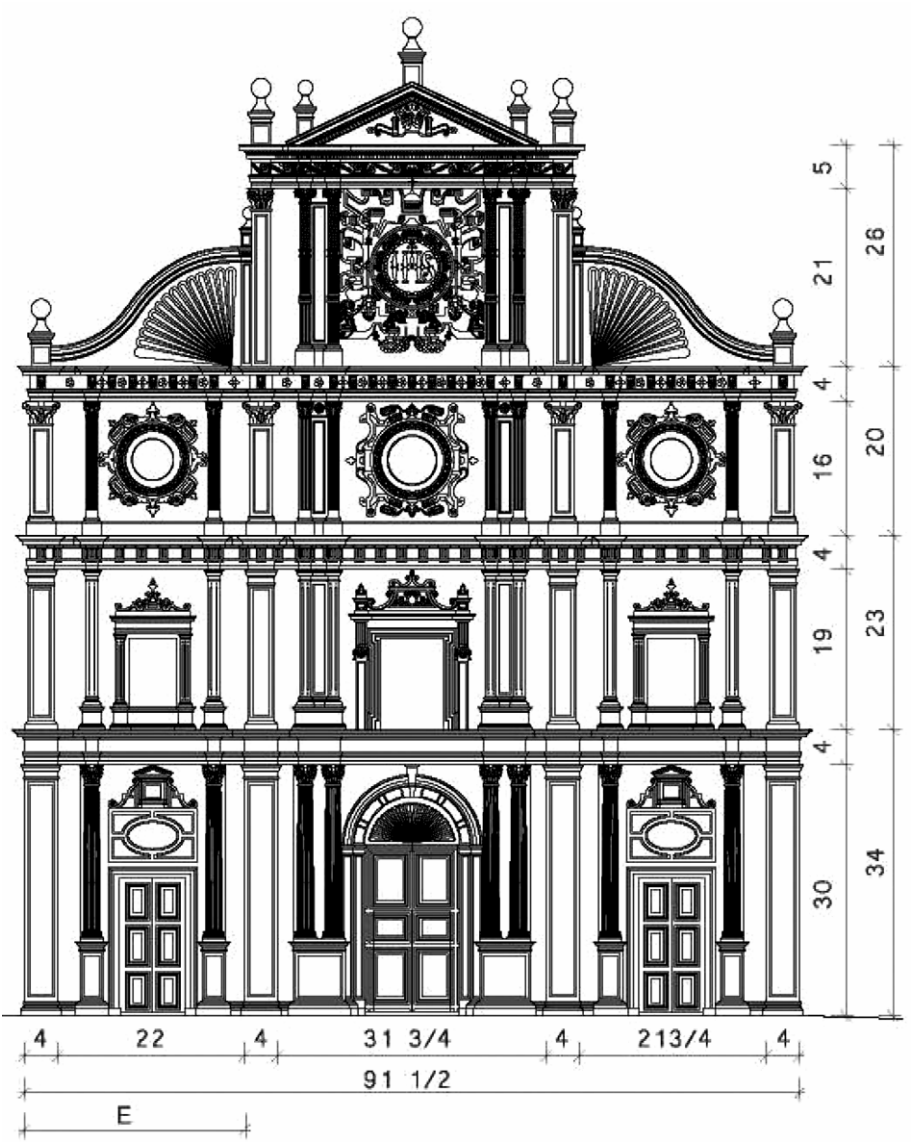

Fig. 11. Façade, Church of Bom Jesus. Dimensions in goa palms

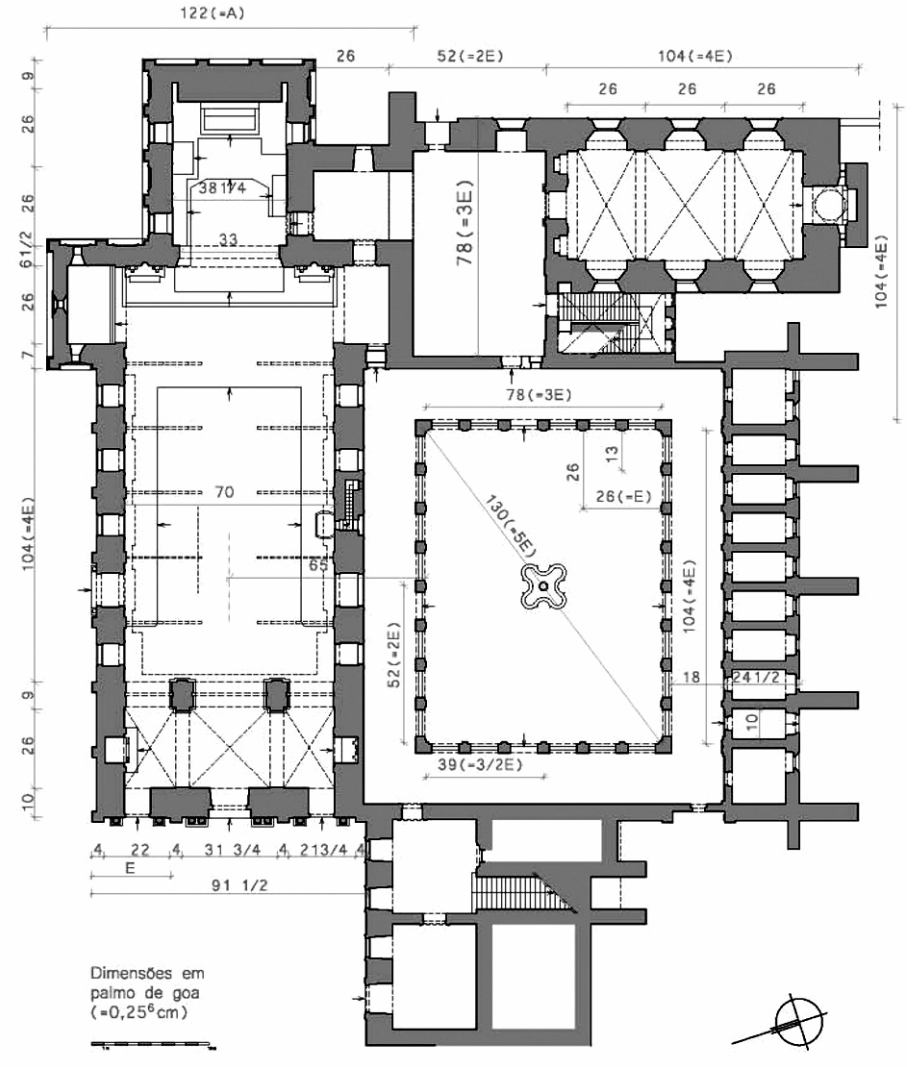

Fig. 12. Plan of the entire complex, Church of Bom Jesus. Dimensions in goa palms

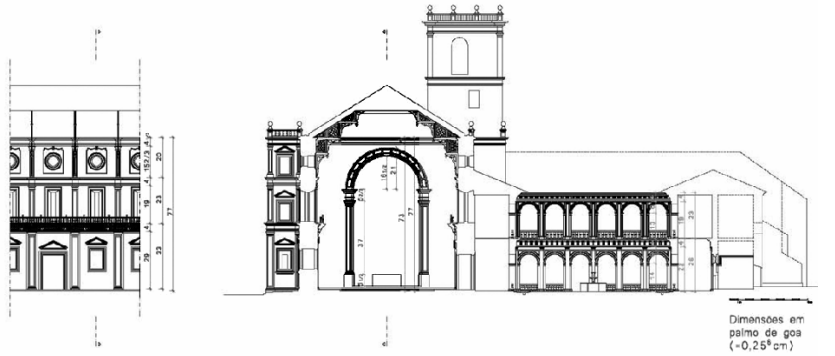

Fig. 13. Interior elevation and section, Church of Bom Jesus. Dimensions in goa palms

Documental evidence indicates that the Bom Jesus's architect was the Jesuit Domingos Fernandes [Wicki and Gomes 1948-1988: XVI, 934], who had most probably already worked with Alessando Valignano in the Professed House [Pereira 2005: 228]. It appears that Domingos Fernandes did not have any special training as an architect, having learned his profession empirically on building sites. Again documental evidence suggests that Fernandes did not design the façade, as it was eventually carried on after 1597 [Wicki and Gomes 1948-1988: XVIII, 808-809]. Its designer, as well as the designers of the seventeenth-century alterations to the Bom Jesus church and sacristy, are unknown.

Architectural proportions in the Church Bom Jesus

As the church Bom Jesus has not a cruciform plan, the approach taken to the proportions analysis was completely different from that of the Cathedral. The starting points were the church's original major width, at the level of the side spaces by the choir, and the form and proportion of the courtyard (fig. 16). Of the two, the church's original major width is no longer evident because of the late 1650s amplifications of the lateral chapel-like spaces by the choir.[6] But the original width is now easily recognizable on the north side, which stands free as it sticks out of the main building. The plaster removal during the 1950s restoration revealed the different stone quality of the seventeenth-century additions, which is more porous and light-coloured than the original laterite stone. This original major width was denominated A and is roughly 122 goa palms.[7] The church's length – excluding the façade's thickness – corresponds to 2A, whereas the choir is A/2 deep. Apparently this simple square construction does not have any further major significance in the plan. But we find it again in the façade (fig. 14). The façade's width corresponds to ¾A and the distances between the portals' axes are ¼A. The ¾A side square inscribing the three main storeys reaches a little above the bases of the spheres' pedestals on both sides of the fourth storey's single bay. A circle inscribing this façade's square – with radius *r* – touches the upper corner of top pediment. The *ad quadratum* progression shows some interesting results: the three inner squares touch some of the façade's structural elements like pilasters and pedestals, which may or not be coincidences. But it is even more interesting to inscribe two rectangles with the proportion 4:3 in the great façade circle (radius *r*), one standing and one lying (fig. 15). The standing rectangle encloses the outer pedestals and columns of the ground floor, whereas the horizontal one touches the top of the spheres over the outer façade pilasters.

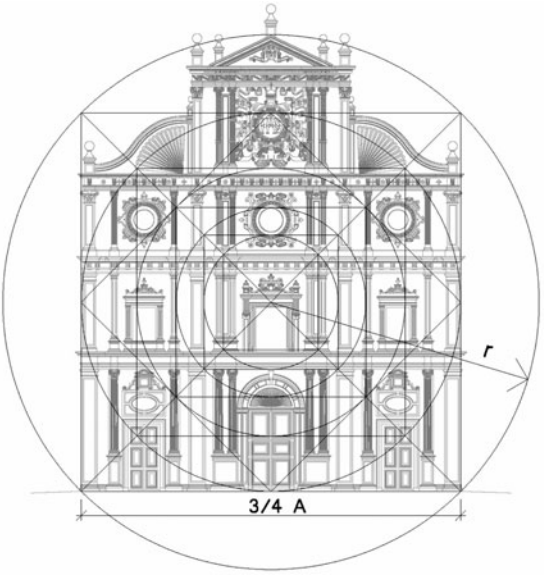

Fig. 14. Proportional analysis of the façade based on module A and radius *r,*
Church of Bom Jesus

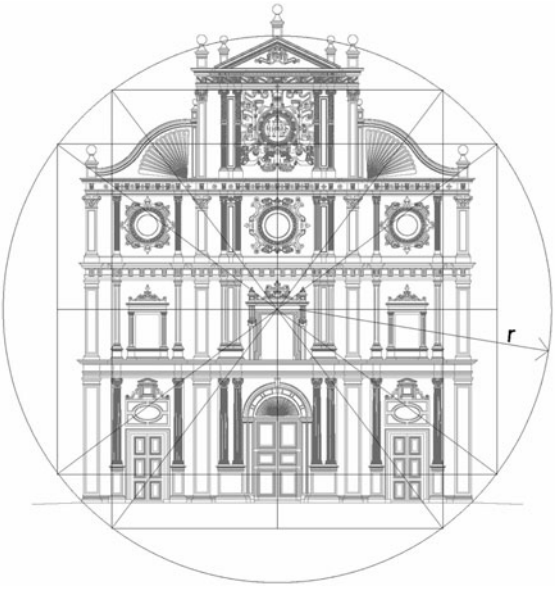

Fig. 15. Proportional analysis of the façade based on radius *r*,
Church of Bom Jesus

But the most astonishing feature is that both coincide in proportion and in dimension with the inner perimeter of the church's courtyard, as seen in plan. So, unlike the Cathedral, it is not the *ad quadratum* progression that causes the proportions of façade and plan to coincide, but a 4:3 rectangle. This courtyard with galleries in two levels has eight arches in each of the longest and six in each of the shortest sides. If we

consider a module of two arches (module E; fig. 19) we get the 4:3 proportion and a courtyard diagonal corresponding to 5 modules. Half of this diagonal is roughly *r* – roughly, because in this case dimensions do not completely coincide. Taken from the cloister, *r* is a commensurable dimension (fig. 17). The arch width – measured from axis to axis of the pilasters – is ca. 3.355 m, i.e., 13 goa palms. A two-arch module is therefore 26 goa palms, which means that the cloister diagonal is 130 goa palms, the half of it corresponding to *r* (65 goa palms). But taken from the façade *r* is an incommensurable dimension (fig. 14). If the main circle inscribes a ¾ A square, *r* is (√2 x ¾ A) : 2, i.e., (√2 x 91½ goa palms) : 2, which is 64.700265… m, a little less than 64¾ goa palms.

Setting the centre of a circle with radius *r* in the north-east corner of the Bom Jesus church courtyard inclosing a 4:3 rectangle – equal to the perimeter of the inner courtyard– the rectangle's right side touches the front wall of the staircase and of the sacristy at the southeast corner of the complex (fig. 17).

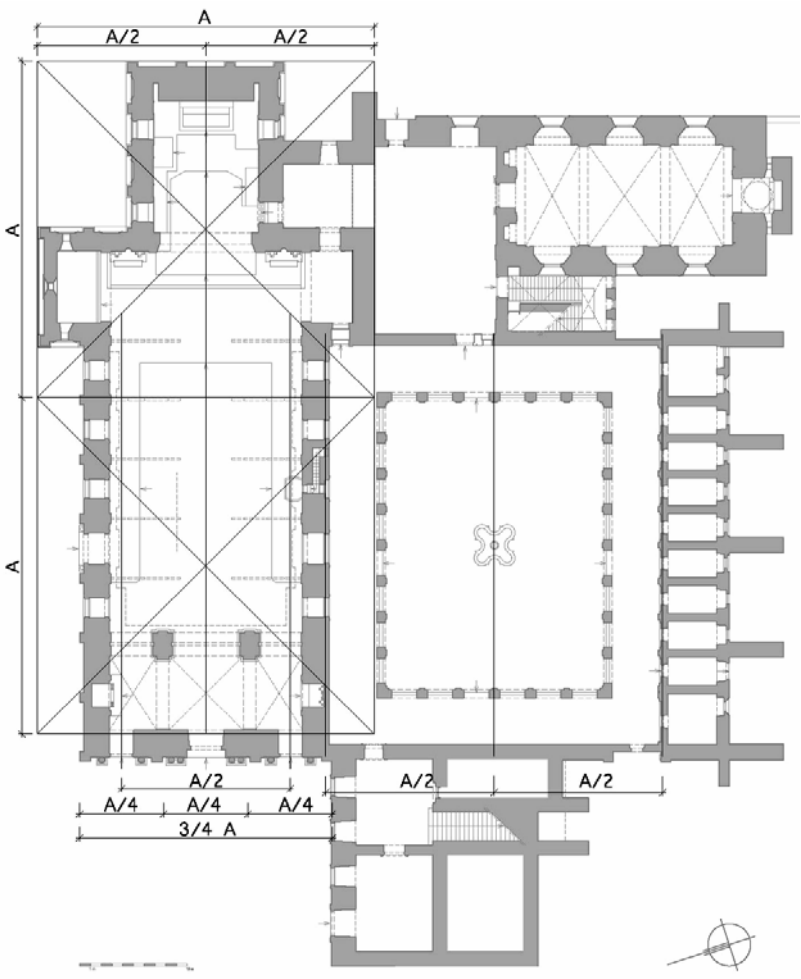

Fig. 16. Proportional analysis of the plan of the complex of the Church of Bom Jesus based on module A

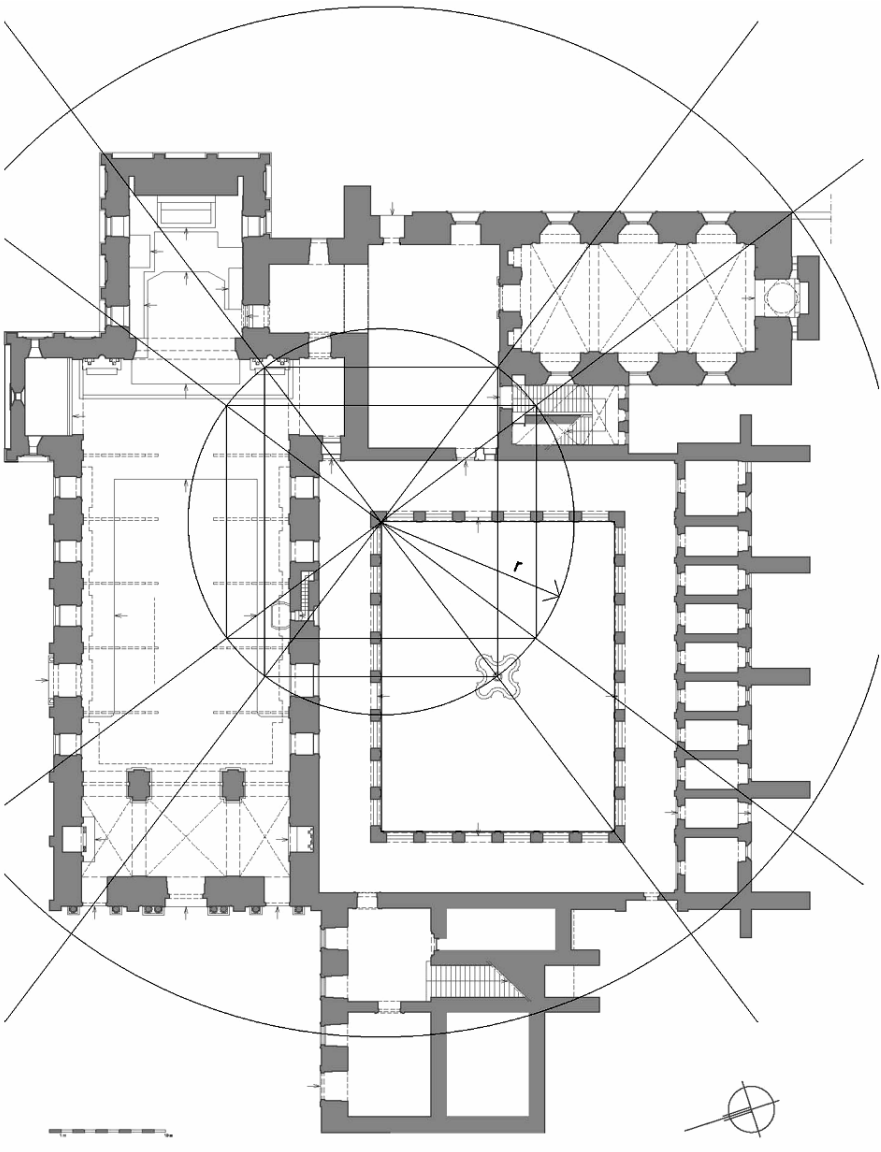

Fig. 17. Proportional analysis of the plan of the complex of the Church of Bom Jesus based on radius *r*

The diagonal of a similar, but transversal 4:3 rectangle coincides with the sacristy's southeast corner. A great circle concentric with the first one and inclosing this point also touches the corner of the Professed House near its staircase and gets very close to the façade's northwest corner. But it is in the case of the sacristy that we find the most interesting coincidences (fig. 18). The very same longitudinal, inner courtyard perimeter defines the sacristy length. Again, a similar but transversal 4:3 rectangle encloses the sacristy and the wall section up to the window opening of the anteroom, but excluding the sacristy's altar volume.

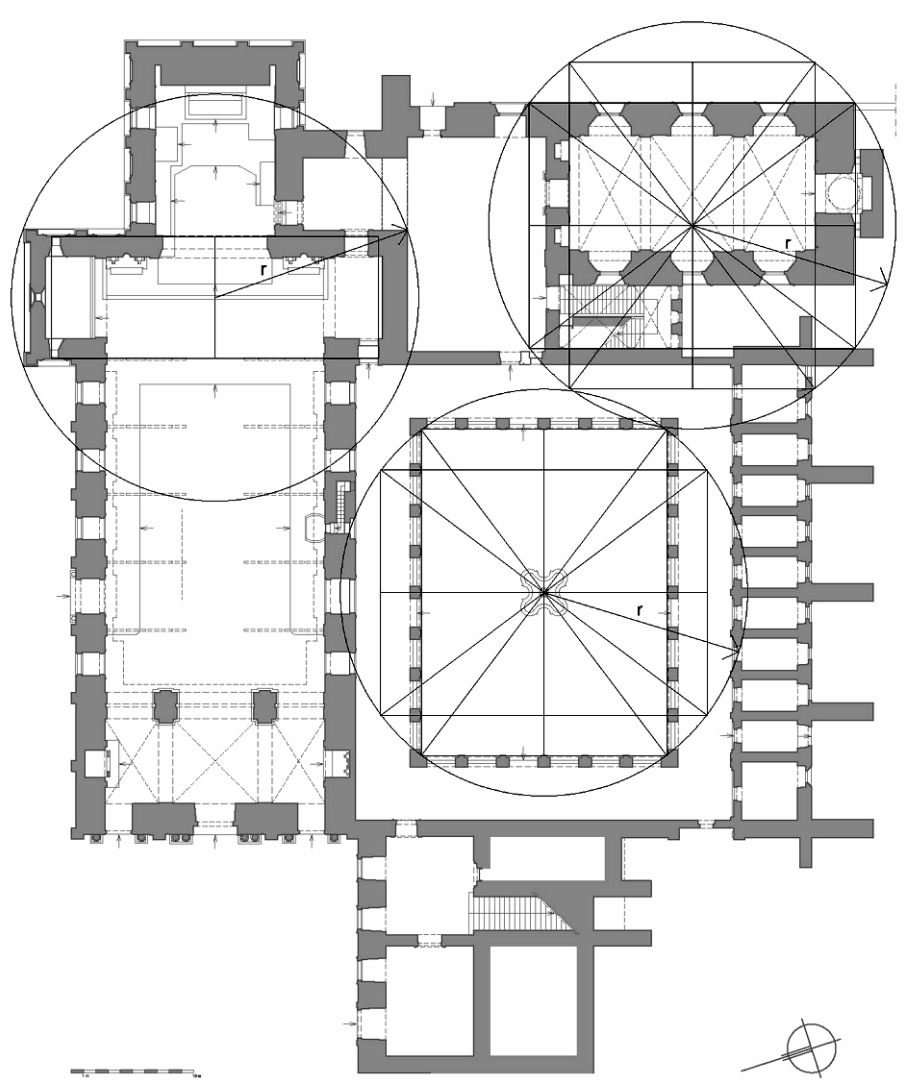

Fig. 18. Proportional analysis of the plan of the complex of the Church of Bom Jesus based on radius *r*

The similarity between the proportional systems of the church and the sacristy proves that the church's proportional system of 1594 was still known around the middle of the following century. It also shows that it was considered important enough to be re-utilized in the construction of the new sacristy. The following drawings show how module E (the unit of the 4:3 rectangle) is present in the entire complex of church, courtyard and sacristy. Spaces like the false transept arms, the choir, the church's nave or the space underneath the tower (between choir and anteroom to the sacristy) were dimensioned according to the module E (fig. 19).

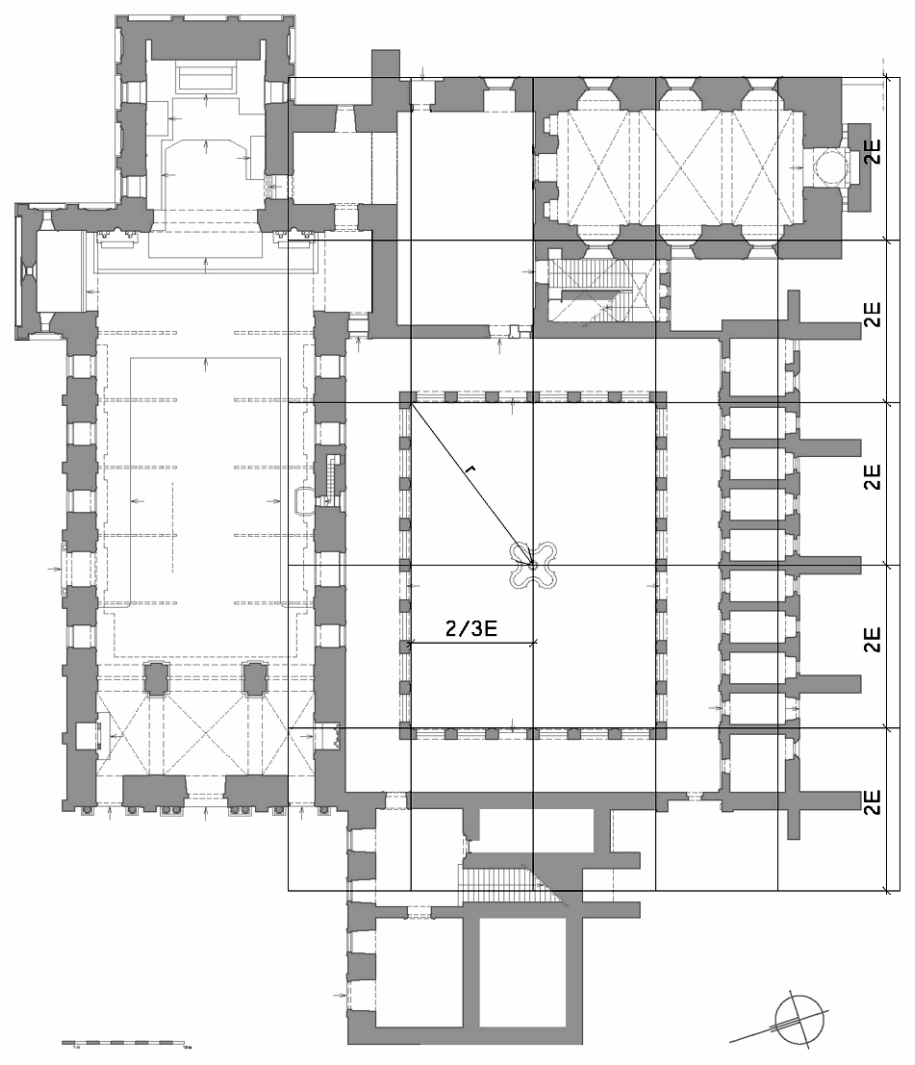

Fig. 19. Proportional analysis of the plan of the complex of the Church of Bom Jesus based on module E

Conclusion: Cathedral versus Bom Jesus

The Cathedral of Goa and the Jesuit church Bom Jesus show two very different proportional systems: the "*ad quadratum*" progression and the 4:3 rectangle. The reason for each respective system must be looked for in the two churches' different status. Nevertheless, the geometric divergence of both systems, the final symbolic significance is the same: the union between Heaven and Earth. In the Cathedral this symbolism has a political overtone, whereas in Bom Jesus it remains mainly religious.

The Goa Cathedral's proportions, mainly based in the Roman *ad quadratum* progression, should be understand as a direct consequence of its double function as a religious and state or royal building. The Cathedral was the principal church in the

Catholic Orient and home of the archbishop and Patriarch of the Orient, but it also was a royal building and therefore an expression of secular, imperial power. The building expresses it most clearly: in the façade's axis there is not only the papal Triple Tiara with Saint Peter's two crossed keys depicted over the main portal, but also the crown of King Sebastião over the central window on the first storey. These two crowns so close to each other in the same building illustrate the entanglement of politics and religion in the process of legitimising the Orient's conquest by Christianity. It is thus not surprising that the chosen proportional system was meant as an intentionally and unequivocal reference to the former Roman Empire the Portuguese were adopting as a model for their colonial expansion in the Orient. With the *ad quadratum* progression the Cathedral's designer achieved the symbolic union between Heaven and Earth as conveyed by a church and simultaneously designed its building form close to the architectural culture of the *romanitas*. Thus, in the Goa Cathedral the ideal of the "Renaissance" as an artistic and political revival of the Roman Empire is materialized to legitimise colonial domination and Christian religious expansion.

In contrast to the Cathedral, the proportional system of Bom Jesus is based on the 4:3 rectangle. Although corresponding to the musical interval of a fourth, I do not believe that this proportion is the result of a "musical" architectural design. The 4:3 rectangle with its diagonal is not only the simplest example of the Pythagorean triangle (3:4:5), but also a rectangle dimensioned on the numbers 3 and 4, whose symbolic meaning in a Catholic missionary context is evident. The number 3 appears for the Divine, not only as a representation of perfection, but also as a symbol of the Holy Trinity [LCI 2004: 1, 524-525]. The number 4 symbolizes the world, referring to the four elements, the four cardinal directions, the four continents[8] and the four seasons of the year[9] [LCI 2004: 4, 459]. Number 4 also has a meaning in the Christian moral, related to the four cardinal virtues, which should conduct the Christian life on Earth – and again the world is symbolised by this concept. The multiplication of 3 by 4 results in 12, a number which has much symbolic significance in many religions and particularly in Christianity [LCI 2004: 4, 582-583]. Its most prominent aspect of interest to Bom Jesus is the meaning of accomplishment of God's Kingdom, which settled on with the announcement of God's word (3) on Earth (4) by the Apostles (12). 12 is therefore a very suitable number for a missionary church as the Bom Jesus. Moreover, we should recall that even before Francis Xavier was canonized in 1622 and his body transferred from old São Paulo to this church in 1624, he was already revered by the Jesuits, who called him the Apostle of India. Those who lived in the Professed House, to which the church Bom Jesus belonged, were following Francis Xavier's steps, that is, they were carrying on the work of the Apostles in a part of the world that the Apostles themselves had never reached (with the exception of Saint Thomas, as was commonly believed amongst the European missionaries). The proportional system of their house church based on the 4:3 rectangle states as much. The missionary aspect is hereby emphasized in opposition to the legitimising of power through the connection between Crown and Church, as it was the case of the Cathedral.

Acknowledgments

The present paper consists in the presentation of research results of a project carried at the Portuguese research centre UNIDCOM / IADE and financed by the Fundação para a Ciência e Tecnologia, Lisbon: "Architecture and Mathematics: proportions systems in two churches of Old Goa of the 16th century."

Notes

1. Like any historical building the Cathedral of Goa is irregular, although the irregularities are very slight for a building of its dimensions. For instances, the crossing's width varies between 11.62 m and 11.67 m, a difference that can be ignored for our purpose.
2. The historical measure unit is the "goa palm", the name of which is not related to the Indian Territory. It is a naval unit of measure of French origin named *goue* that the Portuguese turned into *goa*. It corresponds to 0.256 m [Barata 1965: 155-157; Barata 1989: 191-192].
3. Only the arches' height varies, as those of the side chapels are lower than the central ones, corresponding to the one-storey height of the side chapels.
4. Double pilasters are not a very common feature in Goan Renaissance architecture. They do not appear in Goa before the church of the Carmelite Convent was begun in 1612, which was built by an Italian congregation. Thus, it is very probable that the double corner pilasters at the Cathedral's façade are a later modification of the original project of 1564. Therefore, only the inner row of pilasters is considered in the present analysis.
5. The body of Francis Xavier (1506-1552) was brought two years after his death on Shangchuan Island to São Paulo, the church of the Jesuit College outside Old Goa. After his canonization in 1622 the body was brought to the much more central church of Bom Jesus, where it has rested ever since.
6. This major width is still visible on top of the chapel-like side volumes, as the seventeenth-century amplifications have one fewer storey.
7. Although the same letters are used to nominate geometrical figures in the analysis of both the Cathedral and the Bom Jesus, they are not related with each other.
8. Excluding Australia, which was not officially discovered until 1606, twelve years after the beginning of the Bom Jesus!
9. Ironically, the climate in Goa cannot be divided in four seasons, like those of the European climate. But interestingly enough, in the correspondence of both state clerks and clerics, the terms Summer and Winter are used to distinguish between the Indian dry and rainy seasons.

References

ALBERTI, Leon Battista. 1988. *On the Art of Building in Ten Books*. Joseph Rykwert, Neil Leach and Robert Tavernor, trans. Cambridge, MA: MIT Press.

BARATA, João da Gama Pimentel. 1965. *O «Livro Primeiro da Architectura Naval» de João Baptista Lavanha*. Lisbon: Instituto Português de Arqueologia História e Etnografia.

———. 1989. *Estudos de Arqueologia Naval*. Vol. II. Lisboa: Imprensa Nacional-Casa da Moeda.

BÖCKMANN, Barbara. 2004. *Zahl, Mass und Massbeziehung in Leon Battista Albertis Kirche San Sebastiano zu Mantua*. Hildesheim, Zürich, New York: Georg Olms Verlag.

CALTER, Paul A. 2008. *Squaring the Circle: Geometry in Art and Architecture*. Hoboken: Wiley.

CHICÓ, Mário Tavares. 1956. "Algumas Observações acerca da Arquitectura da Companhia de Jesus no Distrito de Goa". *Garcia de Orta*, número especial, 257-271.

DESWARTE, Sylvie. 1981. "Francisco de Hollanda et les Études Vitruviennes en Italie". In *A Introdução da Arte da Renascença na Península Ibérica*. Coimbra: Epartur. 227-270.

FELLNER, Stefan. 1993. Numerus Sonorus. Musikalische Proportionen und Zahlenästhetik in der Architektur der Jesuitenmissionen Paraguays am Beispiel der Chiquitos-Kirchen des P. Schmid SJ (1694-1772). Diss. Technische Universität Berlin, Berlin.

FRECKMANN, Karl. 1965. *Proportionen in der Architecktur*. München: Verlag Georg D. W. Callwey.

JUNECKE, Hans. 1982. *Die Wohlbemessene Ordnung. Pythagoreische Proportionen in der Historischen Architektur*. Berlin: Verlag der Beeken.

KRUFT, Hanno-Walter. 1995. *Geschichte der Architekturtheorie. Von der Antike bis zur Gegenwart*. 4th ed. Munich: Verlag C. H. Beck.

LEXICON DER CHRISTLICHEN IKONOGRAPHIE (LCI). 2004. Special edition. 8 vols. Rome, Freiburg, Basel, Vienna: Herder.

MOREIRA, Rafael. 1981. "A Arquitectura Militar do Renascimento em Portugal". Pp. 281-305 in *A Introdução da Arte da Renascença na Península Ibérica*. Coimbra: Epartur.
———. 1992. O engenheiro-mór e a circulação das formas no Império Português. Pp. 97-107 in *Portugal e a Flandres. Visões da Europa 1550-1680*. Lisboa: IPPC.
———. 1988. Inofre de Carvalho, a Renaissance Architect in the Gulf. Pp. 85-93 in *Bahrain in the 16th Century. An Impregnable Island*, Monik Kervran, ed. Bahrain.
———. 1993. A Arquitectura Militar. Pp. 137-151 in *História da Arte em Portugal*. Lisbon: Publicações Alfa.
NAREDI-RAINER, Paul von. 1995. *Architektur & Harmonie, Zahl, Maß und Proportion in der abendländischen Baukunst*. 5th ed. Cologne: DuMont.
PEREIRA, António Nunes. 2003. *Die Kirchenbauten in Alt-Goa in der zweiten Hälfte des 16. und in den ersten Jahrzehnten des 17. Jahrhunderts: Zur Entstehung eines Sakralbautypus*. PhD Diss., RWTA Aachen.
———. 2005. *A Arquitectura Religiosa Cristã de Velha Goa. Segunda Metade do Século XVI – Primeiras Décadas do Século XVII*. Orientalia 10. Lisboa: Fundação Oriente.
SERLIO, Sebastiano. 1996. *Sebastiano Serlio on Architecture*, vol. I. Translated from the Italian with an Introduction and Commentary by Vaughan Hart and Peter Hicks. New Haven and London: Yale University Press.
———. 2001. *Sebastiano Serlio on Architecture*, vol. II. Translated from the Italian with an Introduction and Commentary by Vaughan Hart and Peter Hicks. New Haven and London: Yale University Press.
TAVERNOR, Robert. 1998. *On Alberti and the Art of Building*. New Haven, London: Yale University Press.
WICKI, Josef, and John GOMES, eds. 1948-1988. *Documenta Indica*. 18 vols. Roma: Institutum Historicum Societatis Iesu.
WILSON JONES, Mark. 2000. *Principles of Roman Architecture*. New Haven, London: Yale University Press.

About the author

António Nunes Pereira was trained as an architect at the Technical University of Lisbon. He earned his Ph.D. at the Aachen University of Technology, Germany, where he lived for twelve years. His thesis was about the introduction of the European Renaissance in the former Portuguese colony Goa, India, during the sixteenth century, and the development of a specific Goan church architecture. He worked as an assistant to Prof. Dr.-Ing. Hartwig Schmidt at the Department for Conservation of Built Heritage at Aachen University. He also cooperated with several architectural offices in Portugal and Germany. Back in Lisbon since 2003, he has carried on with his research about Renaissance architecture in Goa. With the financial support of the FCT / Lisbon, he has recently concluded a research project about the architectural geometry and proportions of two churches in Goa. In addition to these issues, his research interests include conservation of built heritage, architectural and design theory of the late nineteenth and early twentieth centuries, as well as stage setting in musical dramas. He is currently Associate Professor at the Design University IADE, Lisbon, researcher at UNIDCOM / IADE and, since October 2010, the Director of the Pena Palace in Sintra.