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Keywords: Chichén-Itzá, Kukulcán Pyramid, light and shadow effect, Mayan geometry, perspective

Research

The Sunlight Effect of the Kukulcán Pyramid or The History of a Line

Abstract. When the sunlight bathes the Kukulcán Pyramid in the Mayan city of Chichén-Itzá during the equinox sunset, it casts seven triangles of light and shadow that creep downwards along its northeast stairway. According to the *Popol Vuh*, the effect can be interpreted as the myth of the gods of the Heart of Sky coming to the Sovereign Plumed Serpent. Unfortunately, neither the event nor the kind of geometry used to build the pyramid is reported in the extant Mayan codices. There are two reliable facts: first, a line across the pyramid's base coincides with the orientation of the summer-winter solstice; second, an earlier, smaller pyramid is concealed beneath the current one. Hence the major question here is whether the light and shadow effect was intended or occurs accidentally. Perspective as a surveying method for building provides a key to this riddle, because it accounts for a line according to which the Kukulcán Pyramid was built. To tell the history of this line, we have to bring into context other sunlight effects that take place across the Mayan area, taking into account the question of how the buildings in which such effects occur were built.

Historical context of Chichén-Itzá

In the *Chilam Balam* books [2005], according to the Matichu Chronicle, Part II, Chichén-Itzá was discovered[1] during the Katun 6 Ahau (435-455). In 13 Ahau (495-514), the *esteras* (communities) were organized and Chichén-Itzá was occupied. The Itzáes reigned over Chichén-Itzá for two hundred years, but it was abandoned in 8 Ahau (672-692). The Itzáes went to Chakanputún (Champotón), from Katun 6 Ahau (692-711) until 8 Ahau (928-948), when it too was abandoned. Then, they went astray for 40 years (948-987) until they returned to Chichén Itzá. In Part III of the Matichu Chronicle, the account of disputes, wars, and betrayals among the governors of Uxmal, Mayapán, Itzmal, and Chichén-Itzá are briefly described.

Due to these events, in 8 Ahau (1185-1204), the governor of Chichén-Itzá (of the Itzáes) abandoned "their" homes once again. Although the correlation of events is quite confusing in Part III, Chichén-Itzá was probably abandoned in 6 Ahau (1224-1244). Thereafter, in 11 Ahau (1283-1303), the land of the Ichpá-Mayapán was taken by the "men of the city outside the wells" (the Itzáes) and King Ulmil. Several other dates are mentioned until 8 Ahau (1441-1461), when Ichpá-Mayapán was destroyed and definitively abandoned.

A copy of the so-called Maní manuscripts of the *Chilam Balam* was handed to John Lloyd Stephens by Juan Pío Pérez when they met in a small town called Peto. Stephens and Frederick Catherwood arrived in Chichén-Itzá on March 13, 1842 at 4:30 p.m. and left on March 29. In Stephens' own words: "In half an hour we were among the ruins of

DOI 10.1007/s00004-010-0019-3; *published online* 10 March 2010

this ancient city [they departed from Piste at four o'clock], with all the great buildings in full view, casting prodigious shadows over the plain…" [2000: 326]. When the sunlight bathes the Kukulcán Pyramid (fig. 1) in the Mayan city of Chichén-Itzá during the equinox sunset, it casts seven triangles of light and shadow that creep downwards along its northeast stairway. Ironically, they were there at the right time and season to witness the light and shadow effect on the northeast stairway, but they did not see it because the pyramid was largely concealed by the forest. For sixteen days, Stephens explored the ruins while Catherwood drew views of the buildings with the aid of both his camera lucida [2000: 40-41] and daguerreotype apparatus; the latter was destroyed when the horse carrying it ran away during the journey from Chichén-Itzá to Valladolid.

Fig. 1. A charming little Mayan girl passing by the pyramid enhances its majestic presence. Photograph by the author

Thanks to these instruments, Catherwood's drawings depict many buildings in perspective, stone by stone down to the smallest detail. In one of these drawings, the serpent heads resting at the foot of the stairway of the Kukulcán Pyramid appear quite different from how they look today, an oddity that we will discuss later. Catherwood also drew a plan of the site with the aid of a compass and a line, noticing that none of the buildings coincided with the cardinal points. His plan indicates the road leading to Valladolid passing by the pyramid, and the nearby Hacienda Chichén where he and Stephens stayed (and where my wife and myself stayed in December 2007), just a ten-minute walk from the ruins. The ruins belonged to the Hacienda; in 1894 they were purchased by Edward Thompson, who dredged a *cenote* (a deep natural well) and smuggled many artefacts out of the country.[2] Later, the staff of Carnegie Institution of Washington, led by Sylvanus G. Morley, stayed at the Hacienda Chichén while, over the course of ten years (1924-1934), they explored the ruins, carrying out extensive

excavations and mapping the site. Thus, the restoration works began in cooperation with the Mexican government, which in particular undertook the restoration of the Kukulcán Pyramid and the Juego de Pelota (the ball court). This was during the same years when the junior Maya archaeologist Eric S. Thomson wrote his contemptuous opinion about the Observatory, making evident that he was not as good a "connoisseur" of architecture as he said his father was of wine and good food.[3]

Besides sunlight effects, acoustic phenomena also occur in several Mayan cities. In Chichén-Itzá, for instance, standing on the great plaza, around 30 m. away from the center of the pyramid's north stairway, two Quetzal chirps followed by an echo can clearly be heard when you clap your hands once (the Quetzal is the famous long-tailed bird typical of Yucatán). Specialists in acoustics have investigated this effect and believe that the Quetzal's echo could be a sort of incident sound [Declercq et al. 2004]. Another beguiling effect can be experienced when you stand midway in the Juego de Pelota, between its two immense walls, while another person at the end of the field can hear the words you whisper.

Other sunlight effects and building alignments

I would like to bring into context other Mayan cities where significant sunlight effects also take place, to emphasize that those of Chichén-Itzá are not unique. Many buildings across the Mayan area seems to have a line oriented to the summer-winter solstice.

Fig. 2. a, top left) The House of the Seven Dolls in Dzibilchaltun; b, top right) A view of the alignment of the stelas; c, bottom) Schematic plan of the Sacbé 1. Photographs by the author, drawing by Ambar Hernádez

In Dzibilchaltun, 16 km. north of Mérida, capital of the State of Yucatán, when at sunrise, the sun's rays pass through the central door and windows of the House of the Seven Dolls (HSD) during the equinox, an effect is produced resembling a lighthouse

(fig. 2a). The HSD is at the extreme east of the sacbé 1 (white road 1), which is oriented 94 az-274 az (east-west),[4] and measures 17 m. wide by 400 m. long. As the sacbé axis and that of the HSD are parallel but not collinear, the sunlight also falls alongside the sacbé, thus separating the path of the men from that of the gods (fig. 2b).

The lighthouse effect also takes place during the solstice sunrise, although less intensely, and depending on the solstice season only one of the windows is illuminated. In turn, on the night of the first full moon following the solstice, moonlight through the central door of the HSD illuminates the sacbé. On the left side of the sacbé's extreme west, there are three stelas perfectly aligned at 190 az (south), coincidentally with the same orientation as the HSD's southern upper window (fig. 2c).

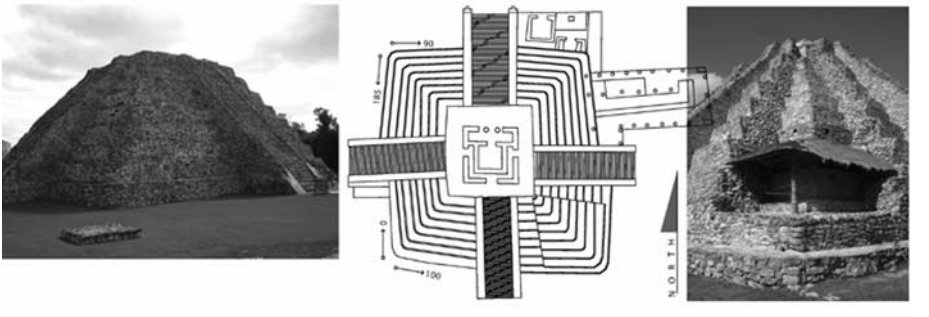

Fig. 3. a, left) The Kukulcán Pyramid in Mayapán; b, center) Notice the lack of symmetry of the plan; c, right) An older substructure is exposed on the southeast corner of the pyramid. Photographs by the author, drawing by Ambar Hernández

In Mayapán, built during the Post-Classical Period (1250-1450 AD), after the fall of Chichén-Itzá, there is another pyramid called Kukulcán, with a light and shadow effect similar to that of Chichén, although it takes place during the winter solstice when the sunlight bathes its southern stairway. This effect is barely perceptible due to the misalignment of the stairways, the inaccurate angles of the pyramid's base (which are off by as much as 10°; its southwest angle points 0-100 az), and the platforms' divergence (figs. 3a and 3b). What we learn from this is that a stepped pyramid produces the effect because of its form; in fact, latitude and orientation can vary while the effect is still produced. Archaeological exploration on the southeast corner of the pyramid shows an older substructure, as in Chichén-Itzá, although here no vault beneath the stairways was left (fig. 3c). Like Chichén-Itzá, Mayapán also had an Observatory, a circular structure whose upper part is missing; it was elevated from the ground to gain view from its four doors, of which the west door is oriented (275 az) toward the northern third platform of the pyramid. It is said that Mayapán is Chichén-Itzá in miniature, but Mayapán was built in a rough way since one can hardly find two parallel lines there.

The architecture of Uxmal, 78 km. south of Mérida, is the most graceful of the Mayan world.

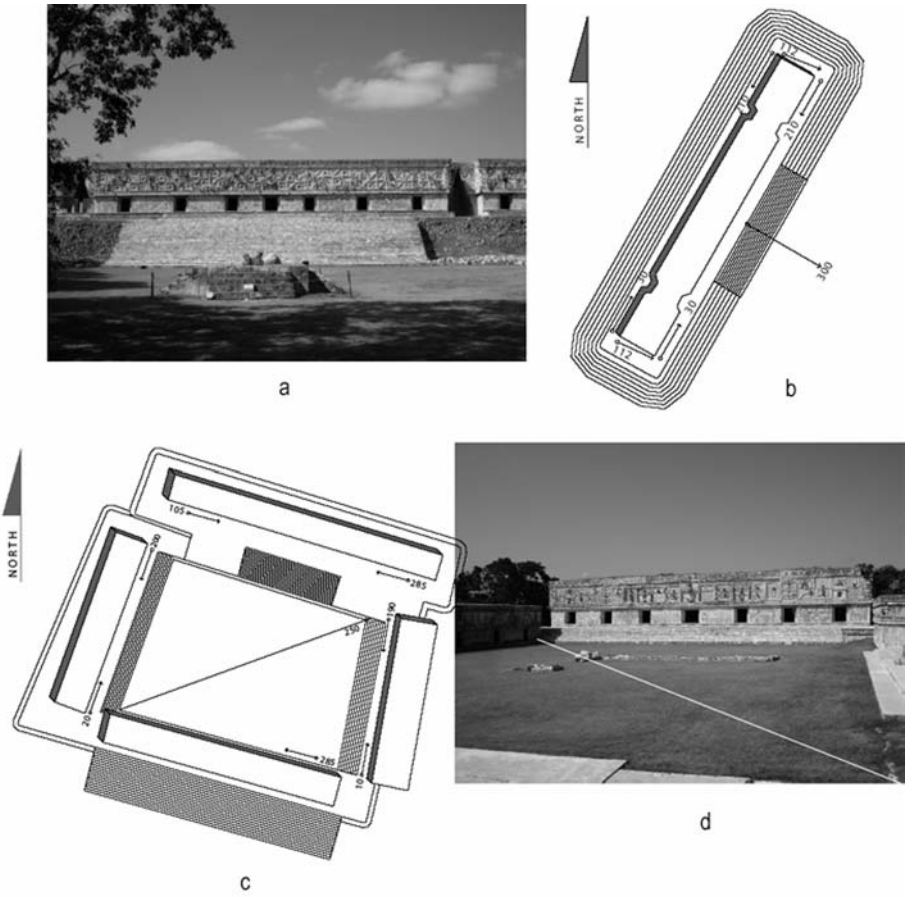

Fig. 4. a, top left) The Governor's palace in Uxmal viewed from the plaza. A broken stone-pole remains on the floor of the small platform at the midst of the plaza, facing the palace; b, top right) Plan of the palace; c, bottom left) Plan of the Nunnery Quadrangle courtyard with its approximated solstice line. d, bottom right) Standing on the courtyard, you cannot tell whether the buildings conform a perfect rectangle or not. Photographs by the author, drawings by Ambar Hernández

The two longest façades of the Governor's Palace (figs. 4a-b) run perfectly parallel to one another, while the two shortest ones deviate 2° from one another. The Nunnery Quadrangle (figs. 4c-d) does not form a perfect rectangle since its shortest sides deviate by 10°. As I expected, the orientation of a diagonal line across the courtyard is very nearly aligned with that of the solstice (70 az-250 az), and can be made equal to that of the base of the pyramid at Chichén-Itzá by slightly moving the farthest point of reference. This line is highly significant, as we will see later. The great pyramid of Uxmal also has nine platforms, still largely covered by the forest (evoking Catherwood's drawings), and despite the fact that it has a orientation similar to that of Kukulcán, the effect is not perceptible on its northern stairway due to its uneven contour.

Fig. 5. a, left) The Five-Storied Building viewed from the plaza. In front of it appears the platform where the missing pole stood; b, center) A view of the rounded eastern platforms. c, right) A view of the fly stairs. Photographs by the author

I expected to see the famous pole (or gnomon) of Edzná, 61 km. southeast of the city of Campeche, standing on its platform base (fig. 5a)and facing the Five-Storied Building (5-SB), but it was inexplicably removed.[5] Most scholars agree that the pole was used to calibrate the Mayan Calendar Long Count, when the zenith passage of noonday would fall on its surmounted capital-like top, casting a complete shadow on its body. Looking from the Nohochná stairs, the sun rises at both sides of the 5-SB, left and right, during the summer and winter solstices, respectively. In turn, the stela placed at the rear of the temple's second room (atop the 5-SB) is illuminated during the equinox sunset, a phenomenon that is still perceptible even though the vault of the first room has collapsed. Edzná overcame the lack of natural deposits of water by inventing the so-called *aguadas* system (washes). Water was collected from everywhere, even from the eastern side of the 5-SB, whose platform surfaces were rounded for this purpose (fig. 5b). This was an early manifestation of the architectural gospel, "Form follows function." Likewise, the innovative ramped stairs connecting the fifty rooms of the 5-SB suggest that the Maya were transforming the massive pyramids into habitable buildings (fig. 5c).

The constructive geometry of the Kukulcán pyramid

The extant Maya codices[6] contain calendar accounts, astronomical observations, social precepts, and political events, but no references to geometry. Unfortunately, we will never know if any texts on geometry had once existed, like those collected in the *Chilam Balam* for medicine.[7] As is well known, Diego de Landa ignominiously burned many Mayan writings after the Franciscan friars landed in the Yucatán Peninsula [de Landa 1986], an event that took place in Maní on July 12, 1562.[8] In spite of this shameful event, there are many unexplored places across the peninsula and, therefore, the birth certificate of our line could still be hidden somewhere. Meanwhile, only the buildings themselves can tell us what geometry lay behind their conception.

We have rounded off the measurements of the Kukulcán Pyramid because it is not perfectly symmetrical; all its sides and slopes deviate a little everywhere (fig. 6). The pyramid is comprised of nine platforms, each about 2.57 m. high (the actual height of the first one). Atop the last platform is a two-story temple 6 m. high, and thus the total height of the pyramid is 30 m. Each side of the pyramid's base is about 55.3 m. The average inclination of the platforms is 53°. The individual walls, or taluds,[9] incline by 72°-74°, and the slope of the stairways is 45°. Obviously, we cannot translate our metric measurements and degrees to the Mayan system because this is unknown. Yet we can wonder if some measurements were encoded in the pyramid, since numbers are everywhere. For instance, the number of steps of the pyramid totalled 365, corresponding to the number of days in a year,[10] while in turn, the 52 ornamental stone

boards (all around the pyramid) coincide with the Mayan Calendar Round.[11] An architectural stone calendar seems to have ruled the program for the current pyramid.

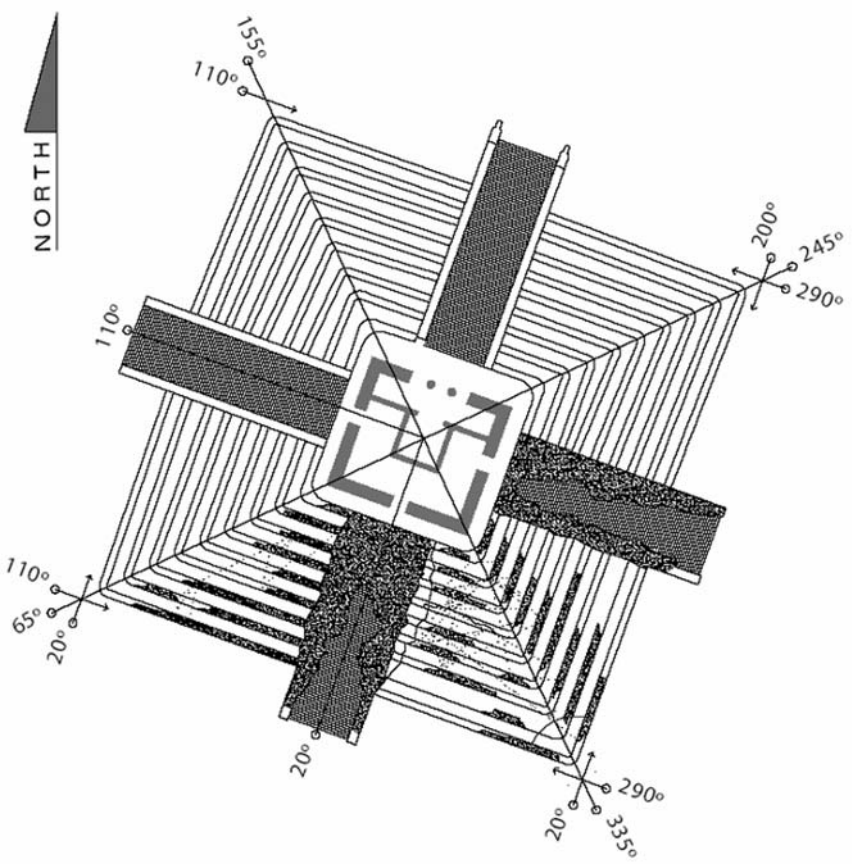

Fig. 6. Plan of the pyramid of Kukulcán in Chichen-Itzá. Drawing by Ambar Hernández based on the author's measurements

The sidewalks around the pyramid suggest that they were used to supply and handle all the materials needed to build the platforms progressively. During this process, low scaffolds were required by the stonemasons to give the platforms their typical talud form. In turn, as the size of the step's tread and riser is equal, the resultant slope of the stairways is 45°, which is unlikely to be calculated by an angular scale. If such a scale were used, why then do the platforms not show the same perfection of slope as the stairways? Standing atop the pyramid and looking downwards, it is evident that its four rounded corners do not align correctly, and that none the borderlines of the sidewalks line up tangentially either. This last phenomenon is a notorious mismatch since the borderlines do not recede (from the top to the base) accordingly in perspective (figs. 7a-b). Looking at the temple from the great plaza, it is quite intriguing to observe how its outer walls appear vertical when they actually slope outwards as they rise. Could this be an intentional visual refinement? It might be, or at least it suggests the use of perspective as a surveying method for building, in a way similar to what the Greeks did in the Parthenon.

It is widely believed that visual refinements were made in ancient cultures to keep the form of a building consistent.

Fig. 7. a, left) Disregarding the natural distortion of the camera lens, it is evident how the platforms' borderlines do not run parallel, and do not recede accordingly in perspective as well; b, right) The same is true in this other view. Photographs by the author

Fig. 8. a, left) The stairway of the older pyramid vaulted by a Mayan vault; b, center) At the entrance of the upper temple, a Chac-mool seems to be wondering, "Who dares to come here", as it protects the jaguar throne (at the rear) from unwanted visitors; c, right) A view of the stairway's right-hand balustrade without a serpent head at its foot. Photographs by the author

As we have mentioned, an older pyramid concealed beneath the current one, whose entrance is located to the northwest side of its stairway, was discovered during the 1930s. Literally, we have to go inside the present-day pyramid to find the main stairway of the older one. The "interior" stairway is roofed by a Mayan vault that leads to an upper sacred chamber, which remains intact just as it was originally found (figs. 8a-b). There, the silence of the past is a breathtaking experience. Even though the older pyramid has no serpent heads at its foot, the orientation of its base turns out to be almost the same as that of the current pyramid. The minimal deviation between the older stairway (110-112 az) and the current one (110 az) would not have nullified the effect, which leads me to hypothesize that the earlier pyramid exhibited the same effect of light and shadow (fig. 8c). In any case, a line of the new pyramid's base (what we call a diagonal) is oriented to the summer-winter solstice (summer solstice sunrise – winter solstice sunset, 65 az-245 az), as I confirmed by measurements on the site.[12]

The light and shadow effect

Were the Mayan builders capable of solving in 3D the projection between two non-coplanar planes? Was the light and shadow effect learned from the older pyramid, or was it noticed and reinforced during the construction of the current one? To solve a projection between two non-coplanar planes in 3D, one needs to have advanced knowledge of geometry. There is no evidence of such knowledge among the Mayans, not even that they were familiar with drawing instruments. This is why I am sceptical of all the scholars who believe that the effect was intentional, just as I am of the theory which suggests that the Quetzal chirps were engineered. This leads us to speculate if by chance the builders paid attention to the shadows cast when they were building the first or second platform of the new pyramid. If this actually happened, then no advanced geometry was necessary, but rather they worked directly with the real thing. This leads us to ask whether the platforms' deviation was a constructive defect or if the platforms were adjusted to line up with the triangular shadows.

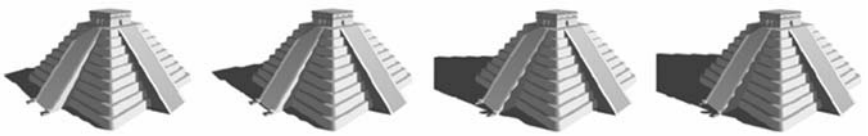

Fig. 9. A 3D computer model of the pyramid showing four stages of the light-and-shadow effect during the equinoctial season. Drawings by Ambar Hernández

What we are certain is that the inevitable effect is produced by the interplay of two non-coplanar planes, namely: the northwest balustrade and the northwest dihedral angle of the pyramid (fig. 9). The inclination of each of these planes (45° and 43.5°, respectively) seems to have been determined independently. The fact is that the angular difference of 1.5° between the planes is what produces the illusion of the seven triangles moving downwards, an illusion that reaches its climax when the sun's rays hit the serpent's head at the foot of the stairway. If such a dazzling moment was indeed devoted to Kukulcán, then March 21 should be his celebration day. Kukulcán (also identified with Venus as coming down from heaven) was a god celebrated across the Yucatán Peninsula until the fall of Mayapán, a tradition thereafter held in Maní. Here, the people made a celebration lasting five days – from 16-Xul through 1-Yaxkin –, waiting for the last day when Kukulcán would come from heaven to receive their offerings [De Landa 1986: 98-99]. Such days, as deduced from Landa's correlation, would correspond to the early days of November, not March when the myth takes place in Chichén-Itzá. Aldana [2003: 33-51] correlates the celebration day with the first appearance of Venus in the evening sky of October 25, 1552, pretty close to Landa's correlation within a few days of difference.

Fig. 10. The southwest stairway of the pyramid of Kukulcán in precarious conditions. Photograph by the author

The effect should also be visible during the equinox sunrise on the pyramid's southwest stairway, but its deteriorated conditions do not allow for this (fig. 10). Theoretically, it should be a perfect symmetrical effect: the morning ascent of the Plumed Serpent throughout the nine layers (represented by the 9 platforms) of the underworld (Xibalba), and its descent on the evening of the same day. However, neither the *Chilam Balam* nor the *Popul Vuh* can help to support this theory. Besides, why does not the southwest stairway have serpent heads, or should we be asking why those of the northeast stairway look mutilated?

The serpent heads at the foot of the northeast stairway were cut atypically, and look mutilated and superimposed (fig. 11a) instead of functioning as a true cornerstone like those of the Osario Pyramid, where the serpent heads are the original ones, and its balustrades lavishly ornamented (fig. 11b). It can be argued that both pyramids belong to different periods, despite the fact that they are about 350 meters away from each other, but it also can be argued that a constructive deflect cannot be justified by the use of a particular style. Landa reported having seen the serpent heads in place,[13] presumably those that Catherwood faithfully drew 300 years later, when they appear as cornerstones (fig. 11c).[14] Whether the original serpent heads were removed and then replaced with others (after the Stephens-Catherwood expedition) or whether they were removed and erroneously reinstalled during the stairway's restoration are hypotheses we should be examining.

Fig. 11. a, left) Architects and laymen alike can notice the wrong way in which the serpent heads were placed in Kukulcán's stairway; b, right) In contrast, the dexterous stonework of the Osario Pyramid's serpent heads, well integrated to the balustrades, makes evident the erroneous placement of those of Kukulcán. Photographs by the author

Fig. 11c. Catherwood's engraving of the serpents' heads

To conclude this section, the use of a rudimentary model to copy the older pyramid's effect, or engineering a new one, could be another theory, although remote. Disregarding the exact date on which the god Kukulcán was honored, such a model could have helped to visualize the pyramid and eventually learn from it the light and shadow effect. If such a model ever existed, it probably is buried somewhere in the pyramid.

A hypothesis about orienting the pyramid

The Ground-Penetrating Radar study conducted by Desmond [1996:23-30] has detected beneath the plaza a subsurface cultural world of which little is known. An extension of 250 m. by 400 m. of bedrock was filled up over a long period, ultimately buring a sacbé, several caverns, and previously laid down floors, all predating the

construction of the plaza, an impressive undertaking for the time. The great plaza of Chichén-Itzá has an imperceptible slope, just enough to evacuate rainwater, which suggests that some surveying techniques were used for levelling and elevating it. Three main buildings occupy the great plaza: the Ball Court at the northwest, the Warrior's Temple at the northeast, and the Kukulcán Pyramid in the center. An Olmec hematite (a kind of lodestone compass), found in San Lorenzo (Veracruz), suggests the hypothesis of its use in planning Mayan centers, as Klokconík et al. pointed out [2007: 515-533].

However, the summer-winter solstice orientation given to the pyramid could have been intended to alert the people regarding the arrival of the season for sowing and harvest, and a lodestone compass would be useless for this purpose. Instead, I believe that a pole was used, since many buildings across the Mayan area seems to have a line oriented to the summer-winter solstice. Besides, it is easy to determine both the summer solstice sunrise – winter solstice sunset and winter solstice sunrise – summer solstice sunset orientations by observing the projecting shadow of a pole planted in the ground over the course an entire year – laying the equinox's orientation at the middle of the solstices' crossing shadows (see fig. 12).

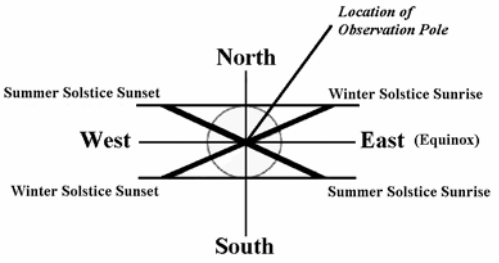

Fig. 12. Using the shadow cast by a pole to find the equinox orientations

Moreover, the pole was among the instruments for surveying that the Mayans presumably were familiar with; similar instruments are illustrated in the *Bodley Codice*.[15] A pole of wood rather than of stone (like that of Edzná, or that in front of the Governor's Palace in Uxmal) would be easier to construct, but less durable over time.

Let us suppose that the current pyramid's base was verified according to the summer solstice sunrise – winter solstice sunset orientation. In that case, three procedures could have been used to construct it: a) planting a pole on the temple's roof; b) placing a cross-staff (a surveyor's instrument for measuring offsets) at each corner of the older pyramid's summit; or c) using the older pyramid as a reference to lay out the new one. Let us examine each of these possibilities. A pole atop the roof of the older temple would have been useless because its shadow would not reach the ground at sunset. On the other hand, the cross-staffs could have helped surveyors prolong the base's diagonals at more distant points.

The third procedure seems the most logical one, based on the geometrical coincidences I found when superimposing the plans of both pyramids (the present-day one and the substructure). Thus, I decided to join the midpoints of the stairways' base of the present-day pyramid and see what happened; to my surprise, I found that the corners of the older pyramid's base perfectly circumscribe an auxiliary square (fig. 13).

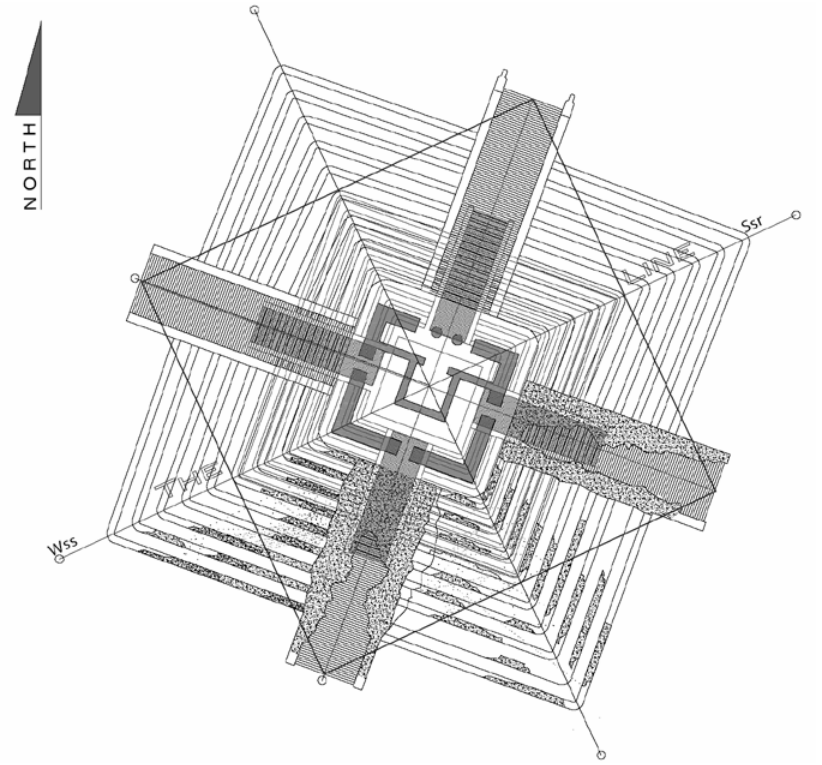

Fig. 13. The plan of the actual pyramid and that of the older one superimposed. Notice how an (imaginary) auxiliary square circumscribes the plan of the older pyramid. "The Line" is oriented according to the winter solstice sunset (wss) and summer solstice sunrise (ssr) orientation. Drawing by Ambar Hernández based on the author's measurements

What these matching plans are telling us is how the first auxiliary outline for the new pyramid's base was executed. Nothing else but simple tools such as henequen ropes (rope made from the fibre of the agave plant), several cross-staffs, and dexterous surveyors, were required to lay out the auxiliary square; but how? Let us imagine a team of surveyors standing atop the upper platform of the older pyramid, placing a cross-staff at each midpoint of the stairways' borderline, while on the ground, another team would do exactly the same at each base of the stairways. Thus, by lining up the upper cross-staff with the one on the ground, so as to have two visual points of reference, the surveyors would prolong lines on the ground until they intersected the auxiliary square that circumscribes the older pyramid's base.

In turn, the vertices of the auxiliary square turn out to be the midpoints for the new stairways. A trial and error process would lead both teams to equalize the sizes of the auxiliary square by manipulating the henequen ropes. In the same manner, the diagonals of the older pyramid could have helped to complete the layout of the new pyramid base. But how did the builders estimate the size of the temple's platform and the stairways' length? Re-examining fig. 13, it seems logical that an axis of the stairways was used for this purpose. Thus, tracing either the NE-SW or SE-NW axis on the ground, they could have divided it into three parts to set the dimensions of the stairways and the temple platform. In other words, the builders worked out the form of the pyramid on the ground.

A hypothesis about how the pyramid's base was laid out

The urban layout of some Mayan centers, such as Dzibilchaltun, Mayapán, Uxmal, Edzná, and Chichén-Itzá, seems not to follow a pre-established pattern, for all of them are different; in contrast, the buildings seem to be rationally oriented.

To start the construction of the pyramid, two basic things were needed: a chosen site, and a line of reference, our line. In practice, as we know, a square can be drawn from any line as long as one knows how to construct right angles. But if the Mayan builders did not possess the notion of a right angle, how did they construct a square from a single line without employing right angles? Most likely, they could have used what we call a diagonal as the prime line to set in place their buildings' foundation. It is not surprising to find such a line precisely across the Nunnery Quadrangle in Uxmal. In my opinion, the Mayan builders invented their own method for laying out a perfect square on the ground. Otherwise, how can we explain the fact that Kukulcán's base is almost a perfect square? (Three corners of the pyramid's base render perfect right angles, as I confirmed with measurements on site.)

As we already have pointed out, the summer-winter solstice orientation of a line of Kukulcán's base was most likely used as the prime line to lay out its foundation. In other words, what we called a diagonal (in the broadest sense of the term used to indicate a line joining two opposite points of a given figure) turns out to be the first laying out line by which the square base of the pyramid was laid on the ground. To sustain this conjecture, which in its own right is the history of our line, we have to prove that a square form can be laid out from a line without using right angles.

When I was inspecting the Governor's Palace in Uxmal, an idea came to me about constructing parallel lines without using right angles. Let us suppose that an oriented line L1 is laid on the ground where points a and b are marked by a rod planted at each one of them. Then, arcs of radius r1 are swinging by a rope attached to the foot of each rod, and where a line passes tangentially to the arcs, a parallel L2 to L1 is constructed.

Fig. 14. Experiment to prove the feasibility of what I call the Mayan method for laying out a square. We laid out the base of the older pyramid, on campus, without using right angles. Photographs by Ambar Hernández

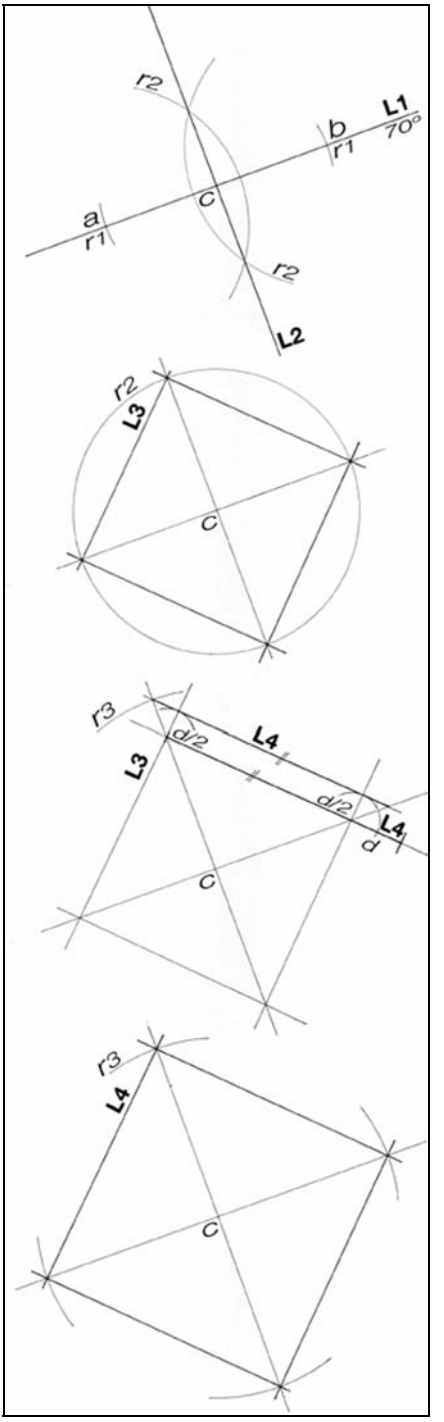

Fig. 14. The Mayan method for laying out a square. Geometric procedure by the author

In order to prove it, my students and I carried out an experiment in front of the Rectory building at the National Autonomous University of Mexico. We chose a spacious pavement to lay out the base of the older pyramid in its actual size. After four hours of work, using only ropes, chalk, and masking tape, we succeeded in constructing an almost perfect square of 33 x 33 m. (fig. 14).

Of course, we assumed the 33 m. as a given datum, since we cannot know how it was determined or what it means in Mayan measurements. The experiment was based on what I have called the *Mayan method for laying out a square*. This is a method that can be useful for explaining the layout that other Mayan buildings have as well. Its general statement and solution are explained next.

Problem: Given an oriented line L1, construct a square of a pre-established side L4 without using the right-angle method.

Solution: Let line L1 be a straight line set out on the ground (oriented by a pole according to the summer-winter solstice), and let c be a point randomly set along it. Carrying out radius r1 from c, points a and b are marked out along L1. Then, swinging arcs from a and b, with a radius r2 > ab/2, a straight line L2 is found where these arcs intersect each other. As L1 and L2 intersect at c orthogonally, by dropping a circle of radius r2, a L3 square is conformed where r2 intersects with lines L1 and L2. Now, to construct from the random L3 a given L4, we proceed to overlap L4 onto L3. In doing this, we would have three options: L4 > L3, L4 < L3, or L4 = L3. Naturally, the latter would occur once in a thousand. Taking the first one, which is our figure's case, carrying out L4 onto L3 and subtracting L3 from L4, a remaining segment (d) would result, which in turn is divided in half. Again, swinging arcs but now from the extreme points of L3, of a radius r3 = d/2, and sliding L4 tangentially to these arcs, it would fit exactly at the intersection points with L1 and L2, thus determining one L4 side of the L4 square. To trace the remaining three L4 sides of the L4 square, it would suffice to carry out the new radius r3 until it crosses L1 and L2. Finally, by joining the found points, a perfect square of the given side L4 is determined (fig. 14: steps 1 through 4).

Due to the restricted length of this paper, both the corollary (L4 < L3) and the proof of the theorem behind this problem will be treated formally in another paper.

Conclusions

Perspective as representation, in my opinion, was preceded by perspective as a surveying method for building. This latter is the underlying issue we have speculated about in this article, in the attempt to find out how it was employed in the Mayan world. We have proved that a square can be laid out from a given line by a simple surveying method. Most likely the base of the Kukulcán Pyramid was laid out in that way. It is precisely the application of such a method that makes the orientation of the pyramid's base so unique. And, in turn, the inclination of the platforms and stairways, ruled by the constructive process of the pyramid, introduced another singularity. On the other hand, the special effect of light and shadow seems to take place as a result of these singularities; there is no evidence to indicate that it could have been engineered by geometrical means at the time. Furthermore, by modelling the pyramid in 3D, we can observe that the effect is still visible even when its orientation is varied within certain parameters. This explains why seven triangles can be observed on March 21, while the number of triangles seen varies between three and nine during some days before and after the equinox. Lastly, thinking as a builder, the most plausible hypothesis is that the effect was noticed and reinforced during the pyramid's construction (as opposed to having been pre-planned). At least, that is the one I am in favor of, until it can be proved otherwise.

Notes

1. Here, the word 'discovered' most likely indicates 'a place to settle was found'.
2. See http://en.wikipedia.org/wiki/Chichen_Itza.
3. Eric S. Thomson boasted that his knowledge of Maya was self-taught. At least he has only himself to blame for what he wrote: "I was given charge of the excavations of the Caracol (the Observatory), that queer ugly round building which…" See *Maya Archaeologist* (1963, University of Oklahoma Press, p. 37).
4. Azimuth (az): In plane surveying, a horizontal angle measured clockwise from north meridian to the direction of an object or fixed point. All azimuth measurements quoted in this paper are the author's.
5. In December 2007, the pole was not there. This pole is a tapered shaft of stone surmounted by a capital-like top. According to the National Institute of Anthropology (INAH) of Mexico, it was mistakenly aligned at the northeast façade of the Five-Storied Building during the restoration of the site, and for this reason, it was put away to avoid erroneous interpretations. This was communicated by INAH authorities in an official letter addressed to the author (Feb. 26, 2008/ Of. Núm. 401-7/333).
6. The only extant Mayan Codices (folding-screen books) are: the Dresden, Paris, Madrid, and Grolier.
7. There are at least three books on medicine: The *Chilam Balam* of Káua, the *Chilam Balam* of Tekax, and the *Chilam Balam* of Nah.
8. *Usaba también esta gente de ciertos caracteres o letras con las cuales escribían en sus libros sus cosas antiguas o ciencias, y con estas figuras y algunas señales de las mismas, entendían sus cosas y las daban a entender y enseñaban. Hallámosles gran número de libros de estas sus letras, y porque no tenían cosa en que no hubiese superstición y falsedades del demonio, se los quemamos todos, lo cual sintieron a maravilla y les dio mucha pena* [De Landa 1986: 104-105].
9. A talud is an outer wall that slopes inward as it rises. The stonework in flint and obsidian is remarkable.
10. By adding the steps of the four stairways plus the step of the temple's base, the result would be [(4 x 91) + (1)] = 365.
11. It was a Mayan custom to renew almost everything every 52 years, which explains the existence of many superimposed structures everywhere in their cities.
12. The azimuthal alignments presented here were taken in situ by the author with a Brunton Transit (surveying compass adjustable for magnetic declination; azimuth accuracy of ± ½° with 1° graduations). To sustain the theory of the solstice line being used by the Mayan to either build squares (Kukulcán) or rectangles (Uxmal), the author needed to rely on his own architectural data of the places he visited. Despite the fact that the author's alignments might differ from other alignments already published, they are relatively congruent among themselves, and therefore they are to be considered accurate. Chichén Itzá location: latitude 20° 40′ 56‴ N, longitude 88° 34′ 05″ W. As it is known, the magnetic declination (MD) varies with the passage of time all over the world. The author estimated a MD of 1° 30′ E, based in the 2007 isogonic chart for North America, while for December of the same year, the National Geophysical Data Center (NGDC) estimated a MD of 0° 30′ E.
13. *Había, cuando yo le vi, al pie de cada pasamano, una fiera boca de sierpe de una pieza bien curiosamente labrada. Acabadas de esta manera las escaleras…* [De Landa 1986: 113].
14. Catherwood's engraving depicts a neat stone cut, perpendicular to the neck of the serpent heads, not in diagonal position as present-day heads have; cfr. [2000: 357, fig. 14].
15. The *Codex Bodley* [c. 1500] is a pictographic manuscript of the Mixtec culture (south-west from the Maya area). Among the illuminated glyphs it contains (carefully organized on a deer skin of 22 feet long by 10 inches wide), there is a "X" conformed by crossed sticks, and a "V" with an eye in the middle, suggesting the presence of surveying instruments to build. See [Codex Bodley: 15, 19, 21, 32].

References

ALDANA, Gerardo. 2003. K'uk'ulkan at Mayapán: Venus and postclassic Maya statecraft. *Journal for the History of Astronomy* **34** (114): 33-51.

BARRERA Vázquez, Alfredo and Silvia RENDÓN, trans. 2005. *El libro de los Libros de Chilam Balam*. Mexico, Fondo de Cultura Económica.

CODEX BODLEY. c. 1500. Bodleian Library, University of Oxford, Codex Bodley 2858. http://www.famsi.org/spanish/research/pohl/jpcodices/bodley/index.html

DE LANDA, Diego. 1986. *Relación de las Cosas de Yucatán*. Mexico, Ed. Porrúa.

DECLERCQ, Nico F., Joris DEGRIECK, R. BRIERS, and O. LEROY. 2004. A theoretical study of special acoustic effects caused by the staircase of the El Castillo pyramid at the Maya ruins of Chichen-Itza in Mexico, in: *The Journal of the Acoustical Society of America* **116** (6):3328-3335.

DESMOND, Lawrence G. and William A. SAUCK. 1996. Entering the Maya underworld: A ground penetrating radar survey at Chichen Itza, Yucatan, Mexico. Pp. 23-30 in *Innovation et Technologie au Service de Patrimoine de l'Humanite*, Actes du colloque organise par Admitech en collaboration avec l'Unesco, Paris, June 24, 1996.

GARCÍA-SALGADO, Tomás. 2004. *Instrumentos para la Geometría Perspectiva* (*Instruments for Perspective Geometry*). Mexico, FA, UNAM, pp. 40-41.

KLOKOČNÍK, Jaroslav, Jan KOSTELECKÝ, and František VÍTEK. 2007. Pyramids and Ceremonial Centers in Mesoamerica: Were They Oriented Using a Magnetic Compass? *Prague, Studies in Geophysics and Geodesy* **51**: 515-533.

LUBMAN, David. "The Mayan Pyramid". http://www.ocasa.org/MayanPyramid2.htm.

STEPHENS, John L. 2000. *Incidents of Travel in Yucatan*. Mexico, Ed. San Fernando.

TEDLOCK, Dennis, trans. 1996. *Popol Vuh: The Mayan Book of the Dawn of Life*. New York, Touchstone.

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Tomás García-Salgado received his professional degree (1968), Master's degree and Ph.D. (1981-1987) in architecture. He is a formal researcher in the Faculty of Architecture of the UNAM (México), and holds the distinction as National Researcher, at level III. Since the late 1960s, he has devoted his time to research in perspective geometry, his main achievement being the theory of Modular Perspective. He also has several works of art, architecture, and urban design. More information regarding his work is available at http://perspectivegeometry.com.