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Research

Giotto and Renaissance Perspective

Abstract. A careful geometric examination of the blind arcades (*coretti*) depicted in Giotto's fresco on the choir wall in the Arena Chapel in Padua shows that they were designed and painted according to the rules of what I term "progressive *costruzione legittima*" and thus represent simulations of visual images. Because no images of this type have come down from Classical Antiquity and because the literary references remain silent in this respect, the *coretti* must be considered, according to today's knowledge, the oldest monuments manifesting the application of the *costruzione legittima*. This means the history of the central (linear) perspective must be rewritten. In any case it was not a Renaissance invention. I expressly agree with the researchers who see Giotto's painting in conjunction with the findings of the Scholastic "optics specialists" (such as Grosseteste, Witelo, Bacon), who all stood with their feet firmly planted on the ground of Euclid's rigidly geometrically conceived visual theory and its Arab commentators.

Introduction

My work on the history of perspective has already been summarized in an essay on Brunelleschi [Hoffmann 1990-1992] as well as one on Masaccio [Hoffmann 1996]: They form the basis for this article on Giotto (ca. 1266-1337). In those earlier essays I made a critical study of the academic literature and do not need to repeat that here. My article on Giotto will provide findings that cannot be gained from the literature, so that I will restrict myself to just a few bibliographical references. Apart from that I refer to the well-known bibliographies on the topic of perspective [Vagnetti 1979; Veltman 1986; Alberti 2000; Sinisgalli 2006].

If one defines "perspective" as a method of representing a three-dimensional object on a plane, one comes to the conclusion, in agreement with known works in art history, that there are many perspectives: parallel and central perspective, axonometry, vanishing point perspective, reverse perspective and many others, above all, however, the mixed forms. Here we are dealing with the (monocular) central perspective, also called linear perspective, which can be geometrically constructed. This central perspective – this is lexicon knowledge – was invented in Florence in the early fifteenth century. The key works are 1) Brunelleschi's (1377-1446) *vedute* of the Florentine Baptistery and the Piazza della Signoria. These panels have not survived, but we know of them from the report by Antonio Manetti, Brunelleschi's first biographer, in which he writes, inter alia, that Brunelleschi painted them in his youth, which perhaps makes reference to the years before rather than after 1400; 2) Masaccio's (1401-1428) *Trinity* fresco in S. Maria Novella in Florence (after 1426) (fig. 1). From the point of perspective the work is divided into two: the tomb of Adam below and the donor couple kneeling on a platform above it lie in front of the wall; behind the wall a barrel vault canopy opens up above, where God the Father appears with the figures of the Holy Trinity. (It is likely that Brunelleschi was consulted in the conception of this work.)

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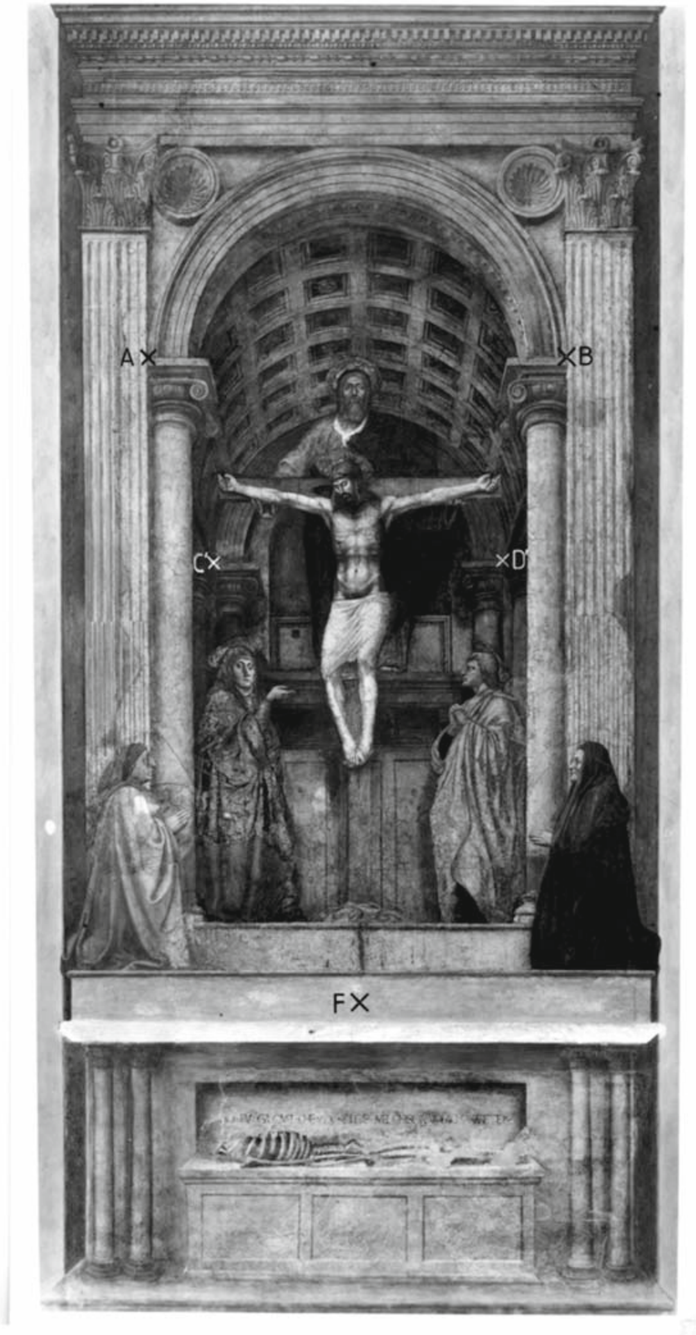

Fig. 1. Masaccio, Trinity fresco (Florence, S. Maria Novella)

What is the difference between these two prototypes of central perspective painting? Brunelleschi represents real bodies on the plane; Masaccio simulates bodies on the plane, as if they were real. This difference is fundamental, yet is scarcely mentioned in the literature on perspective. It is important now to ask how Brunelleschi produced his *vedute*. I have answered this question with the comment that this was only possible with a perspective apparatus and have decided in favor of the apparatus that Alberti first described (but incompletely) in 1435 as the “velo” (a grid of threads), and Albrecht Dürer first illustrated in a print in 1538 (fig. 2).

Fig. 2. Albrecht Dürer, *Der Zeichner des liegenden Weibes*, woodcut 1538

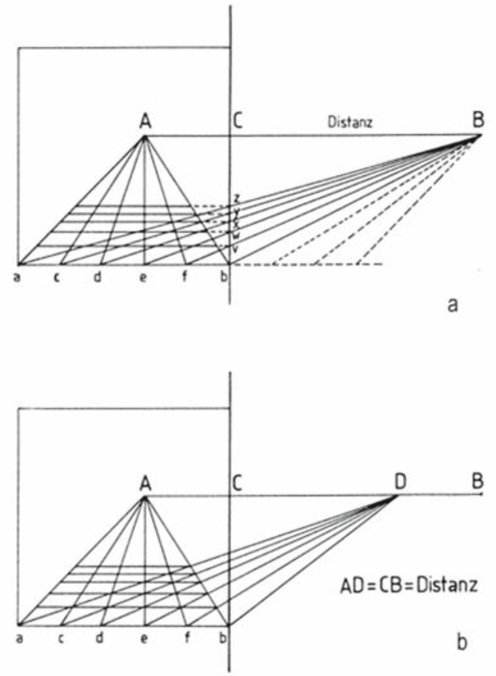

Fig. 3. The *costruzione legittima*: a, above) visual ray construction; b, below) distance point construction

And how did Masaccio construct the perspective of the *Trinity*? My answer: according to the rules of the *costruzione legittima*. This is not a fifteenth-century term, but rather appears for the first time in the seventeenth century (according to Bätschmann [Alberti 2000: 127, note 192]), despite the fact that up to today it still has no clear and generally recognized definition. Fig. 3 displays the two commonly represented variants of the *costruzione legittima*: a) the so-called "visual ray method", which Panofsky [1915] reconstructed graphically from Alberti's description[1]; and b) its simplification and transformation into the "distance point construction".

Both methods solve the task of perspectively deforming a square, represented by the line segment ab in fig. 3, as it would be seen in reality from the eye level and eye distance of a specified observer. The relationship between these two methods were first described exactly by Vignola [1583],[2] who called them *prima regola* and *seconda regola* and determined that the same result could be achieved with both. Therefore, it is nonsensical to declare one method *costruzione legittima* but not the other.[3] No, both constructions are *legittime* and deserve this attribute because they represent geometric methods that simulate the natural (monocular) visual image.

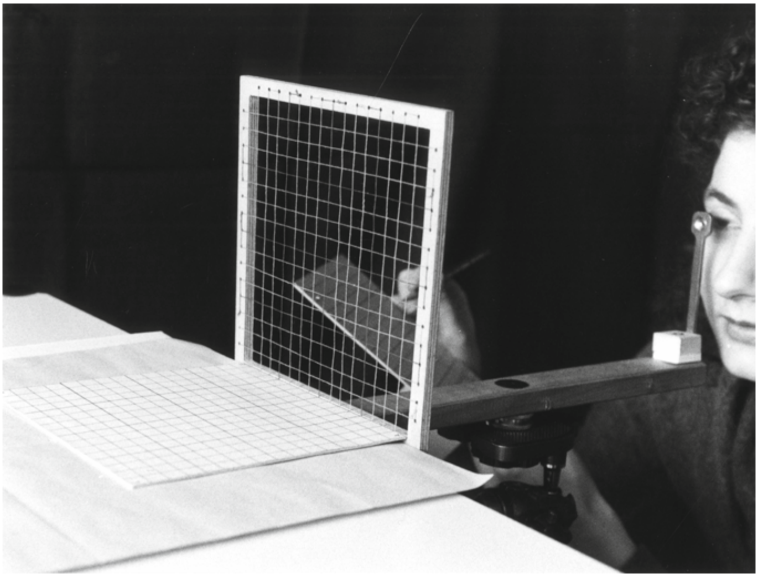

Fig. 4. The perspective apparatu: fixing the visual image

This can be demonstrated with an experimental setup (fig. 4). A perspective apparatus of the kind used by Brunelleschi, and depicted by Dürer (fig. 2), consisting of a frame with a grid of threads (*velo*) and an eyepoint, here represented by a small stand perforated with a hole securely mounted in front of it, concretely illustrate important theorems of the Euclidean theory of optics: visual ray, visual pyramid, cross-section through the visual pyramid, eyepoint and distance. The observer looks through the eyepoint (= apex of the visual pyramid) and has the task of drawing the outline of the square board lying behind the *velo* (= cross-section through the visual pyramid) onto a little panel with the same, but graphically drawn grid. Through the *velo* he sees the real square board as a trapezoid and can transfer this visual image point by point to his little panel by using the threads as reference lines: This is fixing of the visual image based on

Euclid's geometric *Optics*. As I demonstrated elsewhere [Hoffmann 1990-1992], and as far as we can know at present, Brunelleschi was the first to have the idea of intersecting the Euclidean visual pyramid, with its base on the seen object and its apex in the eye of the observer, by using the grid of threads and fixing the visual image with the threads (reference lines). In addition, the perspective apparatus can serve as a measuring device [Hoffmann 2002]. For example, if I know the width of the object seen, I can determine its height and distance. The perspective apparatus does in principle the same thing that today's photography and photogrammetry now do much, much better, namely they fix and measure the monocular visual image.

One can now replace the "visual ray" in the experiment that falls from the eyepoint onto the top edge of the square board by a thread, trace it through the *velo* and transfer this position by hand to the little square panel; the trapezoid will then have the same outline as described above. There is an even easier method (fig. 5).

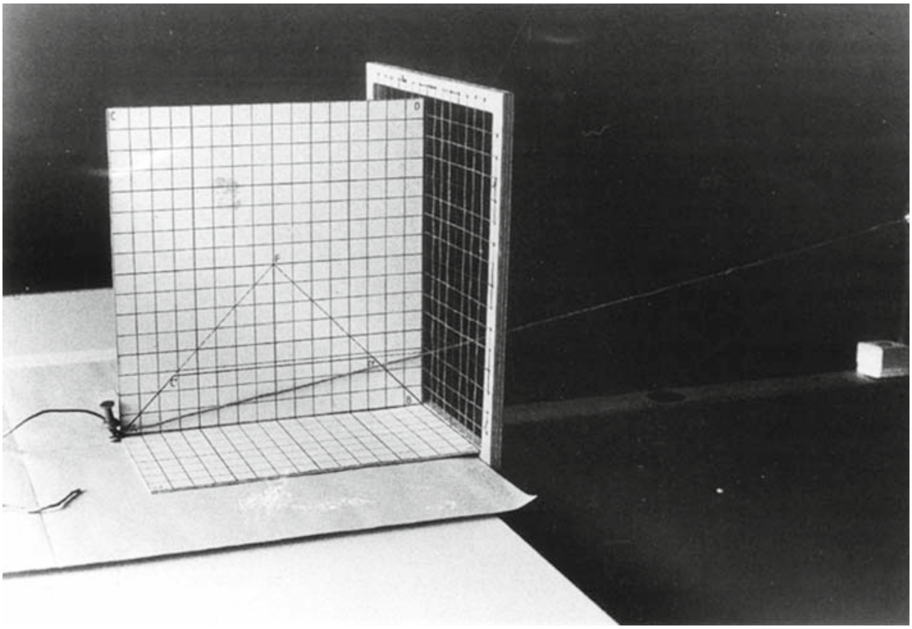

Fig. 5. The perspective apparatus: simulating the visual image ("visual ray construction")

If one places the little panel at a right angle to the grid of threads, marks a point on it on which the thread (visual ray) penetrates the grid, and draws a horizontal line through this point, then the top edge of the square board is represented in perspective. A "visual image" has been created without using the eye. The experimental arrangement corresponds exactly to fig. 3a (and hence Vignola's *regola prima*), whereby the perpendicular line through C represents the grid of threads. This geometric construction in which the genetic relationship to the perspective apparatus remains recognizable enables one to simulate a visual image; fig. 6 and fig. 3b show the method simplified into the "distance point construction" (Vignola's *regola seconda*). The grid of threads is turned around its central axis by 45° and replaced by the little square panel. The (thread) visual ray intersects the triangle ABF directly at the level of the apex of the trapezoid. (We will disregard the geometrical markings for the moment.)

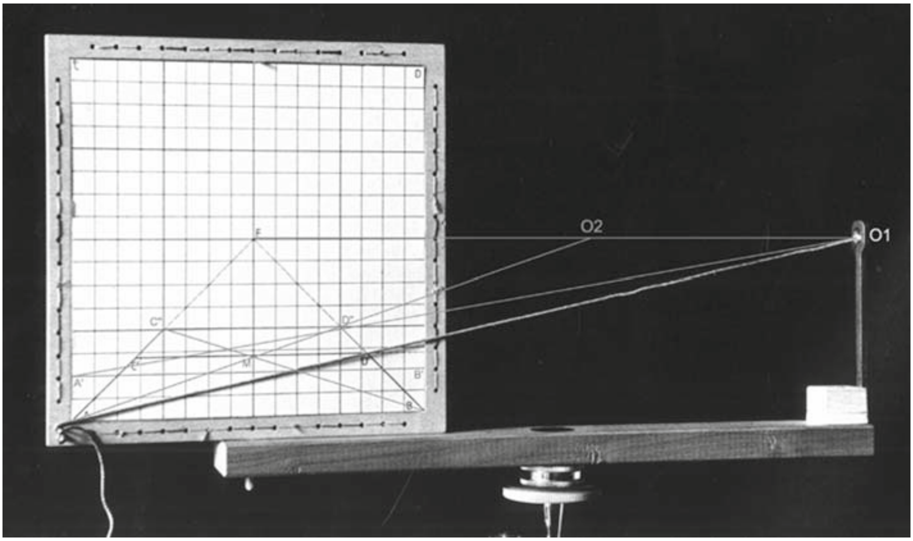

Fig. 6. The perspective apparatus: simulating the visual image ("distance point construction")

To summarize, the perspective constructions in accordance with figs. 3a and 3b (Vignola's *regola prima e seconda*) are of particular cultural historical importance in that they enable one to simulate the monocular visual image according to Euclidean optics; they are thus also the basis for illusionist painting. The term *costruzione legittima* should be reserved for this type of painting (and for it alone). This construction is *legittima*, however, only then when the task is to deform a square perspectively. We will explain below how a rectangle can also be deformed perspectively according to optic-geometric rules, despite these universal laws.

In my article in 1996 I demonstrated that Masaccio designed and painted his *Trinity* based on the *costruzione legittima*. I envisaged the historical procedure to have been as follows: First Brunelleschi invented the perspective apparatus, developed the *costruzione legittima* from it and then constructed the *Trinity* together with Masaccio. I now see that this was not at all as clear and simple as I thought; the following analysis should demonstrate this. In the Arena Chapel at Padua, which Giotto painted with his assistants from 1303-1305, the choir wall is decorated with perspectively painted architecture in the form of a "Syrian arch" above four pilasters. In the lower zone theatre-box-like blind arcades have been placed on the left and right, and are called *coretti* (little choirs). I have a very good picture of the left *coretto*, which the *Museo civico* at Padua gave me many years ago. The following geometric-perspective analysis is based on this one photo (see fig. 7).

Above a parapet panel a Gothic arcade reveals a view into a room with a Gothic ribbed vault that is apparently supported by four narrow corner columns and from whose boss a large chandelier hangs by a rope. The back wall displays a pointed arch like the front arcade and opens up into a window with a slender, square-edged central column. The left side wall is overlapped by a rounded arch and is thus lower than the back wall. The front arcade consists of two short piers that nestle into the pilasters of the Syrian arch and support the pointed arch. Its front face is smooth but bordered by a profile molding.

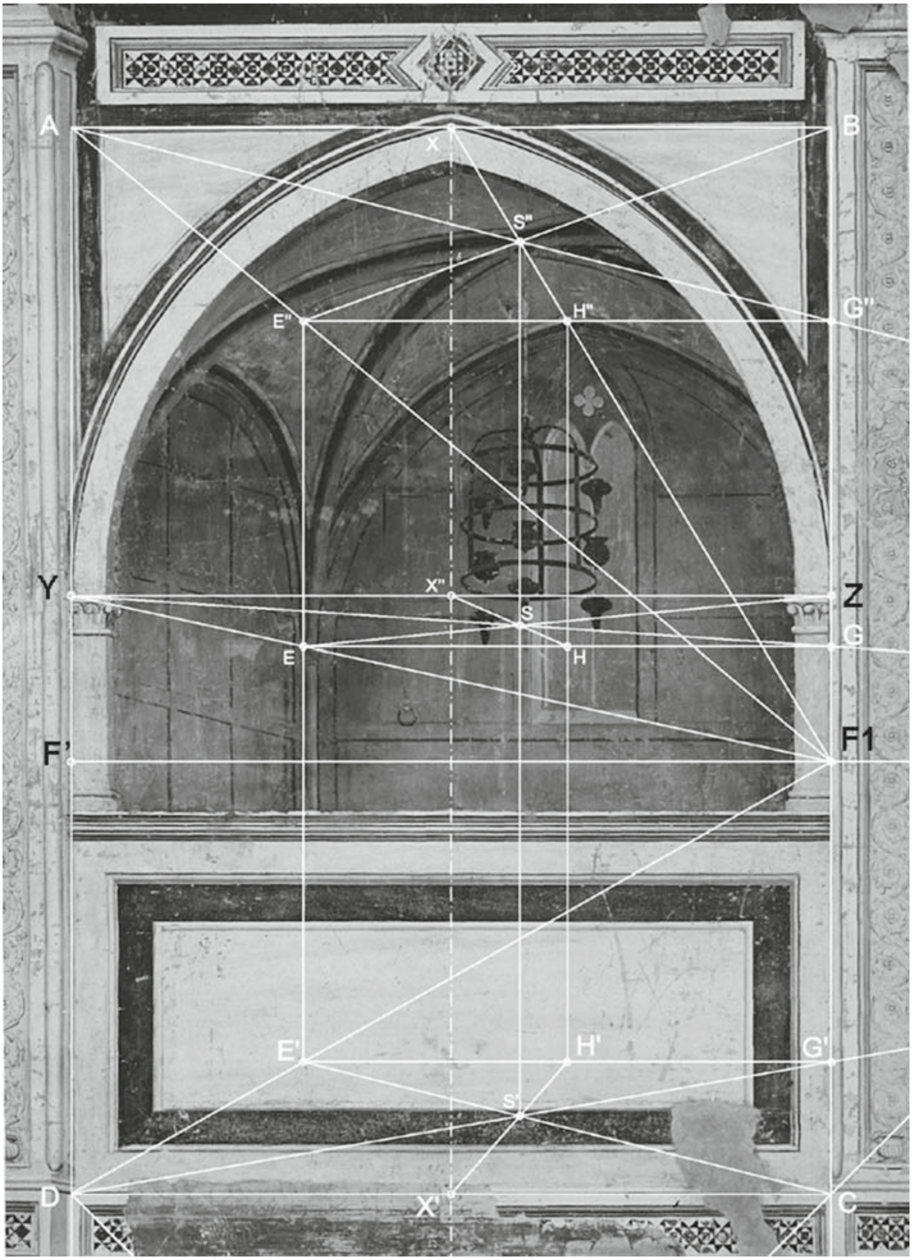

Fig. 7. Arena Chapel, *coretto* and static *costruzione legittima* (Underlying photo reproduced courtesy of the Museo civico, Padua)

Let us examine this molding more closely (fig. 8). It is strong and sculpted on the left side (where a subtle intersection with the pilaster occurs), whereas on the right it becomes much narrower and finally turns into just a line (fig. 8, right). The molding is therefore represented in perspective and that implies the finest observations of real constructed architecture, which is expertly simulated as real here. The astonishing perspectival realism of this detail now arouses interest in the overall perspectival-geometric design of the *coretto* (fig. 7). My analytical tool is the *costruzione legittima*, which I place like a template on the painted *coretto* to measure its construction.

Fig. 8. Details from fig. 7

The first step is to reduce the image field, whose one side must be equal to the side of the square to be foreshortened. Because the *coretto* is placed between the large pilasters, which have been painted according to another perspective construction, it is sensible to divide the width of the entire front arcade, that is, the line segment YZ, which is then at the same time the length of the side of the basic square. I look for the lower boundary of the image field at the foot of the parapet panel on line CD, the top boundary at the crown of the arch (X) of the inner line of the profile surrounding the pointed arch, because this line still just barely exists at Y and Z. The image field is thus rectangular and defined by the vertices ABCD. (Whether these boundaries are correct or not will be proved by the later reconstruction of the perspective design.)

Now we must find the vanishing point (F), that is, the correct position of the eye, from which the *coretto*, if it were real, could be seen from F exactly as it appears on the picture. But because we are looking into the flank of the left pilaster but not into that of the right pilaster, it seems likely that the vanishing point is to be found along the vertical BC line. Assuming (correctly) that the capitals of the piers in the front arcade lie at the same height as the capital (E) of the slender column in the left corner of the *coretto*, I draw a straight line from Y over the abacus of that capital (E) (fig. 9b) to the right until it intersects BC at F1, the vanishing point. What is special about this is that F lies on one side (BC) of the frame of the image, so that the "visual ray" FC thus forms one side of the visual triangle, making a differentiation between a "visual ray method" and a "distance point construction" (Vignola's *due regole*) unnecessary and impossible. If in fig. 3a one imagines the vanishing point A in place of C, then one sees the perspective construction of the *coretto*. To create a perspective according to the *costruzione legittima* we would need the distance point; for the present we do not need it, because we are simply analyzing an existing perspective construction. Without deviating in the least from the rules of the *costruzione legittima* we now lay three planes in the image field ABCD (fig. 7), namely through AB, YZ and CD, which as trapezoids each represent the basic square (a) with the side length CD.

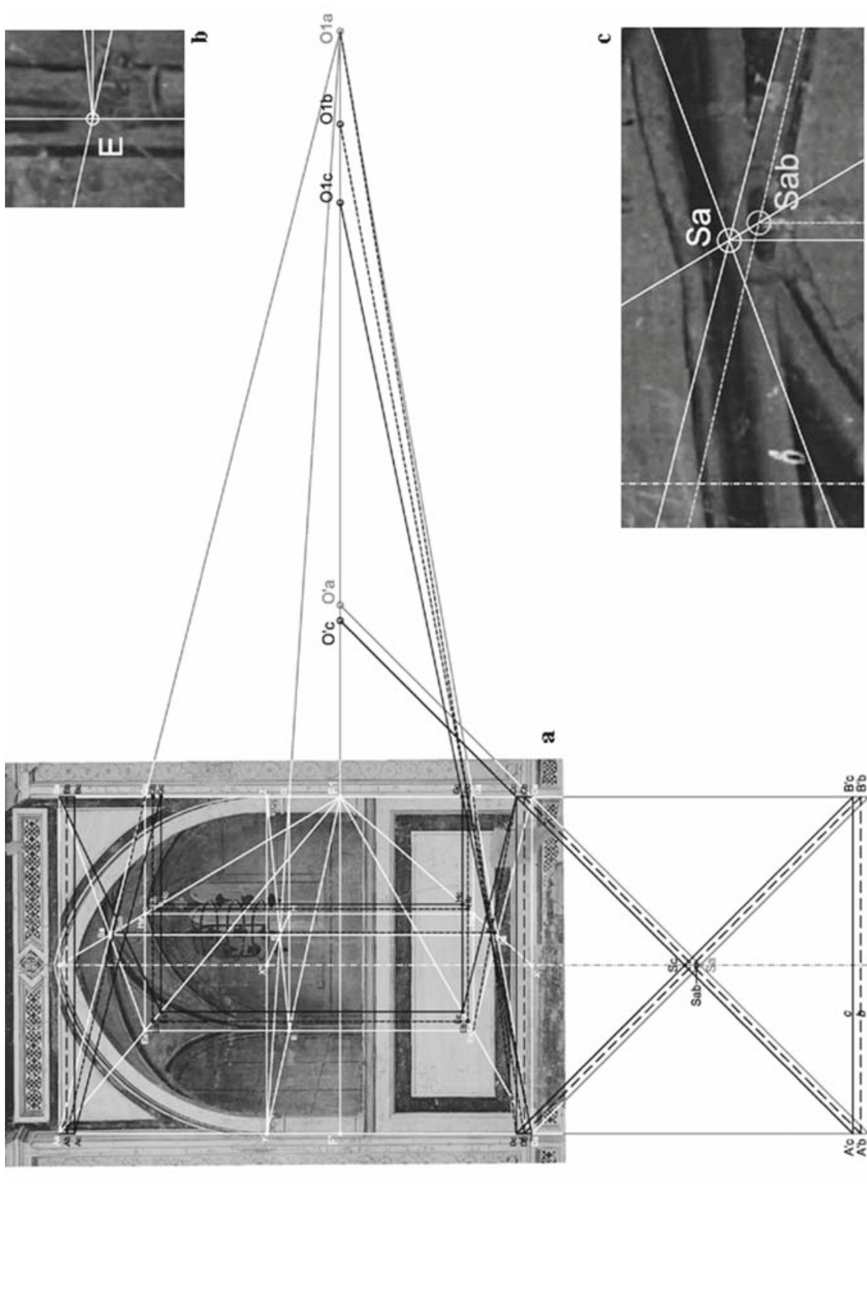

Fig. 9. *Coretto* and progressive *costruzione legittima*

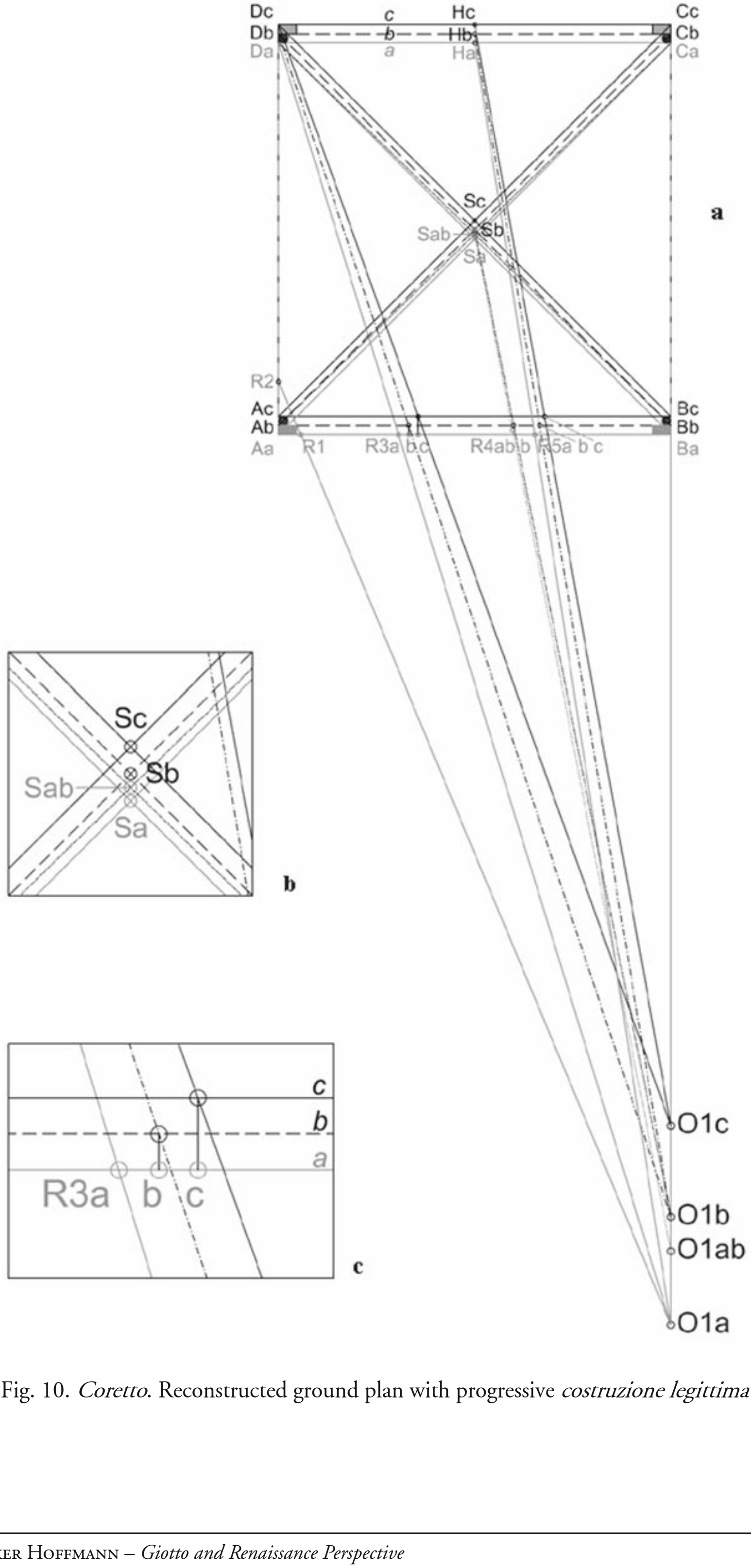

Fig. 10. *Coretto*. Reconstructed ground plan with progressive *costruzione legittima*

The line parallel to YZ through E intersects the perpendicular BC at G; YZGE is the deformed basic square (a) represented as a trapezoid. In the trapezoid one can draw in the diagonals or the central bisecting line to determine the midpoint S. The central axis XX′ intersects YZ at X″, and the vanishing point through X″ (X″F1) intersects EG at H. The planes through AB and CD are constructed in the same fashion. As fig. 9 shows, the projections of the line segments AG″, YG and DG′ then intersect the horizon at O1, the distance point.

We now remind ourselves that the *costruzione legittima*, i.e., the geometric simulation of the visual image, always involves the perspectival deformation of a square. If so, however, in the image shown in fig. 7 the intersection point S″ must fall exactly in the middle of the boss of the ribbed vault, and the crown of the arch on the back wall of the *coretto* must lie exactly on H″ (and not just to the right of it) and the perpendicular from H″ must coincide closely with the window column. But this is not the case, and the hypothesis that the perspective of the *coretti* is based on the *costruzione legittima* does not appear to be confirmed.

The problem we are presented with here will be better understood when we take a look at the picture of the reconstructed ground plan of the *coretto* (fig. 10). It shows that its perspectively relevant components cannot fit into one square, but rather are distributed among three squares: the basic square (a) and two other squares identified as (b) and (c). These three squares are partially displaced against each other in the same direction – by the value of half of the width of a pier in the front arcade – so that together they form a rectangle. This is the solution to the problem of the perspectively *legittimo* deformation of a rectangle as well: It is divided up into individual squares, which then, each one for itself, is foreshortened according to the rules of the *costruzione legittima*. Three didactic drawings should help to make the method clear (fig. 11a-c).

In fig. 11a, ABCD represents the basic square that is to be deformed, ADF the visual triangle (= two-dimensional visual pyramid), F the vanishing point, O1 the distance point and FO1 the horizon. The straight line AO1 intersects BF at C′, the parallel line to AB through C′ intersects AF at D′. The trapezoid ABC′D′ is then the perspectively deformed square ABCD, to which we now attach a second square at AD with the vertices A′ and A″. The straight line A′O1 intersects AF at D′ and BF at C″. D′C′C″D″ are now the vertices of the trapezoid that has resulted from the deformation of the square A′ADA″. Both squares together appear deformed as the trapezoid ABC″D″ [Vignola 1583: 68-69]. If one connects the vertices by diagonals, the geometric center of the trapezoid is determined by their point of intersection and the extension of AC” intersects the horizon FO1 at O2. Such a “wandering distance point” would be mapped to each further square that was to be foreshortened, i.e., square 3 → O3, square 4 → O4, etc. In reality this means that the double square, seen from the distance FO1, as well as the basic square ABCD, seen from the distance FO2, both appear as the same trapezoid ABC″D″. This construction is *legittimo* because the perspectival deformation of squares lined up adjacently takes place in the same visual triangle, that is, in the same visual pyramid, whose apex lies at F or O1. The “wandering distance points” are not needed for this construction, but they can be applied in the simulation of a basic square mutated to a rectangle (see fig. 11c). This construction of the foreshortening of a series of squares was first described by Serlio and Vignola [Vignola 1583].

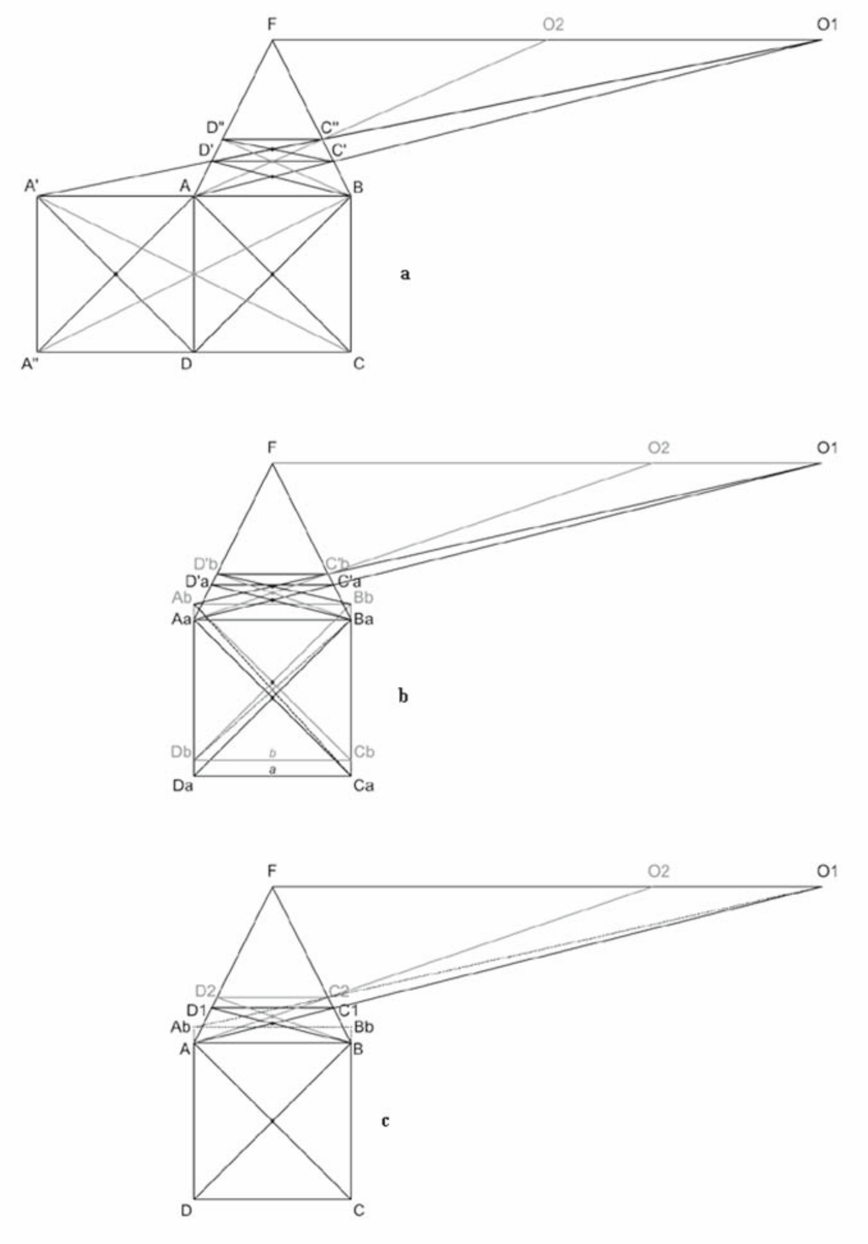

Fig. 11. The progressive *costruzione legittima*: didactic drawings

Fig. 11b and fig. 6 show that not only series of squares, but also congruent squares, partially displaced against each other in the same direction, (that is, together forming a rectangle) can be *legittimo* deformed. Connecting Aa with O1 results in the trapezoid AaBaC′aD′a at the points of intersection with the visual triangle; connecting Ab with O1 results in D′aC′aC′bD′b. The trapezoid AaBaC′bD′b represents the rectangle AbBbCaDa. The construction principle is that the *costruzione legittima* is applied separately to each of the two squares, but these must be mutually interlocked with each other. This is also seen at the points of intersection of the diagonals; (a) and (b) have separate midpoints, but they both share the gray point in between: the point of intersection of the diagonals of the rectangle. The same applies to the trapezoid. – Whether this construction has already been described before, I don't know. Fig. 11b is at the same time the model of the perspective construction of the *coretti* of the Arena Chapel. Fig. 6 demonstrates the construction on the perspective apparatus; the gray lines simulate the displacement of the square and the geometrical consequences.

Fig. 11c shows how an arbitrarily selected "wandering distance point", here O2, can help determine the extent of the displacement of the square. AO2 intersects BF at C2; the extension of O1C2 intersects the extension of DA at Ab. The line segment AAb represents the displacement.

Fig. 3 represents the two methods (Vignola's *due regole*) of the *costruzione legittima*, fig. 11 their two variants, which I would like to call static and progressive. I call static the deformation of the individual square; progressive a series of squares or their partial and monoaxial displacement to the figure of a rectangle.

Now we can better understand the ground plan of fig. 10a. The squares (a), (b) and (c) have been displaced against each other exactly by the value corresponding to half the width of the pillars of the front arcade; yes, the cross-sections of these piers themselves could be considered rectangular in the ratio of 2:1 (which can be easily measured and calculated). The round columns have the same ratio, one of which can be seen in the back corner of the *coretto*, the diameter of half the width of the pier. Hence the piers stand at the bottom between square (a) and square (b) and at the top (as suggested by Giotto) between square (b) and square (c); the columns stand accordingly between square (b) and (c) as well as between square (a) and (b). Their diagonals are drawn in with the points of intersection of Sa, Sb and Sc. Point Sab results from the interlocking of squares (a) and (b) through the diagonals AbCa and BbDa (fig. 10b). The ribs of the Gothic vault lie between AaCc and AcCa as well as BaDc and BcDa.

We now transfer the "displaced squares" to the "image" of the *coretto* (fig. 9a and fig. 12). Below the horizon the squares are pushed upwards by CaDa (Da, Db, Dc); above the horizon they are pulled down by AaBa (Aa, Ab, Ac). The straight lines of Db and Dc to the distance point O1a intersect the perpendicular line CaF1 at Gb and Gc, and the horizontals of these points intersect DaF1 at Eb and Ec. Squares (b) and (c) are also interlocked with square (a). The trapezoid DaCaGcEc is thus the *legittimo* foreshortened rectangle A′aB′aGcDc. If one connects Da with Gb and Gc, the extensions of these straight lines intersect the horizon (through F1) at O1b and O1c: These are the "wandering distance points" of the interlocked squares deformed into trapezoids.

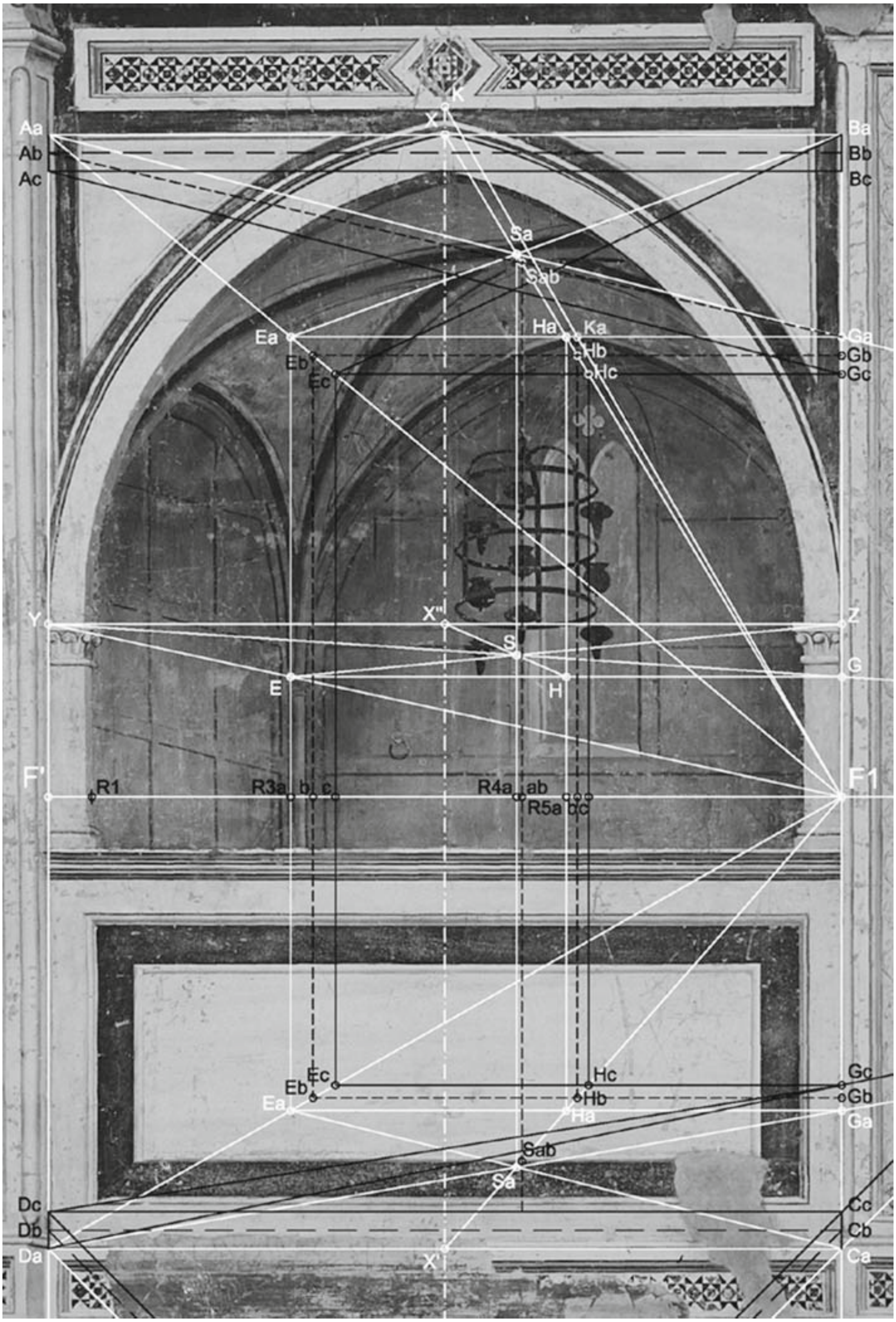

Fig. 12. *Coretto* and progressive *costruzione legittima*

These same operations are performed in the upper plane through AaBa. The straight line AbGa intersects F1X at Sab, and this point thus lies exactly at the point of intersection of the diagonals of the interlocked squares (a) and (b), that is, the rectangle AaBaCbDb (figs. 10a and 10b). In addition, Sab lies exactly along the rope holding the chandelier that hangs from the perforated boss of the ribbed vault (fig. 9c, fig. 12). The ground plan fig. 10a shows the interior organization of the *coretto*: The side of square (a) lies in front of the corner columns, (b) on the back wall, and in the center of (c) stands the window column. The three deeper planes – a sequence of planes stacked behind each other – of the perspective construction are the reason why the crown of the back wall arcade and the window column do not coincide with the perpendicular line through H. As shown in fig. 12, they should instead lay on the points of intersection of F1X with EbGb and EcGc, that is, Hb and Hc. The perpendicular of Hc falls indeed through the central axis of the window column, whereas Hb lies too deep, and the real crown (Ka) can not be determined until the perpendicular line intersects Hb with EaGa. The explanation for this apparent violation of the *costruzione legittima* can be found in the fact that the plane AaBaGaEa lies at the level of the boss of the ribbed vault; the crown Ka, however, represents the height of the vault caps, which appear to be supported by the sturdy ribs. The extension of F1Ka intersects the central perpendicular line through X at K; the line segment KX then corresponds to the "real" thickness of the ribs.

The distance points O1a, O1b and O1c on fig. 9a can be obtained by linearly connecting Da with the trapezoid vertices Ga, Gb and Gc (that is the foreshortened squares (a), (b), and (c) interlocked with each other) and projecting these straight lines to the horizon. The distance points are then transferred proportionally to the ground plan fig. 10a, as well as the distance point O1ab. In fig. 9a this point is obtained by laying a straight line from Aa through Sab, allowing its extension to intersect the horizon (not marked in fig. 9a; only in fig. 10a). Each of the distance points belongs to one of the squares (a), (b), and (c); only O1ab has already been created by interlocking square (a) with square (b).

Let us now compare the ground plan fig. 10a with the geometric construction in fig. 12. In fig. 10a the striking points of the perspective from fig. 12 with the related distance points are connected with each other through straight lines, for example, Da with O1a, Db with O1b and Dc with O1c.

The points of intersection of the straight lines with the bottom sides of the square are marked with a point on each of the corresponding sides a, b, c. If one wants to know at what location these points appear in fig. 12, one must first orthogonally project them to the line segment AaBa (fig. 10c). Why? We must imagine the image plane to be perpendicular above AaBa, the "intersection through the visual pyramid", on which all points that should be seen must be present. The progressive *costruzione legittima* works with several planes stacked behind each other, forming the projection surfaces of the squares mapped to them. Each point on these planes stacked behind each other, must then be projected onto the image plane a above the basic square (a), to become at all visible and measurable on the image. The orthogonal projection here leads to the interlocking of the projection surfaces of the squares (b) and (c) with the image plane of square (a). Their points of intersection on AaBa are denoted R, but for reasons of readability, not all the orthogonal projections are shown in fig. 10a. The position of the points of intersection to (a) can be measured equally well on (b) and (c). In fig. 12 the R points are marked on the horizontals F′F1 and admittedly not by transfer of the correct dimensions of those points from fig. 10a, but rather by orthogonal projection of the spatial points, for example, Ea, Sab, and Hc to F′F1. If one now wants to compare the position of the R points in fig. 10a with those in fig. 12, then one divides the length of the line segment F′F1 by that of AaBa, yielding the conversion factor n: One will then see that the positions of the R points coincide exactly in both figures. (The precision of the design and its pictorial execution is much greater than that in Masaccio's *Trinity* fresco.)

That means that the reconstruction of the ground plan (fig. 10a) must be just as correct as the reconstruction of the *costruzione legittima* in fig. 12. Furthermore, that means that we have here a mapped image of this *coretto*. Just as we can establish the real measurements of a body from the copy of it obtained with the help of the perspective apparatus, so, too, can we obtain all the measurements from the painted *coretto*, which would allow us to convert the painting into real architecture, albeit, mind you, Lilliputian architecture. The width AB of the *coretto* is ca. 146 cm, the height of AD 204 cm; the width of the piers in the front arcade is each the 22nd part of AB and amounts to 6.64 cm. The vaulted room with the vertices Ab, Db, Cb, Bb (see fig. 10a) "is" square and "has" a side length AB of 146 cm; the boss of the ribbed vaulting "hovers" at the height AD of 204 cm above the floor. (Here I am still stating the "real" position of the capital of the window column, whereby in fig. 12 I am selecting Da as the origin of an imaginary orthogonal coordinate system: height 176 cm, width 73 cm, depth 152.6 cm.)

The perspective, the "illusionism" of the *coretti* is often seen by art historians, but – as far as I know – has never been precisely geometrically analyzed.[4] By way of example I quote here Roberto Longhi, Samuel Edgerton und Walter Euler.

Longhi wrote:

> *All' effetto di veridica illusione convengono le due volte gotiche concorrendo ad un solo centro che è sull'asse della chiesa e cioè nella profondità 'reale', esistenziale dell'abside …* [Longhi 1952: 20].

We have seen that each *coretto* has its own perspectival vanishing point. Edgerton wrote:

> On the same triumphal arch wall Giotto painted yet another trompe l'oeil masterpiece, this time entirely in convergent perspective. … He apparently intended both of them to look as if their painted pointed arch frames continued the rib-vault construction in the illusionist architectural spaces. This may be the first postclassic example of what was to become the most popular perspective tour de force in all subsequent Renaissance painting: the illusion that the frame around the painting is not only fixed in the viewer's actual space but also integral to the imaginary structure depicted in the virtual space [Edgerton 1991: 80-81].

That is correct in principle. However, it is somewhat strange that the scholar, who has devoted himself entirely to researching perspective, in this book (on Giotto's geometry) dedicates only a few pages to the Arena Chapel, only a few lines to the *coretti* and avoids a clarifying geometric analysis; the *coretti* are apparently foreign to him.

Euler wrote:

> *Die Grenzen dieses Illusionismus fallen jedoch bald auf: nicht allein die unpräzis durchführte Linearperspektive – die Tiefenlinien der Wandgliederung verlaufen fast im Sinne der umgekehrten Perspektive –*

sondern überhaupt die Diskrepanz zwischen der Ausdehnung auf der Fläche und der dargestellten Räumlichkeit… [Euler 1967: 68-69].

It is correct that the (imaginary horizontal) stone bands on the walls of the left wall of the *coretto* take a course that is not at all consistent with the perspective construction of the whole, and cause confusion at the same time. But the "reverse perspective", which Giotto liked using in other places in the Arena Chapel (see fig. 13), does not come into play here either. This clearly shows the exactly constructed reversal of the *costruzione legittima* of the *coretto* in fig. 14. If Euler is correct, the stone bands would have to have lain on straight lines ("visual rays"), emanating from the vanishing points F4, F5 or F6 – but they don't do that!

Fig. 13. Arena Chapel. Detail of the arcade of The Last Judgment. "Reverse perspective" (Photo reproduced courtesy of the Museo civico, Padua)

In fact, the stone bands follow an entirely different geometrical construction which has nothing to do with perspective (fig. 15). We draw horizontal straight lines along the lower edge of the horizontal stone bands of the back wall, until they hit the oblique stone bands on the left and denote these endpoints with a, b, c. The next step is to plot the line segment G′Z from G′ with the compass to G′O1 at a. After dividing by the square root we obtain the following geometrical series: G′a : √2 = ab; ab : √2 = bc. If one connects these points (a, b, c) with the a, b, c points in the corner of the *coretto* by a straight line and extend this to the perpendicular line AD, these straight lines run exactly along the bottom edge of the oblique stone bands. We end up with the points of intersection a, b, c on AD, and they are equidistant from each other: ab = bc. As mentioned earlier, the strange construction has nothing to do with the *costruzione legittima*; at first sight it looks like the solution to a difficult geometrical task – but it is actually quite simple. It is enough to fix the points a, b, c equidistantly along AD and G′O1 in the sense of the √2 progression. By linearly connecting the points and after arbitrarily intersecting this great number of straight lines to the corner of the *coretto,* the height of the stone bands is produced as a secondary result. It is self-evident that Giotto, instead of orienting these stone bands central perspectively to F, consciously created a striking disruption in the perspective construction. Why? That will remain a resolvable enigma for the time being. In any case these apparently rather awkwardly drawn stone bands have prevented the art and perspective researchers from considering the *coretti* to be worthy of more detailed examination.

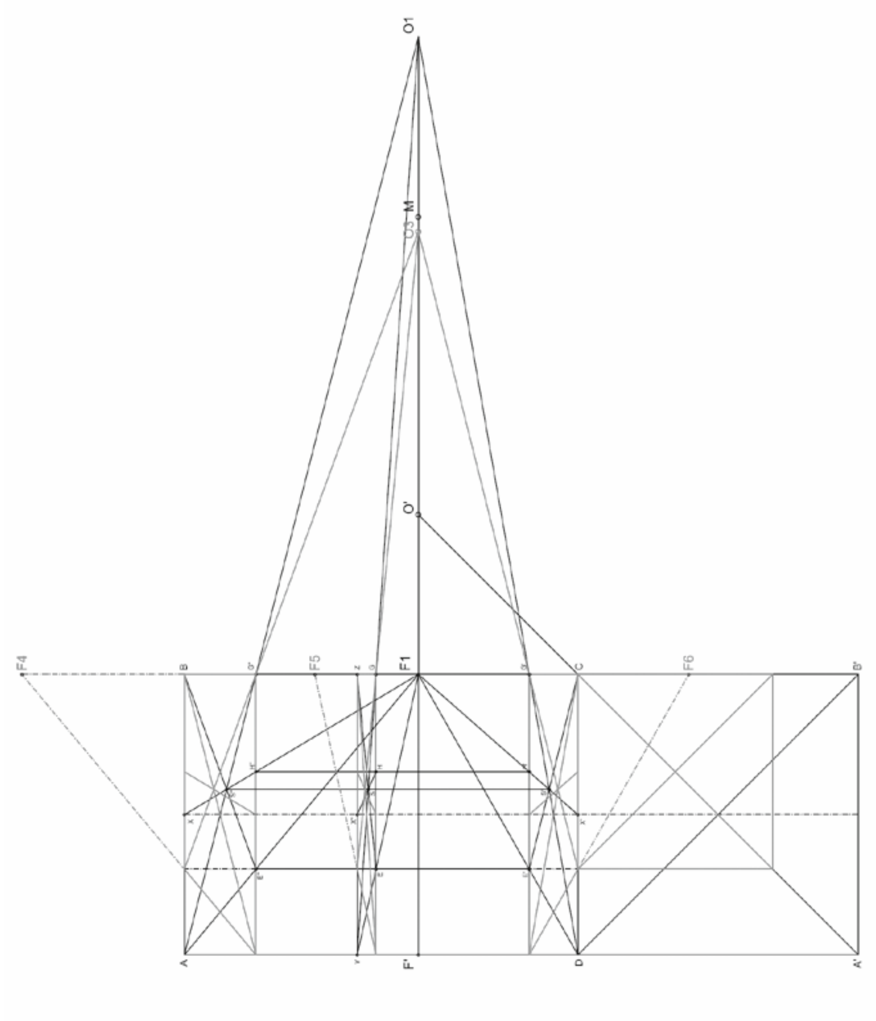

Fig. 14. Static *costruzione legittima* of the *coretto* and construction of “reverse perspective”

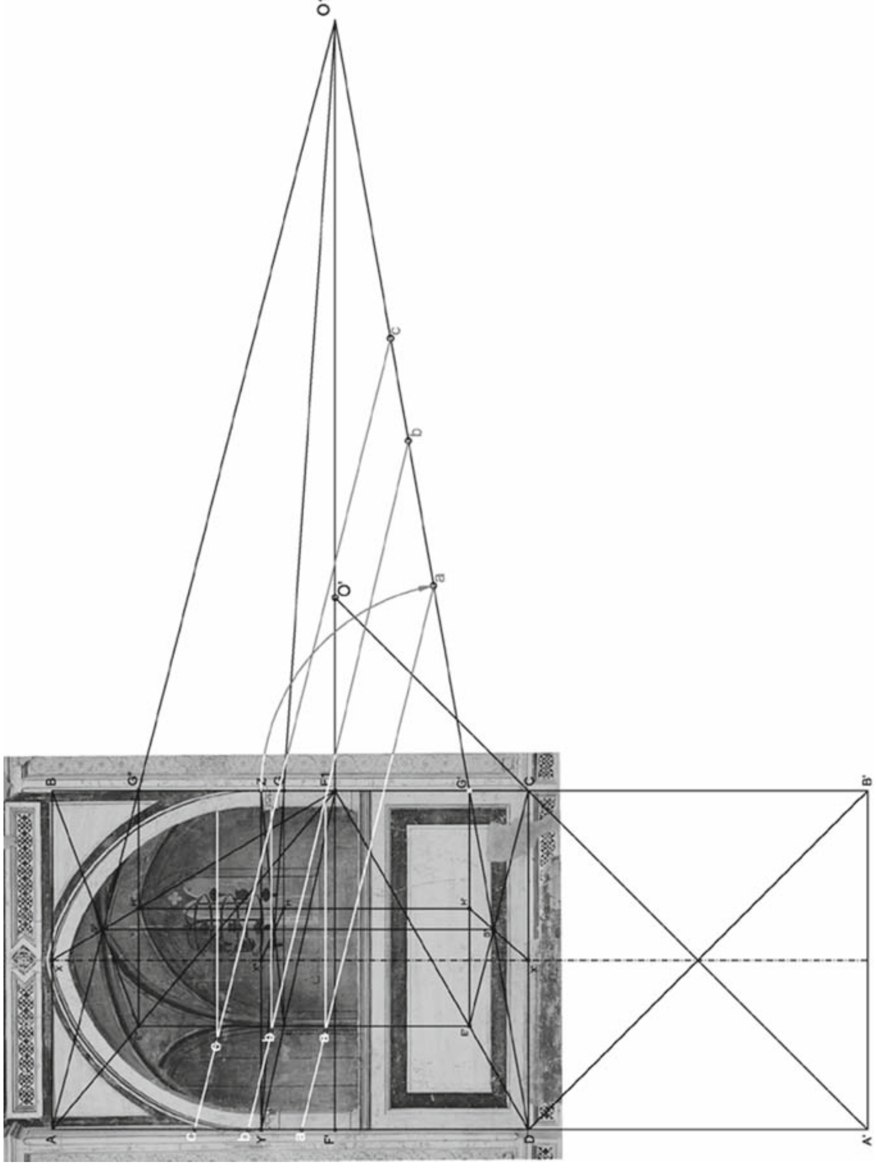

Fig. 15. . Coretto. Geometric construction of the oblique stone bands

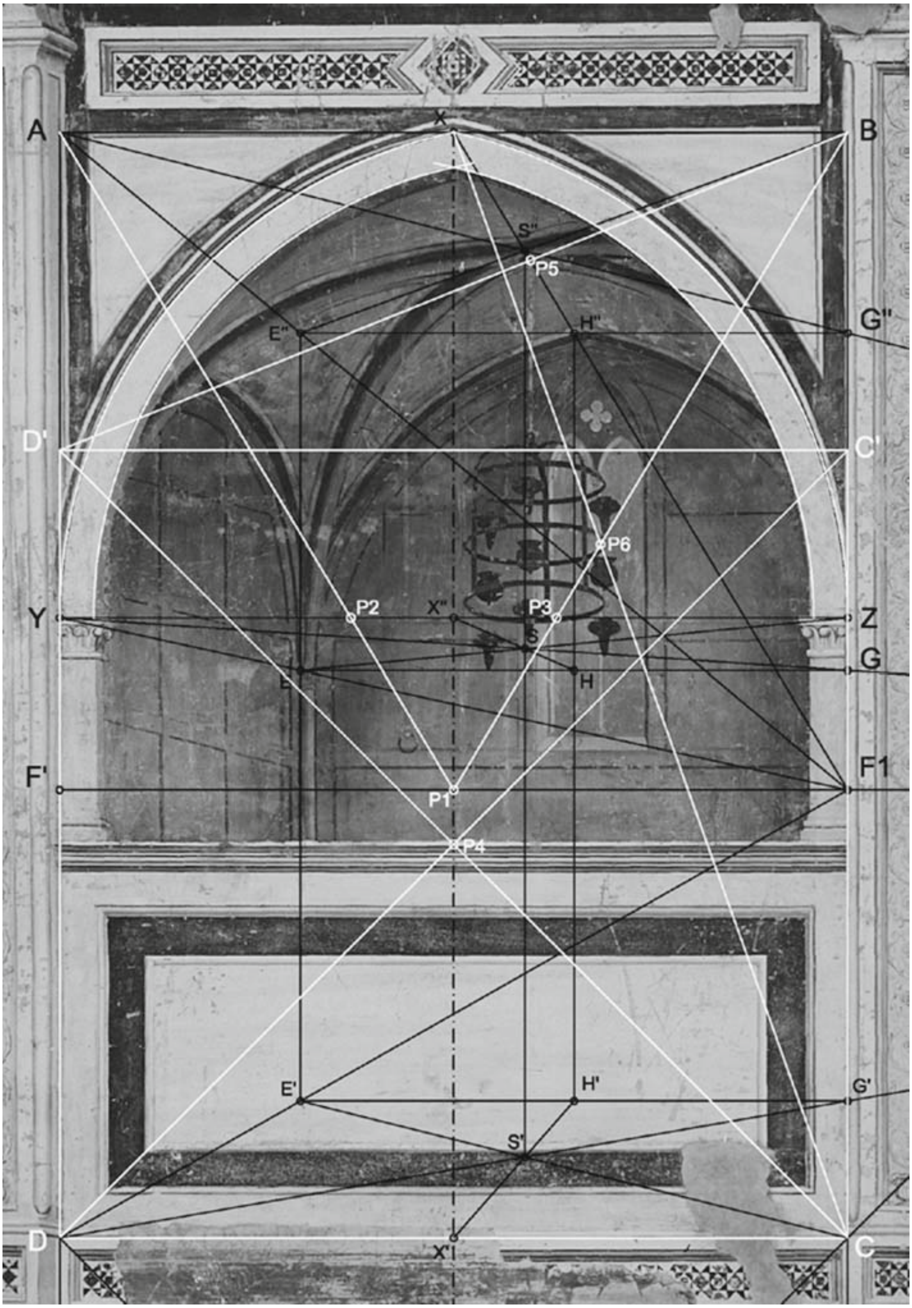

Fig. 16. *Coretto* : Static *costruzione legittima* and surface geometry

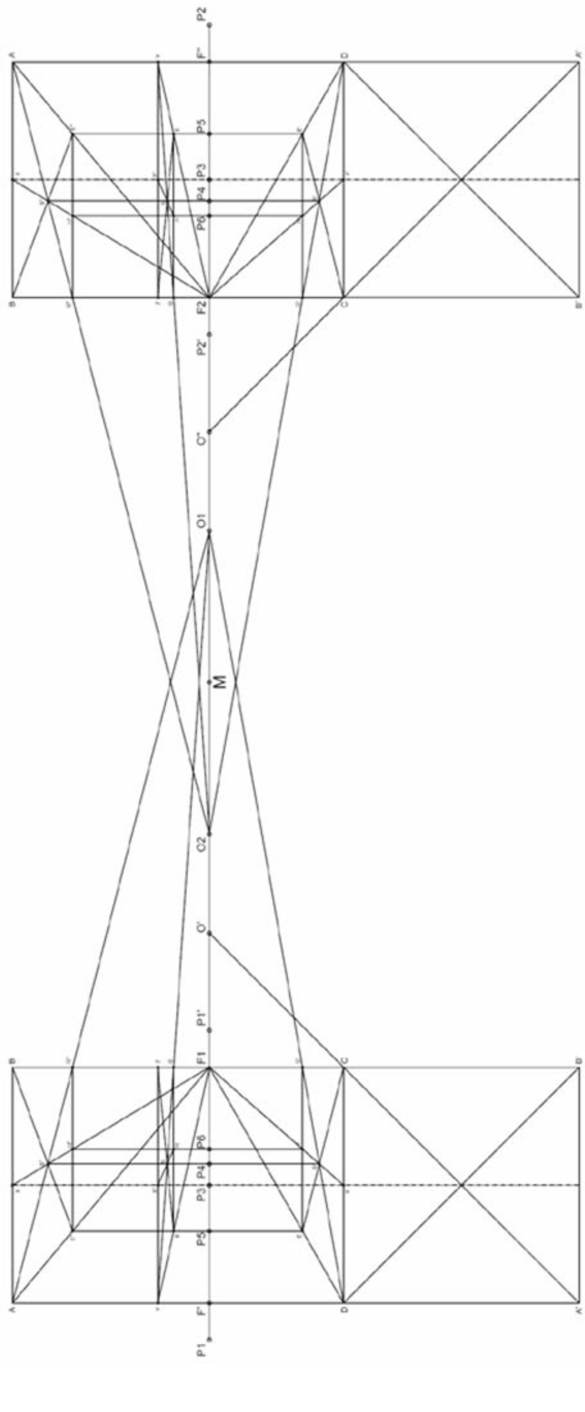

Fig. 17. The reflected static *costruzione legittima* of the two *coretti*

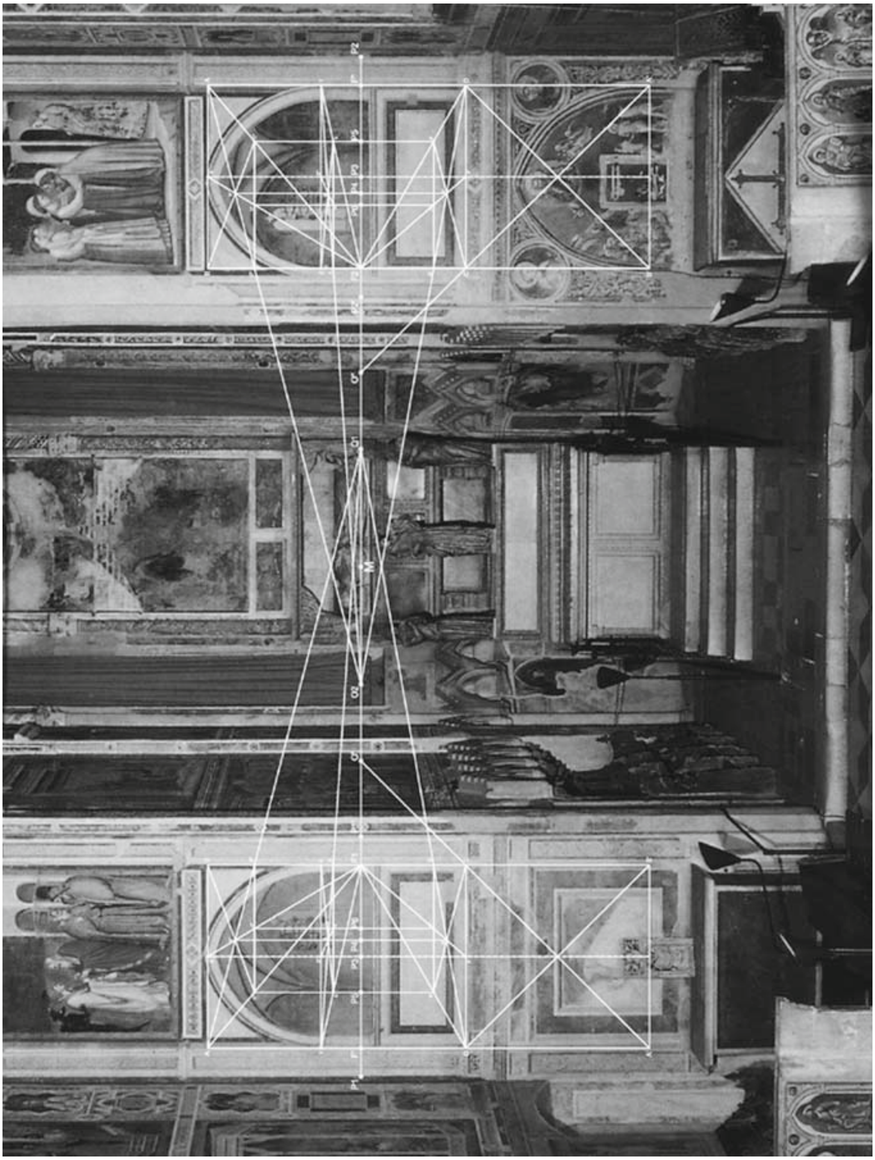

Fig. 18. As fig. 17, projected into a photo of the choir wall of the Arena Chapel

Using surface geometry, independent of perspective construction but still linked to it, Giotto was not sparing in any of the design of the *coretti* – see fig. 16. If one draws the square DCC′D′ over DC, then its diagonals intersect at P4, that is, exactly at the height of the parapet. If one connects P1 (the point of intersection of the central perpendicular lines XX′ with F′F1) with A and B, then P1A and P1B intersect the horizontal YZ at P2 and P3. These are the exact midpoints of the arc of the front arcade: D′B intersects F1X at P5, that is, the point Sab that resulted from the *costruzione legittima*; P1B intersects CX at P6, that is, at the place where the window column stands in terms of surface geometry; the arc around C with CD passes through E. The analysis could go on like this.

The right *coretto*, so it seems, is the mirror image of the left one and vice versa. By reflecting the left perspectival basic construction on the horizon around the midpoint M, a symmetrical line drawing results (fig. 17). If one projects this drawing, whose left half is totally adapted to the image of the left *coretto* (because it was developed from it), onto a photo of the lower section of the chancel arch (fig. 18) it can be seen that on the right side there is a slight displacement between the "image" and the drawing. This displacement may be due to the photo, which was not taken exactly along the central axis of the chapel; however, it could also be that the choir wall is not entirely symmetrically built. Final clarity can be established only by a photogrammetric photograph. So, for the time being, it must remain open as to whether the perspective constructions of the two *coretti* coincide so exactly with each other that one could speak of a mirror image. Yet this last statement is what I consider a prerequisite for a working hypothesis in the following analysis.

Fig. 17 shows the *costruzione legittima* of the left *coretto* and its reflection. It is striking that quite a number of partial line segments along the line segment P1P2 exhibit the golden ratio *(sectio aurea)* to each other. I measured them with proportional dividers and listed them with the *minor* on the left, the *maior* on the right.

MINOR	*MAIOR*
F′O1	F′F″
P1O′	P1M
P1P1′	P3M = P1′O1 = O′O″ = P1O2
P1P6	P1P1′
F1O1	F′O″
F′F1	F1M
P1P3	P3O′ = MO′
BZ (=ZG′)	YZ

The divisions listed here are subject to the above-mentioned admissibility of the "mirror image", and they were certainly not all intentionally made: many may have been a secondary result. However, I have no doubt that Giotto applied the *sectio aurea* here, as an old and noble proportioning method. I forgo further conjectures and am confident that it will soon be possible to photograph the choir wall of the Arena Chapel photogrammetrically and analyze it more precisely.

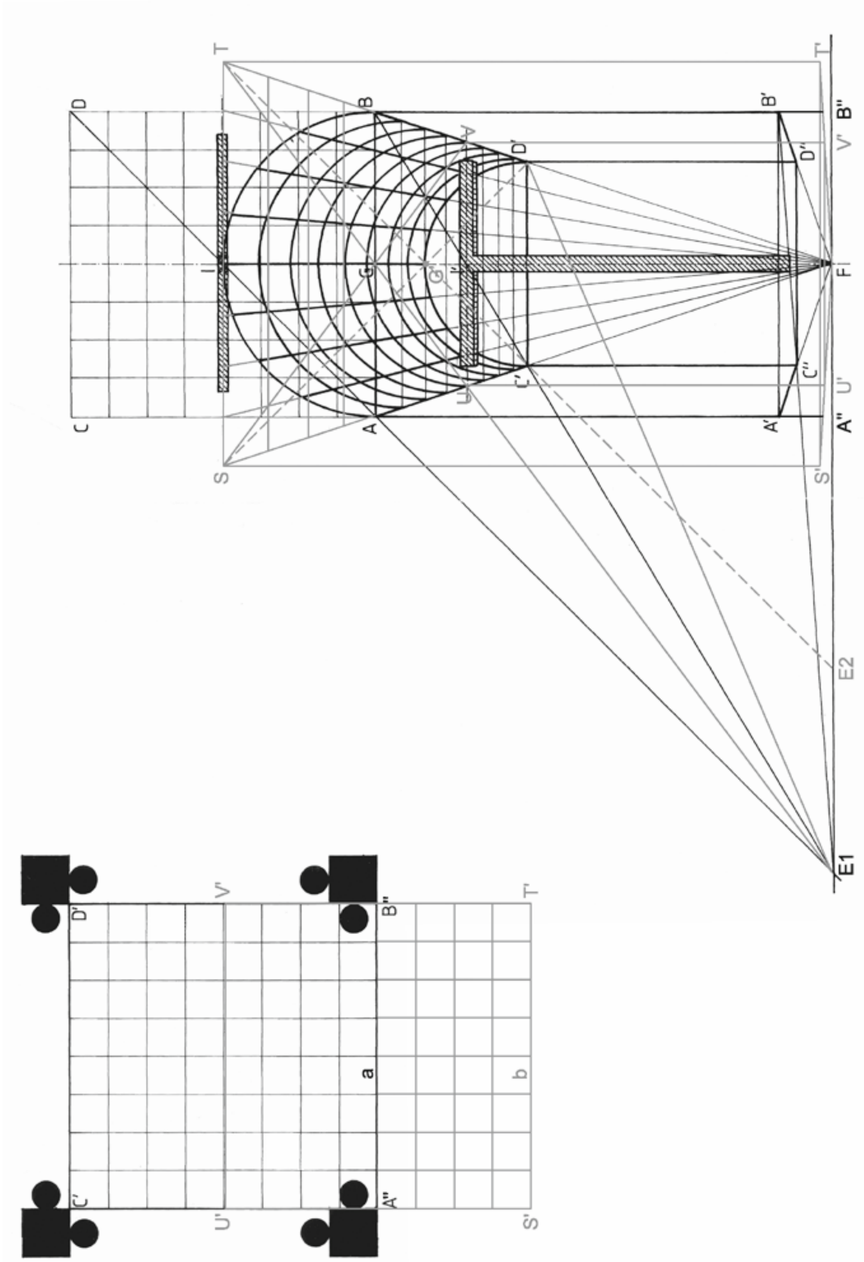

Fig. 19. Masaccio, Trinity fresco: Ground plan and frontal perspective with displacement of basic square

Let me add a comment on the "illusionism" of the *coretti*. Despite elements of surface geometry, they actually represent central perspective: they have been designed and painted according to the *costruzione legittima*, that is, as visual images. Yet the two *coretti* have no common vanishing point and their vanishing points F1 / F2 lie at more than twice the eye level of an observer. Let us image that the *coretti* are constructed architecture: we would then have to climb scaffolding to bring our eye to the position of F1 / F2 and the distance of O1 / O2 to see the constructed *coretti* exactly as Giotto painted them. From the floor of the chapel we would have a different visual image of these constructed *coretti* and, similarly, the painted *coretti* appear distorted to us from the floor. However, this distortion is so slight that it is hardly noticeable. We are well aware that Giotto's *coretti* are simulations of visual images based on the *costruzione legittima*, but these simulated visual images have no natural – that is, no geometric – reference to the observer: vanishing point and eyepoint do no coincide at one point. The term "illusionism" should be avoided here. However, it is striking that Masaccio in his *Trinity* fresco (fig. 1) places the eyepoint at the eye level of a person of normal height (167 cm) and a few years later Alberti in his *Pittura* (1435) makes precisely this – the identification of eyepoint and vanishing point in wall painting – a formal principle.[5] This is the evolutionary step the "Renaissance" took beyond Giotto in the direction of illusionist painting.

Finally, I would like to demonstrate that the method of "progressive *costruzione legittima"* which I have described in the *coretti* was also used by Masaccio in his *Trinity* 130 years later (figs. 1, 19), not to expand the pictorial space to the back, but rather to the front, to the observer, beyond the wall defined as the projection surface. My brief demonstration will not be readily understood by those readers who are not familiar with my Masaccio article [Hoffmann 1996].

Fig. 1 shows the entire fresco, heavily restored in many sections. The points marked F (= vanishing point), A and B (AB = width of the basic square a) coincide with the corresponding points on fig. 19.[6] The frontal perspective is created as follows: The basic square (a) with the vertices ABDC is drawn along the line segment AB and divided into 8 x 8 = 64 squares corresponding to the 64 coffers of the barrel vaulting (see fig. 1). Linearly connecting these panels to AB with F produces the trapezoid ABD′C′, through which the coffered barrel vaulting is represented by semicircles. In front of the "wall" and in front of the vaulted canopy with the "Trinity" inside the donor pair kneels on a platform, which lies just slightly above F. The question is then, what "real" depth must this platform have to provide room for the donor pair? Because the space above the basic square (a) is already filled with the figures of the Holy Trinity, we must create additional perspective space (fig. 19, left) by displacing the square a to the front (and then calling it b). On the frontal perspective (fig. 19, right) we (mentally) displace the a-square (= ABDC) upwards by half its width, so that its base lies on I. The extensions of the horizontal lines through I and the orthogonals FA / FB intersect at S and T. ABTS is then the half basic square (a) deformed into a trapezoid, which – projected onto the legs of the visual triangle FT′S′ – yields the donor platform with the vertices S′T′B″A″. Because the side AB of the real basic square is 210.5 cm long, the "real" depth of the platform is then 105.3 cm. The life-sized donor figures thus have enough room to kneel. On the frontal perspective it can readily be seen that the expansion of the space gained by the displacement of the square will also always produce a homogeneous perspective of the whole space.

Conclusion

I believe I have proved that the *coretti* were designed and painted according to the rules of the progressive *costruzione legittima* and thus represent simulations of visual images. Because no images of this type have come down from Classical Antiquity and because the literary references remain silent in this respect – as Berthold Hub demonstrated recently in a comprehensive study [2008] –, the *coretti* must be considered, according to today's knowledge, as the oldest monuments manifesting the application of the *costruzione legittima*; that means the history of the central (linear) perspective must be rewritten. In any case it was not a Renaissance invention. I expressly agree with the researchers[7] who see Giotto's painting in conjunction with the findings of the Scholastic "optics specialists" (such as Grosseteste, Witelo, Bacon), all of whom stood with their feet firmly planted on the ground of Euclid's rigidly geometrically conceived visual theory and its Arab commentators.

Proof is still missing, unless one would allow the *coretti* to constitute the proof of Giotto's intimate knowledge of the Euclidean-scholastic visual theories. I have distinguished between the image producing perspective (simulating the visual image) and the copying perspective (fixing the visual image). The invention of the perspective apparatus by the young Brunelleschi (ca. 1400), in which the cross-section through the visual pyramid was laid by means of the grid of threads, thus yielding the perspectival copy of a real body also assumes knowledge of the Euclidean visual theory. Looking at it like this, it also seems to me that this invention fits better to the years around 1300 than to the later years around 1400. That must be clarified more precisely.

Acknowledgment

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Notes

1. Now the clearest explanation of Alberti's method is that given by Tomás García-Salgado [1998].
2. Cf. [Vignola 1583], in particular on p. 18 the ingenious figure under *teorema terza*, depicting the *due regole* and proving at the same time that they lead to the same result; Vignola-Danti also mention that Serlio had already described but incorrectly depicted the *due regole*, a mistake, *il quale nasca dalla stampa*. See also [Sinisgalli 1978: 62-63].
3. H. Wieleitner [1920], for example, would like to see the term reserved for the "visual ray construction" (Vignola's *regola prima*); see [Hoffmann 1996: 76]. See also [Grayson 1964] and [Parronchi 1964].
4. The most important writings on this are mentioned in [Kohnen 2004].
5. Alberti, Della *Pittura*, Libro Primo, 19: "*…però che così e chi vede e le dipinte cose vedute paiono medesimo in suo uno piano".* Tomás García-Salgado particularly emphasized the significance of this passage and correctly interpreted it as follows: "The observer and the central vanishing point are the extreme points of the symmetrical line of sight: In the Albertian model, the central vanishing point is given by the central point, which must correspond to the

observer's eye level so that what the observer paints is on the same floor as the painter" [García-Salgado 1998: 120].

6. The vertices C′ and D′ in fig. 1 lie a little higher than in fig. 19, because when Masaccio painted his frescoes he deviated by 4.5 cm from his design. The black lines in fig. 19 show the reconstruction of this former design. This is of no importance here for the interesting question of "displacement of the square".
7. Specifically, see [Bergdolt 2007]. In general, see [Lindberg 1976].

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