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Research

The N4C Joint

Abstract. This paper discusses the discovery that four oblique prisms with a square cross section can intersect, forming a joint by means of a notch parallel to the horizontal plane. The use of this joint in new constructions is explored.

Introduction

Whilst drawing, I discovered by chance that four oblique prisms with a square cross section can intersect, forming a joint by means of a notch parallel to the horizontal plane. I am a sculptor, but my love of architecture led me to develop this joint for use in new constructions; having drawn and executed a large number of sculptures, I felt the need to change the direction of my work and direct it towards the 'useful', and the sculptures were what led me in this direction. Reading and studying the principles of analysis of shape, and the philosophical texts on light and space, and on interior and exterior spaces, written by the great architects of the first half of last century, along with my need to build rather than model, led me to bind together modules and spatial networks based on this joint.

The demonstration of this regularity had to obey one of the elemental theorems of plane geometry, that is, a triangle inscribed inside a semi-circumference whose longest side is equal to the diameter, is a right triangle (Euclid's *Elements*, Book IV, Proposition 5). The longest leg of the triangle and the angle it forms with the hypotenuse is the line sought, that is, the inclination of the members forming the joint. But another condition had to be fulfilled to make the joint possible: the space separating the two parallel members had to be equal to the length of the side of the cross section of the member (fig. 1).

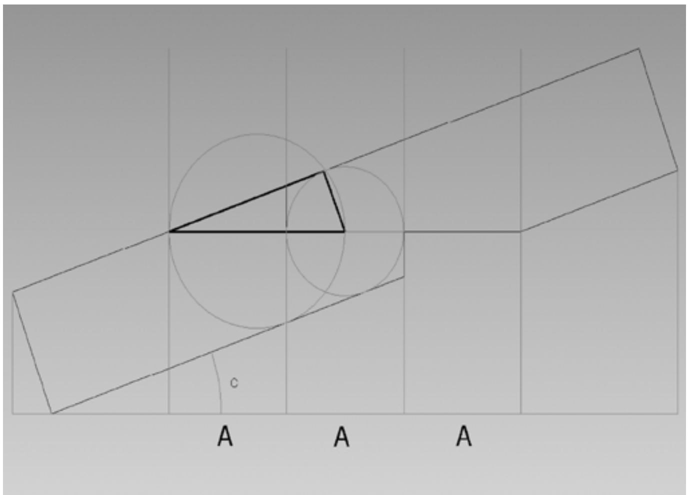

Fig. 1.

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Having discovered the starting point of the oblique line and that the angle it forms with the horizontal is 19°, I then explored the reason for this union. If we look at the projection of one of the members on the vertical plane (in a drawing) and draw a horizontal line which intersects the two oblique lines or edges of the prism, the a-a′ segment which is formed is three times larger than the distance between the two parallels (section of the prism). Dividing this segment into three sections and drawing a perpendicular through these points, two right-angle triangles are formed, and the two legs are the projections of the planes which intersect the prism forming a notch for its assembly; the longest leg is the side of the square and the bearing and contact horizontal plane (fig. 2).

If we now look at the projection of the prisms on the horizontal plane, the horizontal line mentioned previously is the side of a square; this square is equal to a horizontal plane which contains the four bearing surfaces of the four prisms forming the joint, these being separated from each other by another square of the same dimensions (fig. 3).

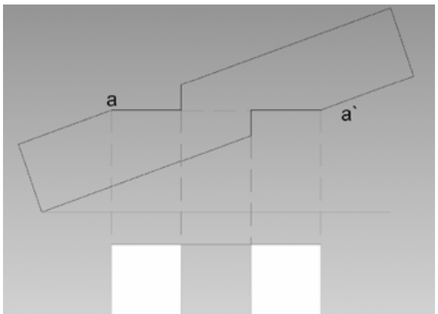


Fig. 2

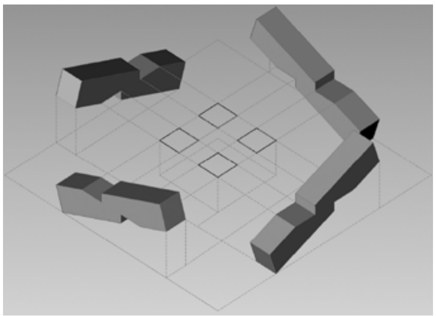


Fig. 3

Once the four bearing surfaces are situated on the same plane, we can repeat these notches all along the prism to form different compositions. If the distance between notches is equal, a three-dimensional space network is formed.

The first piece executed using this joint was "Four squares inscribed within a cube" (fig. 4), a composition formed by two pairs of squares parallel and perpendicular to each other creating two joints aligned on the same vertical line. This sculpture gave the joint its name: N4C, since this closed construction is the smallest one that can be built with two parallel joints; the other pieces are more complex compositions.

One of the complex shapes that can be created using this joint exists in Mandelbrot's fractal geometry. I call it the "cardoon", given its similarity to the artichoke thistle in the way it grows (fig. 5). An N4C joint leaves four ends free; if we extend them in order to make another joint we obtain four more joints, if we repeat this operation on each of these new joints, we will create a geometric progression. Of greatest interest in this progression is that the different joints can be joined together using one of their members, creating a structure that is interlinked and has infinite possibilities for growth (fig. 6).

The application of the N4C joint in architecture

In the construction sector, using this joint to connect up to four members with a rectangular section brings to mind prefabricated elements for the rapid installation of a metallic structure, after the fashion of architecture using steel and glass.

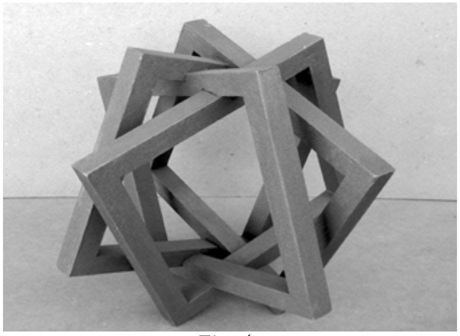


Fig. 4a

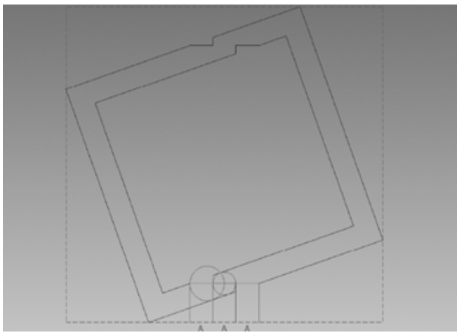


Fig. 4b

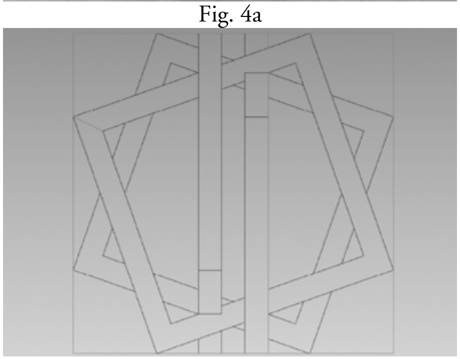


Fig. 4c

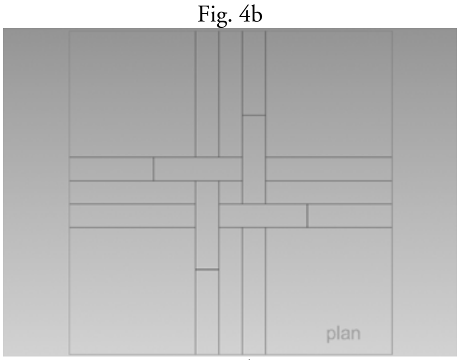


Fig. 4c

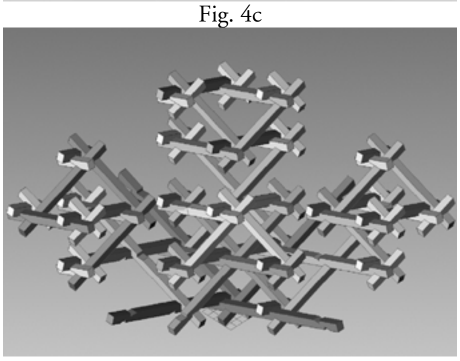
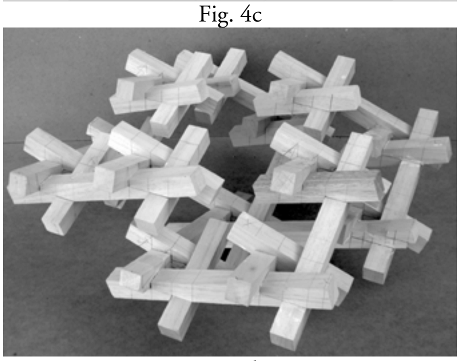


Fig. 5



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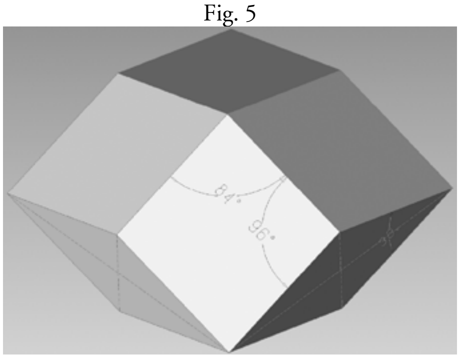


Fig. 7a

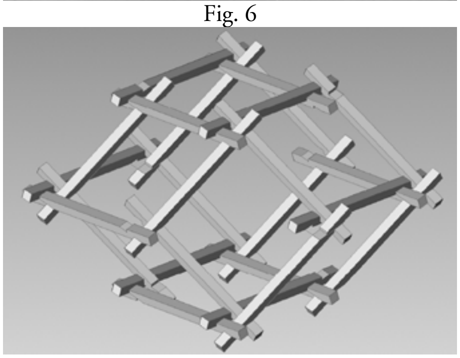


Fig. 7b

The starting point is the rhombic dodecahedron which can be formed using this joint. (fig. 7a,b). Placing the two longest axes of the prism on the horizontal plane, our working plane is the four upper faces (84° and 96° rhombi), which join at their upper vertices, their lower vertices being the bearing points on the horizontal plane, and with a 19° angle. As an alternative, by placing the polyhedron with its two longest axes on the vertical plane, the four aforementioned faces in an oblique position closer to the vertical plane; their angle to the horizontal is 71°, these being supported by one of their vertices on a vertical plane (fig. 8a, b).

If we substitute the arrises of these faces (rhombi) with rectangular prisms with a square cross-section (steel pipe) and we make notches at their ends, we form a frame. If we then make various notches on these four members at equal distances, we create a mesh whose elements are parallel to the sides of the frame (rhombus) (fig. 9).

By taking the four faces of the upper half of the polyhedron in the vertical position, we obtain four frames which are two by two parallel, and the properties of this joint allow any member in one of the planes to be connected to each one of the joints in the other three frames. This can be clearly seen by making a construction similar to the rhombic dodecahedron mentioned above using lengthwise members with a square cross-section (fig.10).

The compositions that result from using the planes of the polyhedron faces in the horizontal position resemble pyramid shaped roofs, the four arrises (beams) joining at the upper vertex in the N4C joint, and supported on the horizontal plane formed by the ground, or completing the entire grid of the face of the rhombic dodecahedron, creating an open structure and giving more floor area (fig. 11a, b).

These four members or arrises must have two notches along their length and at a separation distance equal to that of the crossbeams making the grid. The remaining members merely require a notch on one of their faces to permit intersection.

Using beams with a set number of notches along the length of their faces and the same separation distance between them allows us to alter the surface of the faces and thus the ground plan projection of the roofing formed by the four frames. The relation between the section of the beam and the separation of the notches, and the length, provide us with the formula for calculating the strength of the entire structure.

This joint makes it possible to create compositions with a square ground plan, joining several equal constructions together, rotating them and leaving spaces between them (fig. 12).

In the same way, by using roofing made of beams of different lengths they may be interconnected, thus increasing useable floor space (fig. 13).

Finally, if the stresses acting on the structure make it necessary, this joint makes it possible to reinforce its interior area, strengthening the wall where it joins the horizontal plane of the ground, through the connection of the four members that make up the joint, while at the same time leaving greater height in the access points to the interior (fig. 14). We can also brace the upper vertex of the pyramid on the inside by using a structure that is the inverse of the one built.

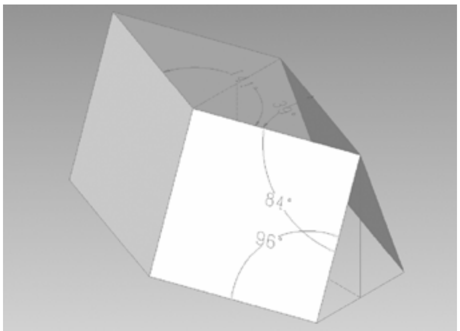


Fig. 8a

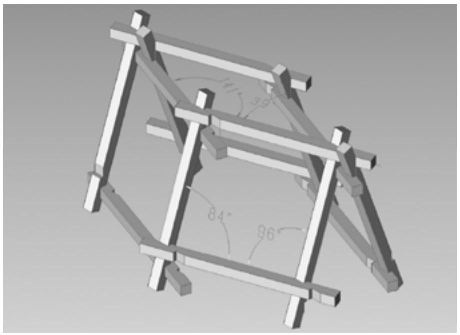


Fig. 8b

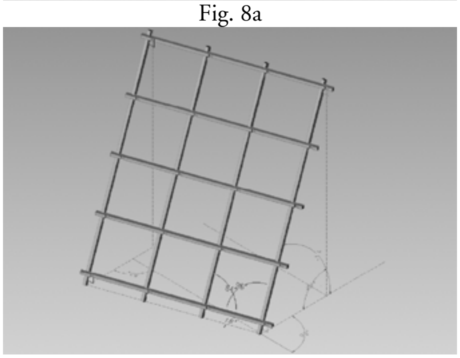


Fig. 9

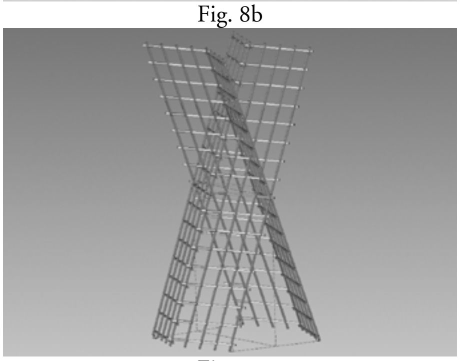


Fig. 10

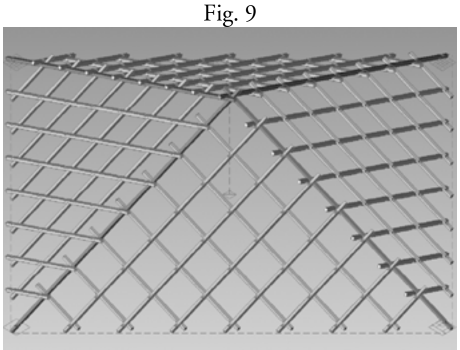


Fig. 11a

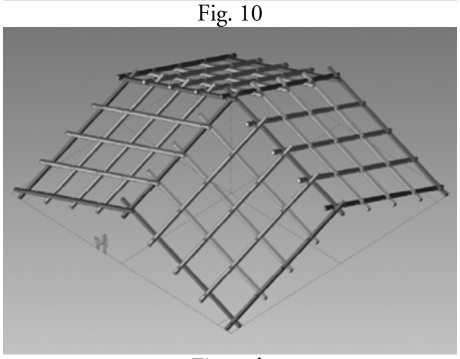
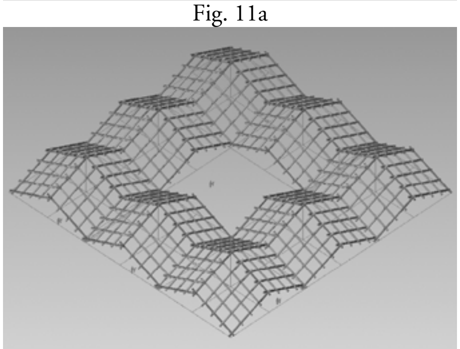


Fig. 11b



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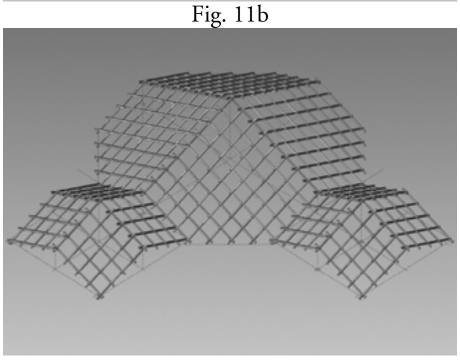

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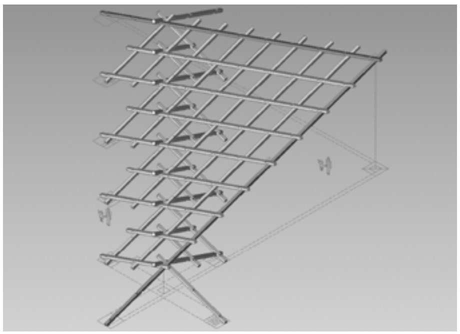

Fig. 14

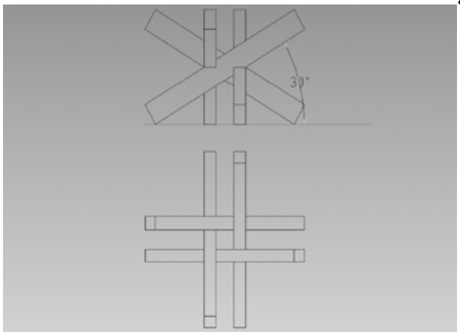

Fig. 15a

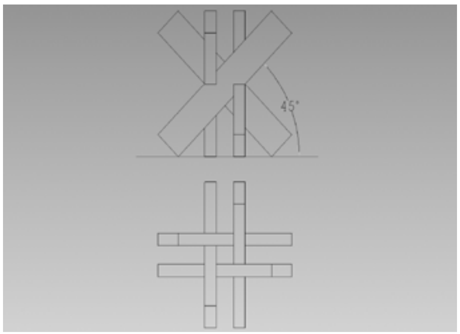

Fig. 15b

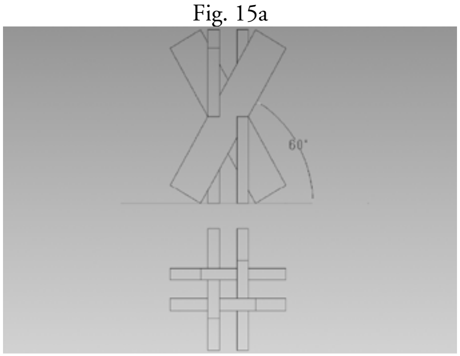

Fig. 15c

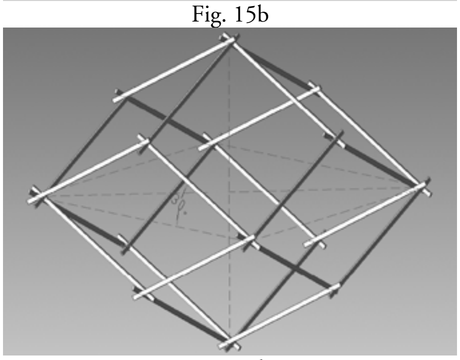

Fig. 16

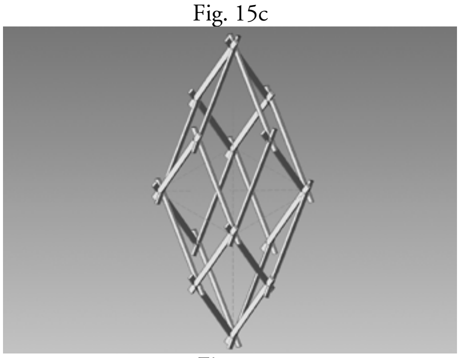

Fig. 17

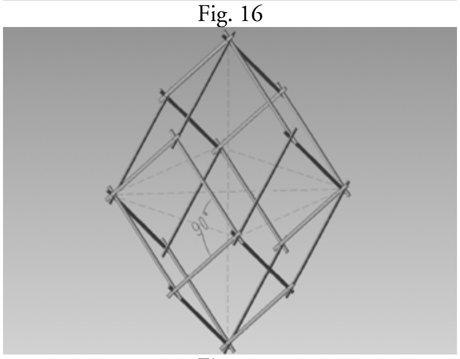

Fig. 18

N4C with different angles of inclination

The essential pre-condition required for the joint is that the four bearing surfaces be on the same horizontal plane. Increasing the angle of inclination makes the horizontal section of the prism smaller, so its width must be increased to allow room for the notches on the one hand, and on the other the separation distance between them must be sufficient from the perspective of beam strength.

Fig. 15 shows three examples of this joint with prism inclinations to the horizontal plane of 30°, 45° and 60°.

For this reason, the section of the prism must be rectangular, with the longer side positioned vertically and the shorter side supported by the other prism. In the case of the 30° and 60° angles, the polyhedron formed is still a rhombic dodecahedron (figs. 16, 17) while in the case of the 45° angle the polyhedron has eight rhombic faces and four squares (fig. 18).

Building with these angles still results in pyramid shapes whose upper vertex becomes more acute as we increase the angle (figs. 19, 20, 21). One of the properties of this joint for configuring different compositions is that the notches of the four members are on the same horizontal plane (fig. 22), which means that a number of these constructions can be joined together on the horizontal plane (see fig. 12).

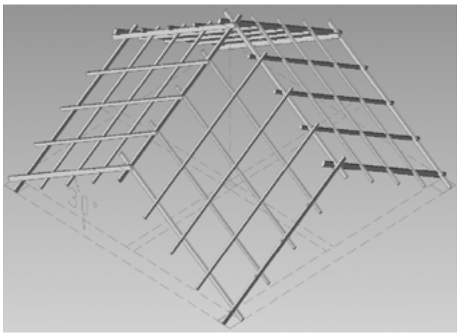

Fig. 19

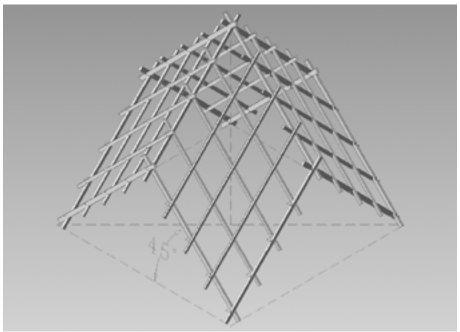

Fig. 20

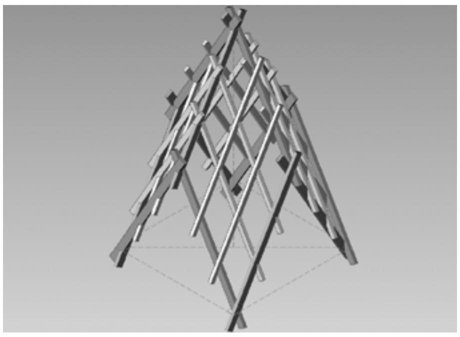

Fig. 21

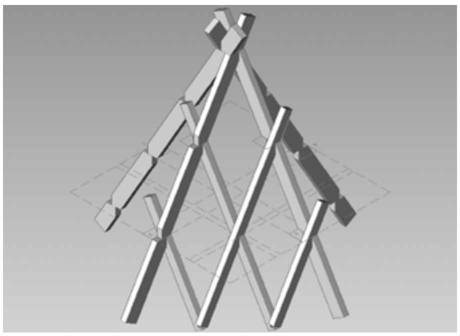

Fig. 22

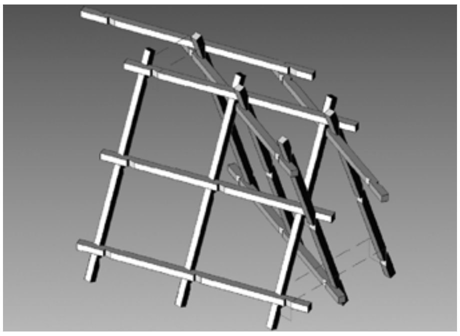

Fig. 23

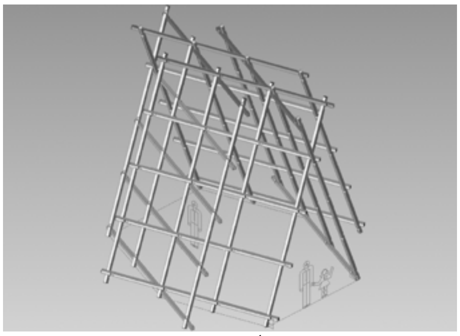

Fig. 24

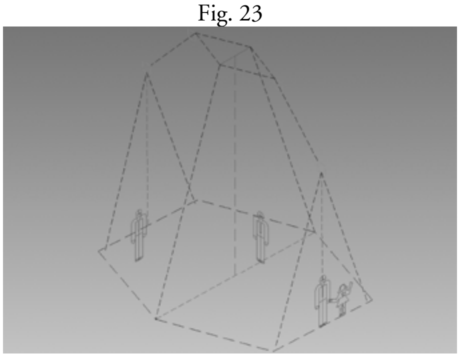

Fig. 25

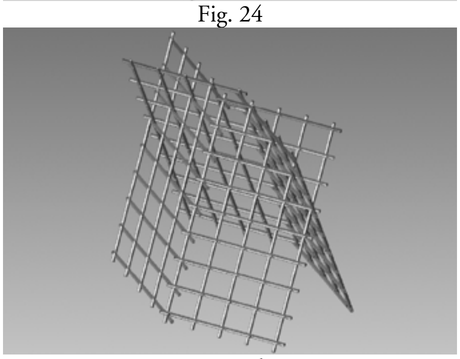

Fig. 26

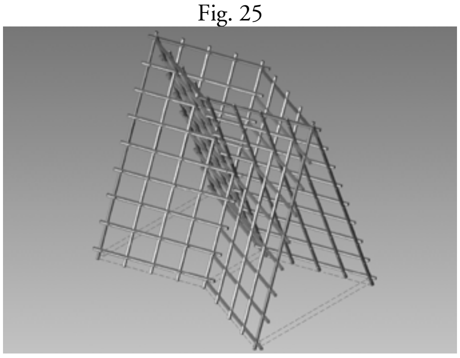

Fig. 27

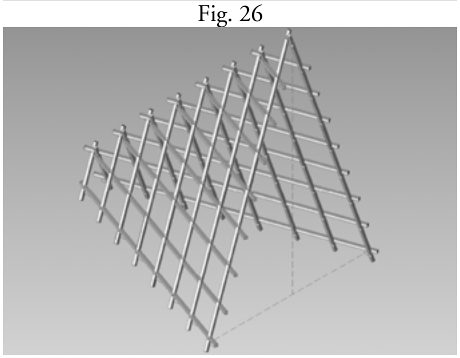

Fig. 28

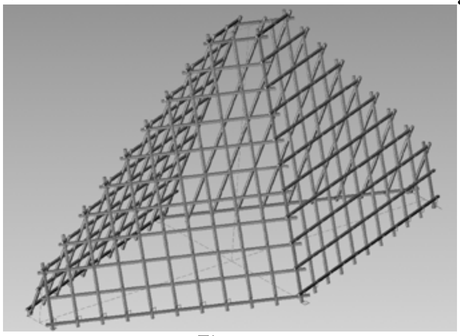

Fig. 29

Taking the working area to be the four faces of the upper half of the polyhedron in the vertical position, the planes offer greater slenderness as their angle of inclination to the horizontal plane is 71°, even though the characteristics of the joints between them are very different. Their arrises intersect, but this is achieved by means of the N4C joint which enables the four members to intersect (fig. 23).

Substituting the arrises of these faces of the polyhedron, forming a frame and creating a structure as in the previous example (the same flat structures, but rotated), four planes which are two by two parallel are formed, which we can imitate and intersect (fig. 24). Let us examine the upper arris of each of the planes: we make them intersect at one point and at the same distance from their ends, the members forming the structure always intersect where the notches which allow intersection and formation of the joint are located. The interior space created by this construction differs from the previous example; its height is three times that of its ground plan projection; and an atypical 'vault' is formed by the junctions or joints where the four planes intersect (fig. 25).

As in the example given previously, using beams of different lengths but with equally spaced notches results in a wide variety of compositions that can be executed.

I call these dihedral plane compositions "vertical architecture", to differentiate them from pyramidal roofs. A greater number of compositions can be created with these dihedral planes. Some examples are: according to the chosen line of intersection between two of its planes (figs. 26, 27); interrupting the continuity of the member at the joining line (figs. 28, 29); positioning the dihedral planes with the concave space on the inside (fig. 30); with this space on the outside (fig. 31); significantly changing the shape of its ground plan (figs. 32, 33). It is clear that the edges of two consecutive dihedral planes cannot be joined by means of the N4C joint (fig. 34a). But if we examine the ground plan projection of the complete joint, the shortest member at one of the ends of the structure is perpendicular to the horizontal member of the other structure located on the perpendicular side of the ground plan. If we extend the two members until they meet, forming an 'L' shape, assembly of the two structures is complete (fig. 34b).

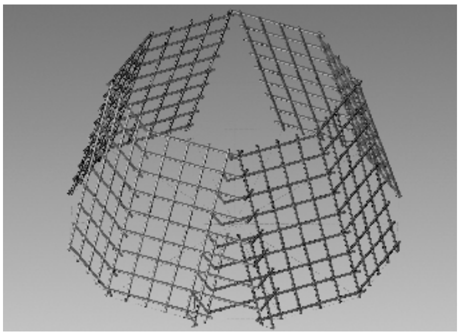

Fig. 30

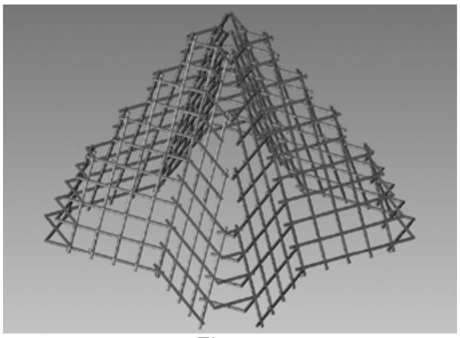

Fig. 31

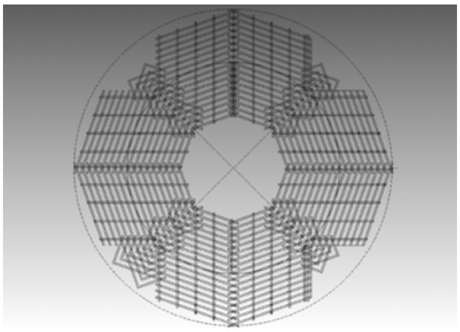

Fig. 32

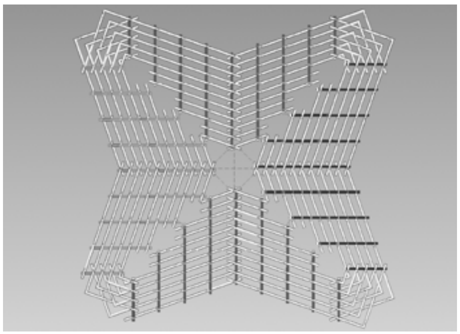

Fig. 33

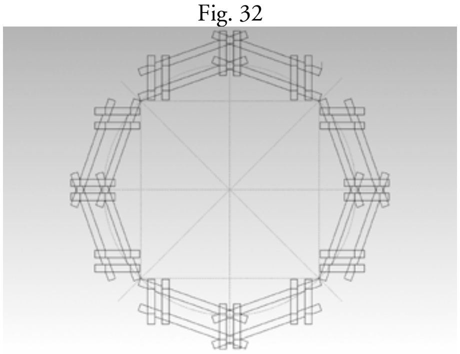

Fig. 34a

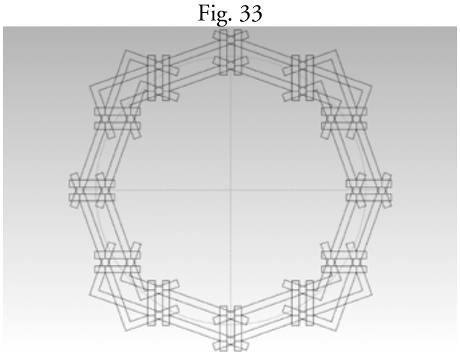

Fig. 34b

Joining four dihedral planes at their non-parallel edges

By placing the rhombic dodecahedron with its two longest axes on the vertical plane, and dividing it into two halves on the same plane, we obtain two equal structures (fig. 35).

If we transpose this division onto our construction with bars joined together using N4C joints, the result is a convex structure whose openings are rhombi with angles equal to the faces of the rhombic dodecahedron, 96° and 84° (fig. 36).

Joining together all the faces using the L-shaped member interlinks the entire construction (fig. 37). If we examine the joints in the upper part of the resulting construction we can see that only three bars intersect, continuing the construction by sliding the same structure vertically, the vertical bars of the upper structure will be those that fill the empty space in the joint. Superimposing these structures creates a tower of infinite size.

Grids

Making a joint with bars which have notches at both ends, and taking two of their parallel elements, we extend them in linear formation, repeating the same operation with the other two elements of the joint (fig. 38). If we keep repeating this assembly we obtain a flat grid which we will position in the vertical plane by rotate through 90° (fig. 39). If we join two of these grids at one of their faces so that the members forming the joint match in a straight line, we create a new grid or a double grid, whose members are fused into a single one; this gives the structure greater rigidity and stability (fig. 40).

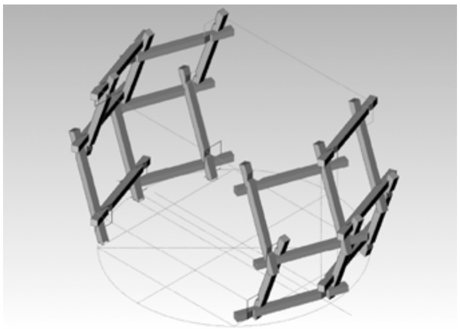

Fig. 35

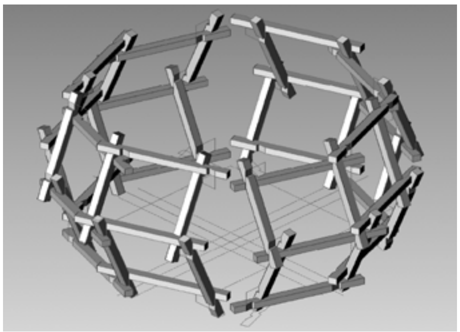

Fig. 36

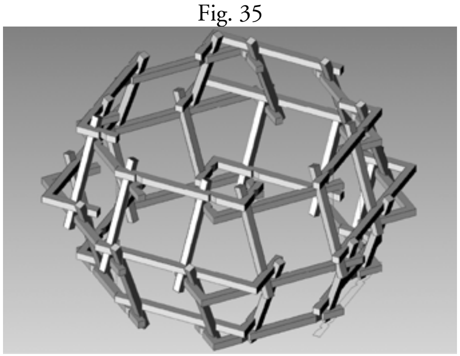

Fig. 37

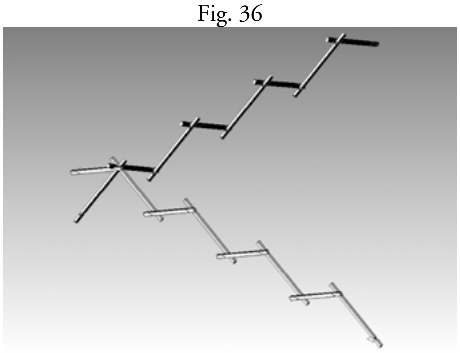

Fig. 38

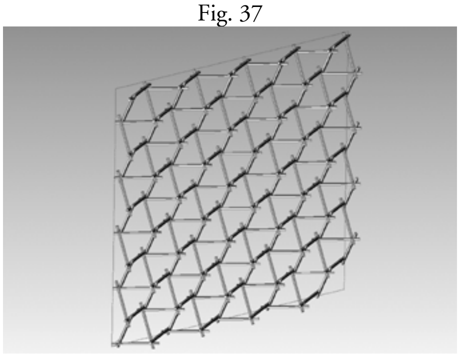

Fig. 39

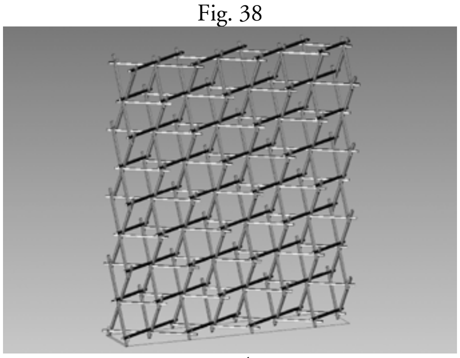

Fig. 40

Joining two of these vertical structures at a right angle can be accomplished in two ways. One way is to position both structures such that their horizontal members are located on the same horizontal plane, then taking two members symmetrical with the bisecting plane and joining them together with a tangent curve, into a single member (fig. 41); this interlinks the two structures. The exception to this is where the joint is 45°, in which case this member will be straight. The other way is by extending a member of each of the structures until they meet at a right angle in a single member (as in fig. 32). Where the grid is a double grid, the two arms of the L will be extended until reaching one or two joints in each grid (fig. 42a, b).

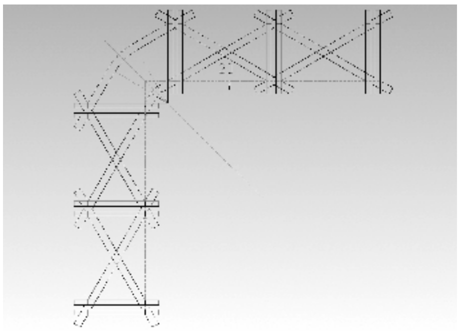

Fig. 41

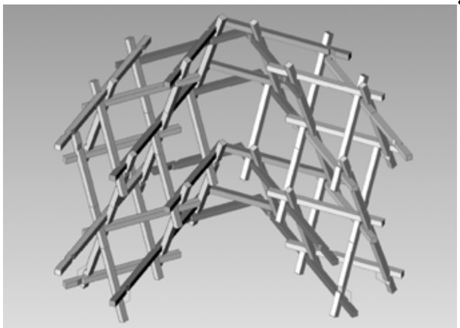

Fig. 42a

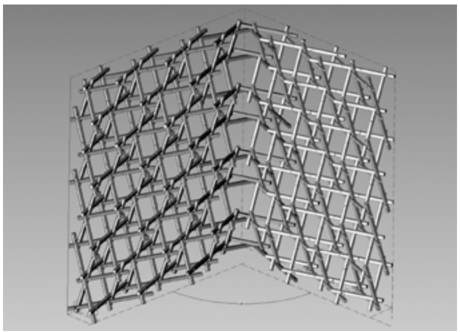

Fig. 42b

Grid on an inclined plane

By making a module with the members of a 30° joint as shown in fig. 43, a grid is created on an inclined plane whose angle is the same as that formed between the central member and the horizontal plane. We create one of these grids in such a way that the starting point is a module; from this first one, two are formed, and from these two three are formed, and so on, creating a triangular structure (fig. 44). This structure is one of the four faces of a pyramid and we can join them together without additional members. When the four faces are joined together, it can be seen that the members making up the arris of this inverted pyramid are one member of each one of the faces fused into a single member, joined together at the notches on one of their ends. This construction can be supported by a pillar whose upper part has a cross section equal to the gap between the members; given its morphology I call it a "tree" (fig. 45).

In the same fashion as these four walls have been joined together, so a number of these trees can be joined together at the upper edge forming a ground plan grid, each supported by a central base acting as a column. In this way, the interior space that is created is similar to pseudo-vaulting formed by intersecting oblique planes (fig. 46).

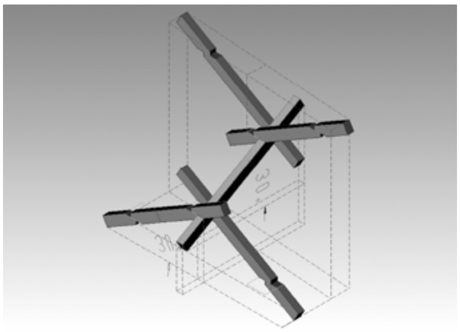
Fig. 43

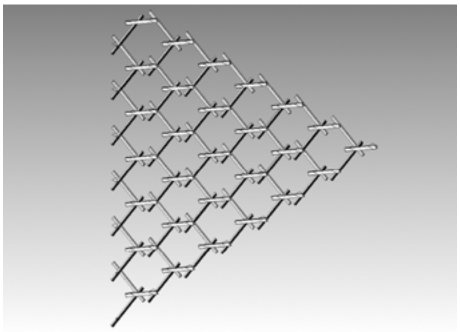
Fig. 44

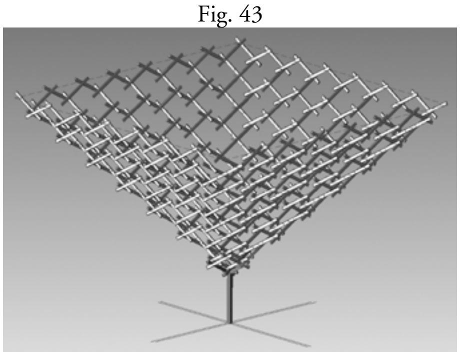
Fig. 45

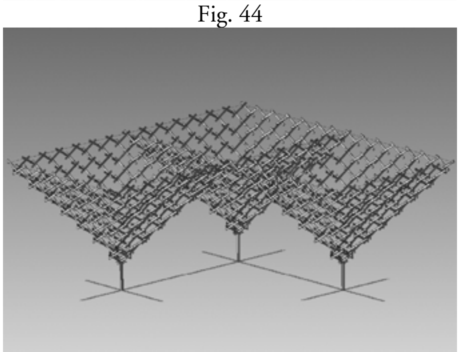
Fig. 46

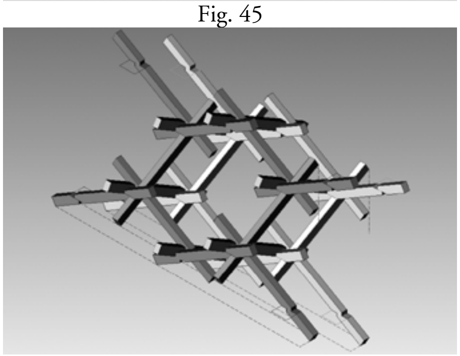
Fig. 47

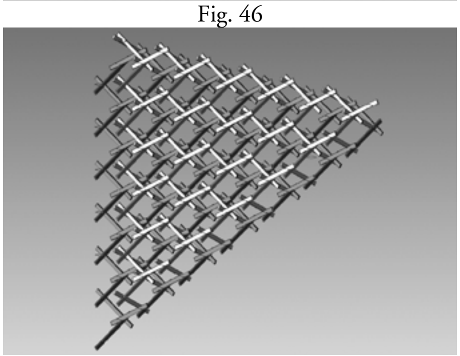
Fig. 48

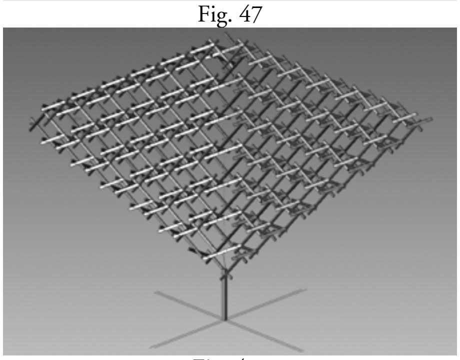
Fig. 49a

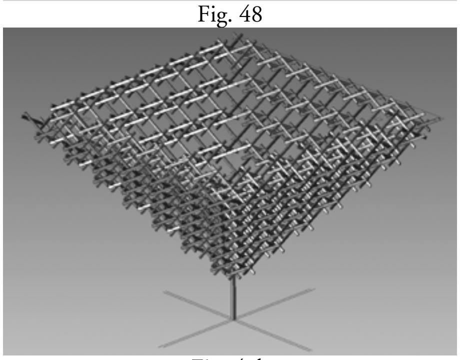
Fig. 49b

Double grid

If we examine the starting module, we can see that the joint is made up of only three members; if we add another member, the free end connects to a new module, creating a double grid following two parallel planes, and thus the structure is reinforced (fig. 47). If we transpose this onto the walls of our tree (fig. 48), these new structures can be joined together in the same way as in the previous case, creating a compact structure (fig. 49a, b).

If we return to the construction shown in fig. 11a and 11b, and examine the joint of the members forming the face of the pyramid, we see that only two members intersect, whereas the joint enables four to intersect. In the case of fig. 11b, positioning one face on top of another following two parallel planes so that their joints are on the same vertical line; these faces can be joined using two shorter members (fig. 50), depending on the separation that we wish to have, thus completing each of the joints with four members (fig. 51). In this example I have used a length equal to the distance between two joints on one of the beams with notches at both ends. If, in addition, we extend the beams from the arris meeting point of the two lower faces, a distance equal to the aforementioned shorter member, until they meet and connect to the upper face in one of their joints, we thus create the most significant linking together of the structure, since all the beams that intersect in the arris, establishing a bearing point, must extend only a very short distance to reach the upper face, thus creating a structure capable of being self-supporting (fig. 52). In the case of fig. 11a, all the members of the upper face will be extended until they reach the horizontal plane of the ground (fig. 53).

Grids formed with intersecting planes

Building flat structures, and considering these as planes which are independent from each other, we can make them intersect in such a way that each of the four members forming the joint belongs to a different plane, the intersecting line being a series of joints in a straight line. To understand this better, we can start construction by joining a member from each one of the planes forming a static joint (fig. 54), then we slide one member on top of another in succession, positioning them in each of the grooves until a lattice is formed (fig. 55). The outcome is the same as would be obtained if four planes intersected, but here it is only possible because of the geometric properties of the N4C joint, the most significant of these being that all the joints of the different members are on the same horizontal plane (the same as shown in fig. 22). The compositions can be more or less complex, depending on the position of the intersecting line in the plane and the number of elements contained in the plane lattice (fig. 56).

Horizontal or vertical linear structures

A linear composition or construction is similar to that of a beam triangulated with pipes. If we construct the four faces of a rectangular prism with members from three joints, one in the centre and two at the ends, and insert into the gap inside the joint a bar of equal cross-sectional dimensions, considered the arris of the prism itself, then where two members of each face of the prism intersect, the four faces are thus interlinked. By forming a core with four members from three joints, the members joined by the central joint and the joint at their ends linked to the prism faces at the central line parallel to the arrises, we thus ensure that the entire assembly is adequately braced (fig. 57). Fig. 58 shows the linear continuity, proving the efficacy of the core joined to the walls.

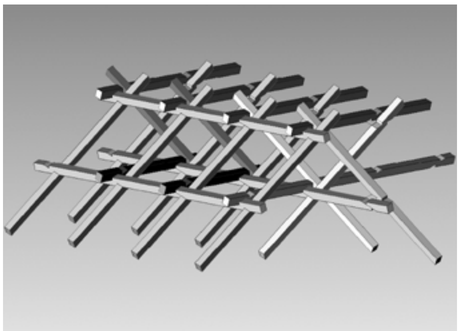
Fig. 50

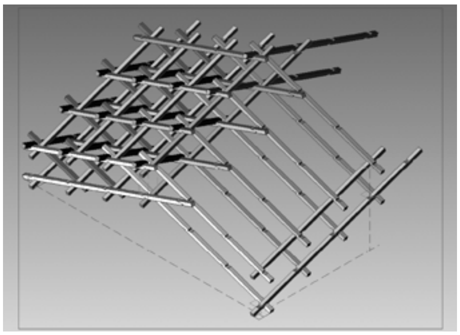
Fig. 51

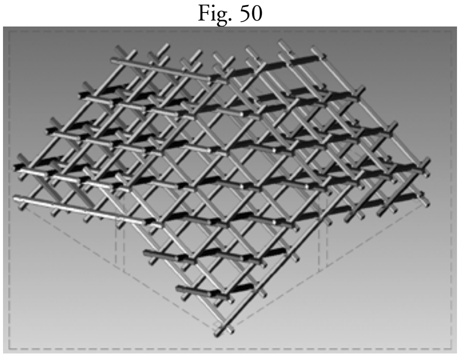
Fig. 52

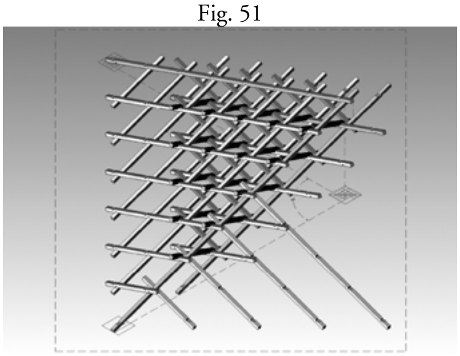
Fig. 53

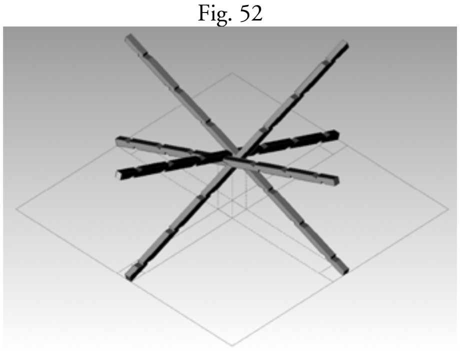
Fig. 54

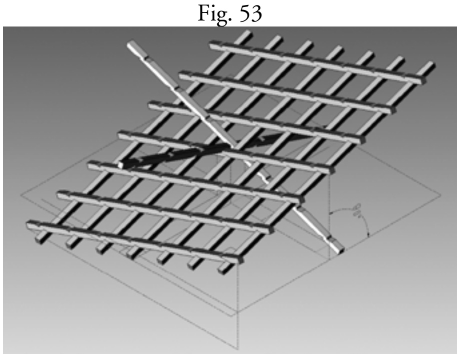
Fig. 55

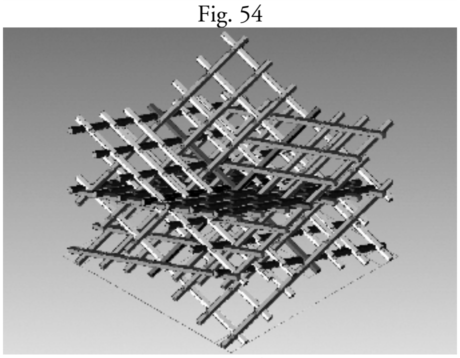
Fig. 56

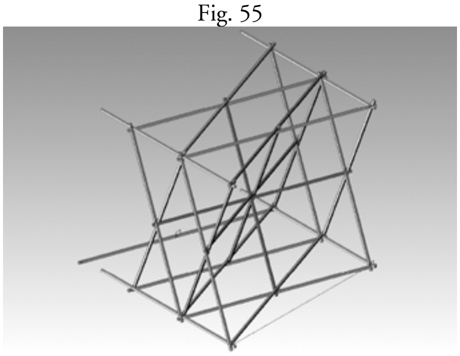
Fig. 57

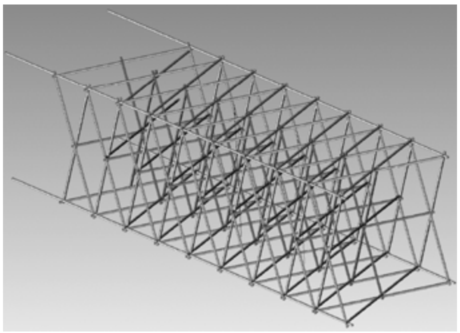

Fig. 58

Conclusion

The main reason I consider the discovery of this joint important is because of its geometric and structural properties; I say geometric, because the conditions it imposes for spanning space are always fulfilled.

As a result of these properties, the structures that can be executed have that idiosyncrasy of shape which makes them new, if we bear in mind that they do not require additional members.

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I would like to thank ALTEC for collaboration in the translations, and J. R. Rolania and Ana Reboredo for their work in the image processing for the drawings. I am also grateful to Santiago Huerta, from the School of Architecture in Madrid, for his interest in the reception of this work.

About the author

Jesús Molina (Spain 1949) is an autodidact. He started his studies to become an Architectural Technician but he did not finish them in order to travel around Europe. In Venice he attended the International University of Art with Professor De Luigi. He stayed at Pulteney College in London. He was awarded the First National Design Prize for the design of a sculptural module with didactic applications in geometry. This work was exhibited in the School of Architecture in Madrid. His work titled "The Public Space" was shown in the Town Planning Course of the Menéndez Pelayo International University, UIMP. His sculptural works are present in several Spanish cities. His work can be seen at http://www.esculturajesusmolina.com.