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Book Review

Leonard K. Eaton

Hardy Cross. American Engineer

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It is a great pleasure for me to review this book, because I was present at its birth and watched it mature. Prof. Eaton's aim, first with an article published in the *NNJ* and then in detail with this fine new book, was to show how American engineer Hardy Cross developed a method for analyzing indeterminate structures that minimized the inconveniences and risks involved in the use and development of reinforced concrete. Along the way, however, Eaton also provides us with a tapestry of other considerations as to the interactions of engineering and architecture, the relationship of engineering and mathematics, and the contrasts between American engineers and their peers overseas. This is as culturally rich a book on a single technical argument as you could wish to find.

Leonard Eaton listened to engineer Holger Falter present a paper entitled "The Influence of Mathematics on the Development of Structural Form" at the Nexus conference on architecture and mathematics which I directed in Mantua in 1998 [Falter 1998], and took exception to one point that Falter made. In order to respond to this statement of Falter's, Prof. Eaton published a paper on the engineering achievements of Hardy Cross in the *Nexus Network Journal* [Eaton 2001]. Research for that paper led to this book.

The subject dealt with here is the problem of analyzing a statically indeterminate system. A structural system is said to be statically determinate when each span can be considered in isolation: an example is a simple beam is supported by a column at each of its ends. An indeterminate system is created by stretching the beam over three or more columns: the forces within each individual span are influenced by those of the spans adjacent to it.¹ The increasing use of reinforced concrete at the end of the nineteenth and the beginning of the twentieth century brought the problem of the analysis of indeterminate structures to the fore: construction in steel favours discontinuous elements, but construction in concrete favours the continuous.

According to engineer Falter, up through the nineteenth century the preferred methods of analyzing structure were graphic, but by that century's end analytic methods began to eclipse the graphic methods. Falter wrote, "Forming hinges was simple with iron, but

difficult with reinforced concrete. It created statically indeterminate frames continuing through several spans, a difficulty surmountable only by using analytical methods" [Falter 1998: 61]. But the analysis itself could be nightmarish: "For a six-story building four bays wide, there will be six unknown x values and thirty unknown y values, or a total of thirty-six simultaneous equations to solve" [Eaton 2006: 19]. Falter's conclusion was that the difficulties inherent in the analytical methods were so enormous that engineers designed mostly simple structures, so as to keep required calculation to a minimum. Prof. Eaton countered this statement with the example of Hardy Cross and his "Moment Distribution Method", which was at least partly graphically based, and much simplified the necessary calculations.

Hardy Cross undertook to solve the problem in the early years of the 1920s. His guiding principles were beautifully simple:

- The visualization of the results of forces acting on a structure;
- The application of the theory of elasticity to structural design;
- An acceptance of approximation in lieu of completely accurate results due to inaccurate data.

Exactly how Cross addressed and solved the problems is fully explained in the book, so there is no need for me to go into that here. What I would like to do, however, is point out some of the very interesting aspects about architecture, engineering and mathematics that Prof. Eaton points out along the way.

It is well known that architects and engineers view each other with suspicion from opposite sides of the same object: that is, both professionals deal with buildings, but from points of view that are felt to be antithetical. Yet formal considerations and developments have historically gone hand-in-hand with technical considerations and developments. There are many examples of this: the great Gothic cathedrals, masterpieces both formally and structurally, is outstanding. The development of analyzing indeterminate structures to permit the use of reinforced concrete is another. Architects in the early years of the twentieth century were intrigued by the formal possibilities of reinforced concrete, but early attempts at large-scale structures were far from elegant. Prof. Eaton writes,

...engineering requirements seemed to dictate that increasing loads would be met by increasing the size of the structural elements. In the period from 1910 to 1930, wrote Carl Condit, "the result, which was on an elephantine scale, especially in buildings with large, open interiors, revealed the diversity of requirements that could be satisfied by reinforced concrete but also the amount of valuable space that was consumed by the sheer quantity of material in long beams and trusses." ... With his usual acuity, Hardy Cross noted that concrete was much heavier in comparison with its strength than timber or steel. "Its weight," he said, "eats up its strength in long span construction." Furthermore, it looked clumsy unless designed with skill.

How clumsy it looked will be recalled by Galileo's famous example of a bone enlarged in scale, demonstrating the "square-cube law":

Galileo had undertaken a comparison of the strength of different-sized beams of uniform material and section. In the absence of other loads, the strength of the beam will increase with the second power, but weight of the beam will increase with the third power of the linear dimensions, that is, as the beam becomes larger, it becomes proportionally weaker, so that for strength to remain constant, it must necessarily grow in size [Huerta 2006: 31].

But *how much* a reinforced concrete beam had to grow in size was unknown. And here Prof. Eaton points out what I thought was one of the most interesting cultural aspects of the problem: American engineers at the time didn't really care about finding the most elegant or efficient solution. American engineers, like their Roman counterparts two millennium earlier, merely threw more material at the problem. Eaton quotes mathematician Felix Klein, visiting the United States in about 1910, as saying,

The United States is such a rich country that it can afford to use twice as much material in a structure as is necessary. Why should a U.S engineer be interested in a theory that tells him how to calculate exactly the amount of steel in a beam needed to carry a given load? If he's worried, all he needs to do is increase the amount of steel and make the beam stronger" [Eaton 2006: viii].

Hardy Cross apparently took up the problem because he found it intellectually intriguing; this in itself was enough to distinguish him from his compatriots.

If we are accustomed to the reciprocal diffidence of the architects and engineers, we are not so used to thinking of a diffidence on the part of engineers towards mathematics. This is the other cultural aspect of the problem that I found particularly interesting. Where theoretical mathematics was strong, so was engineering mathematics. Americans have always generally mistrusted intellectuals. Theoretical mathematics was not particularly strong in the United States, as it was more generally in Europe, and the lack of facility of engineering mathematics in the US consequently reflected that. Eaton writes that "[m]any practicing engineers had dubious mathematical skills in handling simultaneous equations and many had difficulties in visualizing rotations and displacements" [Eaton 2006: 38]. In this too Hardy Cross was distinguished from his compatriots. Cross held geometry to be of the utmost importance for the engineer; to him, the problem of indeterminate structures was geometrical. But it is a surprise to learn that Cross himself remained skeptical about the dependence of structural engineering on complicated numerical analysis. When he comments on Stefan Timoschenko's *Theory of Structures*, Cross says, "another point of view – distinctly European"; for Eaton this means "overly mathematical" [Eaton 2006, 81]. The merit of Cross's solution to the problem lies in its reduction of the necessary calculations to an easily-followed process of "solving a series of normal simultaneous equations by successive approximation" [Eaton 2006, 39].

The use of Cross's easily-applied solution to the problem of indeterminate structure opened the door for the widespread use and development of structures in reinforced concrete in American (and European) architecture. Determining the exact size that beams needed to be, eliminating useless redundancy, resulted in more elegant structures, and this surely provided an incentive to architects intrigued by the new material but concerned with aesthetics. Cross's development of the moment distribution method in the 1920s meant that when Philip Johnson and Henry-Russell Hitchcock made the International Style the new standard for architecture with their exhibit at the Museum of Modern Art in New York in 1932, American engineers were ready. With the passing years, even in the wealthy United States the economy of materials was increasingly appreciated, as during the years of the Korean War when steel was scarce. I agree with Prof. Eaton that "[i]n the field of structural engineering, Hardy Cross was the outstanding American of his time" [Eaton 2006, vii].

Hardy Cross: American Engineer is organized into a Preface, three chronologically ordered chapters dealing with Cross's career, a glossary of technical terms used, a bibliography of the works of Cross, a technical appendix by engineer Emory Kemp with worked examples of Cross's structural theories, notes, and an index. I wish there had been more illustrations. Eaton tells us that Hardy Cross thought that geometry was essential for the structural engineer, that Cross himself was forever making little drawings, and that the visualization of the results of forces on a structure was key to his clear understanding of the problem. Being able to see more of Cross's sketches would have been revealing and helpful.

Notes

1. Mario Salvadori, my mentor in many ways, said more simply that a statically determinate system is one that can be analyzed without having to appeal to any other principle than those of statics, which is the study of equilibrium, while a system that cannot be analyzed by statics alone is said to be statically indeterminate [Salvadori 1971: 4].

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About the reviewer

Kim Williams is the editor-in-chief of the *Nexus Network Journal*.