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Research

A Pyramid Inspired by Mathematics

Abstract. An eighteenth-century pyramid near Brussels contains intriguing mathematical ratios that suggest they were influenced by Egyptomania of the period. Jesuit priest Athanasius Kircher published several books on mysticism and symbolism that were typical of the times. He also wrote a book on acoustics and described the "whispering gallery", an effect which can be observed inside the Wespelaar pyramid.

Introduction

In the village of Wespelaar, 25 km northeast of Brussels (Belgium) stands a curious little pyramid, in the midst of a luscious private park (fig. 1). It is a *folie* or *fabrique* : a little monument that, in an eighteenth-century landscaped park, holds the attention of the strolling visitor and brings him to a certain mood. The park contains several of these: a temple of Flora, an artificial lake with cave, an *Elyseum* around an obelisk.

Fig. 1. The pyramid in Wespelaar, Belgium

The park and the pyramid date from 1797. They belonged to the Louvain brewer Leonardus Artois, who had acquired a mansion and land in Wespelaar and asked the Brussels architect Ghislain Joseph Henry to design a park of 120 hectares and its *fabriques.*

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It is said that the pyramid is a Masonic symbol, but no original plans or documents are extant [Duquenne 2001].

In order to understand the symbolism of the monument we have to keep in mind the general Egyptomania of the period. We shall digress a little on the Jesuit Athanasius Kircher, a noted Egyptomaniac, and then go over to a detailed description and venture some hypotheses on the symbolism of the mathematical ratios hidden in the pyramid.

Egyptomania

Plato, in the dialogue *Timaeus*, attributed to the Egyptian priests the recording of all important events of the past [Plato 1966, §§ 22,23]. Herodotos reinforced the myth that the Egyptians had secret knowledge and the Romans readily believed this. In Imperial Rome (second and third century A.D.) temples to Isis and Serapis were built; at the same time the knowledge of hieroglyphs was being lost in Egypt. Many obelisks were brought to Rome to adorn public squares.

In the Baroque period many of these were rediscovered among the antique ruins and newly erected, often as part of new monuments, in the seventeenth-century papal city. One of the most conspicuous stands on the grotto sculpted by Bernini in the Piazza Navona, the *Fontana dei quattro fiumi.* The concept of the monument was a brainchild of that most memorable man, Athanasius Kircher. This Jesuit was a polyglot, a mathematician and an ethnologist who founded a famous museum. He was very interested in the reports on Buddhism and Confucianism sent home by the Jesuit missionaries and he himself studied the Greek and Arabic sources on the ancient Egyptian civilisation. He is of course only one in a line of scholars who tried to unravel the secrets of the hieroglyphs [Lamy and Bruwier 2005], but his delirious interpretations strongly influenced the esoteric movements of the eighteenth century.

To Kircher, the old pagan sages had inherited, in veiled form, bits of the complete knowledge of the world that Adam had; they were, so to speak, parallel prophets [Pastine 1978]. Of these, he thought the Egyptians had the most profound knowledge and their hieroglyphs were a secret sacred language. He conjectured rightly that the Coptic language was in fact the old Egyptian language, but – as all his contemporaries – he considered hieroglyphs as ideograms, an idea that was reinforced by the then recent discovery of the Chinese system of writing. He followed an intuitive and mystical interpretation of the Egyptian signs, which led him completely astray. He published an extensive work of 1500 pages on the subject [Kircher 1653], which was widely diffused.

Pope Innocent X called on him to interpret the hieroglyphs on the obelisk that was discovered in the circus of Maxentius and Kircher proposed an allegory suggested to him by his interpretation. Bernini executed the monument and when it was completed Kircher wrote an essay where he explained the allegory [Kircher 1650; Rivosecchi 1982]. In short, his views were:

- The small pyramid atop the obelisk symbolizes the Divinity: its apex points to God and the triangular sides refer to the Trinity.
- The obelisk proper represents the world of the angels, through which God communicates with the sublunary world. Its four faces refer to the four elements (fire, air, water, earth) from which all matter is made.

- The obelisk stands on a cave, the symbol of the subterranean world, another pet idea of Kircher, which he published in another memoir [Kircher 1665]. From this world issue the great rivers. In the Bernini fountain the waters flow through four openings that indicate the four great rivers of the four continents: Danube, Nile, Ganges and Rio de la Plata.

A hundred years later, in the "Age of Reason", the belief in secret Egyptian knowledge was greater than ever. Books were published (see, for example, [Pernety 1796]) that were inspired by the Kircher opus. Napoleon Bonaparte believed in the hidden knowledge of the Egyptians and took over a hundred scientists with him on his Egyptian campaign. It was a period where secret societies sprang up throughout Europe and the Masonic lodges borrowed some of their symbols from the Egyptian myths, as well as from the temple of Solomon. To the French Masons the supposed existence of secret knowledge antedating the Bible was an argument for challenging the spiritual dominance of the clerics.

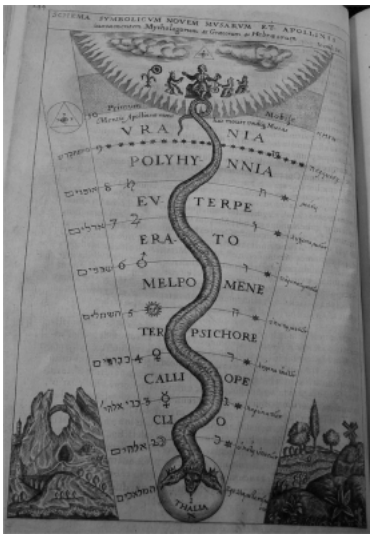
The pyramid : form and orientation

The pyramid is about 7 m high and built from regular rectangular blocks of grey sandstone. The inner chambers and their vaults are in red brick; those were originally plastered, but now the brick is mostly apparent.

There are two striking similarities with the Kircher theories regarding the shape and the position of the building:

- The pyramid is truncated and topped by a little obtuse pyramid, like an obelisk.
- It stands on a cave: the cellar where the ice from the lake was stored.

In that spirit we could look at the pyramid as the symbol of a link between the lower and a higher world



A vertical section through a diagonal is an equilateral triangle (disregarding the truncation). One recalls here that the superimpostion of the equilateral triangle and its reverse form the hexagram, also called Solomon's Seal. To Kircher, probably taking over an older tradition, an equilateral triangle in a circle was the symbol of the "immobile mover", an aspect of the divinity, reproduced in fig. 2.

Fig. 2. The hierarchy of the muses as an alias for the hierarchy of the angels, according to Kircher [1653]

The four corners of the pyramid point approximately to the cardinal points, but not completely: the NS diagonal makes in fact an angle of about 10° to the east with due north (fig. 3).

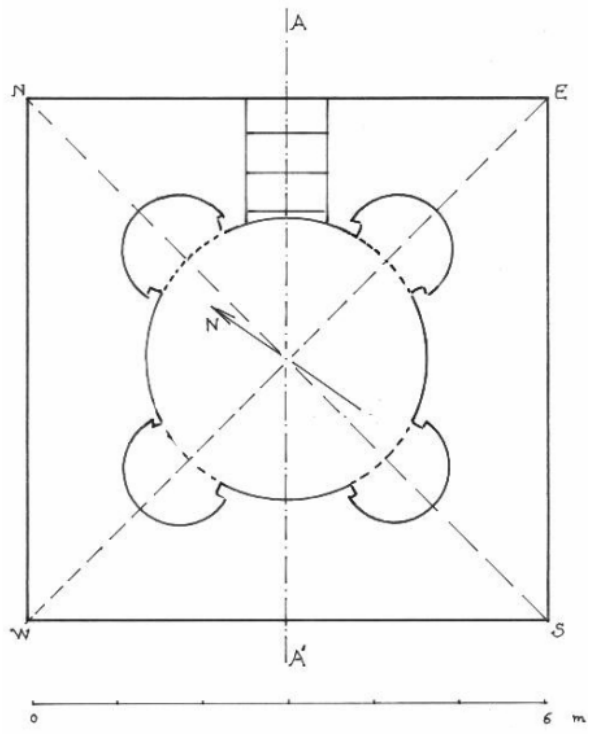


Fig. 3. Base of the pyramid with the projection of the floor of the lower chamber

We offer a tentative explanation. In the pre-Christian era the summer solstice was celebrated, a feast the Church recuperated as St John's day (24 June); in rural areas this is still the occasion to light the St John's fires. In the Masonic tradition, the Orient, the rising sun and even St John play a role. We surmise consequently that the pyramid was in fact oriented with its entrance towards the rising sun on the summer solstice (21 June). At the latitude of Wespelaar (50.53° N) the direction of the rising sun makes an angle of 51.23° with the north, meaning that the NS diagonal should lie 6.23° east of north. This leaves a discrepancy of some 4° to explain. An architect setting out the diagonals on the ground will do so with respect to the north as determined by a compass. He would be aware that there is a local magnetic declination and corrects for it. The most readily available reference on scientific matters at the end of the eighteenth century would have been the *Encyclopédie* of Diderot and d'Alembert, under the voice *aiguille aimantée.* One finds there a table of the declination in Paris for the years 1700 to 1750 when the declination increased linearly from 8°18′ W to 17°15′ W. Extrapolating linearly to 1797 yields an estimated 26° W. The declination, however, was heading for a maximum of 22.5°, which it reached in 1814. Around 1800 it must have been 21.5° with an error of 0.5°. If this hypothesis is true the architect overcorrected the westward declination by some 4.5° to the east, which explains the actual orientation of the pyramid.

The dimensions of the pyramid

The side of the pyramid, near the ground, is 6.10 m. But the actual base is probably larger inasmuch as the level of the soil must have risen through accumulating humus over the last two hundred years, in spite of the excellent maintenance of the park during this time.

At the time of building, the unit of length in Wespelaar was the Brussels foot [Vandewalle 1984]:

$$1 \text{ foot} = 0.27575 \text{ m}$$

Although Belgium had become part of the French republic in 1794, the metric system – barely two years old – was not yet in use.

It is therefore to be expected that the basic measures of the pyramid will be simple numbers when expressed in feet. Everything fits nicely if we assume that the diagonal of the base (which is the side of the equilateral triangle that governs the shape of the pyramid) is 32 feet, i.e. 2^5. For the side of the base this makes

$$32/\sqrt{2} \text{ feet} = 6.24 \text{ m}.$$

The inner chambers. Inside we find a round chamber with slightly conical walls and four niches, which originally held Egyptian vases at the origin. Three steps lead into it, through a small entrance; its floor is 0.74 m above the base (it has recently been re-laid) and has a diameter of 3.35 m. With the original plastering on this was probably 3.31 m, i.e., 12 ft (fig. 4).

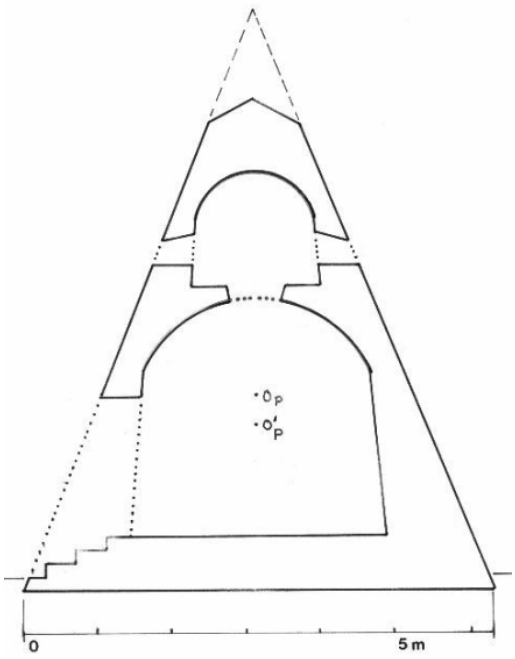


Fig. 4. Vertical section along the line AA′ in fig. 3

Fitting the measured heights of the domed ceiling to a simple curve we find it is spherical (or, less probable, parabolic) and certainly not elliptical nor an inverted catenary; the radius of the dome is 6 ft and its centre lies 2.23 m above the base, i.e., practically 8 ft. In the centre of the dome is an oculus with a lower diameter of 0.67m that gives access to a second chamber, which has a floor with a diameter of 1.70 m and a height of 1.50 m. Because the walls are very rough these measures are approximate. This makes the dimensions of the upper chamber practically one half of those of the lower chamber, with a similar conical shape, topped by a spherical cap. There are no niches nor an oculus in the upper chamber, but there are four semicircular openings in the faces of the pyramid; the radius of the semicircles, measured at the outer surface of the building, is 0.28 m, i.e., 1 ft.

Intriguing ratios. Drawing a vertical section of the pyramid, through its apex and parallel to a side (see figs. 5 and 6), we find that:

1. The pyramid is truncated at the upper plane of the cube erected on its base;
2. The distance of the apex of the actual pyramid to the oculus is equal to the distance of the oculus to the floor of the lower chamber;
3. The position of the oculus is such that its distance to the base relates to the side of the base as 1 to 1.618, i.e. the so-called golden ratio: $1+\sqrt{5}\div 2=1.618\text{K}\equiv\phi$.

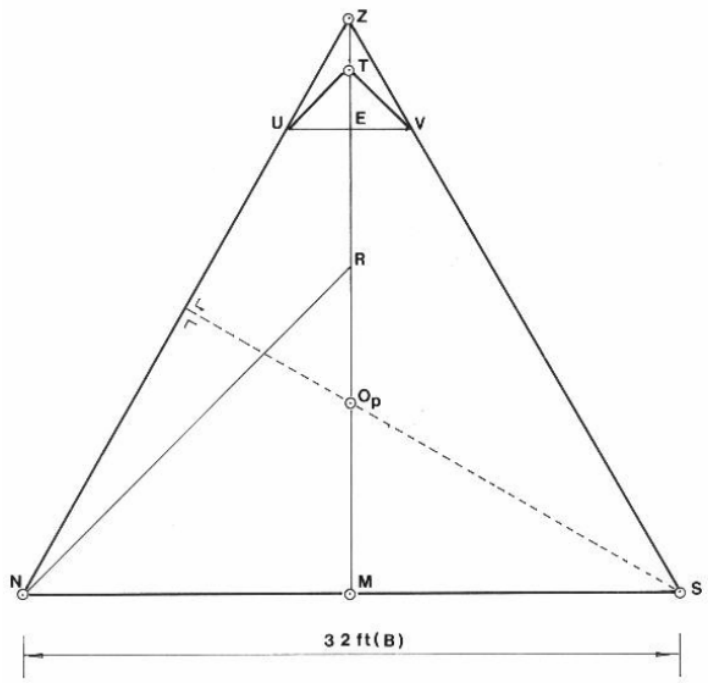


Fig. 5. Design drawing no. 1: section of pyramid through a diagonal, giving an equilateral triangle of 32 Brussels feet; Op is the centre of the circumscribed circle. The faces of the top little pyramid are equilateral triangles.

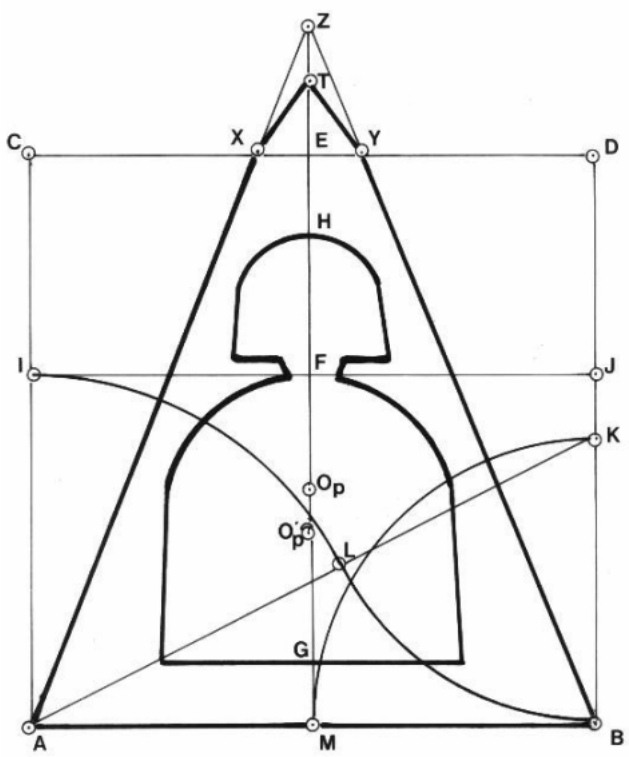


Fig. 6. Design drawing no. 2: vertical section orthogonal to two faces. ABDC is a square. ABJI is a harmonic rectangle. O′p is the centre of the circumscribed sphere to the actual pyramid; this coincides practically with the centre op the spherical cap of the dome

The last characteristic is not surprising. The golden ratio is basic for constructing the pentagram, which we find on the wall of every Masonic lodge or in sorcery manuals as component of a magic circle [Goethe 1808]. It has inspired countless esoteric fantasies since the Renaissance.

The symbolism of the second ratio is less obvious, but let us try one. Let the two chambers represent the onset of the series

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \mathrm{K} = 2$$

Ascending then in spirit by successive steps, in successive chambers, one approaches the ultimate point, which Kircher called God.

We list in Table 1 all the dimensions, in Brussels feet, referring to the segments as defined in figs. 5 and 6.

Name	Segments	Mathematical	Numerical (feet)
Diagonal, rib	NS=NZ=ZS		32
Corner-centre base	NM=MS=MR	NS/2	16
Side base	NR=AB	NS/√2	16√2=22.63
Height	ZM	NM√3	16√3=27.71
Base-centre	MOp	MZ/3	16/√3=9.24
Height top pyramid	UE=EV=ET	XY/√2	16(1-√(2/3)) = 2.94
Half side base	AM=MB=BK=KL	AB/2	8√2 =11.31
Harmonic to side	AL=AI=FM=JB	AB (√5 – 1)/2	(√5-1)8.√2 =13.98
Height cut-off	ZE	ZM-AB	16(√3-√2) = 5.085
Width cut-off	XY		16(√2-2/√3) = 4.15
Height chamber	GF=FT	ET+AB-AI	11.58

Table 1. Dimensions of the pyramid

Acoustics of the lower chamber

Two remarkable properties can be observed, apart from the long reverberation time of sound, characteristic of almost closed rooms with hard walls.

1. A speaker in one niche is more distinctly heard by a listener in the opposite niche than by one in the centre of the room.
2. Speaking in the centre of the room towards the ceiling produces no echo, while speaking sideways or off-centre induces the usual reverberation.

Both effects can be understood by doing geometrical acoustics, the equivalent for sound of geometrical optics; both are good approximations when the wavelength (of sound or light) is short with respect to the characteristic dimensions of the set-up.

In the present case this dimension is about 3 m and the wavelength of speech is around 1 m (frequencies around 300 Hz), making for a tolerable approximation.

The first effect is a clear case of a "whispering dome": the spherical ceiling gathers the sound energy emitted by the speaker and focuses it in a symmetric point, with respect to the axis. The listener on the opposite wall receives all the energy collected by the dome, whereas a person in the centre will only capture the energy falling directly on his auricle. The effect is of course lessened by the reverberating sound where the energy is spread in all directions (fig. 7).

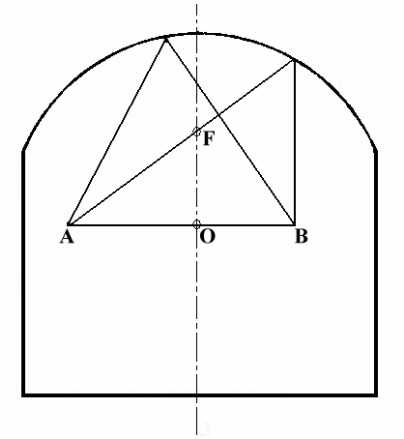


Fig. 7. The rays in a whispering dome. The focus F lies halfway between the centre O and the mirror

The optical analogy of the second effect would be a concave mirror with a hole in its centre (the domed ceiling with oculus) facing a plane mirror (the floor). If the rays are emitted on the axis from a point between the centre of the mirror and the focal point, the concave mirror focuses them in another point further along the axis; but the plane mirror reflects them back and there is a region along the axis from which all rays will be reflected back into the hole. In our case the sound gets into the higher chamber and disappears very soon into the open through the openings in the wall (fig. 8).

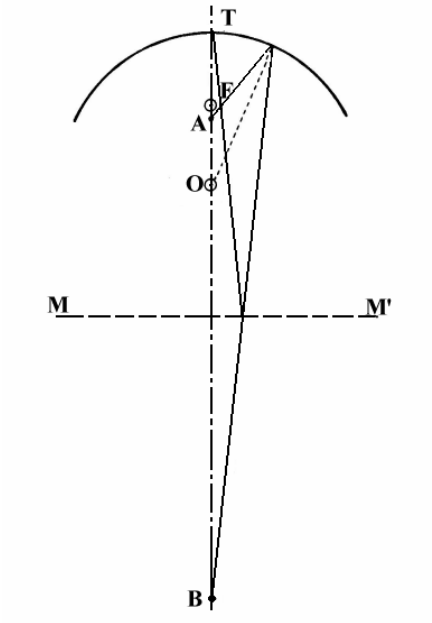


Fig. 8. Sound produced in A is focused in B; the mirror MM′ (floor) reflects it in T; if a hole is present in T the sound disappears without reverberating

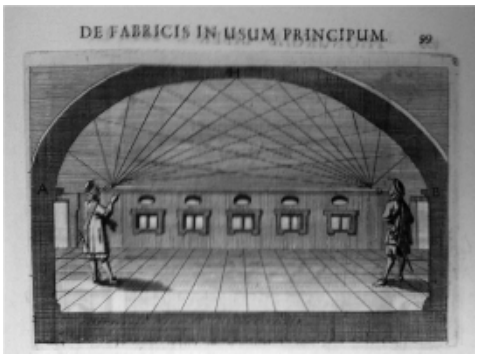


Fig. 9. The whispering dome according to Kircher [12]

Were these effects planned? It was Kircher who introduced geometrical acoustics in his *Phonurgia Nova* [Kircher 1673] and described the whispering dome (see fig. 9). It is important to know in this respect that Mr Artois had a close friend and counsellor who lived in the Wespelaar estate and was a former professor of physics at the University of Louvain: Matthieu Verlat. It is almost certain that he discussed the plans with architect Henry. The phenomenon was explicitly mentioned in the course on sound that Verlat taught at the Arts faculty (see [Godaert 1992, 63]).

Professor Verlat was a priest, canon at St Martin's of Liège, and he may have found a religious symbolism in the effects: wise words are understood by a listener in the right disposition while wasted on others and words spoken upwards, in prayer, reach the top, i.e., the Divinity; words spoken at random contribute only noise.

We mention this last hypothesis only as an example how pyramids tend to induce interpretations in Rorschach fashion.

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Frans Cerulus was born in Ghent (Belgium) in 1927. He studied theoretical physics at the universities of Ghent and Basel (Switzerland). He did research in theoretical particle physics in Copenhagen and Geneva (CERN). He became professor of theoretical physics at the University of Louvain (Belgium) in 1964. Since his retirement he has turned to the history of physics. He is co-editor of the collected works of Daniel Bernoulli.