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Research

Structure of Phenomenological Forms: Morphologic Rhythm

Abstract. The images in architecture are handed down through mathematical forms. The meaning of the plastic value of the forms and the conflict between their visual boundaries are a result of the geometrical composition of the object. Since Stonehenge in Britain, the Egyptian pyramids, the Greek Parthenon or the Roman Pantheon, architecture has been a reflex of simple boundaries without accidental confrontations. Nowadays materials are organised through movement/change in order to represent the required profile. This developed structure emerges in the artistic manifestations according to the theory of continuity. As an expression of the formal quality in opposition to the ancient characteristics of quantity, a new conception of rhythm appears. The concept of a cell as an architectural element that can have any biological form and can be grouped itself according to different ways and functions (such as repetition of floors) is introduced. This concept of cell permits eurhythmia (harmony in the proportion of a building) through the notion of rhythm once all the elements of a building are situated among themselves.

1 Introduction

Throughout the last fifty years of the twentieth century there was been an intense engagement closely connected to the research of artistic composition, in particular architecture, by integrating universal scientific knowledge in fields such as mathematics, physics, and biology into the creative work [Kepes 1968]. Indeed, a change of attitude in Modern Art appeared: to serve society we could no longer understand the laws of modern art without an awareness of notions about space/time [Giedion 1969]. The re-examination of cultural modernity and the influence of architectural linguistics such as semiotics and structuralism increased. Also, sensibilities changed and new cultural forms attempted an approach to other typologies of emotion [Consiglieri 2000]. That transformation of knowledge integrates the contemporary science of composition. Communication theories and phenomenology present additional methods, exhibit alternative solutions based on observation, and offer new thought paradigms for the crisis of meaning within architecture. What structures allow the understanding of a form of expression? Can there be meaning in form or only in content? Can it be sustained through innovation and invention?

In *Gestalttheorie* [Guillaume 1937], an object is developed according to the inherent character of the various surfaces placed in a given relationship to the volume, namely the regular faces, vertices, edges, angles and orientations necessary for the basic views and plans. The phenomenological forms are structures made of

sets of cells grouped together by a process of addition; these arrangements would serve different shapes of rooms or plans of the building [Steadman 1983]. The application of this methodology in the construction of a building corresponds to the production of modular elements and their correlations. The relationship of these small pieces or cells depicted on rhythmic grids is the fundamental element that defines the whole [Bonomi 1974].

In the present work, we consider cells for habitation that are modular structures oriented so as to enclose different spaces and constitute one unity of the space organised in such way that the position of the cell can be recognised, and they are structured in a continuous movement which produces a rhythm.

For a long time rhythm was thought of as a cadence of measures between the different parts and between these and the whole. Rhythm was in relation to the proportion of the human body, which was considered by the Greeks as *analogy*. Hence each part of the building was in fitting proportion to the height, width and depth, and all parts were in their own place within the symmetric whole of the building. This sequence of proportions or figurative relations created the notion of rhythm. The periodic character introduced according to scale defined the rhythm of the composition. Moreover, rhythm was connected with the notions of numbers by their capacity to create an Order [Ghyka 1978]. Thus rhythm was defined as a conceived periodicity [Servien 1930]. Thus, rhythms were considered by means of the regularity relation between space and time.

Through periodic and non-periodic modular concepts introduced by Roger Penrose, the object became based on mechanical strategies, taking into account that the rhythm of an architectural composition is still equilibrium, the element which organizes the relationship between their parts in a harmony or in a dissonance [Darvas 2003]. In recent years, there appeared a new interpretation of the world: the *emergence* [Weinstock 2004]. It is in this state of art that structural morphologies concerning *growing* architecture deal with the organising, shaping, building and transforming new configurations from the manipulation of unit elements, via algorithms [Leoz 1969] (for instance, the Rose and Stafford algorithm [Alsina and Trillas 1984, 154;162]), genetic codes [Lalvani 1982] or group actions [Schattschneider 2004].

The principal aim of the current work is to express the architectural processes through continuum rules. In the next section, the concept of morphologic rhythm as an architectural cell is defined, and some mathematical examples are presented, demonstrating the advantages of open-source computer programs. Section 3 is devoted to a discussion of some architectural examples.

2 Morphologic rhythm

The classical concept of rhythm, which consists in the ordered repetition of isometries (i.e., translations, rotations and reflections), can be generalized to a

morphologic one that emerges as a self-organized life cell. Shape changes also can occur.

Let C be a *cell* defined by

$$C := \{(x, y, z) \in \mathbb{R}^3 : f(x, y, z; t) \leq 1\},$$

where x, y, z represent (as usual) the spatial Cartesian coordinates and f denotes a function defined in the three-dimensional Euclidean space and depending on a time parameter $t \in [0, +\infty[$. For instance, if we take $f(x, y, z; t) = x^2 + y^2 + z^2$, the set C reads

$$C := \{(x, y, z) \in \mathbb{R}^3 : f(x, y, z; t) = x^2 + y^2 + z^2 \leq 1\}.$$

Thus it can be called a *steady state spherical cell* since f is constant as a function of the parameter time t and the set C represents a closed ball of radius 1 that is the union of the boundary (sphere) and its interior (fig. 1).

Fig. 1. Graphical representation of a steady state spherical cell of radius 1. Graphic visualization produced using Maple 6

We say that a cell C has a *morphologic rhythm* if

$$\exists R > 0, \quad C(t) \subseteq B_R(0), \quad \forall t \in [0, T],$$

where $B_R(0)$ denotes an open ball centered at the origin $(0,0,0)$ with radius $R > 0$, f is a continuous function defined on $\mathbb{R}^3 \times [0, T]$ and $T > 0$. The meaning of the above definition can be understood as that the rhythm will take part of the lifetime of the cell. The requirement that the cell belong to an open ball for every time parameter provides the boundedness of the final pattern, since the pattern solutions that blow up have no physical (architectural) meaning.

Let us first consider the kinematics of the center of mass of a cell in order to recognise the classical concept of rhythm, that is, its rigid motion through discrete different instants of time. For the spherical cell of radius 1 (fig. 2) given by

$$C := \{(x, y, z) \in \mathbb{R}^3 : f(x, y, z; t) = x^2 + (y - t)^2 + z^2 \leq 1\} \quad (1)$$

Fig. 2. Graphical representation of a spherical cell of radius 1 at instants of time $t = 0$ and $t = 2$ when $f(x, y, z; t) = x^2 + (y - t)^2 + z^2$. Graphic visualization produced using Maple 6

the classical notion of rhythm corresponds to consider instants of time whose the difference between them is greater or equal than twice the radius of the cell, that means, the cell behaves like a rigid ball which moves by translation. Here the cell appears as invariant under translation. Indeed a rigid and uniform motion in which each point of the cell traverses a line segment with the same length and direction. For instance, taking in equation (1) the instants of time $t=0$ and $t=2$, the translation has the algebraic representation [Alsina and Trillas 1984, 142]

$$\left(\begin{array}{c|c} 100 & 0 \\ 010 & 2 \\ \hline 001 & 0 \\ 000 & 1 \end{array} \right).$$

Fig. 3. Graphical representation of a spherical cell of radius 1 at instants of time $t = 0$ and $t = 1$ when $f(x, y, z; t) = x^2 + (y - t)^2 + z^2$. Graphic visualization produced using Maple 6

However, if different instants of time can be taken into account, a translation in the broad sense can be stated. Indeed, fig. 3 illustrates the spherical cell of radius 1 defined as in (1), that is, it can be regard as moving through the same trajectory $u : \mathbb{R} \rightarrow \mathbb{R}^3$ defined by

$$u(t) = t(0, 1, 0) = (0, t, 0),$$

but now at instants of time $t=0$ and $t=1$. Then there exists an intersection of the cell with itself. Fig. 4 illustrates the spherical cell of radius 1 moving through the trajectory $u: \mathbb{R} \rightarrow \mathbb{R}^3$ defined by $u(t) = t(-1, 1, 1) = (-t, t, t)$.

Fig. 4. Graphical representation of a spherical cell of radius 1 at instants of time $t = 0$ and $t = 1$ when $f(x, y, z; t) = (x + t)^2 + (y - t)^2 + (z - t)^2$. Graphic visualization produced using Maple 6

Other modular coordination created with isometries as generators can be performed for generating decorative patterns as well as architectural phenomenological forms.

If the center of the spherical cell at instant time $t = 0$ is at the origin and its movement describes the parabola $p(t) = -2t^2 + 4t$, in the plane Oyz , then the visual interpretation of the morphologic rhythm corresponds to the movement through a nonlinear trajectory (fig. 5, at three distinct instants of time).

Fig. 5. Graphical representation of a spherical cell of radius 1 at instants of time $t=0$, $t=1$ and $t=2$ when $f(x, y, z; t) = x^2 + (y - t)^2 + (z + 2t^2 - 4t)^2$ for $0 \leq t \leq 2$.

This movement cannot be performed even more by a simple composition of translations and rotations. Only when using discrete instants of time, nonlinear trajectories can be reduced by plotting to linear functions at the chosen discrete instants of time. For instance, the above parabola can be interpolated by two

different linear functions $u: [0,1] \rightarrow \mathbb{R}^3$, $u(t)=(0,1,2)$, and $v: [1,2] \rightarrow \mathbb{R}^3$, $v(t)=(0, 1, 2) + (t - 1)(0, 1, -2) = (0, t, -2t + 4)$ and the cell is then defined by the function

$$f(x, y, z; t) = \begin{cases} x^2 + (y - t)^2 + (z - 2t)^2 & \text{if } 0 \leq t \leq 1 \\ x^2 + (y - t)^2 + (z - 2t - 4)^2 & \text{if } 1 < t \leq 2. \end{cases}$$

Here the shape and the size of the configuration of the cell are not altered, only its position is changed.

This new concept of rhythm also includes the similarity notion, i.e., the result of contractions or dilatations which alters all lengths by a same factor but preserves the measures of angles between lines. Indeed the present concept allows the alteration of the location, the orientation, the size (stretching or thinning) and the shape of the configuration of the cell, yielding the total change of the initial format of the cell. Fig. 6 shows the spherical cell both thinning and shape changing at the instant of time $t = 1$. Finally, the cell motility involving translocation of the cell centroid as well as changes or distortions in the cell shape is shown at fig. 7. Then the outcome of the morphologic rhythm is the continuum process set up by the cell as a function of the parameter time and its connection with the biological forms.

Fig. 6. Graphical representation of a spherical cell of radius 1 at instant of time $t = 1$ when $f(x, y, z; t) = |x|^{2-t} + |y|^{2-t} + |z|^{2-t}$. Graphic visualization produced using Maple 6

Fig. 7. Graphical representation of a cell of radius 1 at instants of time $t = 0$, $t = 1$ and $t = 2$ when

$$f(x, y, z; t) = \left(|x|^{t^2/2-3t/2+2} + |(2t+1)y - 5t^2|^{t^2/2-3t/2+2} + |z - 3t(2-t)|^{t^2/2-3t/2+2} \right) / (1 + t(t-1))$$

3 Discussion and conclusions

Taking a cell as the basic object from which are developed different variations, we can coordinate the compositional process so as to obtain a determined grid that allows the effective rhythmic continuation and a flexible figurative relationship between the different cellular components (fig. 8). Thus the cell is the architectural measure denoting both discipline and conceptual freedom.

Fig. 8: Morphologic rhythm/Alvar Aalto, MIT, Cambridge USA

Upon these co-ordinator modulations, where an element assumes different directions in plan or generally in 3D space, the movements of growth are articulated such that they can be conditioned to have no boundary (figs. 9 and 10). This growth in composition that is not subjected to any modular limit is a solution for an open, free space building that expands through axial elements.

Contrary to this open scheme, Frank Lloyd Wright studied grids of rotative symmetrical cells in order to obtain a dense building (fig. 11). Since the process of grids to form composition in space by Wright rose from a constant geometrical modulation on which the form develops, it also can be assumed to be formed from morphologic rhythm. If those habitational cells of one floor were to have a standard plan shape, the habitability is restricted to the situations of fulfilment of the interiors as well as the systems of lighting and ventilation. If those habitational cells were to have a plan shape which is made by a continuous rhythmic system, the fulfilment of habitat housing would be totally different of the former, affording more interior spaces.

Fig. 9: Boffil, study for Gaudi Quarter

Fig. 10: Frank Ghery, EMR, Germany

Fig. 11: F.L. Wright: Suntop House, Pennsylvania / St. Marks Tower, NY

Fig. 12: Arata Isozaki, study for a spatial city

To comprehend the great complexity of the rhythm under one only element, consider the study of Arata Isozaki for a spatial city: repeatedly fitting the given element results in a (non-unique) solution in the set of the potential solutions of different symmetric and asymmetric shapes (fig. 12).

Despite the cells being conditioning elements on the organization of the spaces, the house must not be thought as a machine of habitat, but constitute a private place inserted in an urban space, where the family is joined in a community. This may be called the *poetics of habitat*, where the closeness allows intimate relationships within the basic laws of proximity and centralization. Moreover, like biological cells (Lat. *cella* “concealed” storage or living room) the architectural cells can perform several kinds of motion that continuously shape their structure in relation to and in interaction with a varying environment. It is the dichotomy of cells, existing and moving either as autonomous units or as components within a multicellular tissue, that leads to a final question: what are the conditions and principles of tissue formation for eurhythmy?

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