Rachel Fletcher

# *The Square*

Geometer Rachel Fletcher explores the 1: √2 ratio associated with the regular quadrilateral figure known as the square, looking at the square's inherent symbolism and the four-ness of the cross and the tetractys, as she constructs *ad quadratum* and other geometric techniques.

## *I Introduction*

Geometric constructions offer specific techniques for spatial composition, from the overall plan to minute details, while sensitizing designers to the experience of spatial harmony. In earlier columns, we considered the 1 : √3 proportions inherent in the *vesica piscis* and the triangle. Here we explore the 1: √2 ratio associated with the regular quadrilateral figure known as the square. We look as well at the square's inherent symbolism and the four-ness of the cross and the tetractys, as we construct *ad quadratum* and other geometric techniques.

## *II Symbolism of the square*

The inherent three-ness of the triangle conveys the mediation of different entities. The four-ness of the square illustrates the dynamic crossing of opposing elements. This meaning is demonstrated in the square's construction, which is built on the crossing of vertical and horizontal axes and symbolizes polarities such as time and space, male and female, expansion and contraction, and heaven and earth. The vertical axis may represent humanity's path of spiritual communication with the divine; the horizontal axis may represent pathways of social communication with one another. The crossing of one element against another suggests a matrix whereby energy is fixed materially, just as the warp and weft threads of the spider's web and the net may symbolize the overall pattern of physical creation.

## *III The square*

- With a compass, draw a circle.
- Draw the horizontal diameter AB through the center of the circle.
- Set the compass at an opening that is slightly smaller than half the radius of the circle.
- Place the compass point at O. Draw arcs that cross the horizontal diameter on the left and right, at points C and D.
- Set the compass at an opening that is slightly larger than before. Place the compass point at C. Draw an arc above and below, as shown.
- With the compass at the same opening, place the compass point at D. Draw an arc above and below, as shown.
- Locate points E and F where the two arcs intersect.
- Draw the line EF through the center of the circle.
- Extend the line EF in both directions to the circumference of the large circle (points G and H).

Lines AB and GH locate the horizontal and vertical diameters of the circle (Fig. 1).

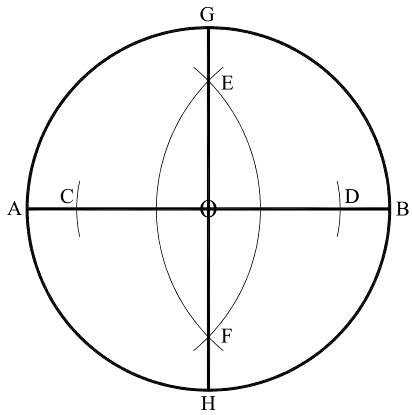



Fig. 1

- Locate point G at the top of the vertical diameter (GH).
- Place the compass point at G. Draw a half-circle of radius GO through the center of the circle (point O), as shown.
- Locate point B at the right end of the horizontal diameter (AB).
- Place the compass point at B. Draw a half-circle of radius BO through the center of the circle (point O), as shown.
- Locate point H at the bottom of the vertical diameter (GH).
- Place the compass point at H. Draw a half-circle of radius HO through the center of the circle (point O), as shown.
- Locate point A at the left end of the horizontal diameter (AB).
- Place the compass point at A. Draw a half-circle of radius AO through the center of the circle (point O), as shown.

The four half-circles are of equal radius and intersect at points I, J, K and L (Fig. 2).

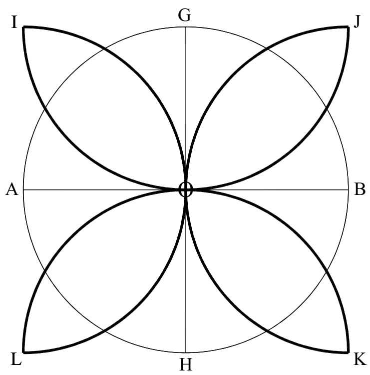



Fig. 2

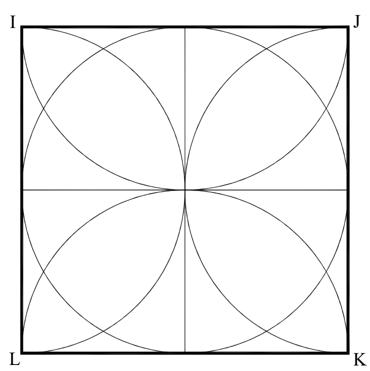



Fig. 3

- Connect points I, J, K and L.

The result is a square (Fig. 3).

**Definition:**

The **square** is a closed plane figure of four equal sides and four 90° angles. "Square" is an adaptation of the Old French *esquare* (based on the Latin *ex-* "out, utterly" + *quadra* "square," which is from *quattuor* "four") [Harper 2001, Simpson 1989].

*IV The cross of time and space*

The square, like the cube, appears to be massive and solid, but in fact is unstable. In Platonic philosophy, the square may symbolize material or "earthly" reality, in part to distinguish the apparent and transitory reality of physical creation from the permanent, intelligible reality of ideal forms [Plato 1961: Timaeus 30c-d, 55d-e, 1163, 1181; Republic, VI, 509d-510b, 745]. In this context, ideal reality may be symbolized by the triangle, the most inherently stable of all geometric shapes.

We traditionally associate the square's four corners with patterns of orientation that guide our daily lives: the four cardinal points on the horizon; the unfolding of the year through four distinct seasons; the turning of the day through sunrise, midday, sunset and midnight. Let us begin with the four corners of the world-the cardinal directions of north, south, east and west-drawn according to modern convention, with north, situated above, as if aligned with the North Star (Fig. 4a).[1]

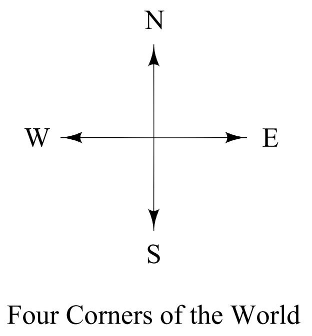



Fig. 4a

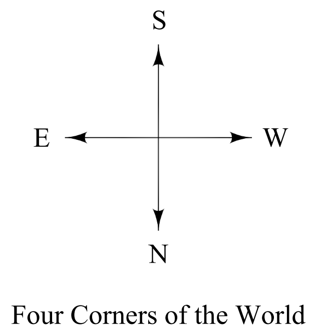



Fig. 4b

Next, the directions are inverted-south above, north below, and west and east on the right and left, respectively, in the manner of maps of the sixteenth century [Heninger 1977, 140-141]. Possibly, this arrangement depicts the ideal form of which our common experience is but a mirror reflection. (Fig. 4b).

The year is born in the spring, flourishes through summer, recedes in autumn, and prepares for new birth in winter. The summer and winter solstices are days of greatest light and greatest dark, and occupy south and north, respectively. The vernal and autumnal equinoxes are days of equal light and dark, and occupy east and west. In this context, the vertical axis signifies extreme tendencies, while the horizontal axis signifies balancing forces. (Fig. 4c).

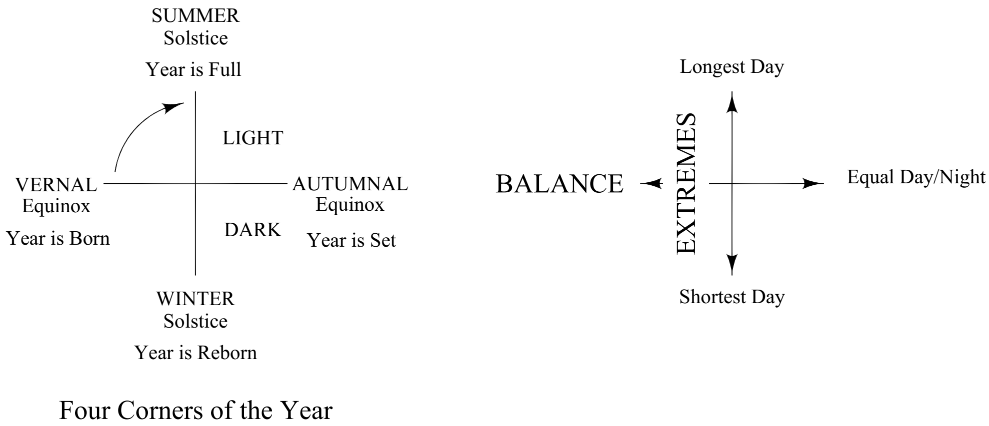



Fig. 4c

In tropical astrology, the signs of the Zodiac are based on the path of the sun relative to the turning of the seasons. The new year begins when the sun crosses the vernal equinox, where the ecliptic intersects the celestial equator, on the first day of spring. Around 125 B.C., the Greek astronomer Hipparchus deduced the precession of the equinoxes, observing the gradual westerly motion of the vernal point through the constellations, resulting in earlier occurrences of the equinoxes each successive sidereal year. Since Hipparchus made this discovery when the vernal point was in the constellation Aries, 0° Aries has become the accepted vernal point and the start of the topical new year [Fenna 2002, Soanes 2003].

Reckoning by the tropical year, the Sun enters the constellation Aries (♈) on the first day of spring and the new year. The Sun enters Cancer (♋) on the first day of summer; Libra (♎) on the first day of autumn; and Capricorn (♑) on the first day of winter (Fig. 4d).

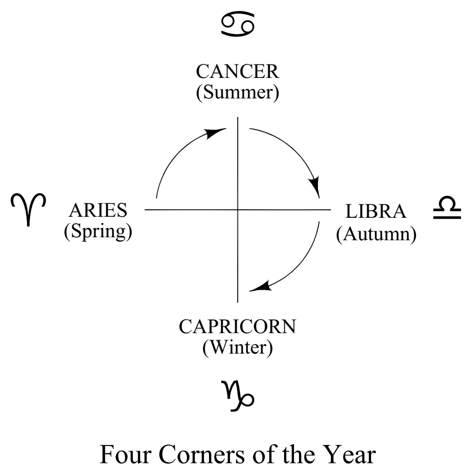



Fig. 4d

**Definitions:** [2]

**Precession of the equinoxes** occurs as the earth rotates about its axis, in response to the gravitational pull of the sun on the earth's equatorial bulge. The result is that the earth's axis of rotation describes a small circle in the sky over a period of approximately 25,800 years. Hence, the signs of the Zodiac no longer coincide with the constellations for which they were named. The Tropical Zodiac originated when the vernal point was in Aries. Because of precession, this point has since traveled across the constellation Pisces, and will enter Aquarius about the year 2500 [Fenna 2002, Soanes 2003].

The **tropical year**, or **solar year**, is measured relative to the sun, and is the period between successive vernal equinoxes (approximately 365 days, 5 hours, 48 minutes, and 46 seconds in length). The **sidereal year** is based on the rotation of the earth relative to the fixed stars and constellations (approximately 365 days, 6 hours, 9 minutes, and 1 second in length) [Fenna 2002].

Two **equinoxes** occur each year at the precise moment when the sun crosses the celestial equator and the days and nights are equal in length; the first day of spring (March 20 or 21) and the first of autumn (September 22 or 23). The equinoxes are known as the first point of Aries and the first point of Libra. "Equinox" is from the Old French *equinoxe* and the Latin *aequinoctium* (from *aequi-* "equal" + *noct-*, stem of *nox* "night") [Ridpath 2003, Simpson 1989].

Two **solstices** occur each year when the sun is furthest north or south of the celestial equator, and appears to stand still. In the northern hemisphere, the summer solstice occurs on the longest day of the year, the first day of summer (June 21), when the sun appears at the point on the ecliptic that is furthest above the celestial equator, intersecting the Tropic of Cancer. The winter solstice occurs on the shortest day of the year, the first day of winter (December 21 or 22), when the sun appears at the point on the ecliptic that is furthest below the celestial equator, intersecting the Tropic of Capricorn. In the southern hemisphere, these positions are reversed. The Latin for "solstice" is *solstitium* (from *sol* "sun" + *sistere* "to stand still"), which means "the time when the sun seems to stand still." [Lewis 1879, Nave 2001, Simpson 1989].

In astronomy, the **Zodiac** is a band of the celestial sphere that extends approximately 8 or 9 degrees on either side of the ecliptic and locates the apparent motions of the sun, moon and principal planets. In astrology, the Zodiac is divided into twelve equal parts or "signs," each bearing the name of a constellation for which it was originally named. "Zodiac" is from the old French *zodiaque*, by way of the Latin *zodiacus* "zodiac," from the Greek phrase *zodiakos kuklos* (from *zôion* "living being, animal," "figure, image" or "sign of the Zodiac" and *kuklos* "a round" or "a ring"), which means "circle of little animals" [Harper 2001, Liddle 1940, Simpson 1989].

"**Aries**," the Latin word for "ram," is the first sign of the Zodiac (♈), which the sun enters at the vernal equinox, about March 20. The constellation Aries (the Ram) is said to represent the ram whose Golden Fleece is sought by Jason and the Argonauts [Lewis 1879, Simpson 1989, Soanes 2003].

"**Cancer**," the Latin word for "crab" or "tumour," is the fourth sign of the Zodiac (♋), which the sun enters at the summer solstice, about June 21. The constellation Cancer (the Crab) is said to represent a crab that is crushed beneath the foot of Hercules. *Karkinos*, the

Greek for the sign of Cancer, means "tumour" or "crab," so named, according to Galen, because the swollen veins surrounding a malignancy resemble the limbs of a crab [Lewis 1879, Liddell 1940, Simpson 1989, Soanes 2003].

"**Libra**," the Latin word for "pound" or "balance," is the seventh sign of the Zodiac (♎), which the sun enters at the autumnal equinox, about September 22. The constellation Libra (the Scales or Balance) is said to represent a pair of scales representing justice [Lewis 1879, Simpson 1989, Soanes 2003].

"**Capricorn**" (the Goat) is the tenth sign of the Zodiac (♑), which the sun enters at the winter solstice, about December 21. The Latin for "Capricorn" is Capricornus (from caper "goat" + cornu "horn), a literal translation of the Greek aigokerôs, which means "goat-horned" (from aix "goat" + keras "animal horn") [Lewis 1879, Liddell 1889, Liddell 1940, Simpson 1989].

Each season of the year corresponds to a period in the twenty-four hour day. The day is born in the east at sunrise and is full at midday, when the sun shines overhead. The day sets in the west at sunset. At midnight, the sun travels below the horizon (Fig. 4e).

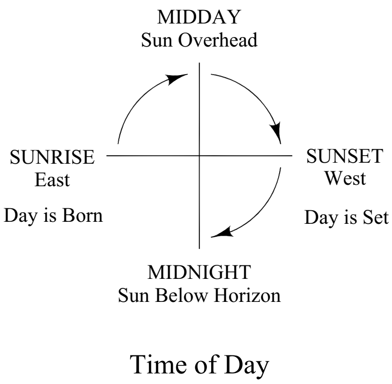



Fig. 4e

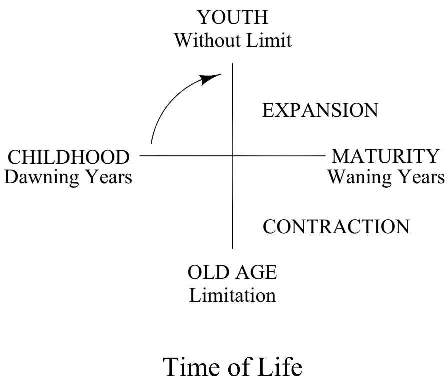



Fig. 4f

The periods of a twenty-four hour day correspond to the phases in a person's lifetime: the dawning moments of childhood, the limitless possibility of youth, the waning years of maturity, and the limitation of old age. (Fig. 4f).

### *V The cross of elements*

The cross of four elements may be arranged in various ways. Here, the elements of fire and earth mark the vertical axis, above and below. Fire may be compared with the force of levity and radiant spirit. Earth may be compared with the pull of gravity and the physical body. Along the horizontal axis are mediating elements of air and water. Water, like our tears, conveys feeling and emotion. Air, like the breath that produces the spoken word, gives voice to intellect and thought. In this arrangement, the elements descend in a spiral of decreasing density, from fire through air and water to earth (Fig 5a).

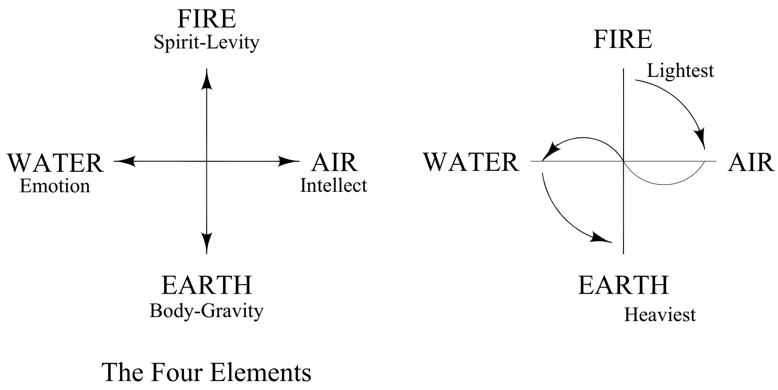



Fig. 5a

Here, the elements are characterized by qualities of expansion and contraction, or hot and cold; and solution and fixation, or moist and dry. Fire is hot and dry. Water is cold and moist. Fire and water share no qualities, and are contrary. Earth is cold and dry. Water is cold and moist. Water and earth share the quality of cold, and are compatible (Fig. 5b).

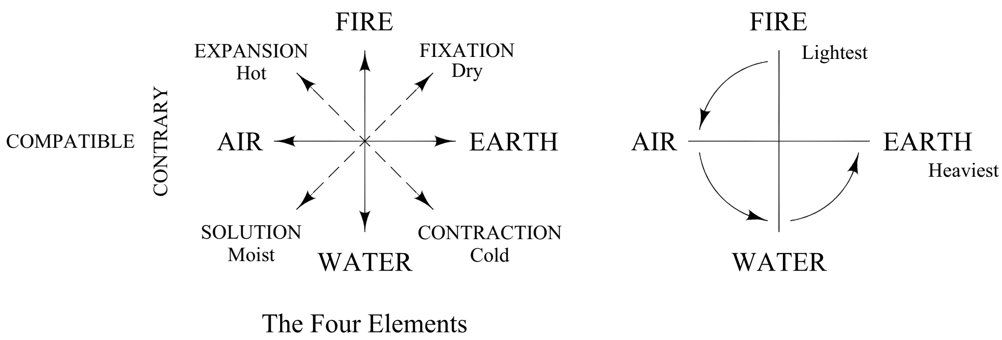



Fig. 5b

In this arrangement, the elements correspond to the Cardinal signs of the Tropical Zodiac, which appear chronologically in clockwise motion. Aries (♈) is characterized by the element of fire and marks the first day of spring and the new year. Cancer (♋) is characterized by the element of water and marks the first day of summer. Libra (♎) is characterized by the element of air and marks the first day of autumn. Capricorn (♑) is characterized by the element of earth and marks the first day of winter. Aries is ruled by the planet Mars and is the archetypal image of the Individual. Aries opposes Libra, which is ruled by the planet Venus and is the archetypal image of the Partner or Other. Cancer is ruled by the Moon and is the archetypal image of the Mother. Cancer opposes Capricorn, which is ruled by the planet Saturn and is the archetypal image of the Father (Fig. 5c).

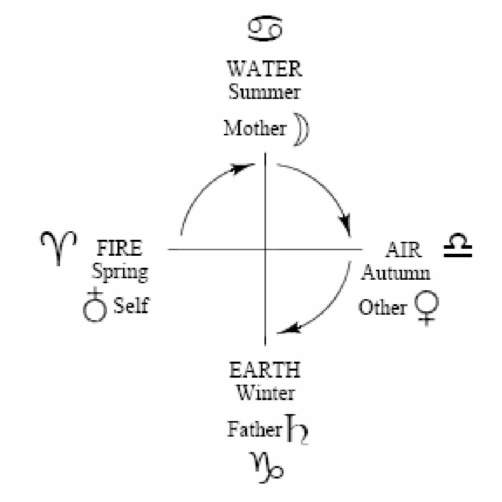
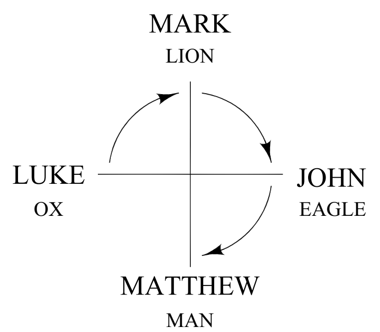


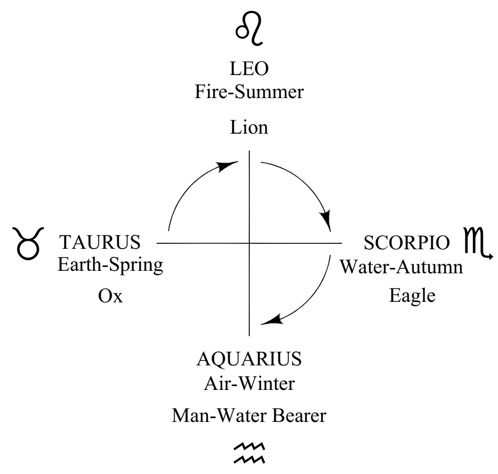

Fig. 5c

**Definitions:**

Each of twelve signs of the Zodiac is characterized by one of four elements (fire, water, air or earth) and one of three qualities (cardinal, mutable or fixed). The **Cardinal** signs in astrology mark the beginnings of the seasons (Aries–spring, Cancer-summer, Libra–autumn and Capricorn–winter). The **Fixed** signs mark the middle of the seasons (Taurus–spring, Leo–summer, Scorpio–autumn, and Aquarius–winter). The **Mutable** signs mark the end of the seasons (Gemini–spring, Virgo–summer, Sagittarius–autumn, and Pisces–winter).



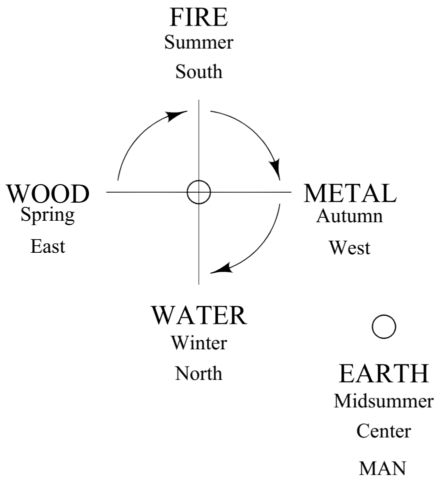

The Four Apostles




Pagan Origins of Christianity

Fig. 5d

Chinese tradition identifies five elements, but these, too, may be represented on a cross. The element of fire corresponds to the direction of south and the season of summer. Water corresponds to the direction of north and the season of winter. Wood corresponds to the direction of east and the season of spring. Metal corresponds to the direction of west and the season of autumn. The element of earth corresponds to the midsummer season, at the center of the cross (Fig. 5e).

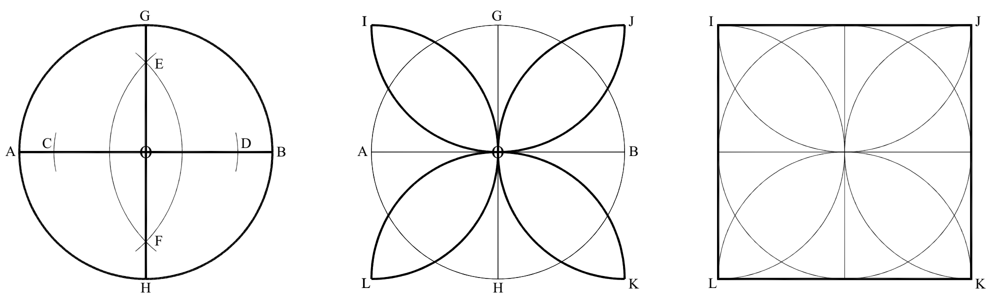



The Five Elements

Fig. 5e

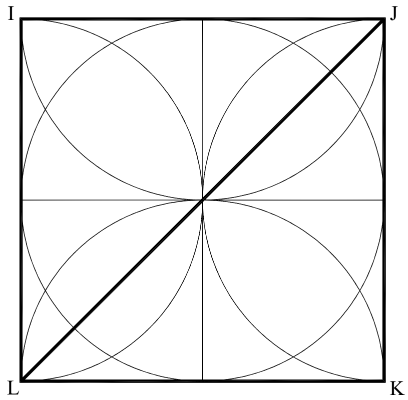
### *VI The ratio l : √2*

Repeat Figures 1, 2 and 3, as shown, to draw a square IJKL.




- Draw the diagonal JL through the square (IJKL).

If the side (IJ) of the square is 1, the diagonal (JL) equals √2, or 1.4142135.... The side and diagonal of any square are in the ratio l : √2 (Fig. 6).




IJ:JL :: 1:√2

Fig. 6

*VII Proof by the Pythagorean Theorum*

The Greek mathematician and mystic Pythagoras is credited with discovering the theorem of right-angled triangles that bears his name. The **Pythagorean Theorum** states that in any right triangle, the area of the square drawn on the hypotenuse ($c$) is equal to the sum of the areas of squares drawn on the triangle's remaining two sides ($a$ and $b$), in other words, $a^2 + b^2 = c^2$.

**Definitions:**

**Hypotenuse** is the side of a right-angled triangle that is opposite to, or subtends, the right angle. "Hypotenuse" is via the Late Latin hypotenusa from the Greek *hupoteinousa* (from *hupo* "from under" + *teinô* "to stretch" or "to stretch out in length"), which means "line subtending" [Liddle 1940, Simpson 1989].

The **square root** is a number or quantity that produces a given number when multiplied by itself. For example, the square root of 25 (√25) is 5 and the square root of 2 (√2) is 1.4142135…. Square roots of integers, or whole numbers, are often incommensurable.

The Pythagorean Theorum is demonstrated by the fact that a triangle of sides 3, 4, and 5 is a right triangle.

The square on the side of 3 contains 9 squares of side 1.

The square on the side of 4 contains 16 squares of side 1.

The square on the hypotenuse of 5 contains 9 + 16 or 25 squares of side 1 (Fig. 7).

A string of twelve equally spaced knots may be used to construct a right angle, when made into a 3:4:5 triangle.

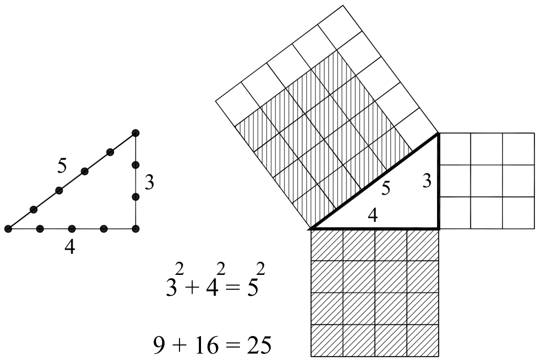



Fig. 7

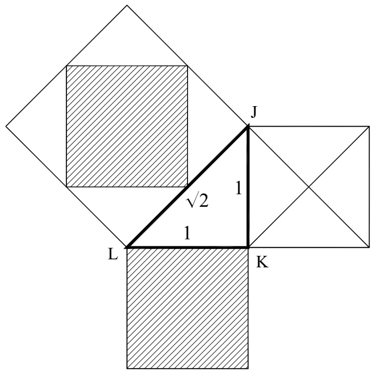



Fig. 8

Locate the right triangle JKL that exists within the square IJKL (see Fig. 6.)

Sides JK and KL each equal 1.

- Draw a square on side JK.
- Draw the square's two diagonals.

The square on side JK divides into four isosceles triangles.

- Draw a square on side KL.
- Draw a square on hypotenuse LJ.

The square on hypotenuse LJ contains the square on side KL and the four isosceles triangles within the square on side JK.

The area of the square on side JK is 1.

The area of the square on side KL is 1.

By the Pythagorean Theorum, the area of the square on hypotenuse LJ is 1 + 1 = 2.

Therefore, the side of the hypotenuse LJ is √2 (Fig. 8).

### VIII *Ad quadratum* constructions

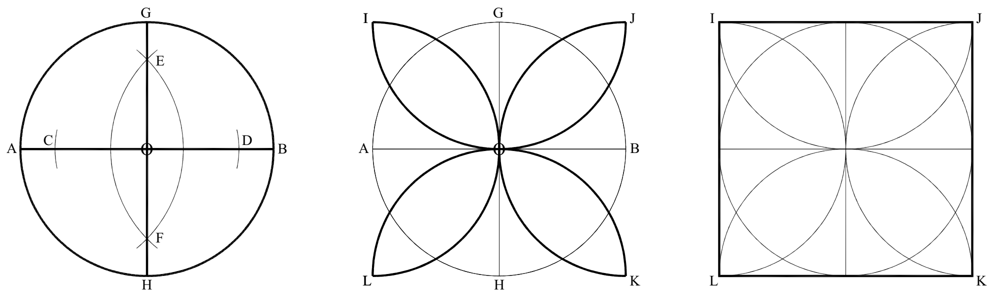
**Definition:**

***Ad quadratum*** means "to the square." *Quadratum* is the Latin for "square" or "quadrate." The suffix *ad* means "to" or "toward." [Lewis 1890].

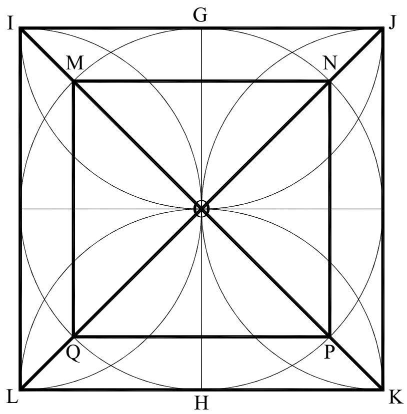
The 1 : √2 relationship between the side and diagonal of a square is intrinsic to the *ad quadratum* geometric construction. This is a series of squares in which the side of a larger square equals the diagonal of the next smaller, while the area of the smaller square is halved.[3] Inscribing a circle within a given square, then inscribing a new, smaller square within the circle may achieve the *ad quadratum* construction. Another method connects the midpoints of a square's four edges in order to set a smaller square diagonally within. In both instances, the edge lengths of successive squares decrease in the ratio l : 1/√2 or √2 : 1 [Watts 1996, 171 and Orrell 1988, 142-149].

METHOD 1: Alternating Squares and Circles

Repeat Figures 1, 2 and 3, as shown, to draw a square IJKL:




If the side (IJ) of the square is 1, the diagonal (JL) equals √2, or 1.4142135....

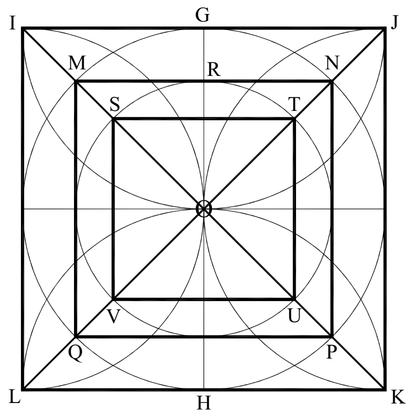



IJ:MN :: 1:1/√2

JL:NQ ::√2:1

IJ:JL :: MN:NQ :: 1:√2

Fig. 9




IJ:MN :: MN: ST :: 1:1/√2

JL:NQ :: NQ:TV ::√2:1

IJ:JL :: MN:NQ :: ST:TV :: 1:√2

Fig. 10

- Locate points M, N, P and Q where the diagonals (IK and JL) intersect the circle.
- Connect points M, N, P and Q.

The result is a smaller square (MNPQ).

The side (IJ) of the larger square (IJKL) equals the diagonal (NQ) of the smaller square (MNPQ).

The area of the larger square (IJKL) is double the area of the smaller square (MNPQ) (Fig. 9).

- Locate point R where the vertical diameter (GH) intersects the side (MN) of the smaller square (MNPQ).
- Place the compass point at O. Draw a circle of radius OR.
- Locate points S, T, U and V where the diagonals (IK and JL) intersect the circle of radius OR.
- Connect points S, T, U and V.

The result is a smaller square (STUV).

The side (MN) of the larger square (MNPQ) equals the diagonal (TV) of the smaller square (STUV).

The area of the larger square (MNPQ) is double the area of the smaller square (STUV) (Fig. 10).

### METHOD 2: Alternating Squares in Active and Passive Positions

Repeat Figures 1, 2 and 3, as shown, to draw a square IJKL:

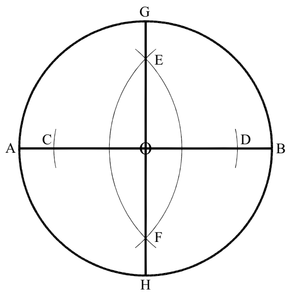



Fig. 1

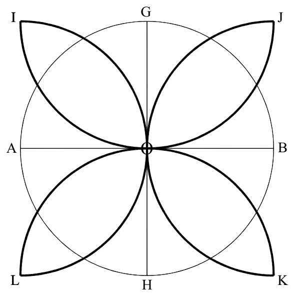



Fig. 2

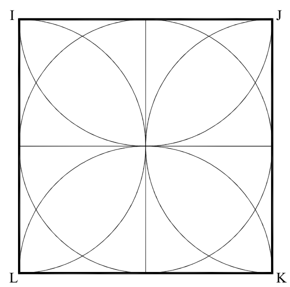
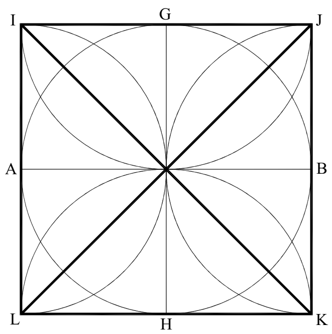
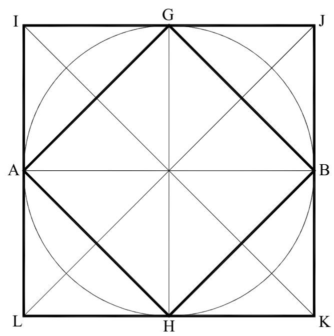



Fig. 3




IJ:JL :: 1:$\sqrt{2}$

Fig. 11.




IJ:GB :: 1:1/$\sqrt{2}$
JL:BA :: $\sqrt{2}$:1
IJ:JL :: GB:BA :: 1:$\sqrt{2}$

Fig. 12

- Draw the diagonals (IK and JL) through the square (IJKL) (Fig. 11).

Locate the midpoints (G, B, H and A) where the vertical and horizontal diameters (GH and BA) intersect the square.

- Connect points G, B, H and A.

The result is a smaller square (GBHA).

The side (IJ) of the larger square (IJKL) equals the diagonal (BA) of the smaller square (GBHA).

The area of the larger square (IJKL) is double the area of the smaller square (GBHA) (Fig. 12).

Locate the midpoints (N, P, Q and M) of the square (GBHA) where the diagonals (IK and JL) intersect the square.

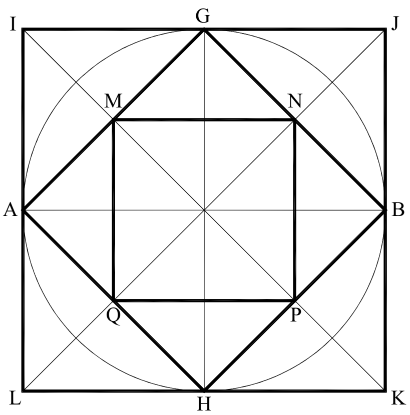



Fig. 13

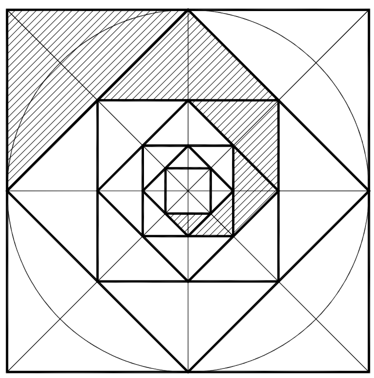


Fig. 14

- Connect points N, P, Q and M.

The result is a smaller square (NPQM).

The side (GB) of the larger square (GBHA) equals the diagonal (PM) of the smaller square (NPQM).

The area of the larger square (GBHA) is double the area of the smaller square (NPQM) (Fig. 13).

Repeat the process, as shown, alternating the squares in active (point up) and passive (base down) positions.

- Locate a 1 : √2 spiral composed of six isosceles triangles, in succession (Fig. 14).
- Locate two 1 : √2 spirals composed of six isosceles triangles each (Fig. 15).

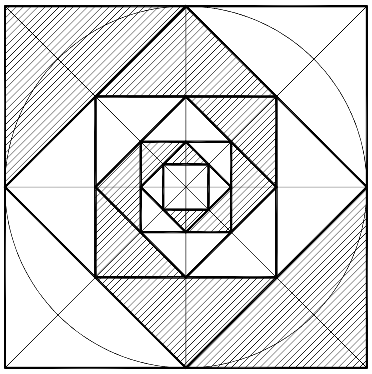


Fig. 15

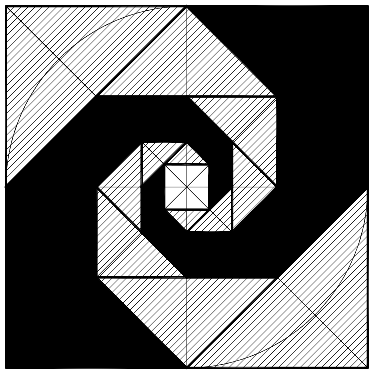


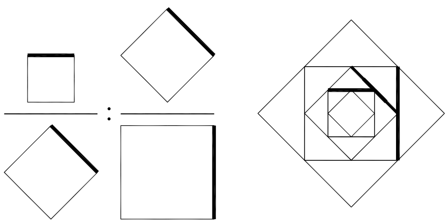
Fig. 16

- Locate four 1 : √2 spirals composed of six isosceles triangles each.

Note that the spirals decrease continuously toward a fixed point of origin (the pole or eye), but never touch it (Fig. 16).

*IX Three- and four- term proportions*

Geometric constructions can visualize abstract, algebraic statements, in spatial terms.[4] For example, the three-term geometric proportion 1 : √2 :: √2 : 2 can be expressed as a progression of squares, in which each successive term of proportion is the edge of a proportionally larger square (Fig. 17).



1:√2 :: √2:2

Fig. 17

The three-term geometric proportion 1 : 2 :: 2 : 4 can be expressed as a progression of squares, in which each successive term of proportion is the area of a proportionally larger square. Dividing each square into isosceles triangles of equal size makes this apparent (Fig. 18).

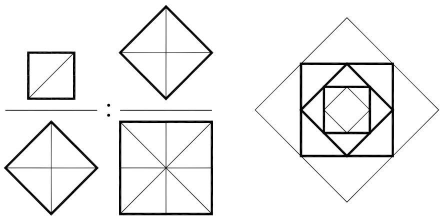



Fig. 18

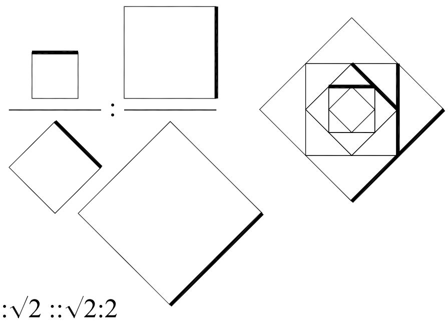



Fig. 19

The four-term proportion 1 : √2 :: 2 : 2√2 can be expressed as a progression of squares, in which each successive term of proportion is the edge of a proportionally larger square (Fig. 19).

**Definitions:**

In mathematics, **ratio** is the comparison of one quantity to another, as in *a* : *b* or *a* / *b*, signifying that "*a* is to *b*." "Ratio" is from the Latin *ratio* (from *rat-* "reckoned," from the verb *reri* "to think"), which means "reckoning, numbering," "calculation," and also "reason" or "relation" [Hoad 1996, Lewis 1879, Simpson 1989].

**Proportion** expresses similitude or likeness between two or more ratios. Proportion is the "due relation of one part to another…as renders the whole harmonious." "Proportion" is from the Latin *proportionem*, "comparative relation, analogy," which is adapted from *proportione*, "in respect of one's share." [Liddell 1940, Simpson 1989]. [See Fletcher 2004a, 94.]

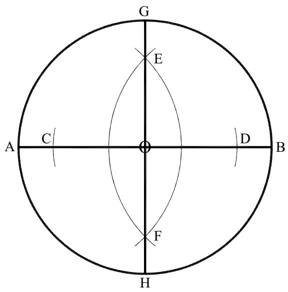
Mathematical ratios are signified by ":" which can mean "relates or compares to." Proportions are signified by "::" which can mean "in the same way as." Thus, the proportion *a* : *b* :: *b* : *c* is a comparison of the ratios *a* : *b* and *b* : *c*, such that "*a* relates to *b* in the same way as *b* relates to *c*."

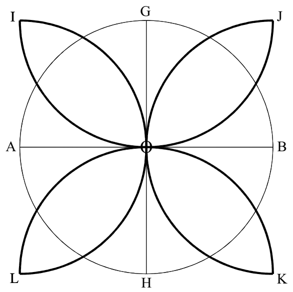
**Equation** expresses equality or sameness among two quantities or expressions. "Equation" is from the Latin *aequatio* (from *aequare* "make equal," from *aequus* "even, plain, level, flat"), which means "equal distribution." Equations are signified by "=" which means "is equal to" or "is the same as" [Hoad 1996, Lewis 1879, Soanes 2003].

Proportion and equation are distinct processes that reflect different premises. The sign "::" for proportion presumes the uniqueness of individual elements. The sign "=" for equation presumes that individual elements can be made equal or the same.

## *X The 1 : √2 rectangle*

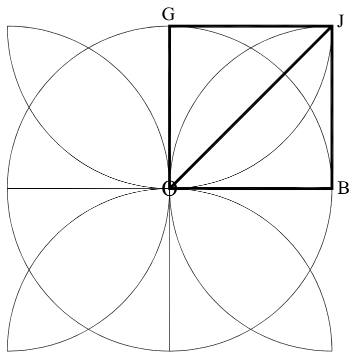
Repeat Figures 1 and 2, as shown.



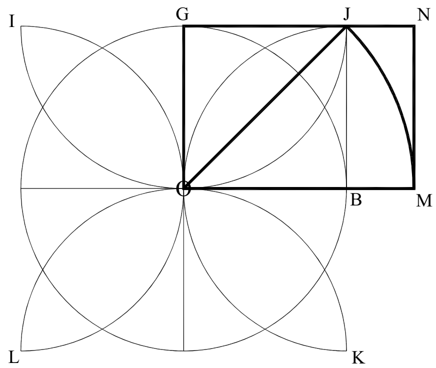

- Connect points O, G, J and B.

The result is a square (OGJB).




OG:OJ :: 1:√2

Fig. 20




OG:GN :: 1:√2

Fig. 21

- Draw the diagonal OJ through the square (OGJB).

The side (OG) and the diagonal (OJ) are in the ratio l : √2 (Fig. 20).

- Place the compass point at O. Draw an arc of radius OJ that intersects the extension of line OB at point M.
- From point M, draw a line perpendicular to line MO that intersects the extension of line GJ at point N.

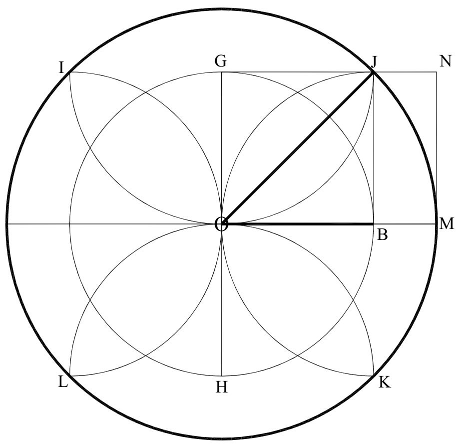
The result is a rectangle (OGNM) with short and long sides in the ratio 1: √2 (Fig. 21).

- Place the compass point at O. Draw a circle of radius OJ that intersects the points J, K, L and I.

The radius (OG) of the original circle and the radius (OJ) of the larger circle are in the ratio l:√2.

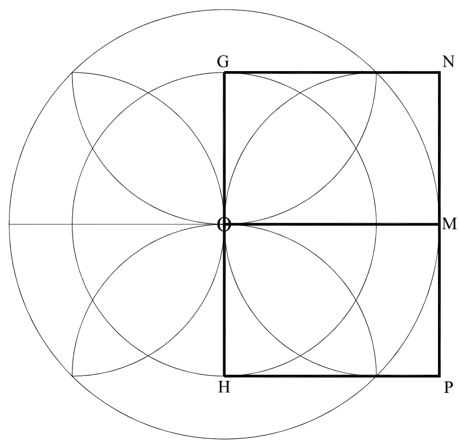
The perimeters of the two circles are in the ratio l : √2.

The areas of the two circles are in the ratio 1: √2 (Fig. 22).




OG:OJ :: 1:√2

Fig. 22




OG:GN :: GN: NP
1:√2 ::√2:2

Fig. 23

- From point H, draw a line perpendicular to line HG that intersects the extension of line NM at point P.

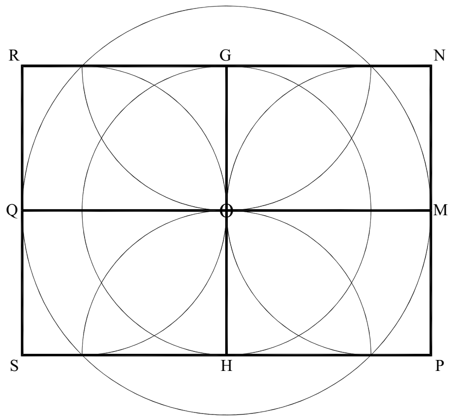
The result is a rectangle (GNPH) with short and long sides in the ratio √2 : 2 or 1: √2.

The major 1: √2 rectangle GNPH divides into two reciprocals (OGNM and HOMP) that are proportionally smaller in the ratio 1:√2 (Fig. 23).

- Extend the line MO to point Q, as shown.
- From point Q, draw a line perpendicular to line QM that intersects the extensions of lines NG and PH at points R and S.

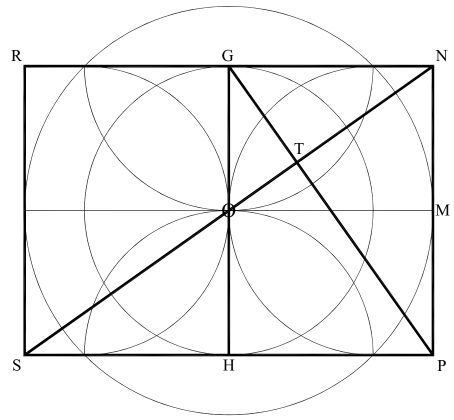
The result is a rectangle (NPSR) with short and long sides in the ratio 2: 2√2 or 1: √2.

The major 1: √2 rectangle NPSR divides into two reciprocals (GNPH and RGHS) that are proportionally smaller in the ratio 1:√2 (Fig. 24).




OG:GN :: GN:NP :: NP:PS
1:√2 ::√2:2 :: 2:2√2

Fig. 24




GP:NS :: 1:√2

Fig. 25

- Draw the diagonal NS of the major rectangle NPSR.
- Draw the diagonal GP of the reciprocal GNPH.

The diagonals (NS and GP) intersect at 90° at point T (Fig. 25).

- Locate the l : √2 rectangle OGNM.

The side MO intersects the diagonal GP at point U.

- From point U, draw a line perpendicular to line MO that intersects line GN at point V.

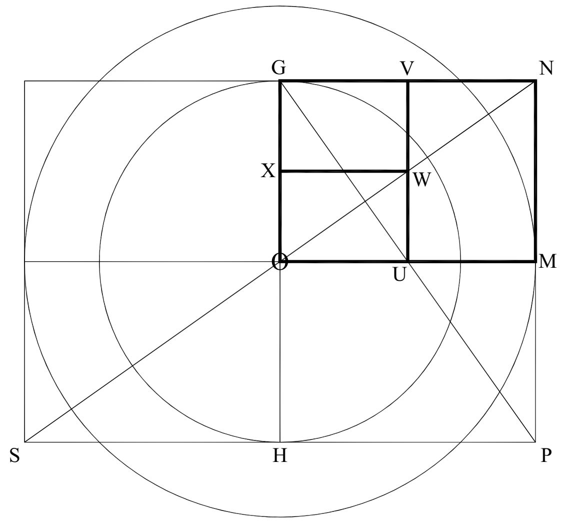
The result is a rectangle (UOGV) with short and long sides in the ratio 1/√2 : 1 or 1:√2. The major 1: √2 rectangle OGNM divides into two reciprocals (UOGV and MUVN) that are proportionally smaller in the ratio 1:√2.

- Locate the l : √2 rectangle UOGV.

The side VU intersects the diagonal NS at point W.

- From point W, draw a line perpendicular to line VU that intersects line OG at point X.

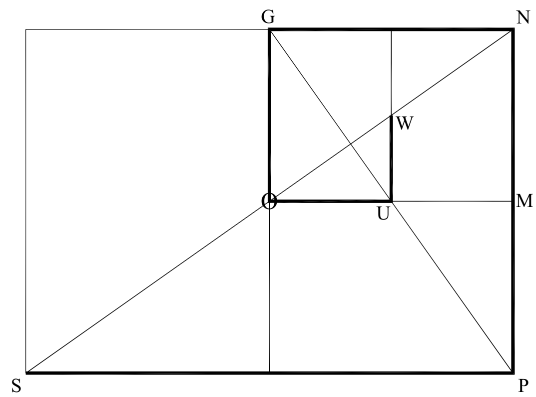
The result is a rectangle (WUOX) with short and long sides in the ratio 1/2 : 1/√2 or 1:√2. The major 1:√2 rectangle UOGV divides into two reciprocals (WUOX and VWXG) that are proportionally smaller in the ratio 1:√2 (Fig. 26).




WU:UO :: UO:OG:: OG:GN
1/2:1/√2 :: 1/√2:1 :: 1:√2

Fig. 26

- Locate the equiangular spiral of straight-line segments WU, UO. OG, GN, NP and PS (Fig. 27).




WU:UO :: UO:OG:: OG:GN :: GN:NP :: NP:PS
1/2:1/√2 :: 1/√2:1 :: 1:√2 ::√2:2 :: 2:2√2

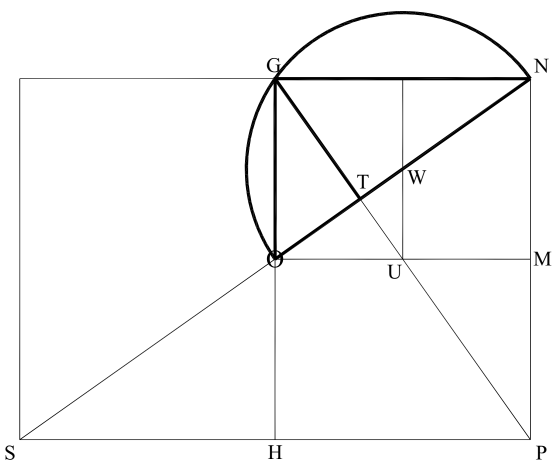
Fig. 27

- Locate the diagonal ON of the l : √2 rectangle OGNM.
- On the diagonal ON construct a semi-circle. (Place the compass point at W. Draw a semi-circle of radius WO, as shown.)
- Locate point G on the perimeter of the semi-circle.
- From point G, draw lines to points O and N.

According to the Theorum of Thales, the triangle OGN is a right triangle.

- From point G on the semi-circle, draw a line GT perpendicular to line ON.

According to the Law of Similar Triangles, triangles OTG, GTN and OGN are similar. Line TG is the mean proportional or geometric mean of lines TO and TN. [See Fletcher 2004b, 106-107.] (Fig. 28.)




TO:TG:: TG:TN

1:√2 ::√2:2

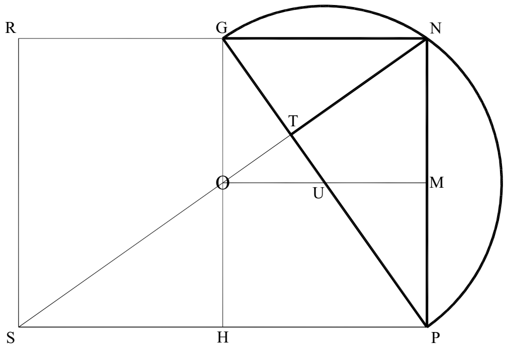
Fig. 28

- Locate the diagonal GP of the l : √2 rectangle GNPH.
- On the diagonal GP construct a semi-circle. (Place the compass point at U. Draw a semi-circle of radius UG, as shown.)
- Locate point N on the perimeter of the semi-circle.
- From point N, draw lines to points G and P.

The triangle GNP is a right triangle.

- From point N on the semi-circle, draw a line NT perpendicular to line GP.

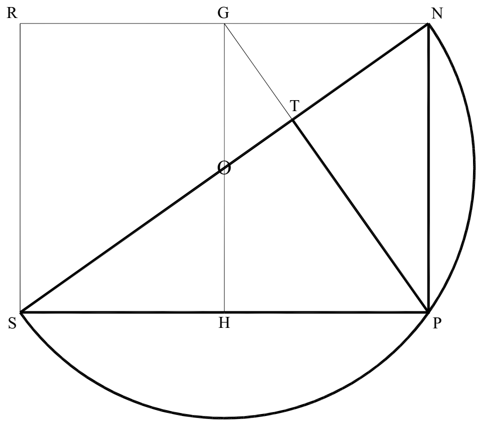
Triangles GTN, NTP and GNP are similar. Line TN is the mean proportional or geometric mean of lines TG and TP (Fig. 29).




TG:TN:: TN:TP

$\sqrt{2}$:2 :: 2:2$\sqrt{2}$

Fig. 29




TN:TP:: TP:TS

2:2$\sqrt{2}$ :: 2$\sqrt{2}$:4

Fig. 30

- Locate the diagonal NS of the l : √2 rectangle NPSR.
- On the diagonal NS construct a semi-circle. (Place the compass point at O. Draw a semi-circle of radius ON, as shown.)
- Locate point P on the perimeter of the semi-circle.
- From point P, draw lines to points N and S.

The triangle NPS is a right triangle.

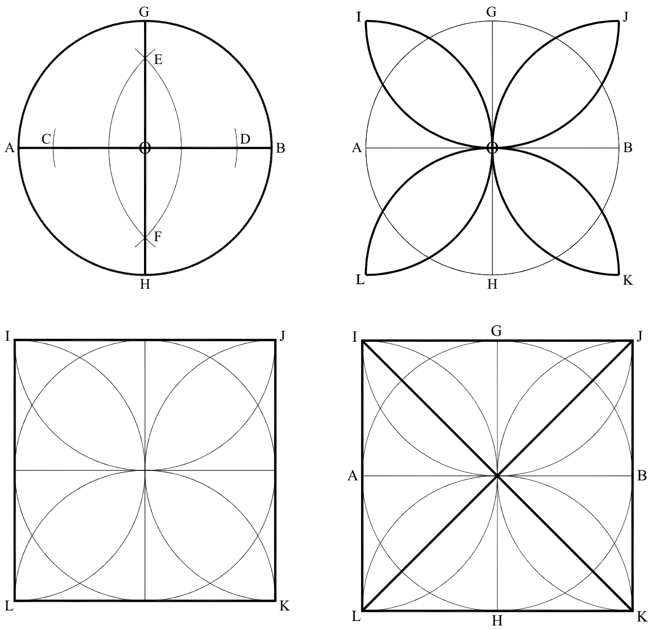
- From point P on the semi-circle, draw a line PT perpendicular to line NS.

Triangles NTP, PTS and NPS are similar. Line TP is the mean proportional or geometric mean of lines TN and TS (Fig. 30).

### *XI The Sacred Cut*

A unique 1 x √2 geometric construction derives from the square and its division by four arcs equal in radius to the square's half-diagonal. The result is a composition of one center square, four smaller corner squares, and four 1 : √2 rectangles. Tons Brunés names this the "Sacred Cut" for its ability to generate a circle and square of almost precisely equal perimeters and to divide the side of a square almost precisely into seven equal parts.[5]

Repeat Figures 1, 2, 3 and 11 as shown, to draw a square IJKL and its two diagonals.




IJ:JL :: 1:√2

The side and diagonal of any square are in the ratio l : √2.

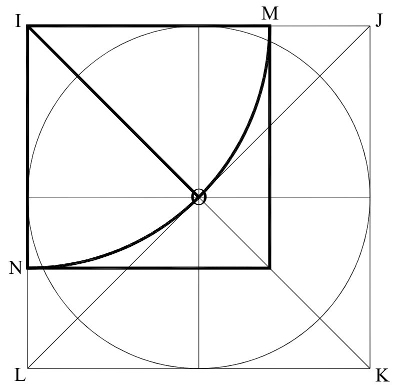
IJ : JL :: 1: √2

- Locate the half-diagonal IO.
- Place the compass point at I. Draw a quarter-arc of radius IO, intersecting the original square at points M and N.

If the side (IJ) of the original square is 1, lines IM and IN each equal 1/√2 and divide the square at sacred cuts.

IM : IJ :: 1: √2

- Draw a square on lines IM and IN, as shown (Fig. 31).




IM:IJ :: 1:√2

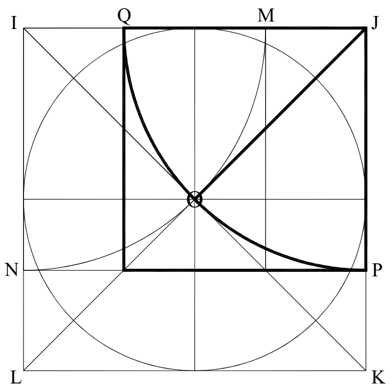
Fig. 31

- Locate the half-diagonal JO.
- Place the compass point at J. Draw a quarter-arc of radius JO, intersecting the original square at points P and Q.

If the side (JK) of the original square is 1, lines JP and JQ each equal 1/√2 and divide the square at sacred cuts.

In addition, IQ : QM :: 1: √2

- Draw a square on lines JP and JQ, as shown (Fig. 32).




IQ:QM :: 1:√ 2

Fig. 32

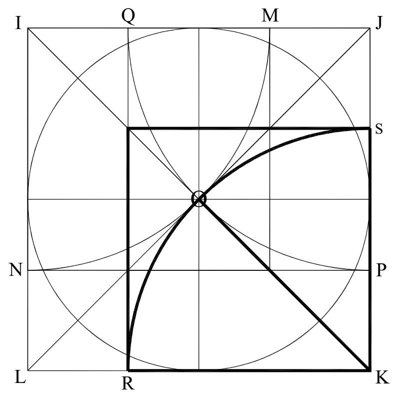



Fig. 33

-

- Locate the half-diagonal KO.
- Place the compass point at K. Draw a quarter-arc of radius KO, intersecting the original square at points R and S.

If the side (KL) of the original square is 1, lines KR and KS each equal 1/√2 and divide the square at sacred cuts.

- Draw a square on lines KR and KS, as shown (Fig. 33).
- Locate the half-diagonal LO.
- Place the compass point at L. Draw a quarter-arc of radius LO, intersecting the original square at points T and U.

If the side (LI) of the original square is 1, lines LT and LU each equal 1/√2 and divide the square at sacred cuts.

- Draw a square on lines LT and LU, as shown (Fig. 34).

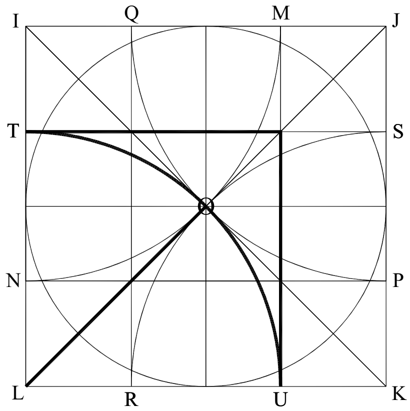



Fig. 34

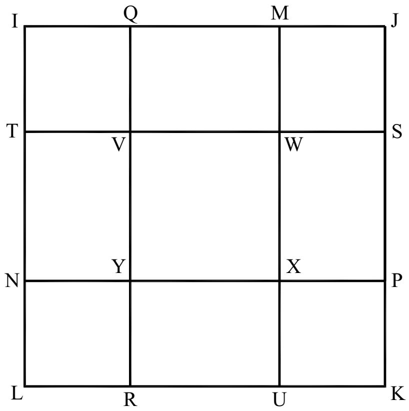



Fig. 35

- Remove all construction lines, except the original square (IJKL) and the lines (QR, MU, TS and NP) that divide the square at sacred cuts.

The grid that remains contains a center square (VWXY), four smaller corner squares (such as IQVT), and four 1 : √2 rectangles (such as QMWV), as shown (Fig. 35).

- Reintroduce the diagonals IK and JL.

The diagonal (IV) of the corner square (IQVT) is equal in length to the side (VW) of the center square (VWXY) and to the long length (VW) of the 1 : √2 rectangle (QMWV). The side (IQ) of the corner square (IQVT) and the side (VW) of the center square (VWXY) are in the ratio 1 : √2.

Each corner square (such as IQVT) contains two isosceles right triangles. The center square (VWXY) contains four identical isosceles right triangles.

The area of each corner square (such as IQVT) is half the area of the center square (VWXY). The diagonal (VX) of the center square (VWXY) is equal in length to the half-diagonal (IO) of the original square. The half-diagonal (IO) generates the sacred cut of the original square (IJKL) (Fig. 36).

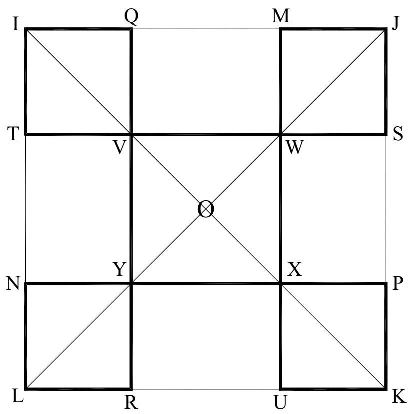



Fig. 36

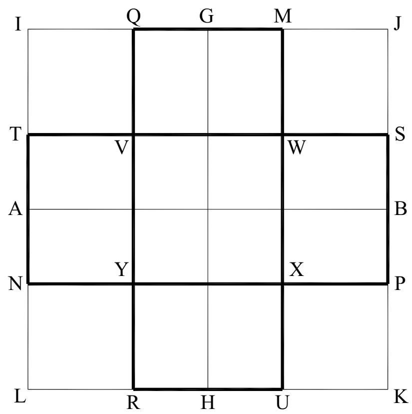



Fig. 37

- Reintroduce the horizontal and vertical axes (AB and GH).

The four rectangles (such as QMWV) are each in the ratio 1 : √2. The horizontal and vertical axes (AB and GH) divide the rectangles into two proportionally smaller 1 : √2 rectangles (Fig 37).

- From Figure 34, remove all construction lines, except the original square (IJKL), the diagonals (IK and JL), and the four quarter-arcs, as shown (Fig. 38).

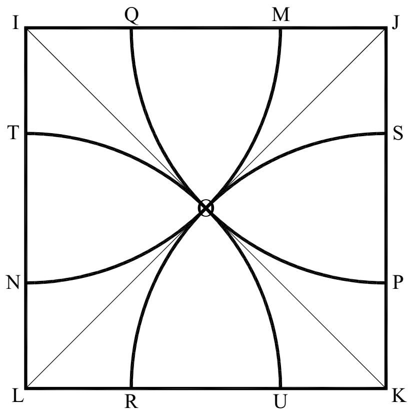



Fig. 38

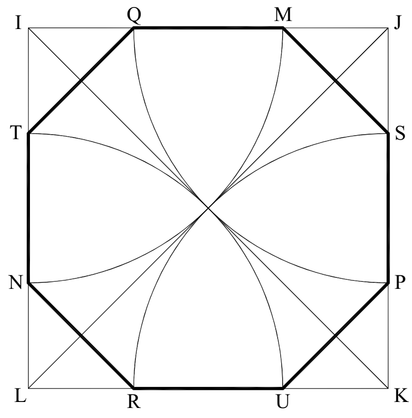



Fig. 39

- Connect the points where the quarter-arcs intersect the original square (Q, M, S, P, U, R, N and T).

The result is a regular octagon that inscribes the original square (Fig. 39).[6]

The sacred cut construction illustrates how successive squares increase in geometrical progression by the addition of L-shaped gnomons.[7]

- From Figure 34, remove all construction lines, except the original square (IJKL), the diagonal IK, the midpoint (G) of line IJ, and the midpoint (O) of the diagonal IK.
- Place the compass point at I. Draw a quarter-arc of radius IG, intersecting the original square at points G and A.
- Connect points I, G, O and A.

The result is a square (IGOA).

- Place the compass point at I. Draw a quarter-arc of radius IO, intersecting the original square at points M and N.
- From point M draw a line perpendicular to line IJ, intersecting the diagonal (IK) at point Z.
- Connect points I, M, Z and N.

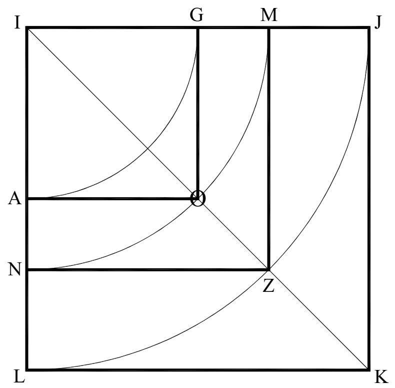
The result is a square (IMZN).

- Place the compass point at I. Draw a quarter-arc of radius IJ, intersecting the original square at points J and L.

The diagonal (IO) of square IGOA is equal in length to the side (IM) of square IMZN.

The diagonal (IZ) of square IMZN is equal in length to the side (IJ) of square IJKL.

The three squares progress in the ratio $1 : \sqrt{2}$.




$$IG : IM :: IM : IJ :: 1 : \sqrt{2}$$
$$IO : IZ :: IZ : IK :: 1 : \sqrt{2}$$

Fig. 40

Line OG divides line IM at a sacred cut.

Line ZM divides line IJ at a sacred cut.

The area of square IGOA is half the area of square IMZN.

The area of square IMZN is half the area of square IJKL (Fig. 40).

### *XII The tetractys*

The tetractys, thought to be invented by Pythagoras, is an equilateral triangle composed of ten dots. It builds on the notion that the sum of the first four numbers (1 + 2 + 3 + 4) expresses totality and perfection, signified by the decad or number "10." The tetractys conveys that all things are conceived in unity (One), proceed through four levels of manifestation, and return to unity (Ten), once again. The mathematician and Platonic philosopher Theon of Smyrna, among others, interpreted various principles of natural and cosmic order through the tetractys model [Guthrie 1987, 28-30; Theon of Smyrna 1979, 62-66].

**Definition:**

"**Tetractys**" is the Greek *tetraktus*, from *tetras*, which means "the number four" or "the fourth day." It is the Pythagorean name for the number figure that expresses the sum of the first four numbers (1 + 2 + 3 + 4 = 10) and is understood to mean the source of all things. Another name for tetractys is "quaternary" [Liddell 1940, Simpson 1989].

### Number

The original tetractys is formed by the addition of the numbers 1, 2, 3 and 4. These correspond to the archetypes of Monad, Dyad, Triad and Tetrad and their qualities of Unity, Multiplicity, Harmony and Body or Form (Fig. 41a).

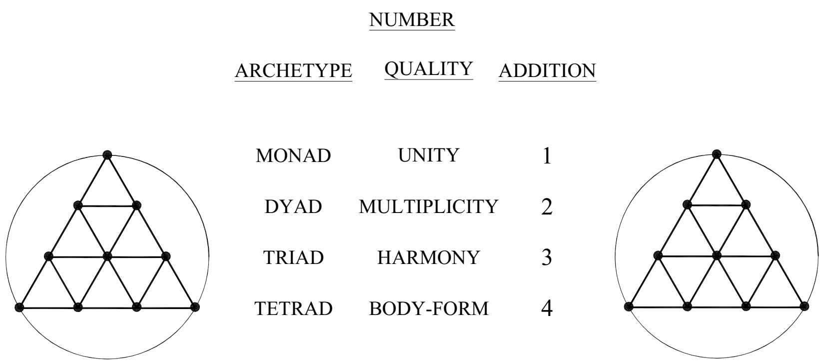



Fig. 41a

### Geometry or Magnitude

In the physical space of Euclidean geometry, the numbers 1, 2, 3 and 4 may be compared to the point, line, plane and solid. A minimum of two points is required to make a line. A minimum of three is required to make a plane figure (the triangle). A minimum of four is required to make a solid body (the tetrahedron). Points are places or locations without dimension. Lines, planes and solids delineate the first, second and third spatial dimensions of length, width and depth. Geometric figures contain vertices, edges, surfaces and volumes. (Fig. 41b).

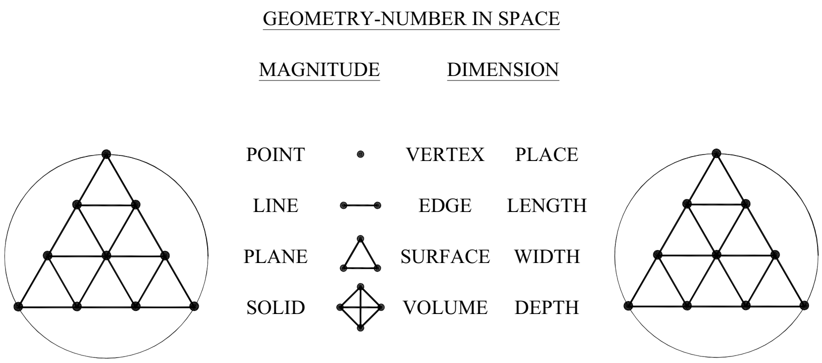



Fig. 41b

## Music

Antiquity credits Pythagoras with associating the length of a vibrating body with the sound of its tone or pitch, and consequently with expressing the most elementary musical concords in simple whole number ratios. The ratios 1 : 1, 2 : 1, 3 : 2, and 4 : 3 describe the relative lengths of two vibrating strings that sound the fundamental, the octave, and the perfect fifth and fourth musical intervals, respectively. Inversely, the ratios 1 : 1, 1 : 2, 2 : 3, and 3 : 4 describe the same musical concords relative to their rate of frequency, or the numbers of cycles the two strings vibrate per unit of time (Fig. 41c).[8]

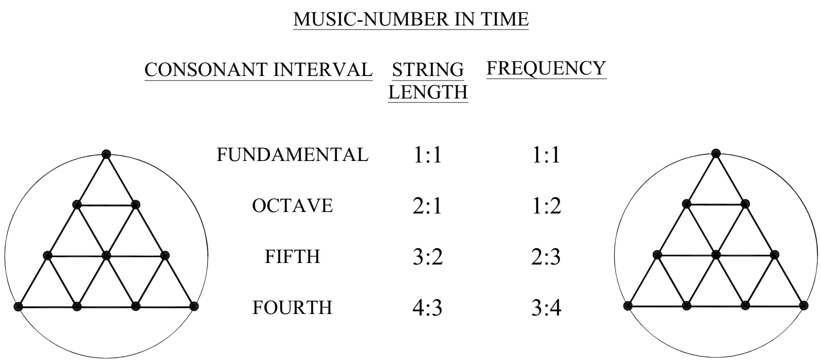


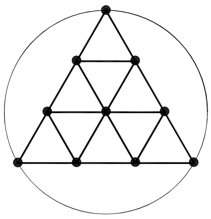

Fig. 41c

## Platonic lambda

The Platonic *lambda*, so called because it resembles the Greek letter of that name (Λ), advances the first odd and even numbers (2 and 3) through their square and cubic powers. After the number 1, the first even number (2) and its multiples of 4 and 8 make up the series on the left. The first odd number (3) and its multiples of 9 and 27 make up the series on the right. Both series signify the development of form through point (place), side (length), surface (length x width) and solid (length x width x depth). The linear numbers 2 and 3 represent straight and curved sides, respectively. The squared numbers 4 and 9 represent plane and curved surfaces, respectively. The cubed number 8 represents solids of planar surfaces, such as the cube. The cubed number 27 represents solids of curved surfaces, such as spheres and cylinders [Theon of Smyrna 1979, 62-63].

The *Timaeus* of Plato attributes the numbers of the *lambda* to the structure of our three-dimensional universe. [See Plato 1961: *Timaeus* 35a-36c, 1165-1166.] F. M. Cornford offers a compelling interpretation that incorporates the insertion of arithmetic and harmonic means between the numbers 1, 2, 3, 4, 8, 9 and 27 (Fig. 41d).[9]

PLATONIC LAMBDA Λ



| MULTIPLICATION EVEN | MULTIPLICATION ODD | LINEAR FORM |
|---|---|---|
| 1 | 1 | UNITY-POINT |
| 2 | 3 | LINEAR-SIDE |
| 4 | 9 | SQUARE-SURFACE |
| 8 | 27 | CUBIC-SOLID |

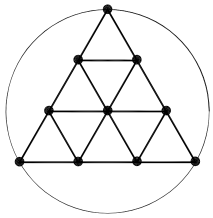
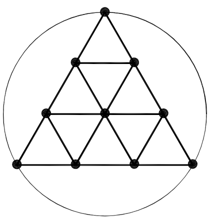


Fig. 41d

## Cosmos

The tetractys may signify elements of the cosmos such as: the four cardinal points of east, south, west and north; the four seasons of spring, summer, autumn and winter; and the four periods of sunrise, midday, sunset and midnight, in a solar day (Fig. 41e).

COSMOS



| CARDINAL POINT | SEASON | SOLAR DAY |
|---|---|---|
| EAST | SPRING ♈ | SUNRISE |
| SOUTH | SUMMER ♋ | MIDDAY |
| WEST | AUTUMN ♎ | SUNSET |
| NORTH | WINTER ♑ | MIDNIGHT |

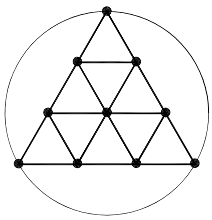


Fig. 41e

## Nature

Plato says the natural world is composed of four elements of fire, air, water and earth, which he compares to four simple volumes or bodies-tetrahedron, octahedron, icosahedron and cube, respectively. In fact, these comprise four of the five elementary or "regular" solids, whose surfaces are composed entirely of equilateral triangles, regular pentagons, or perfect squares. Plato alludes to a fifth regular volume, the dodecahedron of twelve pentagonal faces, which is understood to represent the zodiac or totality [1961: *Timaeus* 54d-56c, 1181-1182].

### Definitions:

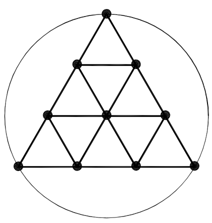
**Platonic** or **regular solids** are convex polyhedra (solid bodies comprised of polygonal faces), such that:

- all of the faces are the same;
- all of the faces are regular polygons-squares, triangles or pentagons;
- the same number of edges meet at each vertex; and
- all of the vertices lie on the surface of a circumscribing sphere.

The five solid bodies that meet these criteria are the tetrahedron, octahedron, hexahedron (or cube), icosahedron and dodecahedron. Tetrahedrons are contained by four equilateral triangles. Octahedrons are contained by eight equilateral triangles. Hexahedrons or cubes are contained by six squares. Icosahedrons are contained by twenty equilateral triangles. Dodecahedrons are contained by twelve pentagons.

"Tetrahedron" is from the Greek *tetraedros* (from *tetra* "four" + *hedra* "sitting-place" "seat, base" or "face of a regular solid"), which means "having four faces." "Octahedron" is from the Greek *oktaedros* (from *okta* "eight" + *hedra*), which means "eight-faced." "Hexahedron" is from the Greek *hexaedros* (from *hex* "six" + *hedra*), which means "six-faced." "Cube" is from the Greek *kubos*, which means "dice." "Icosahedron" is from the Greek *eikosaedros* (from *eikosi* "twenty" + *hedra*), which means "twenty-faced." "Dodecahedron" is from the Greek *dôdekaedros* (from *dôdekas* "twelve" + *hedra*), which means "twelve-faced." [Liddell 1940, Simpson 1989, Soanes 2003].

NATURE AND MATTER



| ELEMENT | SIMPLE BODY | GROWTH |
|---|---|---|
| FIRE | TETRAHEDRON | SEED |
| AIR | OCTAHEDRON | ROOT-STEM |
| WATER | ICOSAHEDRON | BLOSSOM |
| EARTH | CUBE | FRUIT |

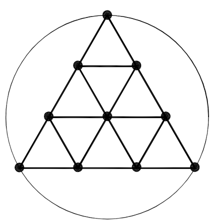


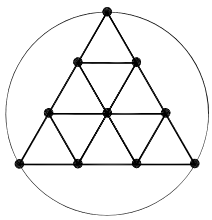
Fig. 41f

In the botanical world, organisms develop through four distinct phases of: point, or seed; growth in length, through the root and stem; growth in width or breadth, through blossoms and flowers; and solid growth in thickness or depth, in the fruit. (Fig. 41f).

### Human Society

Theon says that human beings exhibit four faculties of spirit, intellect, emotion and sense, while making judgments based on thought, science, opinion and feeling. Human lives proceed through four distinct ages of childhood, youth, maturity and old age, recognizing in turn that which is "mine," "yours," "ours" and "thine." Human societies evolve from individuals; to families with "*line*ages;" to villages that expand across planar surfaces; and ultimately to dense cities that "solidify" in three dimensions [Theon of Smyrna 1979, 64-65] (Fig. 41g).

MAN AND SOCIETY



| FACULTY | AGE OF MAN | | SOCIETY |
|---|---|---|---|
| SPIRIT | CHILDHOOD | MINE | SELF |
| INTELLECT | YOUTH | YOURS | FAMILY |
| EMOTION | MATURITY | OURS | VILLAGE |
| SENSE | OLD AGE | THINE | CITY |

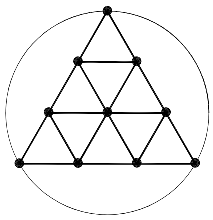


Fig. 41g

Theon concludes that all the world is based on such quaternaries and therefore "is perfect because everything is part of it, and it is itself a part of nothing else" [Theon of Smyrna 1979, 65-66]. Thus, the Pythagoreans swore "by him who into our souls has transmitted the Sacred Tetractys, the spring of eternal Nature" ("The Golden Verses of Pythagoras" [Guthrie 1987, 164]).

### *XIII Application: Bramante's Tempietto*

In a previous *Geometer's Angle*, we examined Donato Bramante's Doric style Tempietto in Rome, as it appears in Sebastiano Serlio's *Trattato di architettura* (*On Architecture*). The elevation of this Renaissance church conforms to the proportions of a l:√3 rectangle and the vesica piscis it encloses. [See Fletcher 2004b, 109.] In similar fashion, the Tempietto in plan emerges from an interplay of circles and squares and the square's inherent l : √2 proportions.

The plan features an inner sanctuary, surrounded by a portico with two rings of coffers and a ring of sixteen columns, culminating in three rings of steps [Serlio 1544, III, xlii, 67v] (Fig. 42).

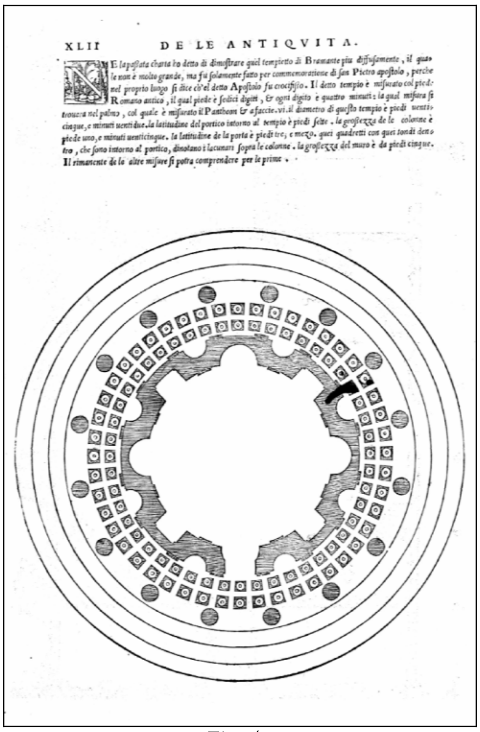


Fig. 42

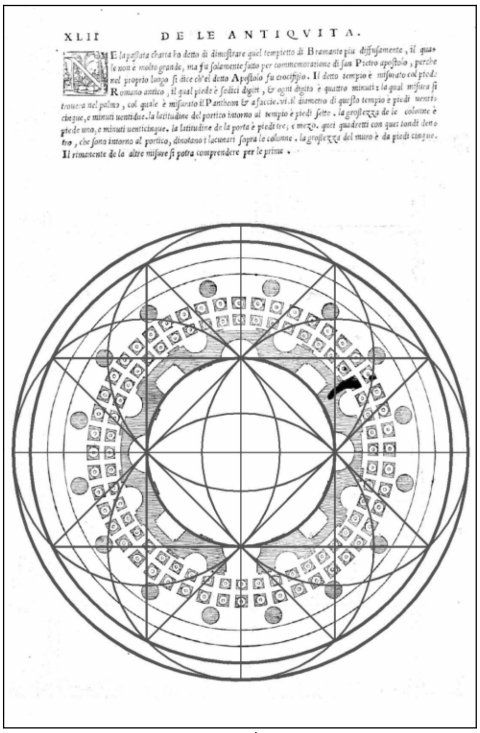


Fig. 43

- Draw a circle that traces the inside of the sanctuary wall.
- Within the circle, draw a square in active position.
- Place the compass point at a corner of the square. Draw a circle whose radius equals the side of the square. Repeat at all four corners.
- Draw vertical and horizontal axes to the outer limit of the construction, as shown.
- Draw a circle that encloses the four circles.

The large circle locates the outside edge of the outermost step.

- Draw a square about the circle that locates the inner sanctuary.
- Extend the sides of the two squares in both directions, until they intersect, as shown.

The result is a star-octagon. Its $1 : \sqrt{2}$ proportions are apparent.

- Draw a circle that encloses the star-octagon.

That circle locates the inside edge the outermost step (Fig. 43).

Variations on the star-octagon may be seen in traditional Amish quilts. Another example appears in a pre-1500 A.D. petroglyph of the Native American Mi'kmaq people, discovered in

Bedford, Nova Scotia. In Mi'kmaq hieroglyphic writing, the star-octagon symbolizes the 'sun.' The knobbed crosses may be part of the hieroglyph for 'star' [Whitehead 1992, 7] (Fig. 44).

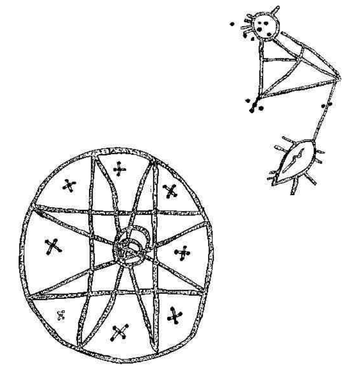


Fig. 44. Image: Petroglyph tracing. R. H. Whitehead, 1983. History Collection, Nova Scotia Museum, Halifax, P179/ N-17, 24

## Notes

1. As viewed from the Northern hemisphere.
2. See Fletcher 2005, 143, for definitions of celestial equator, ecliptic, and Tropics of Cancer and Capricorn.
3. In the *Meno* of Plato, Socrates uses this construction to show that the soul of an unschooled slave boy possesses knowledge, even before birth, of fundamental truths that may be accessed through recollection [Plato 1961, *Meno* 82b-86b, 365-371].
4. An algebraic statement of a three-term geometric proportion is $a : b :: b : c$. An algebraic statement of a four-term proportion is $a : b :: c : d$.
5. [Brunés 1967: I, 54-108.] Brunés' technique for "squaring of the circle" is based on the observation that the quarter-arc drawn on the sacred cut, or half the diagonal of the original square, and the diagonal of half the original square are equal in length within a degree of accuracy of 0.6%. We will explore various techniques for "squaring the circle" in a future column [See Brunés 1967: I, 73-74, 93-94; Watts 1987, 269; Watts 1996, 171-172].
6. This method is the basis of Sebastiano Serlio's construction of the octagon [Serlio 1996: I, 28 (fol. 19)].
7. See Fletcher 2004b, 110, note 5, for the definition of gnomon.
8. For example, a vibrating string of length 1 sounds a perfect octave above the tone sounded by a comparable string of length 2, while the string of length 1 vibrates at double the frequency.
9. In other words, the numbers contained in the two geometric series (1, 2, 4, 8) and (1, 3, 9, 27). [See Cornford 1937, 66-72.] According to Theon, "the arithmetic mean is one in which the mean term is greater than one extreme and less than the other by the same number." (1, 2, 3) "The geometric mean, also called the proportion proper, is the one in which the mean term is greater than one extreme and is less than the other by a multiple or superpartial ratio of the first term to the second or of the second to the third." (1, 2, 4) The harmonic mean occurs when "the mean term is greater than one extreme and is less than the other by the same part of the extremes. Thus in the proportion formed of the numbers 2, 3, and 6, the extreme 6 is greater than 3 by half of 6, and the other extreme 2 is less than 3 by half of 2" [Theon of Smyrna 1979, 76]. If the extreme terms are $a$ and $c$, the arithmetic mean is $(a + c)/2$; the geometric mean is $\sqrt{ac}$, and the harmonic mean is $2ac/(a+c)$. The arithmetic mean is also known as the mathematical average. We will revisit proportional means in a future column.

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### *About the geometer*

Rachel Fletcher is a theatre designer and geometer living in Massachusetts, with degrees from Hofstra University, SUNY Albany and Humboldt State University. She is the creator/curator of two museum exhibits on geometry, "Infinite Measure" and "Design By Nature". She is the co-curator of the exhibit "Harmony by Design: The Golden Mean" and author of its exhibition catalog. In conjunction with these exhibits, which have traveled to Chicago, Washington, and New York, she teaches geometry and proportion to design practitioners. She is an adjunct professor at the New York School of Interior Design. Her essays have appeared in numerous books and journals, including "Design Spirit", "Parabola", and "The Power of Place". She is the founding director of Housatonic River Walk in Great Barrington, Massachusetts, and is currently directing the creation of an African American Heritage Trail in the Upper Housatonic Valley of Connecticut and Massachusetts.