Arzu Gönenç Sorguç

Teaching Mathematics in Architecture

The Department of Architecture of Middle East Technical University offers a course entitled 'Mathematics in Architecture' for the third year students. In the beginning of the term, students are forced to imagine themselves as two-dimensional creatures living in a two-dimensional space. At this point, fundamentals of architectural geometry are introduced first in the plane, simply by employing the set concept; mapping as a general tool is then introduced and students are asked to use mapping in their design to correlate the project requirements and geometry. Following that, the principles of isometries and isometric constructions are introduced. In the second part of the term, students are allowed to think in terms of three-dimensional space and topics related with the principles of similarities and proportions and symmetry are presented. In the final part of the course, students are forced to think themselves as three-dimensional creatures living in a four-dimensional space and this fourth dimension is sought. The last topic of the course is related with biomimicry in architecture and mathematics inherent in bio-forms and man-made structures

1 Introduction

Mathematics may be the only way to understand the 'space' in which we are living. We have been modifying/changing our space but at the same time we have been modified/changed by the space. The architects in 'our space' have been designing 'our sub-spaces' or even it is possible to call 'our sub-universes' which should be coherent with 'our space'. In that sense mathematics is one of the important tools to fulfill this task. Therefore, in the Department of Architecture of Middle East Technical University we decided to propose a course entitled 'Mathematics in Architecture' for the third year students (Arch 333).

In the beginning of the term, students are forced to imagine themselves as two-dimensional creatures living in a two-dimensional space. At this point, fundamentals of architectural geometry are introduced first in the plane, simply by employing the set concept; mapping as a general tool is then introduced and students are asked to use mapping in their design to correlate the project requirements and geometry. Following that, the principles of isometries and isometric constructions are introduced. In the second part of the term, students are allowed to think in terms of three-dimensional space and topics related with the principles of similarities and proportions and symmetry are presented. In the final part of the course, students are forced to think themselves as three-dimensional creatures living in a four-dimensional space and this fourth dimension is sought. The last topic of the course is related with biomimicry in architecture and mathematics inherent in bio-forms and man-made structures.

Throughout the course, the participation of the students is encouraged through team work and projects. The first projects are mostly the analysis of famous buildings and their task is to investigate the mathematics inherent in these buildings. In later projects, students start providing their own proposals. The participation of the students in this course is essential and the examples are chosen according to their interests.

It can be seen that in the beginning of the term students feel uneasy with mathematics but by the end they consider mathematics as a very useful design tool and find that their creativity is improved. Therefore, it is possible to say that this course improves the design skills of the architectural students as well as their awareness of their environment.

2 Mathematics and architecture

Since the very beginning of time, men have been trying to understand the 'universe' and 'space' that we live in by employing multi-disciplinary point of views. Today, we have more questions then ever before, but still these two concepts are can be sources of confusion and every discipline tries to provide an answer for them. Architects and mathematicians are seeking the answers within the domains of knowledge of their disciplines. It is possible to give the definitions of these two interrelated concepts. Webster defines 'universe' as 'the totality of known or supposed objects and phenomena throughout space; the cosmos; macrocosm, the whole world with reference to humanity, a world or sphere in which something exists or prevails, and in logic represents the aggregate of all objects' [Merriam-Webster 2004]. The definition of 'space' is much broader and has several usages in languages; Webster defines it as 'the unlimited three-dimensional realm or expanse in which all material objects are located and all events occur, the portion or extent of this in a given instance, extent or area in two dimensions; a particular extent of surface, a set of points or mathematical elements that fulfills certain prescribed conditions: Euclidean space; vector space, an interval of time' [Merriam-Webster 2004].

Men and space interact with each other in many levels, and today concepts of multi-dimensional and virtual space are increasingly linked to new concepts introduced by computational and information technologies. The involvement of IT in almost every 'knowledge domain' and the interventions of various disciplines provides, on the one hand, solutions and answers to many problems, but on the other hand it results in knowledge domains that are increasingly complicated due to the complexity of the tools/systems employed. In this paradigm, mathematics still plays a unique role providing a universal and pure language shared by all fields. Architects providing designs to create man's own 'sub-spaces' use mathematics to express themselves in a more implicit way in comparison to other disciplines. In every architectural design, mathematics has several manifestations that contribute to the process, relating needs with design, providing an order, and a 'graphical code,' which is a product of geometry or some mathematical functions to allow the integration of other disciplines.

The important role of mathematics has been recognized by all architects in ages, yet architects, and especially students of architecture, feel uneasy about mathematics, which results in some mental blocks which later prevent further inquiries into problems of architecture and concepts belonging to IT. In 2000 a mathematics course entitled 'Mathematics in Architecture' was proposed in the Department of Architecture of Middle East Technical University to address these issues, and since that time this course has been mostly offered to second and third year undergraduate students of architecture. In this paper, course content and teaching methods will be described briefly, in order to share our four years of experience in this field.

3 The course description and methodology

'Mathematics in Architecture' is an elective course that aims to make explicit the relation between architecture and mathematics, thus the role of mathematics is emphasized, with the new age of informatics and so called 'algorithmic thinking' questioned in architectural design. The major challenges in designing the course content are not only to decide on the topics to be covered, but also to overcome the uneasiness of the students. Thus, subjects and examples that students are already familiar with from their basic design education and even from their daily life are chosen. Students are forced to reconsider topics from a mathematical point of view. They are encouraged to use mathematics as a tool to express their perception or solution and to enhance their ability to evaluate several combinations and optimize their choices, as well as to develop the attitude of algorithmic thinking in handling complicated problems. They are also encouraged to integrate IT in their designs to explore new media.

In the very first lectures, students are forced to imagine themselves living in a world of two-dimensions in order to question concepts like 'dimensions', 'reference systems' 'coordinates', and 'space'. Edwin Abbot's famous book *Flatland: A Romance of Many Dimensions* [1952] was chosen as the reading assignment and forms a very important platform for further discussions of problems of *n*-dimensionality. Starting from the very beginning, the concepts of 'sets' and then 'functions-relations' are used in throughout the course in order to establish a base for further discussions.

Following this, issues related to form-geometry and structural stability/materials are discussed. Euclidian forms are also defined as sets and these forms with their geometries are analyzed by revealing their relation with chosen materials and the resulting structural stability. The artifacts representing these relations are chosen from among the most popular ones which are introduced to students in the first year: Roman and Greek columns with their slender forms; the Pantheon with its huge dome and the Coliseum with its curved form; Calatrava's bridges with hyperbolas, etc. In all these examples, in addition to studying the geometries, the students are encouraged to think in terms of stress/strain and geometric relations. It is seen that, after these discussions, concepts such as moment of inertia, polar moment of inertia, neutral axis, etc., which are considered abstract by many students, become concrete. Consequently, it is observed that students feel more comfortable in designing structural systems and their forms due to that awareness. This part is completed by showing many examples both animate and inanimate, and the concept of biomimesis is briefly mentioned. Here, it is important to point out that mathematics is also a tool for biomimesis as long as a proper objective reference is set. Many books are assigned for these studies and some of them are discussed in class [Penrose 1999]. The ancient Japanese art of paper-folding, origami, is also used to study geometry-form-structural stability and the relationship between 2-D and 3-D spaces. Origami, an art that the students mostly consider as toy-making or a hobby, takes on a new meaning when seen in relationship to the issues outlined above. Moreover, students start to look at and question their 'universe,' which they think they know, from the different point of view provided by mathematics, which also reveals other possibilities for thinking with a broader perspective.

Following these topics, the students are required to read *Gulliver's Travels* [Swift 1986] and are asked what they think about that book as architects. Actually, this question is also asked casually at the beginning of this course, as if this question is just a part of daily conversation. It is interesting to see that at the beginning what they say is just about the story of the book, but during the second part of the course, they say that the dominant characteristic of the book is related to 'scaling'. From that point on, isometries and similarities are the focus of the course. In discussing the isometries, students are asked first to 'decipher' an existing 'wallpaper' with the rules of isometries and provide a 'pseudo-code' of it. Next, they are asked to design their own wallpaper, which is an exercise that they did in the first-year basic design course. However, this time, students are asked first to provide rules and describe the constraints. In this exercise, the intention is to discuss the process of 'mathematical optimization' and to introduce the concept of 'algorithmic thinking'. Terms such as design computing, shape grammars, etc., are not mentioned explicitly during these exercises. The reason for this is to provoke students to develop their own attitudes to understand these issues, first by pure mathematics, then with the discussions in literature. Music and dance also employed as tools to teach isometries and to study 'mapping' in terms of functions and relations [Beardon 1983; Hood 1969; March and Steadman 1971].

After studying isometries, the students are asked a simple question: Can you graphically manifest infinity? Then the famous chaos game is played all together without saying what is happening. The purpose of these exercises is to present the principles of 'fractals', 'self-similarity' and then concept of 'fractal architecture'. Many visual examples are shown, and the students are also asked to try to find examples from nature, flowers, pine cones, corns, leaves etc. Students are asked to read Gleick's famous book *Chaos: Making a New Science* [1988]. As the discussions progress as a whole, the examples brought by the students to the class increase in number and become more interesting. Then, similarities and similarity transformations are studied together with scales and proportions. From the Golden Ratio to Le Corbusier's modulor to today's natural logarithm's base are questioned along with several examples of man-made artifacts as well as examples found in nature. The danger in studying these issues is the potential of mystification of similarities and proportions that is encountered from macro- to microcosm. In order to prevent this, effort is to spent to employ mathematics as it is and discussions are carried out accordingly.

Finally, thinking in *n*-dimensional space with algorithmic thinking, which we try to introduce throughout the course, is reconsidered as an introduction to design computing and effect of information technologies in the field of architecture. At this point, the strong relation between mathematics and information technologies is presented together with the brief history of IT. Then the role of IT in architecture and design is questioned, and the mathematics as the keystone of this relation is briefly studied for further inquiries to speculate about architecture in new media [Hillis 1999; Bartle 2004].

It is possible to say at first glance that the course content is very condensed, regarding the topics chosen requiring different domains of knowledge. But the aim of this course is to point out the existence of mathematics—implicitly or explicitly—in various fields and to develop awareness related to the interventions of mathematics and architecture in order to improve students' design ability. Looking at space with a new point of view, through mathematics, helps students to see differently and allows flexibility in design process. Then discussions are carried into the fourth year of their education, in the digital design studio in which architectural design and space is explored with the help of digital media to enhance design ability and to speculate further about all these issues.

4 The evaluation of students and the impact of the course on the formation of students

As the brief outline of the course presented in the previous section illustrates, students are forced to form a link not only between mathematics and their discipline but also with their way of thinking, developing a skill for 'algorithmic thinking'. Thus, the evaluation of the students within the course and then regarding their ability to synthesize their knowledge with other courses and in the long-term, as well as the impact of this course in their formations, is a very difficult task. Therefore different strategies have been discussed and proposed and objective quantitative methods have been sought for long-term evaluation.

The first step is to evaluate the class performance of students, which is realized through a series of assignments related to the topics covered in the course. The problems are specified in such a way that students have already handled them in previous years but are now expected to propose solutions that include the basic knowledge of mathematics or at least an awareness of mathematics. Each assignment ends up with presentations open to all the academic staff and students so that it is possible to carry out multi-dimensional discussions and students can reconsider those problems with their newly acquired knowledge. In all these assignments, it is emphasized that mathematics does not limit their creativity and, on the contrary, mathematics and thus algorithmic thinking, even when many constraints are imposed on the problems, have the potential to improve the solutions. The subjects of the assignments vary from simple wallpapers to the analysis of buildings with complicated geometries and functions.

It is observed that students taking this mathematics course usually also take the courses related to digital and computational design. It is seen that the levels of their design skills and their success in adapting themselves to new design media are comparatively high with respect to other students in these courses with regards to their performance in the class and their final design products. But, since this course is rather new in the Department of Architecture, it is not yet possible to make a statistical quantitative analysis in order to measure the impact of the course. On the other hand, the comments of students and comments of colleagues who are studying with these students are very affirmative about the reasoning presented in the course, and their comments encourage the instructor to improve the course content.

5 Conclusion

The aim of this course is to illustrate the concrete and strong relationships between mathematics and architecture in an explicit way, through subjects and examples with which students are already familiar. In teaching, we try to be as simple as possible in definitions and discussions in mathematical expressions, avoiding complicated terminologies in most cases. Music, origami, and literature, as well as mathematics books, games and anything else believed to be useful in revealing these relations, are all employed as tools to breakdown the mental blocks that exist in many students. We attempt to show through many examples that mathematics is everywhere and can be employed in various ways to provide solutions and answers that can be understood by everyone. In this way, we show that mathematics is not something 'alien' but is a universal code or language that every discipline can use.

Our experience is that students taking this course feel much more comfortable about mathematics, and the awareness that they gain helps them to question the design process from a very broad perspective. It is also seen that students can adapt themselves to IT and can question digital media to learn about the potential it offers. The awareness provided by the course helps students to conceive design solutions from many perspectives and their ability to question what has been or can be done is improved considerably.

References

ABBOTT, E.A. 1952. *Flatland : A Romance of Many Dimensions*. New York: Dover Publications.

BARTLE, R.A. 2004. *Designing Virtual Worlds*. Indianapolis, Ind.: New Riders, 2004.

BEARDON, A.,F. 1983. *The Geometry of Discrete Groups*. New York : Springer-Verlag.

GLEICK, J. 1988. *Chaos: Making a New Science*. Cardinal, 1988.

HILLIS, K. 1999. *Digital Sensations: space, identity and embodiment in virtual reality*. University of Minnesota Press.

HOOD, G.J. 1969. *Geometry of Engineering Drawing*. New York: McGraw-Hill.

MARCH, Lionel and Philip STEADMAN. 1971. *The Geometry of Environment: an Introduction to Spatial Organizations in Design.* London: RIBA Publications.

MERRIAM-WEBSTER ON-LINE. 2004. http:/www.m-w.com.

PENROSE, R. 1999. *The Emperor's New Mind: Concerning computers, minds and laws of physic*s. Oxford: Oxford University Press.

SWIFT, J. 1986. *Gulliver's Travels*. Oxford: Oxford University Press.

About the author

Arzu Gönenç Sorguç graduated from the Mechanical Engineering Department of Middle East Technical University. She received her M.Sc. and Ph.D degrees in Acoustics from the Mechanical Engineering Department of the Middle East Technical University as well. Since 1998, she continues her studies in Department of Architecture as an assistant professor in the same University. She is interested in not only acoustics but innovative structural design; sustainable and light weight structures, solid modeling and mesh enhancement, digital design and software architecture, Biomimesis as an inspiration of structure design and origami. She became interested in Mathematics in Architecture in 1999 as a result of her research in 'algorithmic thinking and design process' and 'new tools for structure design'. Since 1998, she has been also carrying out many researches at the Department of Mechanical Sciences and Engineering of the Tokyo Institute of Technology, collaborating with Prof. Ichiro Hagiwara. She has several publications and projects. Her recent publications are on multi-dimensional thinking and new drafting tools for architectural design (*Japanese Society of Mechanical Engineers International Journal*, special issue on "The Latest Flontiers of CAD/CAE/CG, July 2005) and on mesh generation and enhancement (*Japan Journal of Industrial and Applied Mathematics*, February 2005).