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From Natural Forms to Models

The course presented at the Istituto Statale d'Arte (ISA), a high school for visual and design arts, is a guided tour of the world of forms of seashells. The main goal of this course is to present to the student prevalently scientific methods of interpretation of forms.

Introduction

> *"…a small collection of curious objects, celebrations of natural and artificial forms, decorated a shelf on the wall. Some ingenious wooden puzzles, some shaped colored stones, some rock crystal spheres, some shaped and smoothed conch shells that appear to the eye as almost the natural origin of architecture, in the same way that once every fossil was considered "nature's joke" and thus an inanimate witness to the creation of the world."*
>
> Manlio Brusatin describing Carlo Scarpa's house in the essay "La casa del architetto" [1984].

Reading this description, many will recall their own the pleasure in arrangements of objects of interesting forms, small visual notes to one's self for who knows what future occasion. Those working in design know the importance of the silent education that these forms offer. The didactic itinerary that we deal with here is, in some way, a kind of reflection on this theme. It is a guided tour of the world of forms of conch shells, conducted within the program of a design school. The main objective of this work is to set forth for the student methods of interpretation that are prevalently scientific.

The material forms of the world speak in a language that is a network of simultaneous stimuli, far from the logical sequences that helps us to assign names to things and to rationalize. It is a process in which the sensorial impressions are first extracted from the continuum and then ordered in itineraries formed of questions and attempted answers. To obtain something that is not merely chance from this flux of information is a difficult task but it is one of the principle objectives of a formation in the world of vision and design. Since we undertake this task in the context of didactics, we can say that that each discipline possesses analytical filters that attempt to translate "continuous" reality into a "discrete" whole, an interpretative structure.

Natural objects, given the extraordinary stratification of meanings, lend themselves nobly to investigations, such as this one, of an interdisciplinary nature. Further, the spontaneous beauty of natural forms is something to aspire to; this beauty is born from the intimate relationships between form, material and function where every layer, every color appears essential, a sobriety made of infinite subtleties, which design theorists hold to be a fundamental quality of formal coherence.

The observance of natural forms has always inspired design choices in architecture; from the classic theme of the spiral staircase, articulated in innumerable examples both ancient and contemporary—the admirable staircase at the Castle of Blois attributed to Leonardo, that of Gaudi for the Sagrada Familia, and Pei's recent museum in Berlin, to name just some—to more subtle and profound relationships between architectural form and natural principles (it suffices to think of the late works of Gaudi based on a vocabulary of static spontaneous forms that implicitly invoke images of natural objects). This kind of research, ever more diffuse, in the course of the twentieth century with the so-called architecture of engineers, has been carried forward, albeit in different languages and with different approaches, by Nervi, Musmeci, Le Ricolais, Candela, Fuller and Calatrava. The German Frei Otto merits a separate acknowledgment, as founder of an interdisciplinary group where architects and engineers work side-by-side with biologists.[1] In terms of research into architecture and natural forms which is decidedly less related to engineering but not for this the less interesting, we recall the work of the American Frank Gehry and the Swiss Jacques Herzog and Pierre De Meuron with their book *Natural History*.

The structure of the work

The first part of our work responds to the questions: What produces conch shells? To what function do they respond? The second part deals with the geometrical-mathematical properties of these forms and how it is possible to describe the great visual variety in terms of just a few variables. The third part attempts a design synthesis: a possible interpretation in geometric forms of the observations gathered during the first two phases, the construction of modules that are capable of effectively reproducing in two or three dimensions the dynamics observed in nature. In the first phase what is determinate is what the natural sciences bring, in terms of design, to what we define as the informational criteria of the project. In the second phase are given their due the disciplines that take into account form in the more abstract sense of the term: mathematics and descriptive geometry. Finally, in the design and execution phase, the design laboratories furnish technical and theoretical tools for effectively extracting forms from concepts.

The choice of this particular argument, besides the desire to create interesting forms, was fundamentally motivated by the following considerations:

1. The descriptive precision to which the forms of conch shells lend themselves that illustrate a process that is easily described in geometrical-mathematical terms but that is also capable to generating a variety of forms that is large enough so as not to seem the product of a single matrix;
2. Conch shells present a case in which geometrical-mathematical requirements (in this case dictated by the necessity of isometric growth) impose certain rules upon nature;[2]
3. The various aspects that this itinerary brings to light concerning modularity, one of the traditional tools of design. The analysis undertaken by means of what we have called 2D and 3D demonstrates how the traditional concept of modularity based on the congruence of parts can be, in some cases, advantageously amplified by means of other geometric transformations.[3]

It should be added that this local kind of approach implied by this work could serve as a concrete introduction to the study of complex systems. This vision inverts the hierarchy of organization of form, introducing the concepts of parallelism and sensitivity that for years has been the patrimony of the more abstract scientific disciplines but that should nevertheless be learned and digested within the context of a design school (to this we shall be dedicating some didactics projects in future years).

The relationship between form and function

This part, amply dealt with by the professor of natural sciences, is here reduced to a few schematic notations of a functional nature, since it is not possible in a limited space to go into them in more detail. The natural sciences, studying the relationships between behaviour, environment and survival strategies, perform, from our point of view, the role of an authentic analysis of the informational criteria of "project conch shell", that is, the requirements to which it must respond.

Schematically we can observe that in all cases the conch shells perform the function of a shield and support for the soft, vulnerable parts of molluscs, even if the behaviour of the various classes of molluscs are rather different. There are essentially two typologies: univalve shells and bivalve shells. Let us consider the classes of subtypes of the conch molluscs:

- **Gastropods**, which are herbivorous and carnivorous, require a portable shell that permits them to move agilely in search of food. The gastropods are the classic univalve conch shells with a spire that is more or less marked (fig. 1).

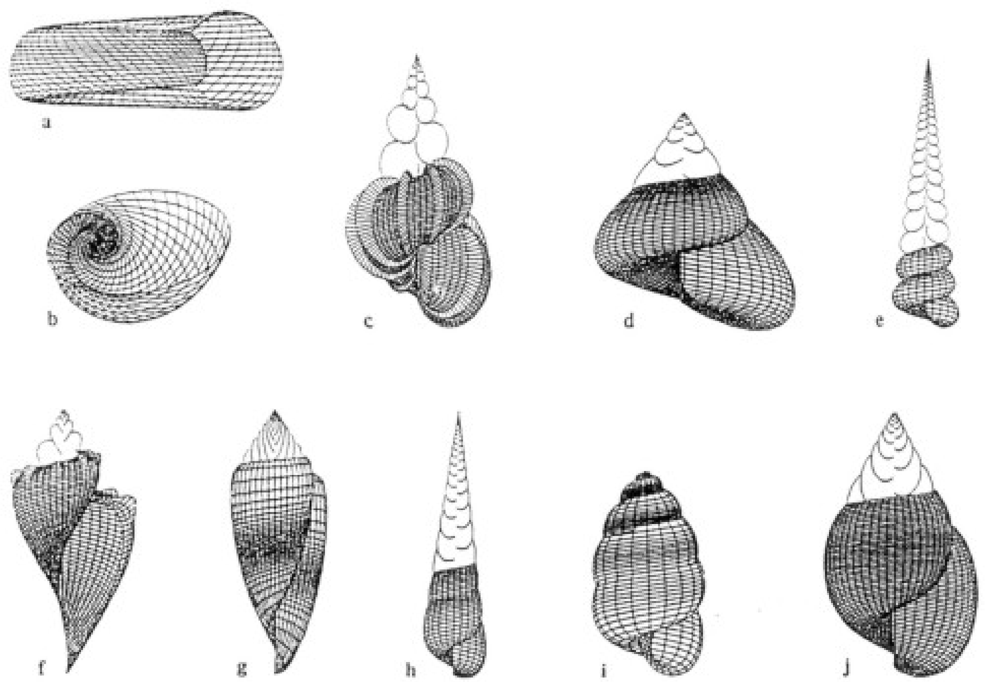


Fig. 1. The shapes of some gastropod conch shells in computer simulations by Cortie [1990]. All of these shapes answer in different ways to the function of a portable shield

- **Bivalves** are sedentary filterers that live in muddy environments and that protect themselves from predators by quickly digging down into the mud thanks to the wedge shape of the shell that is formed of two halves that are hinged together. Also, the bivalve conch affords better protection of the soft parts of the body from the abrasive action of the grains of sand (fig. 2).

- The **cephalopods** are predators for which speed of movement is an essential tool for survival. They have more or less lost their exterior shell, conserving in some cases internal residuals that serve as supports, such as in cuttle fish or squid, or as organs that aid in maintaining a hydrostatic equilibrium, such as in the genus *Spirula*. There are exceptions such as the Nautilus, which produces a very beautiful exterior shell that serves for hydrostatic equilibrium, or as the Argonaut (fig. 3b), which produces through its tentacles a beautiful temporary external structure, used as a egg carrier.

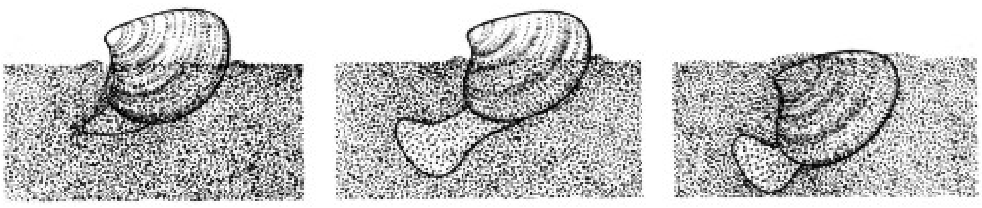


Fig. 2. The sequence of a bivalve digging into the mud. The wedge-shaped the conch permits a great facility of movement [Mezzetti 1987]

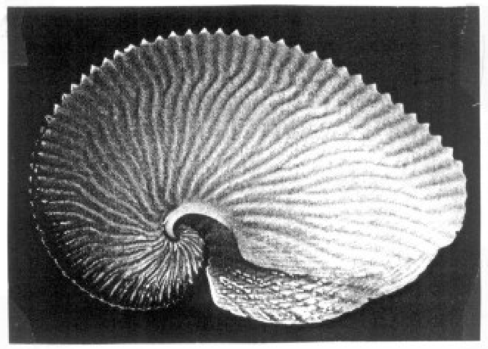


Fig. 3. In a, left) we see the typical tube-shaped conch shell of a scafopode (*Dentalium*) which serves the protective function of a sedentary filter. In b, right) a cephalopod conch Argo Argonaut that is periodically constructed as an egg carrier

Engineer or tinker? The collection of different functions assigned to the conches of different species of molluscs reminds us of what Jacob said in his "Evolution and Tinkering" [1977].[4] He observes that, from the point of view of human design, nature sometimes seems to adopt an approach to engineering that conceives structure rationally in order to respond to the requirements, sometimes even extreme, of the ecosystem in which they have to operate. But more often nature seems to act as would a passionate tinker who finds himself having to adapt to new uses structures that were already conceived for other purposes.

In the case of the conch shells, we can perhaps say that the portable shelters seem conceived by the engineer while the tinker then re-adapted them as egg carriers, internal skeletons, or even as floating devices similar to submarines.

Observations and morphological surveys. Conch shells of almost all molluscs, including bivalves, are characterized by an elegant design and symmetry that is based on a cone that turns in a spiral about an axis.

From Thompson[5] we note the following observations:

1. **growth by addition**: conch shells increase their dimensions by adding new material to what already exists. Thus the forms that we see conserve and contain in themselves all the previous phases of growth. Each adult conch shell conserves at its apex the proto-conch in which it has its origins.
2. **isometric growth**: in their growth conch shells always conserve the same shape. If we take two conches of the same species but of different ages we can see that the one shape is the

enlargement of the second. This characteristic, known as isometric growth, permits the mollusc to increase its own size while maintaining the same proportions among the parts of the body housed in the shell (fig. 4).

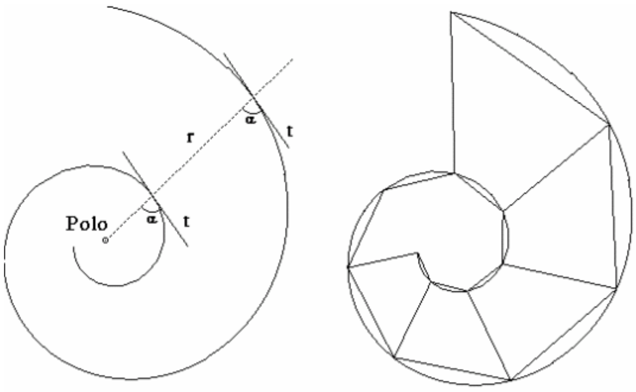


Fig. 4. In the image on the left we see a logarithmic spiral on which is indicated a constant angle a between a radial and a tangent that characterize it. On the right we have the same spiral segmented in quadrants. Imagining that a hypothetical mollusc in the course of its growth lives in the largest segment, we can understand how this habitational strategy permits it to grow in a rigid shell without changing its shape. This strategy, known as isometric growth, is permitted only by shapes that are structured, in the plane and in space, on the logarithmic spiral

It has been known since 1638 that the spiral shape of conch shells has the property of self-similarity during growth (see **isometric growth**). This implies that the projection of any generating spiral onto an orthogonal plane at the axis of symmetry (in reality, as will be explained below, this is an axis of rotational translation) produces a curve that was studied for the first time by Descartes and defined by him as an equiangular or logarithmic spiral.

Geometric analysis. We sharpen the studies undertaken thus far by citing a geometric definition of the morphological characteristics of the conch shell by Cortie.[6] Not wanting going into the details of his mathematical model with its sixteen variables, we limit ourselves to an effective description of its fundamental characteristics.

For the sake of simplicity, let us imagine the surfaces of the conch shell as the results of the **rotation of a plane figure** (a **directrix** curve that represents the shape of the opening out of which the mollusc comes) **about an axis** according to the following procedure:

1. Let us consider a reference plane containing the axis of rotation; the directrix figure (which we imagine to be flat) can lie in that plane or it can make with it an angle that is always constant.
2. The directrix, in the course of its rotation, increases constantly in its linear dimensions, while maintaining invariable the value of its angles. At every successive turn the figure grows uniformly, that is, the rate of growth of the directrix is constant. The distance between the directrix and the axis of rotation also increases according to the same rate of growth.
3. In the course of its rotation, the figure can perform a translational movement along the axis. The vector that describes the resulting translation is directly proportional to the rate of growth of the directrix (fig. 5).

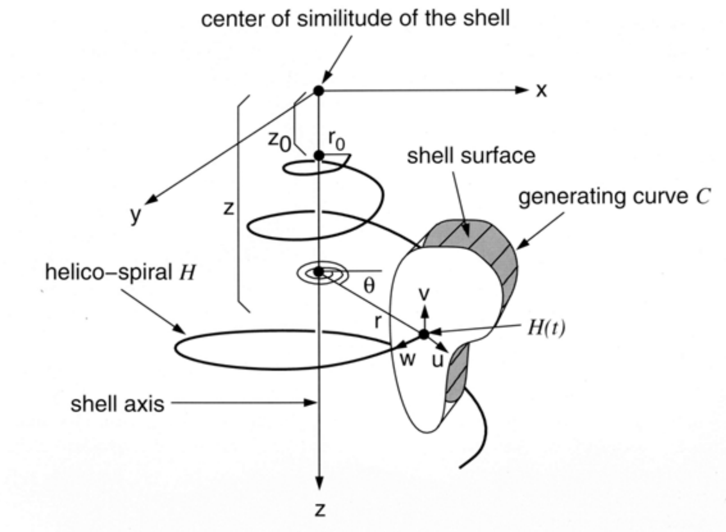


Fig. 5. The generation of the helix by the rotational translation of the shape of the opening of the conch shell, after Meinhardt [1995]

Respect for these conditions produces a solid (which from now on we will call a shell) having as its directrix the shape of the opening and as its generator the helixes which, when projected onto an orthogonal plane, produce, as mentioned, logarithmic or equiangular spirals (fig. 6).[7]

At this point the mathematics teacher—who has already introduced the measure in radians of the angles and the representation of point by means of polar coordinates—completes the investigation introducing the description and the equations for spirals (both Archimedean and logarithmic).

This description is the point of departure for our work. What has just been discussed imply the principle on which our research will hinge:

The complex form of the helix can be subdivided into elementary parts that are self-similar, which we will call modules

By which

By varying the form of the modules, the shape of the helix is varied

This statement implies the assumption of a local kind of approach. As happens sometimes in the context of the science of complexity,[8] we can describe the global shape solely by means of given module and those adjacent to it.

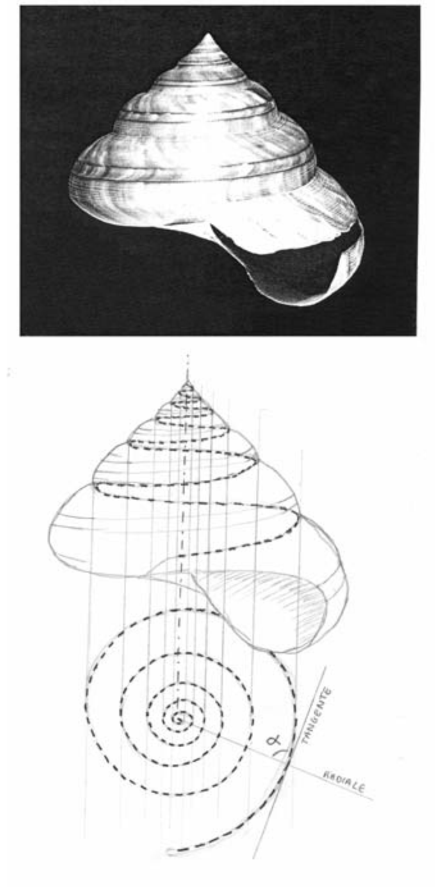


Fig. 6. From a conch shell (in this case a Pleurotomaria) we obtain by projection the corresponding logarithmic spiral. The line of connection is projected by a band of parallel straight lines on a plane that is orthogonal to the axis of the conch. From the spiral thus projected it is possible to calculate the angle a that characterizes it

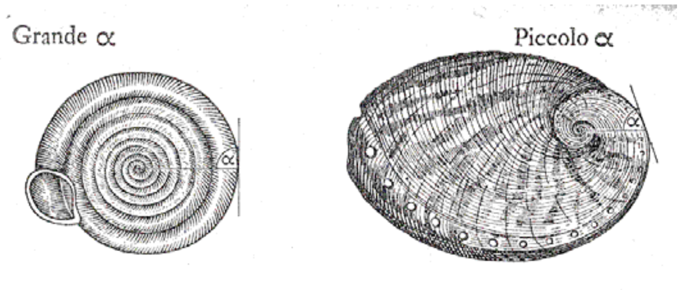


Fig. 7. From Thompson [1961] we have two conches characterized by different values of a. In our 3D model a will be called parameter A

In order to clarify this kind of approach, it seems best to take as the point of departure the study of a situation that is more accessible in terms of three-dimensional intuition. We will deal with one that, perhaps a little pompously, we have called a 2D model, referring to a hypothetical conch sell in a two-dimensional world. Studying this model helps us to better understand the logic behind the isometric growth that will become the nucleus of a 3D model that approximates the dynamics of form that are actually observed.

The 2d Model

The 2D model is a very immediate tool that favours an intuitive approach to the problem of the morphology of structure based on the logarithmic spiral. Let us assume as a beginning module a quadrilateral, and by means of reduction let us make a smaller copy, the rate of reduction such that the measure of the larger side of the copy corresponds exactly to the smaller side of the starting module. Now let us arrange the two forms so that the two sides in question are perfectly adjoined. When we repeat this operation a certain number of times we can observe the growth of a small spiral-like structure similar to an antler or a conch (figs. 8 and 9). In this section we study the theoretical presuppositions of this simple procedure and the analogies between those presuppositions and the geometric mechanisms of growth of a real horn or conch.

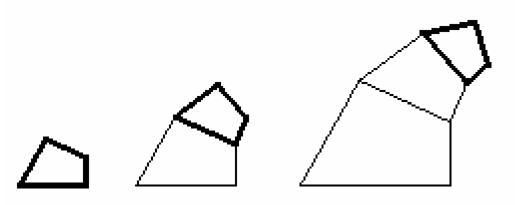


Fig 8. Three instances of aggregation, in one direction, of modules that are quadrilaterals similar to the one indicated

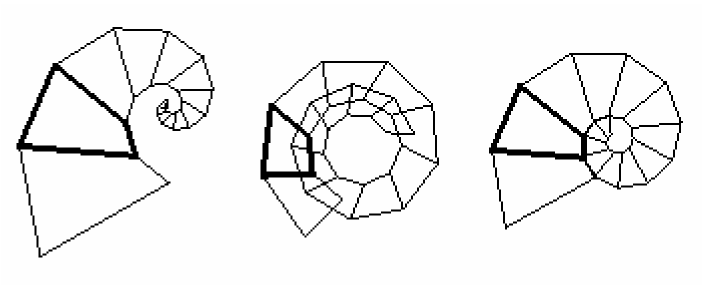


Fig. 9. Three linear aggregations produced by different modules

The aggregation of quadrilaterals. Let us begin our itinerary by observing that any convex quadrilateral—when joined side by side with similar copies of itself—produce a *covering*[9] of an area of the plane in which all the vertexes of the *paving* belong to two distinct families of logarithmic spirals having the same eye but opposite rotational directions.[10] This is a procedure for paving the plane with modules that have the same shape but are of different sizes (fig. 10 shows examples of how, from any given quadrilateral it is possible to develop a covering of the plane).

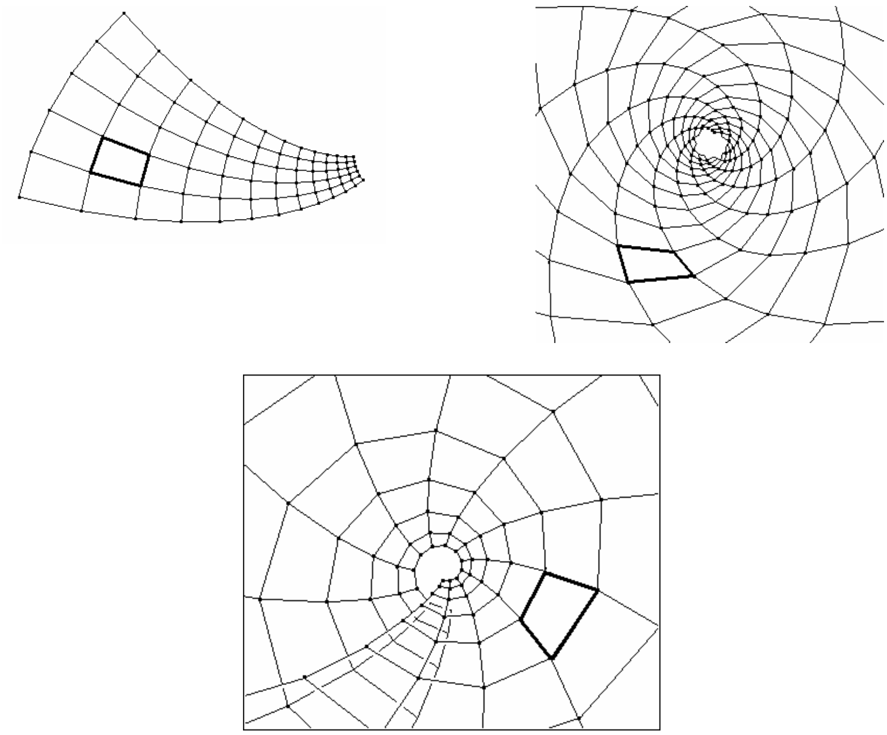


Fig. 10. Aggregation in all directions of modules that are quadrilaterals similar to the one indicated. a, above left) the initial phase, the result of a procedural iteration that is an articulation of the plane that can be developed depending on the characteristics of the model; b, above right) a paving without any overlapping modules; c, below) a covering with overlapping modules

We can confirm that

every quadrilateral is a module in a covering of the plane whose vertexes belong in two families of equiangular spirals with the same eye

In the next section we will deal briefly with the classification of all quadrilaterals, and then analyse the relationships established between each quadrilateral and the family of spirals that it produces.

Classification of quadrilaterals. We have distinguished four parameters, α, β, δ, ϕ, expressed in the terms of angular size, whose variations allow us to identify any family of quadrilaterals—obtained from the intersection of two triangles—and to distinguish every single possible case (see fig. 11 for a description in intuitive terms).

In particular let us consider how the variation of the four parameters characterizes the categories of convex quadrilaterals (recall that, according to this description, when an angle is equal to 0 the two straight lines that go to it are parallel).

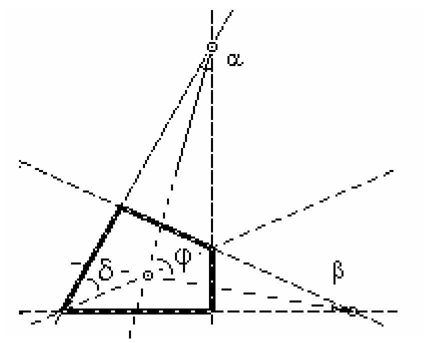


Fig. 11. The four parameters that permit the identification of all convex quadrilaterals: α, β are the angles of the vertexes of the two triangles that overlap to produce the quadrilateral; ϕ is the angle of intersection of the bisectors of α and β, vertex ϕ does not generally lie on the diagonal. δ is the angle of the exterior angle (that opposite the eye) of the quadrilateral, between the incident diagonal and the adjacent side that goes to α

a) with **α, β > 0** we have **generic quadrilaterals**;
b) with **α = β = 0** we have **parallelograms**.

Within the parallelograms:

- when **ϕ = 90°** we have **rectangles**;
- among the rectangles, when **δ = 45°** we have **squares**;
- in all other cases when the relationship between angles **δ** and **ϕ** is **δ = ϕ/2,** we have **rhombi**.

c) with **α** or **β = 0** we have **trapezoids** (with **ϕ = 90°** we have **isosceles trapezoids**).

Quadrilaterals and pavings. In light of the parameters for the classification of quadrilaterals, we reopen discussion on the relationships between them and the coverings that they produce. Considering that "each quadrilateral corresponds to a covering", we can identify the two fundamental relationships between a quadrilateral and the covering it generates:

a) given a quadrilateral, there is a corresponding covering of the plane formed by two families of equiangular spirals with the same eye, characterized respectively by angular values:

$$\rho = 180° - \alpha;\ \tau = 180° - \beta \text{ (fig. 12)};$$

b) given a quadrilateral, the eye of system of spirals—which forms the corresponding covering—is one of the two points of intersection (the other is a vertex of the quadrilateral) of two circumferences circumscribing two triangles that, by overlapping, create the quadrilateral (fig. 13).

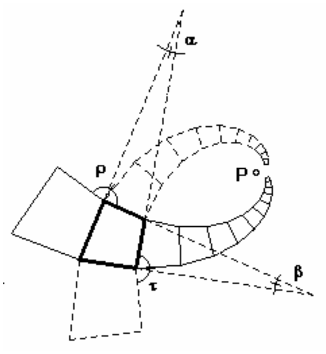


Fig. 12. From the values of the angles α and β that characterize a quadrilateral it is possible to determine the values of angles ρ and τ that characterize the two families of broken spiral-forms that these produce by means of the relations: $\rho = 180° - \alpha$; $\tau = 180° - \beta$

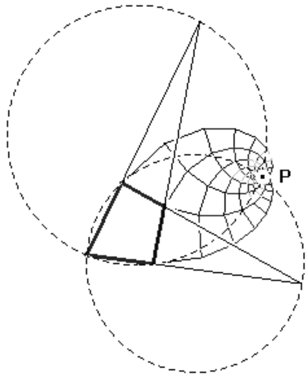


Fig. 13. Given a quadrilateral it is possible to determine the circles circumscribing the triangles that, by overlapping, give rise to the quadrilateral, that is, the two triangles of vertexes α and β that we used for the classification. The intersection of the two circles described identifies two points: one is a vertex of the quadrilateral; the other is the eye of the system of spirals produced by that quadrilateral

Applying these considerations to the classes of quadrilaterals we can see what follows (see the table summarizing these in fig. 14).

Parallelograms, which have values of α and β equal to 0, produce systems of spirals that are straight lines and thus the covering corresponds to a paving of the plane of modules that are congruent. In this case the eye of the system can be considered as infinitely far (a point at infinity).

Trapezoids, which have values of α and β equal to 0, produce systems of spirals in which the sides not parallel to the module are aligned with the eye. In the case of isosceles trapezoids the system of spirals collapses into a system of concentric circles and radial straight lines that pass through the eye.

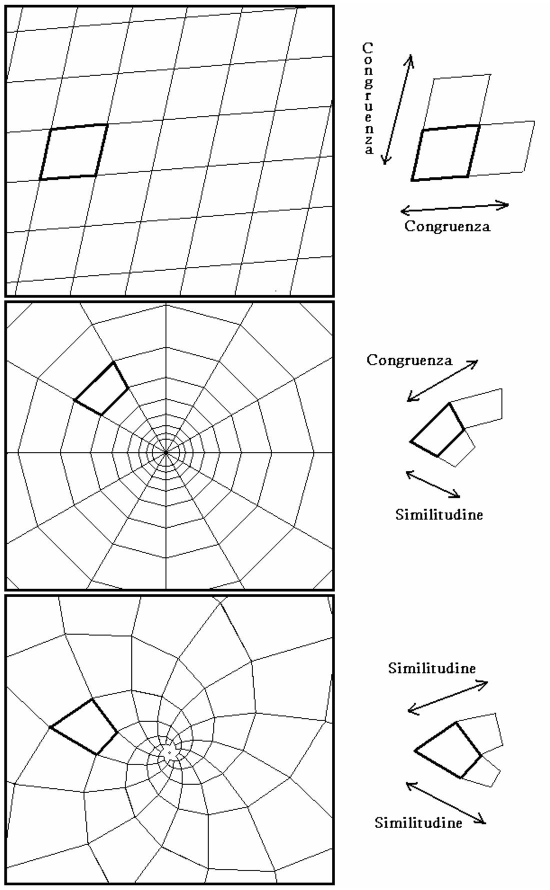
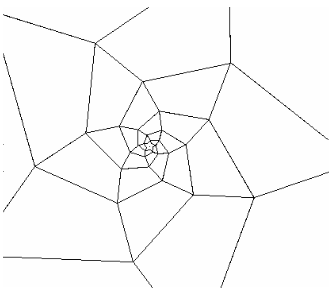
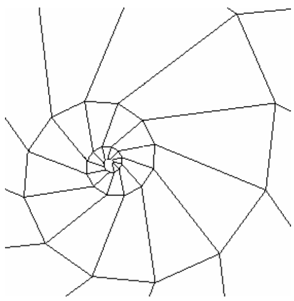


Fig. 14. Paving produced by various typologies of quadrilaterals



(5,5)



(1, 11)

Fig. 15. Numbers in parentheses ( ) indicate the number of clockwise or counter-clockwise spirals generated by the paving

Generic quadrilaterals, having values of α and β that are not equal to 0, produce—generally—systems of spirals in which the sides of a module are not aligned with the eye.

Pavings of quadrilaterals and 2D conch shells. From this overview of coverings produced by similar quadrilaterals we reach the object of this section, which is that of producing shapes that are 2D models of hypothetical conches in two-dimensional space. From a conch shell, even one in 2D, we expect, in the first place, a serrated spire, that is, that between one turn and the next there are not residual spaces or overlaps. We note that the serrated spire is a primary element of identification of a conch shell.

In order to produce the model of the conch shell in 2D, a quadrilateral must tile such that the whole tiling is the result of the spire of one strip only, as we see in fig. 15 (1, 11). The image in fig. 10b, for example, shows 12 strips in one direction and 9 in the other; the lower one of fig. 14c shows 6 and 12; that on the left in fig. 15 shows (5, 5). Now let's describe a geometric construction that allows us to determine the quadrilaterals, whatever they may be, modules that can construct a paving like that in fig. 15b, characterized by the presence of only one strip in one of the two directions. We will not treat in more depth this topic, since its development would require the use of tools that are beyond the simple basic geometry used in this work. It is interesting to note how this aspect of the 2D model leads us to reflect on geometric problems related to the phylotaxis, opening new horizons to quasicrystallography.

Geometric construction of a serrated spire. Let us imagine that we wish to produce a conch shell, that is, a serrated spire of modules starting with any convex quadrilateral, which, as we see in fig. 16a, produces an open spire.

We can find for every quadrilateral in the row the point of intersection of the diagonals, and we then join to them the centres of adjacent modules and those of the modules that are found one in front of the other in the successive turns of the strip in fig. 16b. Once we have identified a quadrilateral we can develop it into a row that is sure to produce a serrated spire or from which we can create a paving of the plane. In fig. 17 we can see a collection of 2D conch shells produced in this way; we see also that they can be interpreted as a projection on a plane of 3D conch shells.

In the next section we will verify a strict analogy between the characters of the systems in two and three dimensions, almost a simple translation from 2D to 3D. To us, these correspondences appear particularly meaningful:

In the plane	*In space*
The structure is always organized about a **point**: the eye of the main spiral.	the structure is always organized about a **straight line**: the axis of the helix generatrix.
The **segments** that represent the sides of the modules form a constant angle with respect to a radius coming out of the eye.	The **polygons** that represent the sides of the modules form a constant angle with respect to a plane that passes through the axis of rotational translation.

This part of the work was made possible thanks to the use of an interactive computer support such as Cabri II. By means of the creation of a macro-construction it is possible to quickly join the similar quadrilaterals of the right dimensions. Without the use of the computer this task would be almost impossible to manage graphically.[11]

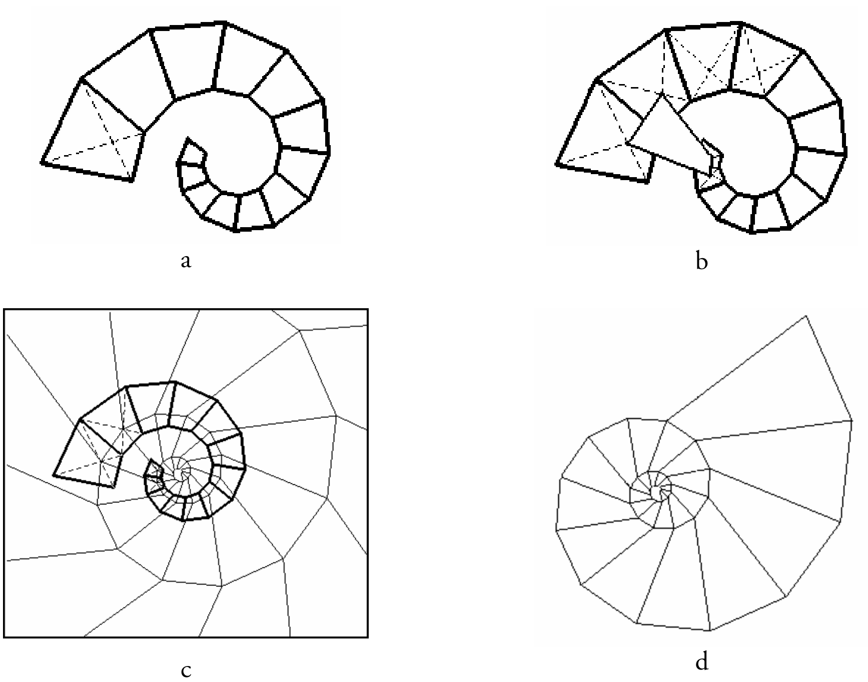


Fig. 16. Construction phases of a serrated spire starting with any module at will: a) the identification of the centres; b) construction of the new module; c) construction of the paving; d) the conch shell

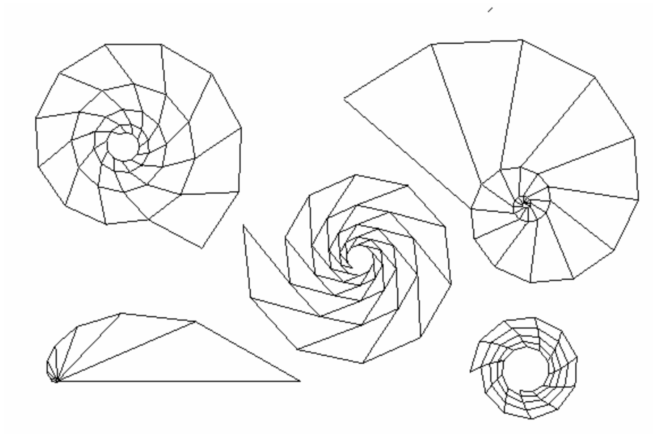


Fig. 17. Typologies of “conch shells” produced by various 2D modules. The variety of shape is evident, as is the similarity to images of real conch shells. The interesting fact is that this similarity marks a strict analogy between the morphological principles; thus, we are not dealing with mere copies but with objects produced (at least in spatial terms) by the same process

The 3d Model

The 3D model is based on a very simple idea, the extension of the 2D model to three dimensions. As we have seen, in the plane it is sufficient to assemble similar copies of a quadrilateral to produce a logarithmic spiral; this is also true in space, where it is possible to identify a base shape that can be aggregated in similar copies to produce a helix.

The idea is that of arbitrarily collocating two similar polygons (which for simplicity we will think of as regular) in space and then to unite by line segments the points that we will identify as corresponding vertexes (in the box we will see this operation conducted on two regular hexagons).

The only restriction that we impose is on the free choice of the positions of the sides in space and that of the absence of reciprocal rotation between them.[12] That means that setting side-by-side their planes of appurtenance of the two bases of the module (as if to close the pages of an imaginary notebook), they form couples as the result of a homotheity. The figure generated by this simple operation is the module (for a more precise definition of its construction, see the box below).

Aggregation. We proceed with the construction of our 3D model by making, by means of a similarity, some copies of the module. The relationship of this similarity will be the same as that between the dimensions of two sides of the module. Proceeding in this way we will produce a series of modules of decreasing sizes. The modules will then be aggregated (one over the other) so as to make couples of coinciding bases with the same dimensions, having taken care to make the vertexes correspond.[13] With this procedure of aggregation we will obtain a spatial structure with a spiraling movement whose characteristics will depend completely on the parameters that determine the form of the module.

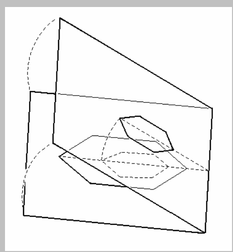
Parameters of the module. At this point in the work, after having set forth the basic operations of construction and aggregation we shall see in an effective way—modifying the critical factors of the shape of the module—how they can reproduce, in an essential form the dynamics that can be observed in nature.[14]

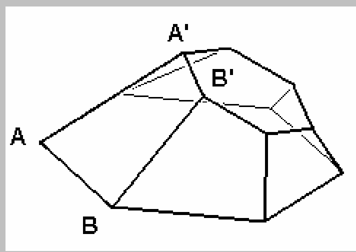
It is possible to demonstrate that with any choice whatsoever of the following parameters one obtains a module that is capable of generating by aggregation a solid that can be inscribed in a shell, that is, a surface having helixes as the generatrixes:

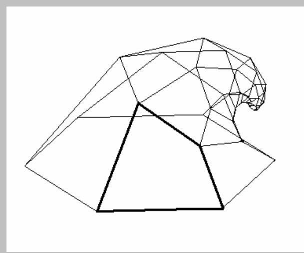
A) the reciprocal inclinations between the sides (dihedral angle value γ);
B) the homothetic relationship that relates the two sides;
C) the proportion between the distances of the two sides with respect to the hinge;
D) the coefficient that expresses the value of the reciprocal translation between the two sides with vectors parallel to the axis of rotational translation.

The construction of the module

1) On a plane we locate a polygon, which will constitute the lower side of our module; the upper side will be constituted of a polygon similar to the first. Let us position arbitrarily in space the upper side of the module, choosing an inclination that is also arbitrary. In any case, as we place the sides in space, we can always imagine them to be lying in two hinged planes (like pages in a notebook).







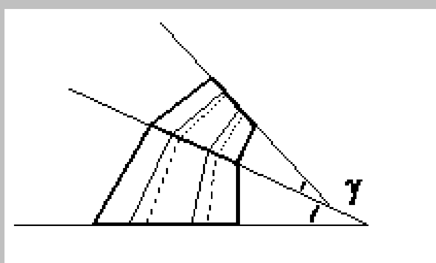
2) We unite their corresponding vertexes, that is, those that correspond to the plane homotheity just cited (AA', BB' in the figure) until they form a figure that resembles a truncated pyramid: this is the module. It can be demonstrated that, generally, the quadrilaterals that make up the lateral faces of this figure are not planar.

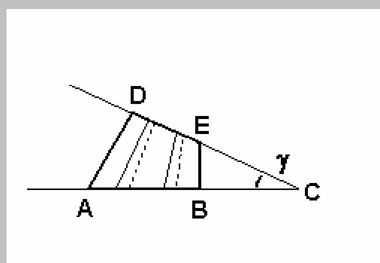
In this orthogonal lateral projection it is possible to evaluate the following three parameters of a given module (we can see that a module with sides that are similar polygons of *n*-sides, constructed in the way we have indicated, always appears as a quadrilateral). The directions have therefore a general validity and can all be related back to the observations made about 2D models:

Parameter A: the value of angle γ, contained between the planes of appurtenance of the sides;

Parameter B: ratio AB/DE, relationship of homotheity between the lateral projections of the sides;

Parameter C: cross ratio AC/AB : CD/DE expresses the positioning relative to the two sides with respect to hinge C and furnishes an indication of the inclination of the sides with respect to the axis of rotational translation (see the box below).





Parameter C. In order to understand better the importance of this factor, let us consider in the first place what happens when two modules are superimposed.

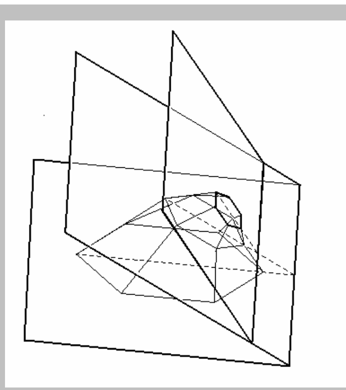


fig. a

In **figure a**, we can see that the superimposition of the two sides implies that of the respective planes of appurtenance: the three planes form a system in which the hinge of the upper module is located at a certain distance from that of the lower module. This distance is established by the value of the cross ratio C:

C = AC/AB : DC/DE

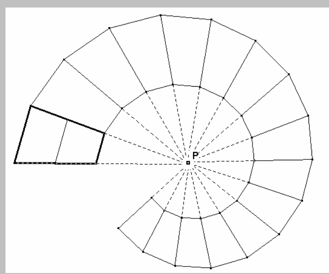


fig. b

In **figure b**, when C is equal to 1 the hinge of the two adjacent modules coincides in a single hinge that constitutes axis p of the rotational translation of the structure.

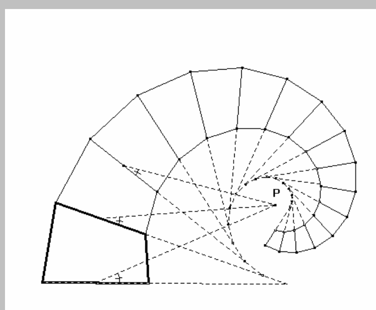
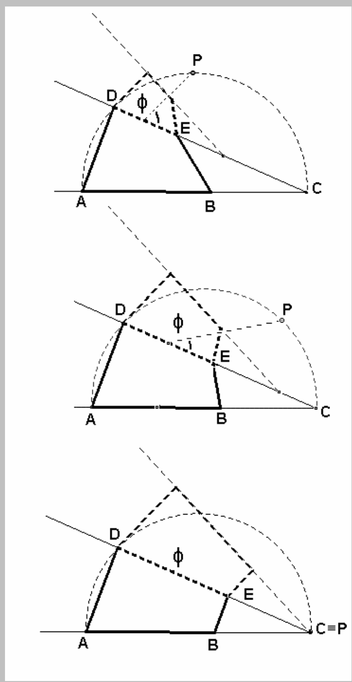


fig. c

In **figure c**, the further the value of C is from 1, the farther the hinges are from each other, in lateral projection, forming a logarithmic spiral about the axis of rotational translation p.

In **figures d, e, and f**, we can see how the three increasing values of cross ratio C influence the position of the axis of rotational translations P with respect to the sides of the two modules. We note in particular that the angle comprised between a plane (in dotted lines) passing through p and the centre of a side tends towards 0 when the cross ratio C tends to 0.



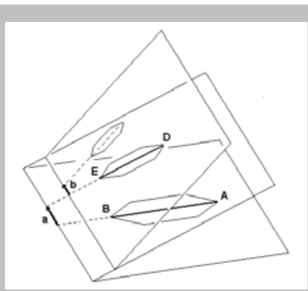
in **d**: C = 0.56, ϕ = 70°

in **e**: C = 0.78, ϕ = 41.1°

in **f**: C = 1 and ϕ = 0°

Parameter D is the only one that can't be described in a lateral projection in as much as it indicates the translational component of the module in a direction parallel to the axis of rotational translation. In other words, it tells us that there exists a lateral movement in the positioning relative to the two sides of the module (in the figure it is identified by vector a).

In **figure g** we see the sides of two consecutive modules connected at their respective hinges of rotation by a dotted line segment that is perpendicular to the hinge and passes through the centre of the sides. The arrows highlighted (vectors) make evident the entity of the two translations, the relationship between the vectors of the two adjacent modules is the same as that between the two sides of a module.



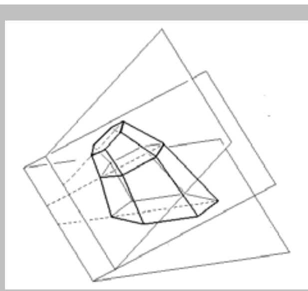


fig. g (left) and **fig. h** (right)

Making reference to **figure g** relative to the two aggregated modules that we see in **figure h**, we can say that:

$$\frac{|\mathrm{a}|}{|\mathrm{b}|} = \frac{AB}{ED}$$

Parameter A directly influences the velocity with which our shell turns on itself about the axis. It is the parameter a traditionally indicated by biologists (see fig. 6).

Parameter B influences the width of the cone that, when it turns, generates the shell (if we imagine that Parameter A is 0, then our shell would be a cone that is more or less pointed, according to the value of Parameter B).

Parameter C, on the other hand, influences the inclination of the sides with respect to the axes, during the rotational translation.

Parameter D is the vector that controls the movement of the two sides in a direction parallel to the hinge of the module and therefore at the axis of rotational translation of the structure. This parameter is that which in reality distinguishes conch shells that are "pointed", such as that of the Turritella, of those "flat" ones such as the Nautilus or the Ammonites. As can be easily intuited, the difference lies in the presence of the translation along the axis (very evident in the first case, and practically nonexistent in the second).

Some graphic examples. Figures 18, 19, and 20 show some "conch shells" side-by-side with the modules that produced them. We can appreciate the effectiveness of these reproductions, which allow us to intuit how, modifying the basic shape, it is possible to simulate many dynamics of the natural forms and beyond. One of the most stimulating aspects of this work is that the notable resemblances between the images produced and the natural objects lies in the close analogy of the constructive principles and not in an attempt at imitation. In fact, some of these images propose forms that, even though the procedures have been correctly followed, generate forms that are rather improbable for a real conch shell.

The images that we see were produced with Cabri II, and even while they correctly illustrate, from the point of view of spatial intuition, the correspondence between the shape of the module and the consequent shape of the shell, they do not precisely represent the true spatial dynamics. This part of the work relative to the graphic representation in 3D is still a work in progress, while at the time of the work that we are presenting here we had developed most of all the part concerning the built models.

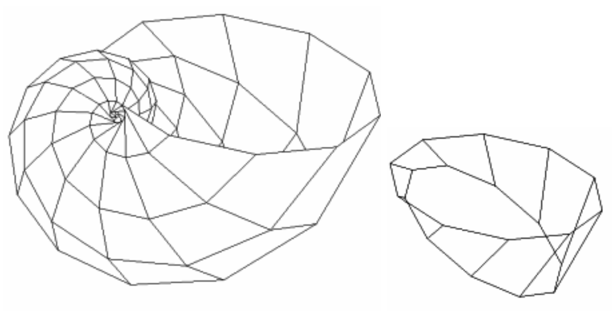


Fig. 18. The construction of a conch shell that is similar to a gastropod. This shell is characterized by: Parameter A, a marked inclination between the sides; Parameter B, a low ratio of homotheity; Parameter C, a marked inclination of the sides with respect to the axis; Parameter D, the absence of translation with respect to the axis. The ridges of the lateral faces of the module highlight the non-planar quality of the quadrilaterals of which it is made

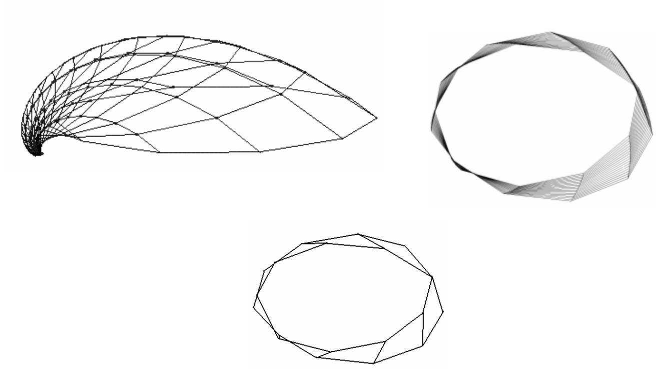
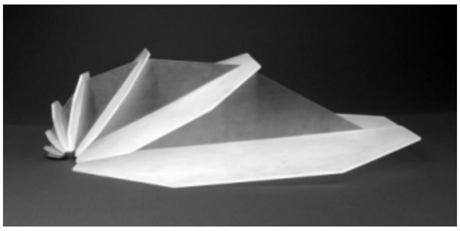


Fig. 19. This "bivalve" is characterized by: Parameter A, a limited inclination between the sides; Parameter B, the low ratio of homotheity; Parameter C, a slight inclination between the sides and the axis; Parameter D, the absence of translation with respect to the axis. The ridges of the lateral faces of the module highlight the non-planar quality of the quadrilaterals of which it is made

Fig. 20. An "impossible" conch shell produced by the module on the right

The built models. Following the studies undertaken we realized a series of models in metal and Plexiglas that are shown in Figs. 21 and 22. The intent was to represent some of the more meaningful categories of the undersea world of mollusc shells.



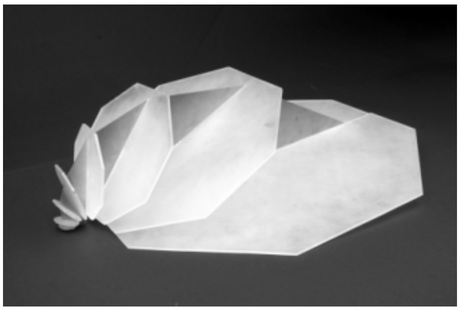


Fig. 21. A bivalve conch shell. The module with an octagonal base has a notable inclination between the sides and a marked ratio of reduction between sides, while translation along the axis is completely absent

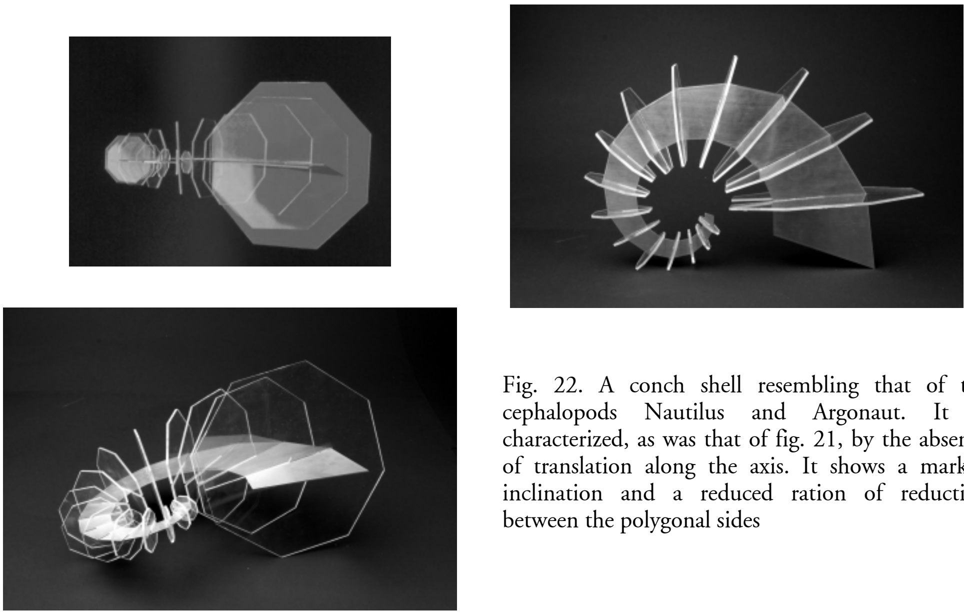
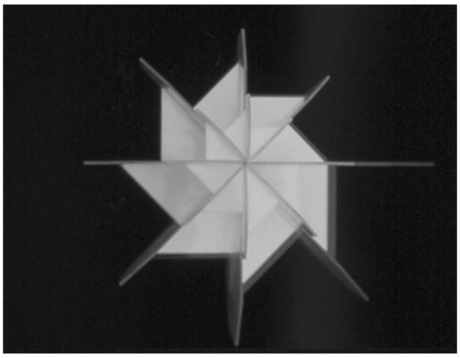
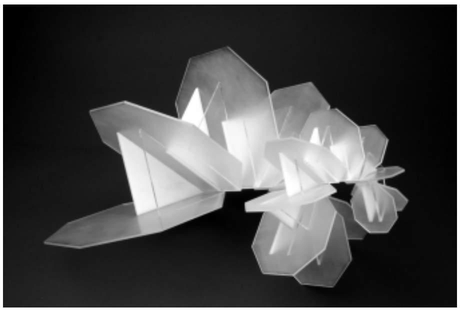


Fig. 22. A conch shell resembling that of the cephalopods Nautilus and Argonaut. It is characterized, as was that of fig. 21, by the absence of translation along the axis. It shows a marked inclination and a reduced ration of reduction between the polygonal sides





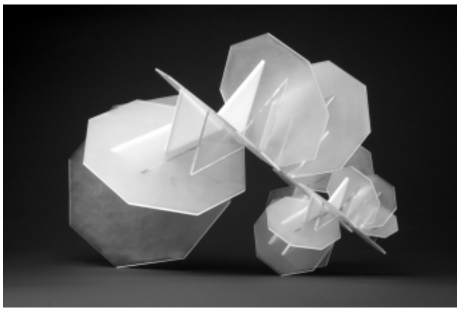


Fig. 23. Conch shell similar to some of the gastropods characterized by a high level of translation of the sides along the axis, a marked inclination and a high ratio of reduction between the sides

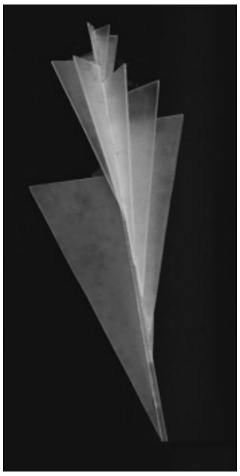


Fig. 24. Conch having triangular sides, a marked translation along the axis, a strong inclination between the sides and, and like all the models built, the absence of inclination of the sides with respect to the axis

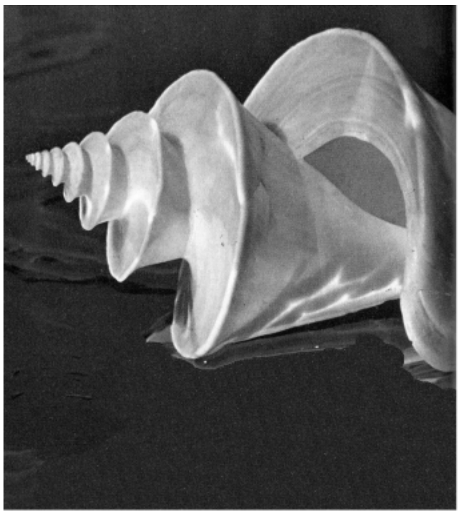


Fig. 25. *Tatcheria mirabilis*

All of the models were based on a module with octagonal sides. With the aim of alleviating the problem of the side faces of the modules that, as mentioned, are curved, we chose to avoid realizing them by connecting the octagonal sides to a continuous metal structure. This choice highlights the shape of the load-bearing structure based on a planar logarithmic spiral and allows us to underline the fundamental role of the this shape, which dictates the rhythm of growth in space as well.

In actual fact, these models should be considered studies in which we have chosen to look at situations that are particularly suitable to a schematic representation.

Future didactic work could develop in at least two directions:

- the attempt to represent a specific type of conch shell identifying—by means of the languages of various disciplines—a module and a constructive strategy that is appropriate to that case;
- the construction of models of "impossible" conch shells of particular beauty and formal interest.

The relationship between possible/impossible is a particularly stimulating theme for research into the fundaments of linguistics such as the one presented here, and is one of the specific themes of research undertaken in accordance with scientific criteria.

Translated from the Italian by Kim Williams

Notes

1. See the fundamental series published by the Instituts für leiche Flachentragwerke of the University of Stockholm [Institute for Lightweight Structures 1977].
2. This is what Peter Stevens in *Les formes dans la nature [*1978] defines with the effective expression "space tyranny", that is, those cases in which spatial imperatives, mostly of a topological nature, impose some of the formal choices.
3. Effectively substituting, within the theory of the tiling of the plane, congruence with other geometric transformations such as affinity or homology gives rise to curious phenomena of visual illusions, one example of which is "false axionometry", which we discuss in the section "Some graphic examples".
4. François Jacob [1978] was the winner of the Nobel prize for medicine with Jacques Monod, author of *Chance and Necessity*, a fundamental thesis on the natural philosophy of contemporary biology.
5. In 1917 D'Arcy Thompson's *On Growth and Form* [1961], a bible of morphological studies of nature republished several times throughout the world, has influenced generations of naturalists, architects, engineers and scholars of shape in the broadest sense of the term.
6. Michael Cortie in [Stevens 1978] presents a mathematical model, visualized by means of computer graphics, of the growth of mollusc shells.
7. The value a of the angle that is characteristic of this spiral shows a particular taxonomic value, tending to remain constant in the course of the evolutionary history of many species; see [Meinhardt 1995].
8. For example, the study of cellular automata.
9. "Side to side" means that the side of a module has to coincide entirely with that of the module adjacent to it. In the case of a tiling with rectangles, an example of side to side tiling is that of a grid made of intersecting straight lines, with a tiling that is not side to side is that, for example, in which the rectangles are arranged like bricks in a wall.
10. By *covering* we mean a tiling of the plane in which a point on the plane can belong to more than one module; by *paving* we mean rather a tiling in which a point on the plane can belong to only one module, the final possibility being that the articulation (or packing) is one in which a point on the plane may not belong to any module.
11. All constructions which by their nature are developed through a chain of local steps, pose the problem of the multiplication of the margin of error at every successive operation.

12. The presence of a rotation between the sides should be excluded since it would induce in the final structure a helical torsion that isn't present in real conch shells.
13. This restriction as well is intended to exclude a helical torsion in the aggregated structure.
14. Those wishing to go into this part of the work in greater depth should contact the authors.

Select Bibliography

BRUSATIN, Manlio. 1984. La casa dell'architetto. In *Carlo Scarpa: opera completa*, F. Dal Co, G. Mazzariol, eds. Milan: Electa.

INSTITUTE FOR LIGHTWEIGHT STRUCTURES. 1977. "Pneus in Natur und Technik / Pneus In Nature and Technics", Vol. 9 of the *Mitteilungsreihe des Instituts für leichte Flächentragwerke*, Institute for Lightweight Structures, Stuttgart. (In particular we cite the section dedicated to conches: "Growth despite hardening").

STEVENS, Peter S. 1978. *Les formes dans la nature*. Seuil Evreux.

CORTIE, M. 1990. The Form, Function, and Synthesis of the Molluscan Shell. In *Spiral Symmetry*, Istvan Hargittai, ed. World Scientific.

MEZZETTI, G. 1987. *L'uomo. Dalla natura alla scienza*. La Nuova Italia.

JACOB, Francois. June 10, 1977. Evolution and tinkering. *Science* **196**: 1161-1166.

———. 1978. *Evoluzione e bricolage*. Turin: Einaudi,1978.

THOMPSON, D'Arcy W. 1992. *On Growth and Form*. Cambridge: Cambridge University Press. (First edition 1917).

MEINHARDT, H.. 1995. *The Algoritmic Beauty of Sea Shells*. Berlin: Springer-Verlag.

COOK, Theodore Andrea. 1978. *The Curves of Life*. New York: Dover Publications. (First edition 1914).

DAWKINS, Richard. 1997. *Climbing Mount Improbable*. Penguin Books.

BARNES, Robert D. 1985. *Zoologia: gli invertebrati*. Padua: Piccin.

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