Mark A. Reynolds

The Bilunabirotunda

Geometer Mark Reynolds explores the Johnson Solid known as the *bilunabirotunda* and illustrates its possible use as an architectural form.

From *Wolfram Online* (http://mathworld.wolfram.com/JohnsonSolid.html), we find the following quote regarding the group of face-regular, convex polyhedra known as The Johnson Solids:

> The Johnson Solids are the convex polyhedra having regular faces and equal edge lengths (with the exception of the completely regular Platonic solids, the "semiregular" Archimedean solids, and the two infinite families of prisms and antiprisms). There are 28 simple (i.e., cannot be dissected into two other regular-faced polyhedra by a plane) regular-faced polyhedra in addition to the prisms and antiprisms (Zalgaller 1969), and Johnson (1966) proposed and Zalgaller (1969) proved that there exist exactly 92 Johnson solids in all. There is a near-Johnson solid which can be constructed by inscribing regular nonagons inside the eight triangular faces of a regular octahedron, then joining the free edges to the 24 triangles and finally the remaining edges of the triangles to six squares, with one square for each octahedral vertex. It turns out that the triangles are not quite equilateral, making the edges that bound the squares a slightly different length from that of the enneagonal edge. However, because the differences in edge lengths are so small, the flexing of an average model allows the solid to be constructed with all edges equal (Olshevsky).

For this Spring column, I would like to present one of these forms, the ***bilunabirotunda***, and some related drawings, including notated conceptual sketches for a piece of architecture based on this form. The bilunabirotunda was named, along with the others, by Viktor Zalgaller, in 1966. Some of the other more fascinating names in this family of forms are: the *Gyrofastigium*, the *Metabidiminished Rhombicosidodecahedron*, the *Gyrobifastigium*, and the *Hebeshenomegacorona*. Fig. 1 shows a linear perspective of the bilunabirotunda.

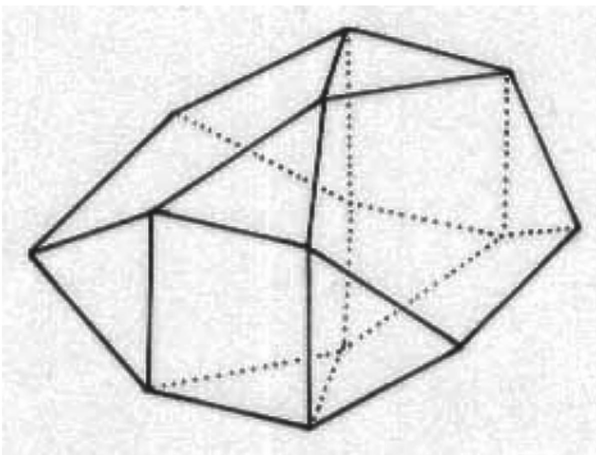

Fig. 1

Part of the bilunabirotunda's beauty is that it uses the same three geometric shapes that are found in the five Platonic Solids: the equilateral triangle, the square, and the regular pentagon. Specifically, the form contains:

- two squares;
- four pentagons;
- eight equilateral triangles;
- four golden section rectangles (internally).

All edges of the faces are equal. The pattern for the form is seen in Fig. 2.

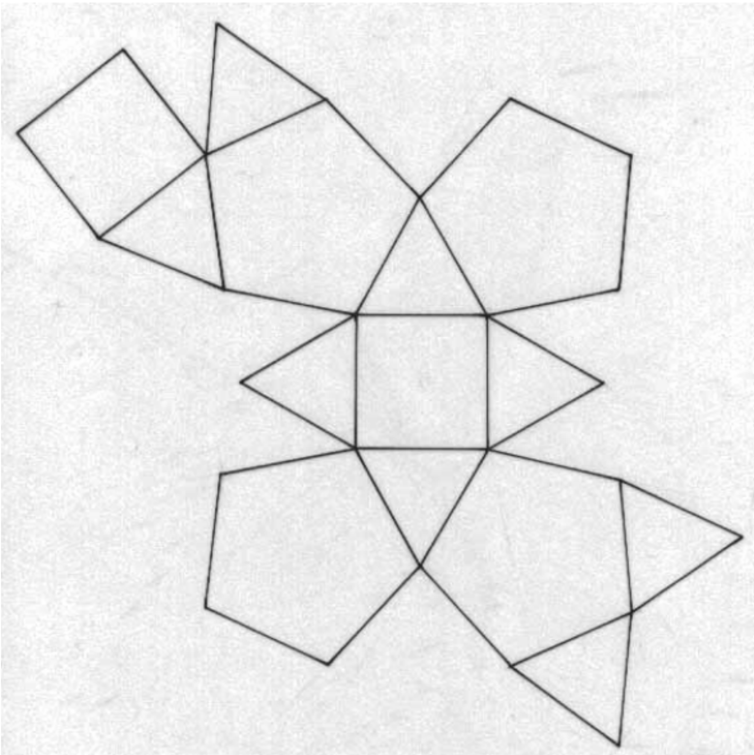

Fig. 2

I will leave the feasibility of the structural stability and building materials of this architectural fantasy to the expert engineers, and its aesthetics, as all geometry has some degree of appeal to the expert architectural historians whose eyes grace the pages of this journal, for I am but a country geometer, prone to the temptations of geometry as art and design, a visually creative device; I am still awed by its mighty power and artistry more than I am about its mathematical certainties. So it is in that spirit that I designed a twenty-first century residence, ideally replacing the domes still occasionally found in the hills and forests of Marin County, California. In Figure 3, I have drawn the three views of the form, choosing to place a square as the "footprint" in plan view.

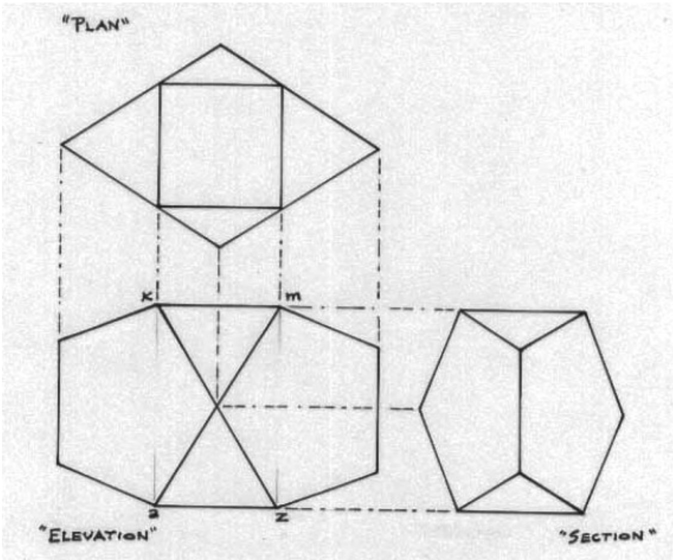
"PLAN"
k
m
z
z
"ELEVATION"
"SECTION"

Fig. 3

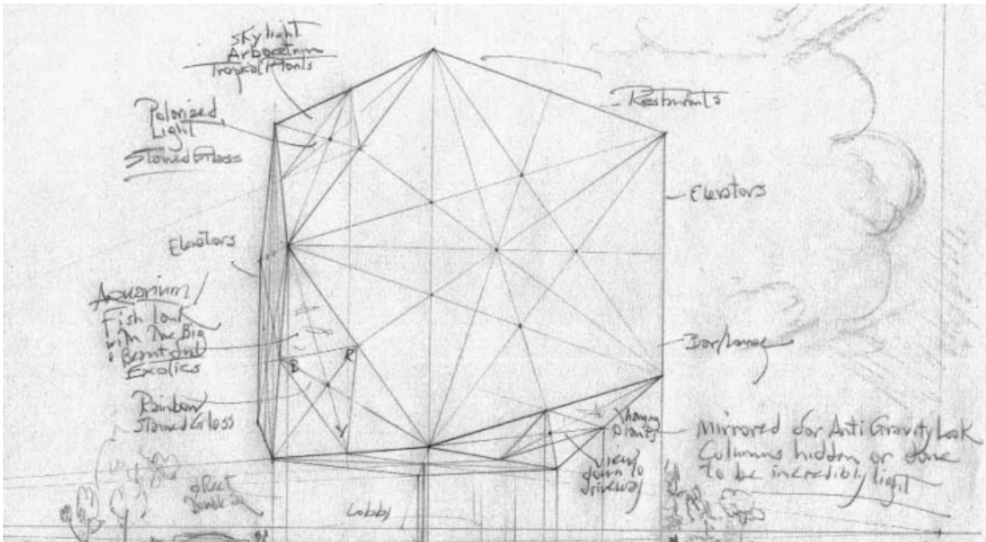
Restaurants
Elevators
Elevators
Lobby
Mirrored for Anti Gravity Look
Bilunabirotunda
Apartment Bldg.
or
Hotel

Fig. 4

This is arbitrary, as the form could stand in a number a ways, but the square base is one of the more stable possibilities. As indicated above, the form also contains a golden section rectangular solid; that is, four golden section rectangles connected on their long sides, and sealed with a square on either end. The chord of the pentagon becomes the long side of the golden section rectangle, and the short side is the edge of the square. There is also a $\phi + 1$ rectangle that runs internally using the edges of the pentagons for its short sides.

I have indicated the ϕ rectangle in Fig. 3 by the noting the vertices a, k, m, and z. The edges of the pentagons appear to become the diagonals of this rectangle in elevation (but not on the plane), which I find a lovely addition to the form's aesthetics.

Figures 4 and 5 are pencil renderings of an architectural fantasy.

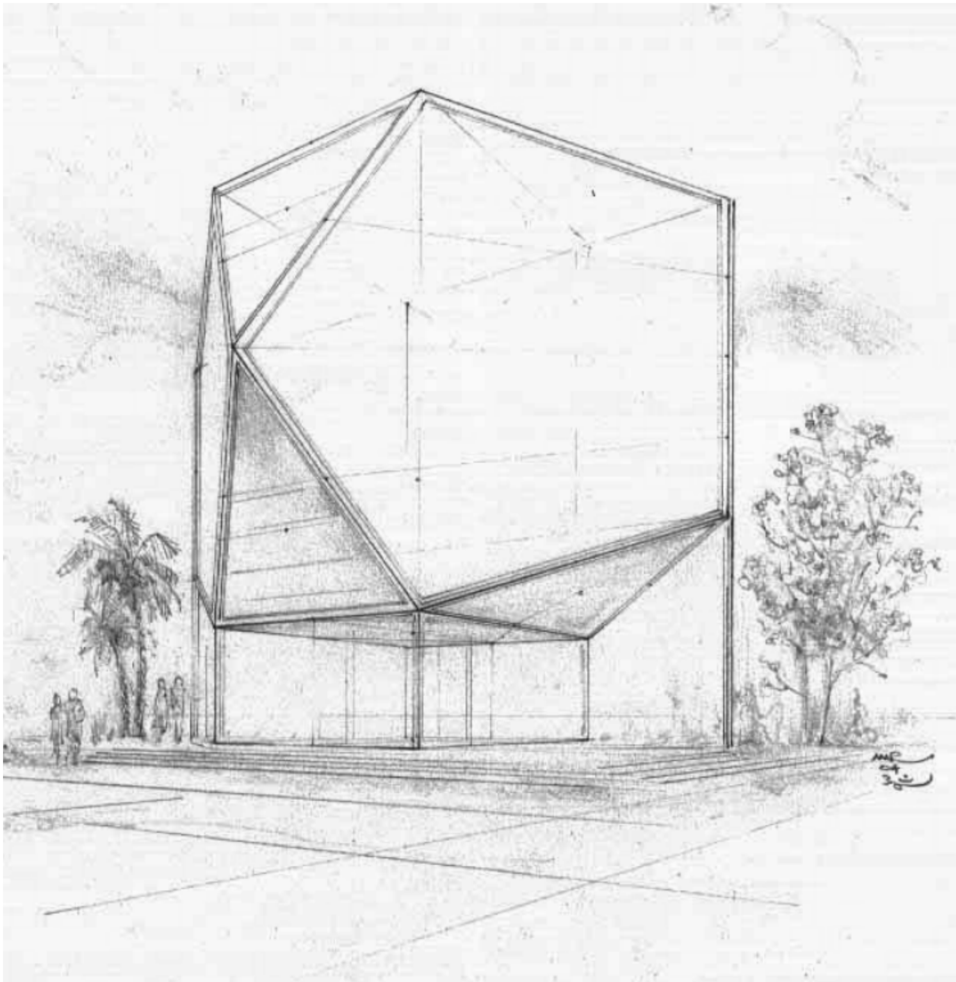

Fig. 5

At first glance, the bilunabirotunda appears to be a dodecahedron from this view; however, because of the presence of the squares, I believe that this form would be more viable as an architectural reality than the dodecahedron would be. For me, the dodecahedron is heavy and bulky; the bilunabirotunda has a gracefulness and intrigue about it. The bilunabirotunda has the added advantage of containing a rectangular solid in its core, yielding a square plan. Although there is a cube in the core of the dodecahedron, the cube is not in parallel alignment with the form's pentagonal faces, nor are the edges. (Vertexes are the only commonality.) This means that the dodecahedron would need a pentagonal plan or somehow to be able to "stand" on an edge or a vertex, and the cube would be skewed, posing incredible difficulties in structuring its interior.

The triangular and pentagonal spaces of the bilunabirotunda lend themselves readily to recreational, novel areas, or they can function as utilitarian spaces. These sloped equilateral triangles and pentagons give opportunities for the introduction of light into the interior in ways that are somewhat different than pyramids and geodesic domes, as these elements stand in high contrast structurally to the rectangular solid. Because the form allows for new perceptual opportunities from the interior, I have suggested some novel ideas, like an aquarium that could be viewed from underneath, stained glass that casts its colors into the interior/exterior spaces from new directions and angles, and views of the outside world that are different from standard windows and wall openings, as well as "floors" of sloped transparent planes. All of these features are securely anchored to a standard rectangular solid, creating a sense of complexity while maintaining a very traditional structure within.

This may be the first concept in a potential new series of architectural studies that I could call Anti-deconstructionism. Of the ninety-one remaining Johnson Solids, most are probably not architecturally feasible. Many are, however, splendid structures for consideration for use in public spaces; as sculptures, all are worthy of study and appreciation. They are a curious and enjoyable addition to the Platonic and Archimedean Solids we grew up with, yet not so infinitely problematic structurally as prisms and anti-prisms eventually become given the opportunity to develop them over time.

Enjoy the lengthening days. Perhaps one of them will actually be 24 hours long!

About the Author

Mark A. Reynolds is a visual artist who works primarily in drawing, printmaking and mixed media. He received his Bachelor's and Master's Degrees in Art and Art Education at Towson University in Maryland. He was awarded the Andelot Fellowship to do post-graduate work in drawing and printmaking at the University of Delaware. For the past decade, Mr. Reynolds has been at work on an extensive body of drawings, paintings and prints that incorporate and explore the ancient science of sacred, or contemplative, geometry. He is widely exhibited, showing his work in group competitions and one person shows, especially in California. Mark's work is in corporate, public, and private collections. Mark is also a member of the California Society of Printmakers (six of his images can be found on their website by clicking on "Galleries" then scrolling down to Mark Reynolds under "Artist Member Porfolios), the Los Angeles Printmaking Society, and the Marin Arts Council.

A born teacher, Mr. Reynolds teaches sacred geometry, linear perspective, drawing, and printmaking to both graduate and undergraduate students in various departments at the Academy of Art College in San Francisco, California. He was voted Outstanding Educator of the Year by the students in 1992.

Additionally, Reynolds is a geometer, and his specialties in this field include doing geometric analyses of architecture, paintings, and design. He presented, "A New Geometric Analysis of the Pazzi Chapel", at the Nexus 2000 conference in Ferrara, Italy, and "A New Geometric Analysis of the Plan of the Teotihuacan Complex in Mexico" at Nexus 2004 in Mexico City. He has published, "A Comparative Geometric Analysis of the Heights and Bases of The Great Pyramid of Khufu and The Pyramid of the Sun in Teotihuacan", in the Nexus Network Journal, vol. 1, no. 4. He lives with his wife and family in Mill Valley, California.