Daniele Capo

The Fractal Nature of the Architectural Orders

Daniele Capo tests certain concepts that are proper to fractal geometry with regards to architectural elements. The purpose is not to show that the architectural orders are true fractal objects, but rather that how fractal "instruments" can be used to approach certain objects and what kinds of information can be gleaned by such an approach. Understanding the orders, which for centuries have provided the basis for Western architecture, in light of the analysis presented above, allows us to observe, through the analysis of numerical data, how small elements are inserted in a continuous and coherent whole. If we interpret this structure fractally we do not distinguish between the essential and the inessential; everything is essential and so creates in this way a greater (fractal) coherence. It could be said, in this light, that the general form is not what counts the most, but rather, what is really important is the way in which parts hold together.

Introduction

A discussion of architecture and fractals can lead to ambiguous territory [Ostwald 2001; Balmond 1997; Jencks 1997a; Jencks 1997b; Eisenman 1986]. The aim of the present paper is to test with regards to architectural elements certain concepts that are proper to fractal geometry. The purpose is not to show that the architectural orders are true fractal objects, but rather that how fractal "instruments" can be used to approach certain objects and what kinds of information can be gleaned by such an approach. It is worthwhile mentioning that an architectural element is only approximately fractal, since it cannot have details that are infinitely small; thus, in this regard, I prefer not to speak of "fractal architecture," but rather of architecture "with a fractal nature."

As a guide, we can take the definition of fractal sets F laid out by Flaconer [1990: xx-xxi]:

i. F has a fine structure, that is, details at an arbitrarily small scale;
ii. F is too irregular to be described in a traditional geometric language, both locally and globally;
iii. F often has some form of self-similarity, perhaps approximate or statistical;
iv. Usually the "fractal dimension" of F (defined in one way or another) is greater than its topological dimension;
v. In the most part of the cases, F can be defined in a very simple way, perhaps recursively.

In the field of architecture, Carl Bovill [1996] performed a fractal analysis by measuring, by means of the method of box-counting, the fractal dimension of some works of Wright and Corbusier. In the present paper I would like to make some observations on the implications of an architecture with a fractal nature, and in particular, to demonstrate how a discussion of this kind is well suited to the architectural orders (it appears that thus far the architectural orders have never been subject to a fractal interpretation).

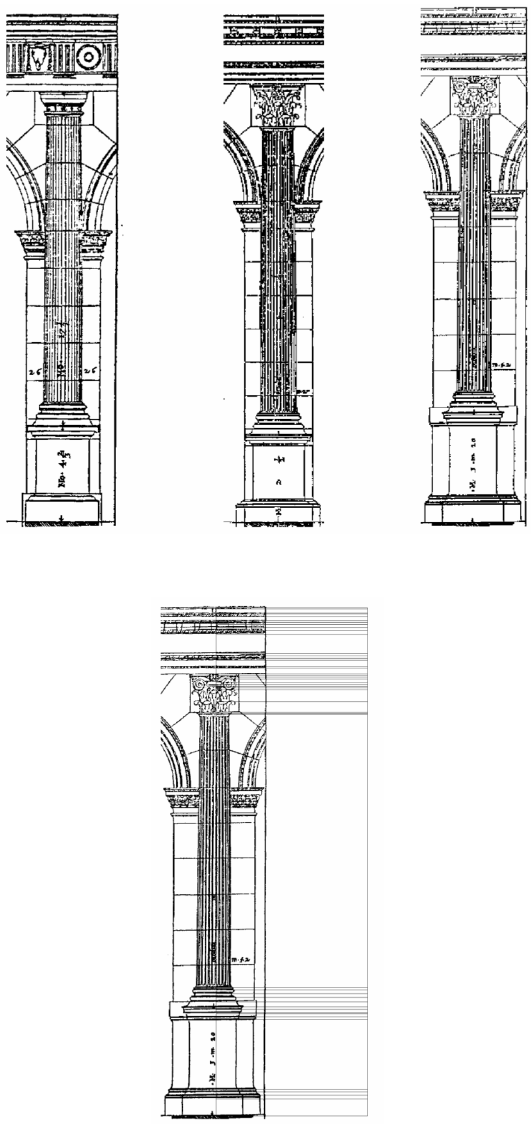

Fig. 1. The analysis in this paper was performed on the sequence of spaces between one molding and the next. a, above) Doric, Corinthian, and Composite orders, from Palladio; b, below) the Composite order and the corresponding set of horizontal lines.

The architectural orders can be taken as meaningful case studies for several reasons. In the first place, the object of study is easily defined; secondly, the analysis can be limited only to vertical successions of elements that are clearly disparate; through focussing on only one dimension, an analysis can be perfomed in a systematic and precise fashion.

In spite of its simplicity, or perhaps because of it, the example of the architectural orders furnishes a very clear image of how fractal analysis can be applied to architecture in general and contributes to the resolution of the ambivalence concerning the meaning of the term "fractal" in the field of architecture.

A Definition of the Object under Examination

The architectural orders taken into consideration are those defined by Palladio in his treatise on architecture (Fig. 1) [Palladio 1992: 30-67].

The analysis will focus on the succession of vertical elements. In essence, one takes the intersection between a straight line and the segments that separate one element from another, and this set of points is considered to be the element in question. This may appear to be an over-simplification, but it is also true that in this way a complete analysis is made possible. Further, the error that might be due to this abstraction can, at the most, result in the de-fractalization of the order. Thus, if we are able to demonstrate a coherence between our abstraction and the fractal hypothesis, then we are justified in saying that the actual order also possesses a fractal nature, perhaps even greater than that which we hypothesized. For now, however, let us accept the idea of investigating the way in which the surface is structured in one dimension.

Methodology

There are two methods by which the investigation is carried out. The first consists in measuring the box-counting dimension of the set of points defined above. The second consists in an analysis of the relationship between number and dimension of the intervals between the points (that is, of the elements that form the architectural order).

With the first method (Fig. 2), we count the number of small squares that are "occupied" by some point of the set being investigated, and at each successive passage, we divide the side of the square by two. Rather than using the classic method of box-counting, we will use a slightly modified version, "the information dimension,"[1] which takes into consideration the number of points that fall in each square. The values obtained are placed on a graph so that the number of squares occupied are laid out on the x-axis, and the logarithm of the inverse of the side of the square are laid out on the y-axis.[2] By means of a statistical analysis the straight line is obtained that best approximates the distribution of points and the coefficient of correlation, which tells how valid it is. The slant of the line represents the fractal dimension of the set of points for the architectural order. In order to obtain the dimension of the architectural order (that is, the dimension of the original drawing, taking into account only the horizontal rows), it is sufficient to add 1 to the result obtained.[3]

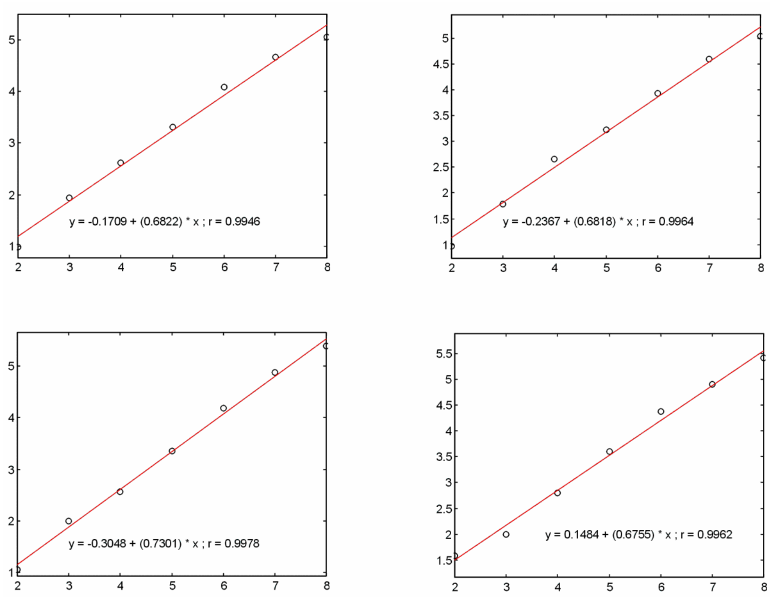

Fig. 2. Logarithmic graphs of the results obtained by the box-counting method. a, top left) Doric order; b, top right) Corinthian order; c, lower left) Composite order; d, lower right) the "control" order, artificially generated as a Cantor set-like fractal.

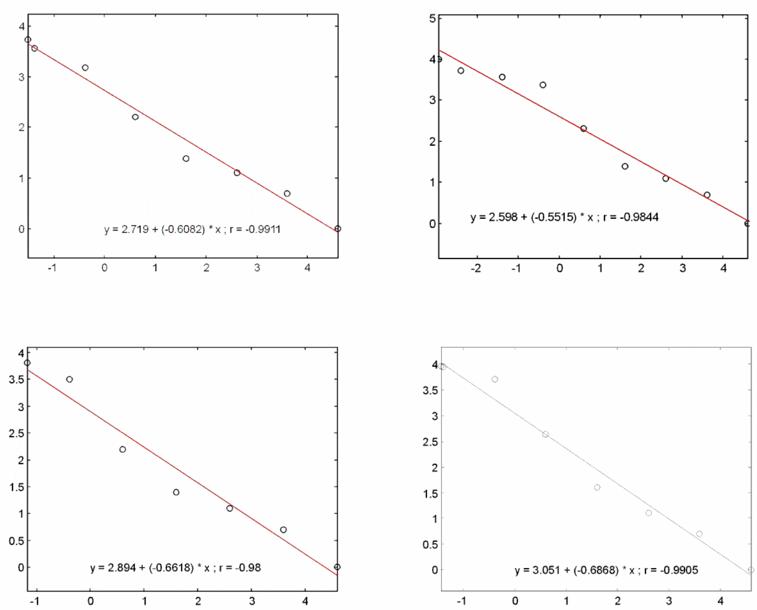

Fig. 3. Logarithmic graphs of the results obtained by counting the spaces with regards to their length. a, top left) Doric order; b, top right) Corinthian order; c, lower left) Composite order; d, lower right) the "control" order, artificially generated as a Cantor set-like fractal.

The second method (Fig. 3) consists in counting the number of spaces with a length greater than length u, which is varied by halving it. The result is placed on a logarithmic graph in which the number of spaces is laid out on the x-axis, and the value of u on the y-axis. This system derives from the kind of analysis suggested by Nikos Salingaros.[4] In this case as well it is necessary to determine the slant of the line that best approximates the data and the degree of correlation between the two.

It should be observed that the advantage of undertaking an analysis of only one dimension is that the trend of the data can be immediately visualized. Given that we are talking about a set of points and not of a true fractal, it can be expected that, when the investigation is refined, the result tends to zero;[5] what counts is how slowly it does so (that is, how much more it tends towards an authentic fractal). This is easily verified from the point where the curve formed by the data flattens out to the point of becoming horizontal. The fractal coherence can be estimated, then, by means of a simple visual comparison of the data, which is not so automatic if the investigation is extended into the plane. In this case we have the values of the dimensions tending towards one, which is more difficult to perceive with the naked eye.[6]

Comparison

The analysis thus conducted leads to a set of results, but it is important that they be compared with a control situation purposely constructed to possess given fractal characteristics. Analyzing our "artificial" order provides us with a kind of litmus paper which allows us to verify the investigation. The limit that we imposed of treating only the succession of vertical elements once again works to our advantage in that we can easily construct a similar paragon.

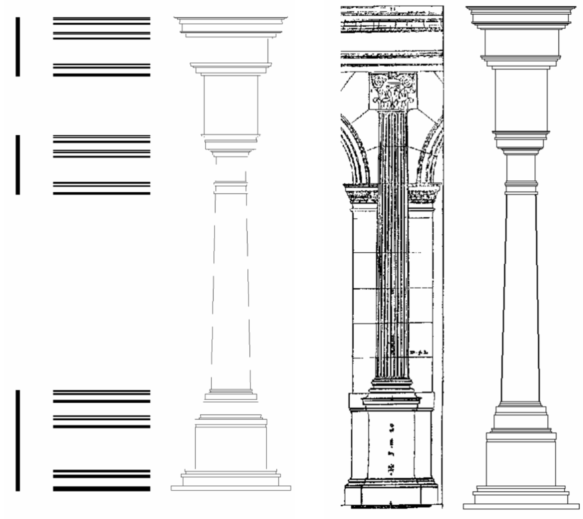

Fig. 4. a, left) A fractal model used as an element for comparison (see Fig. 2 and Fig. 3); b, right) The construction of a fractal model is useful to the analysis in order to be able to compare the results with those obtained on the actual objects under investigation. This model demonstrates the characteristics, in its distribution of parts, common to the architectural orders, even on first inspection.

We can imagine reading the succession of elements typical of the architectural orders as a Cantor set. In fact, in the former as in the latter, we find large spaces surrounded by small spaces that are, in their turn, surrounded by still smaller spaces. At this point we must define a model based on a modification of the traditional Cantor set and subject it to analysis (Fig. 4).

A first check can be effected by a direct visual comparison, drawing an architectural order and putting it next to a real one. When a certain plausibility of the hypothesis on the basis of the model is observed, we go to the same investigation discussed earlier in the section on methodology, and the results are compared.

Results of the Investigation

The fractal analysis conducted on three orders of Palladio—the Corinthian, the Doric, and the Composite—and on the control model, showed a fundamental coherence with a fractal interpretation.

The "information dimension" method shows that all three of Palladio's orders maintain a certain consistency of the data up to the eighth level, indicating that the value of the dimension is demolished only when the count is based on squares with a small side that is equal to 1/256 of the height of the entire order. If we consider a total height of 10 meters, we can conclude that the fractal coherence is maintained down to a detail of 4 centimeters, which is not surprising considering that, in the architectural orders, there are moldings that are exceedingly small.

The second method validates the results of the first, showing how the number of elements continues to increase as their height gradually diminishes, a characteristic that is essential to fractal objects. In this case, as above, the most important result is the large interval on which the fractal interpretation has been effected. The trend not being perfectly linear would seem to deny that this is true, even though the coefficient of correlation is still very high, but if the form of the graph is carefully considered, it can be discerned that the most important element is the tendency of the details to grow as their height decreases.

The "control" order, explicitly constructed with a fractal recursion, furnishes results that are very similar to those obtained from the analysis of Palladio's orders, reinforcing still more this interpretation. This helps us to understand that the jumps present in the graph relative to the second method are inevitable.[7]

The fractal dimension measured is not fixed, as can be easily verified in the graphs, but oscillates. The important thing is to notice that it also oscillates for the control order, and is greater than than the theoretical one that is known in this case. The problem therefore reduces down to two facts: the first is that the method itself has limits, as was noted above; the second is that we must not consider the orders as "simple" fractals, but rather as examples of multi-fractals in which diverse dimensions coexist.[8] Even taking into account these limits, it can be brought out in any case that the dimension runs between 0.6 and 0.7.[9] Knowing that these values are approximate by excess, we can in any case affirm that the dimension can be collocated in a position that tends to mediate between 0 and 1, where 0 represents the null set (a total absence of any element of interest) and 1 the completely full set (visual chaos, where every part is filled) (Fig. 5). The geometry of these architectural objects strike a balance between the two extremes, a fact that is held to be extremely important by both Mandelbrot [1981: 45-47] and Eglash [1999: 171].

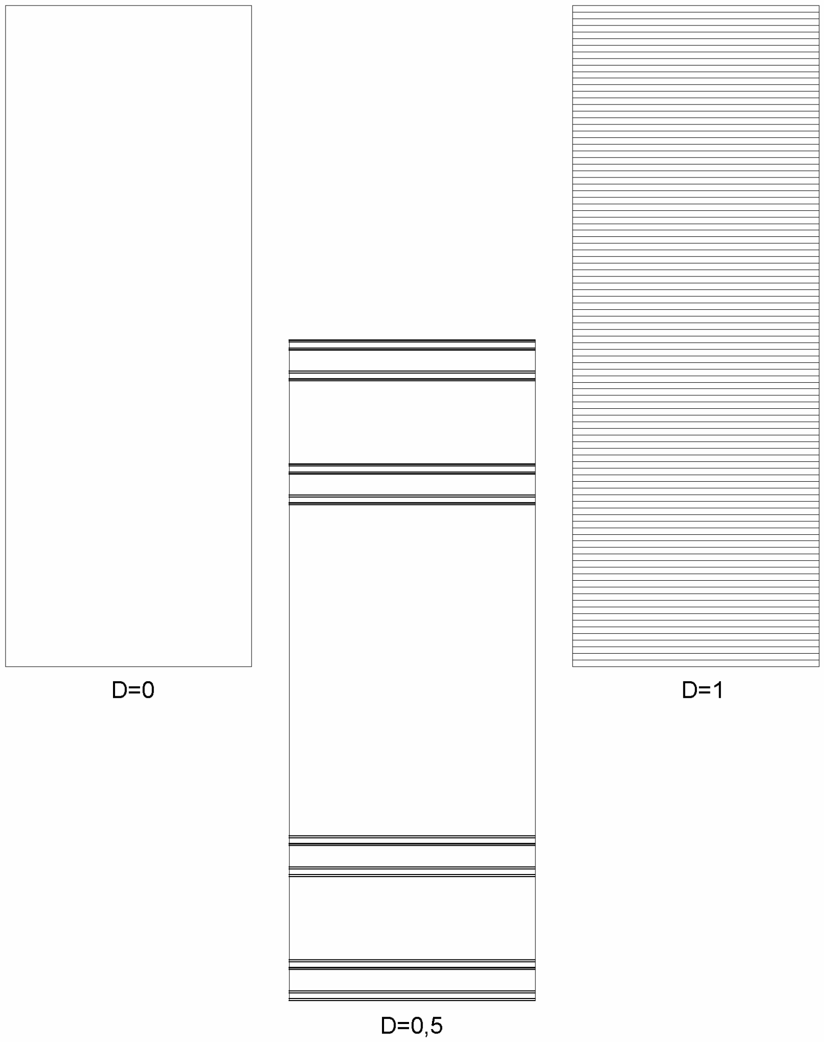

Fig. 5. a, left) Empty set, D=0; b, center) fractal set, D=0.5; c, right) full set, D=1. In the empty set, the lack of any structure whatsoever is observed. In full set, whee the subdivision invades every part, the same effect is obtained "by excess". The fractal set, derived from a "fractal dust" of dimension 0.5, strikes a balance between the two extremes generating a kind of structured hierarchy of focuses of visual interest. There exist a multiplicity of structured zones, but at the same time, the subdivision does not invade the entire field.

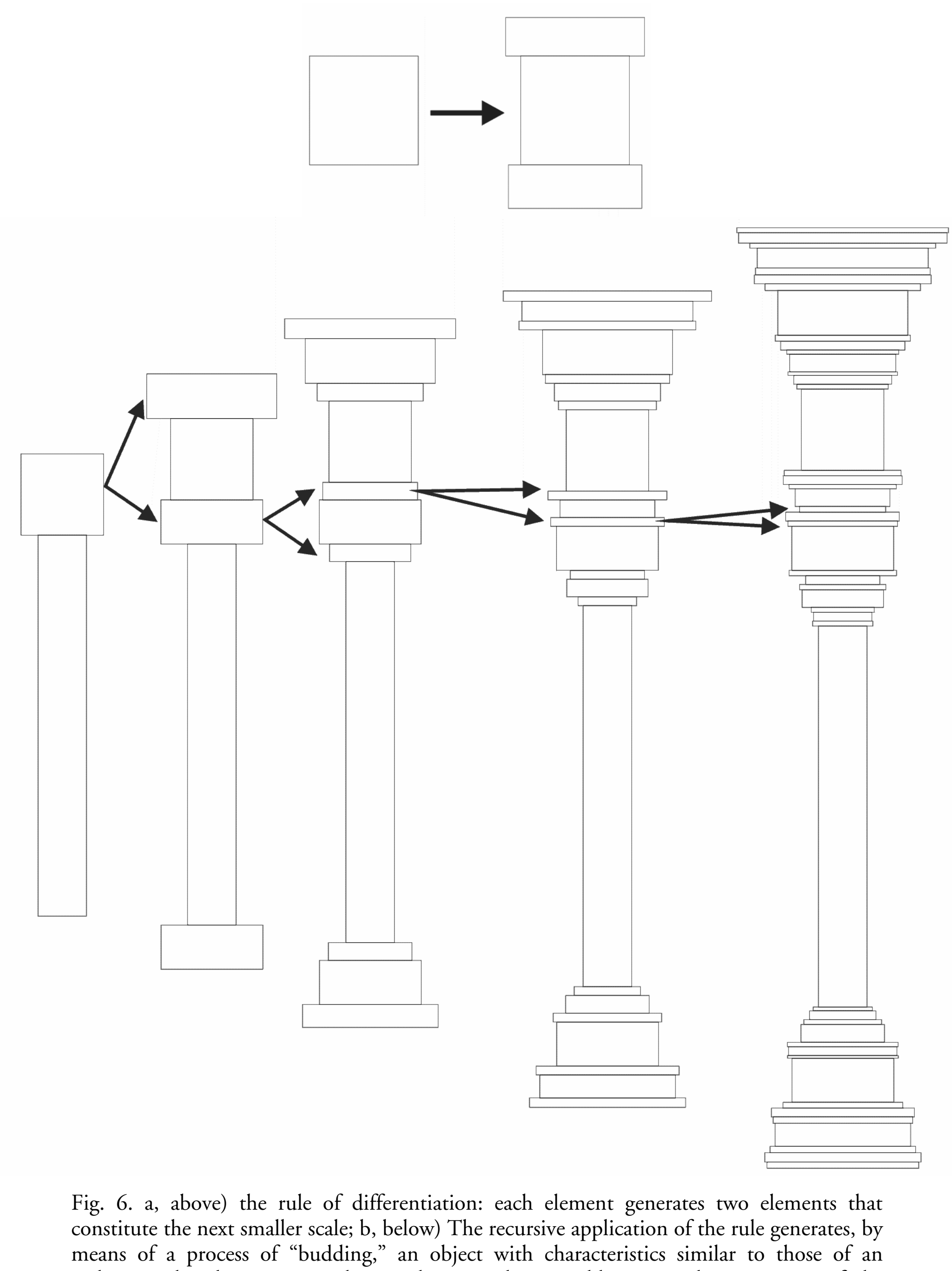

Fig. 6. a, above) the rule of differentiation: each element generates two elements that constitute the next smaller scale; b, below) The recursive application of the rule generates, by means of a process of "budding," an object with characteristics similar to those of an architectural order. In particular can be note the resemblances in the succession of the vertical spaces, which give rise to a similar "weave." This example demonstrates how a complicated and strongly structured system can emerge from a simple rule.

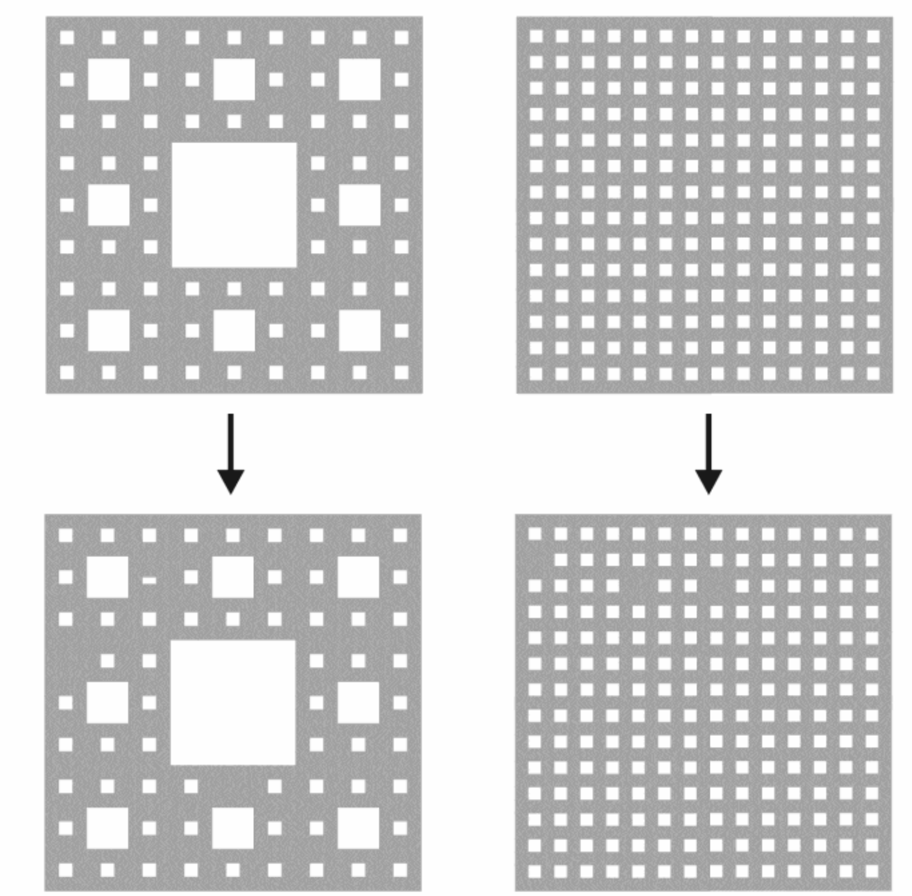

Fig. 7a. The comparison between the two figures, one fractal, the other not, demonstrates how the first is in conditions to support small modifications without dramatically altering its order, which the second, based on the concept of repetition, is immediately affected by the changes

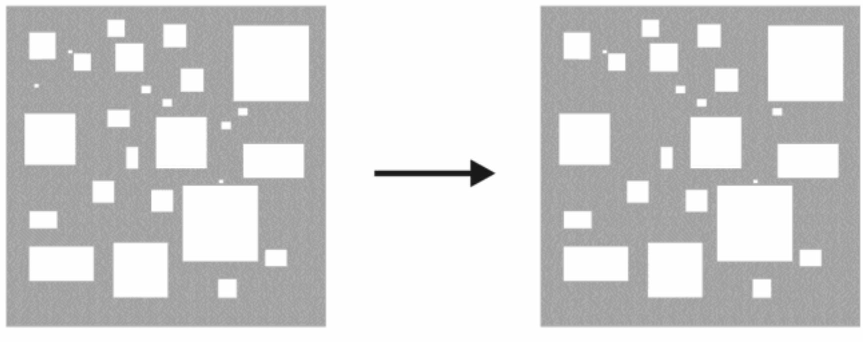

Fig. 7b. Even a figure composed of a chance series of forms without any order is not effected in a particularly evident way by modifications. For this reason it can be said that the fractal order mediates between the two extremes of repetition and pure disorder

Understanding the orders, which for centuries have provided the basis for Western architecture, in light of the analysis presented above, brings us to certain considerations. The first is that it allows us to observe, through the analysis of numerical data, how small elements are inserted in a continuous and coherent whole.

If we interpret this structure fractally we do not distinguish between the essential and the inessential; everything is essential and so creates in this way a greater (fractal) coherence. It could be said, in this light, that the general form is not what counts the most, but rather, what is really important is the way in which parts hold together. For example, by means of this analysis it could be said that the "abstractions" that reduced the architectural orders to their principal elements (as in certain architectures of totalitarian regimes of the first half of the last century) did not grasp this fact, while an architect like Wright[10], even while not replicating the form of the orders, realized an architecture which, from the point of view of fractals, came very close to them.

The second observation is that the Cantor set constitutes an approximate yet realistic model of the kind of fractal geometry exemplified by the architectural orders. This idea could be carried forward by hypothesizing a simple process of "budding out" which gives rise to structural architectural systems of a similar nature (Fig. 6). Starting with few elements, a pier on a plinth, experiments could be undertaken to see what would happen when a greater number of elements was distinguished, introducing others above and beneath, and continuing this operation a certain number of times. It can be readily observed that this means of proceeding, based on a simple logic, is capable of generating a very high level of differentiation, giving rise in the end to something that comes very close to the actual forms taken by the architectural orders through history. The kind of structuring that we have seen presents us then with a peculiarity from the point of view of perception. In fact, we can affirm that structures with fractal natures are visually very robust. We can observe this by comparing two drawings, one representing Sierpinski's carpet, the other a square subdivided into smaller squares (Fig. 7). By slightly modifying the two drawings it can be seen that the second reflects the effects of modification in an accentuated way, while the first seems effected to a lesser degree. This characteristic of fractal figures could account for the fact that the architectural orders, even while subject to modifications in the parts of which they are constituted, maintain their "order;" if their geometry was not of the fractal nature discussed in this paper, the order would diminish.

Translation from the Italian by Kim Williams

Notes

1. The "information dimension" consists in keeping a count of the greater or lesser probability that a square will be "filled" by some part of the figure. For the definition, see [Peitgen, et al. 1998].
2. For our purposes we consider the height of the entire architectural order, from the ground to the uppermost molding, to be one unit.
3. This problem can be taken back to that of the multiplication of two sets with different fractal dimensions, the dimension of which is equal to the sum of the dimensions. Cf. [Falconer 1990].
4. For a discussion of Salingaros's position on "fractal" architecture, see [Salingaros and West 1999]. The method that we suggest can also be extracted from the analysis of Mandelbrot

[1987] of the Cantor set. The links between the architectural orders and the Cantor set will become evident later in the text.

5. A fractal object such as the Cantor set will never tend to zero but, in the case of real objects, we cannot ever arrive at this level of abstraction. For this reason we have introduced the definition of architecture as having a "fractal nature," to indicate that architecture which, within certain limits, behaves in a way that is similar to a fractal.
6. It is possible to propose a different means of representation which would still present the same advantages given above.
7. In essence, the form of the graph is effected by the fact that the length u has been halved each time. The jumps represent the fact that in those points the height of the elements "jumped" in a more rapid manner.
8. We are unable to find actual examples of applications of methods of multi-fractal analysis to architecture. Within the limits of this brief paper we can only advance the hypothesis that a similar approach can furnish new information about the geometry of architecture.
9. 0.6 and 0.7 are the dimensions of the set of points that has been found with our analysis. The dimension of the succession of moldings varies therefore between 1.6 and 1.7.
10. For studies of Wright's architecture with regards to fractals, see [Bovill 1996] and [Eaton 1998].

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Daniele Capo, born in 1972, graduated in Architecture at the University of Florence, in 2002. His thesis was on "Architecture and Fractal Geometry" (with Prof. R. Corazzi and Prof. G. Conti). Presently he lives in Viterbo where he works as an architect and graphic designer. He is studying for a Master's degree in "interior yacht design". A full curriculum is available at http://digilander.libero.it/daniele_capo.