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Palladio's Villa Emo: The Golden Proportion Hypothesis Defended

At Nexus 2000, Rachel Fletcher argued that Palladio may well have made use of the 'golden section', or extreme and mean ratio, in the design of the Villa Emo at Fanzolo. In this issue of *Nexus Network Journal*, Lionel March argued that the Golden Section is nowhere to be found in the Villa Emo as described in *I quattro libri dell'architettura*. In the present paper, Rachel Fletcher defends her original thesis, comparing the Villa Emo as actually built to the project for it that Palladio published in his book.

Introduction

According to conventional wisdom, the Villa Emo at Fanzolo could never have been based on Golden proportions. I could not believe this myself—not, that is, until I saw the entire mathematical scheme for Palladio's elegant Renaissance buildings, which sit on a flat, fertile plain in Treviso, in northern Italy [Fletcher 2000].

In "Palladio's Villa Emo:The Golden Proportion Hypothesis Rebutted" [March 2001], Lionel March argues that the Golden Section, or extreme and mean ratio, is nowhere to be found in the Villa Emo as described in *I quattro libri dell'architettura*. Palladio, he says, "has given the actual measurements" and they simply do not add to a scheme of Golden proportions. He is absolutely right. The extreme and mean ratio is not observed in the Emo plan as it was published. But the villa Palladio described in that publication is not the villa he built and that survives today.

The discrepancy between the two versions was known as early as the 1770s. That was when Ottavio Bertotti Scamozzi published *Le fabbriche e i disegni di Andrea Palladio*, in which he struggled to reconcile numerous inconsistencies between built and published versions of Palladio's works [Scamozzi 1976: 75-76]. Alas, Scamozzi's measurements were not as accurate as we would have liked. Fortunately, a more definitive survey was performed in 1967 by the architects Mario Zocconi and Andrzej Pereswiet Soltan for the Centro Internazionale di Studi di Architettura "Andrea Palladio" (C.I.S.A.) [Rilievi 1972; Favero 1972: 29-32 and scale drawings a-m].

Many believe Palladio's published plans present idealized versions of his buildings, permitting him to make adjustments for the special conditions of specific sites. But perhaps, in some instances, different versions provided options for design and proportional schemes. For example, the published plan for the Villa Emo presents a conventional set of stairs that leads to a south-facing portico. In fact, a unique, elongated ramp was built. Members of the Emo family today believe it served as both an entryway and a threshing floor to meet the villa's agricultural needs. Does it correct the building's proportions to substitute

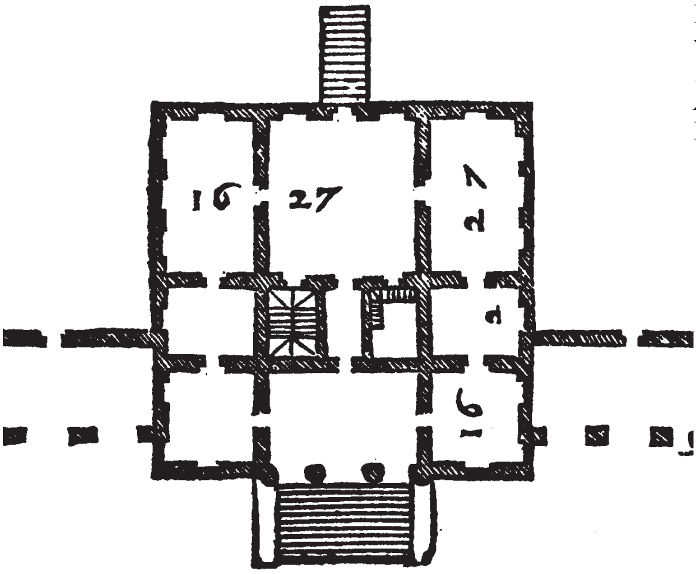


Figure 1. Detail of Palladio's woodcut of Villa Emo. From *The Four Books on Architecture* by Andrea Palladio [Palladio 1997:133].

the ramp with shorter conventional stairs? The Emo family thinks not, and perhaps Palladio did not think so, either, for a corrected set of measurements is not indicated.

Different measures are specified, however, for the plan of rooms on the main floor of the central block, and these are the stuff of musical and mathematical harmonies, as Lionel March so brilliantly demonstrates. The discrepancy is subtle, perhaps too subtle to reflect real versus ideal conditions, but sufficient to suggest a different mathematical interpretation.

Built and published measured compared

Overall length and width of the main floor plan. Compare the built and published versions of the main floor plan, beginning with overall length and width, as published, in Vicentine feet (the Vicentine foot corresponds to 34.75 centimeters [Favero, p. 18]) (Figure 1).

Lionel March calculates total length by adding individual measures along a north-south axis of length, including the lengths of three individual rooms and the thickness of four walls. For the moment, the thickness of any given wall is called x.

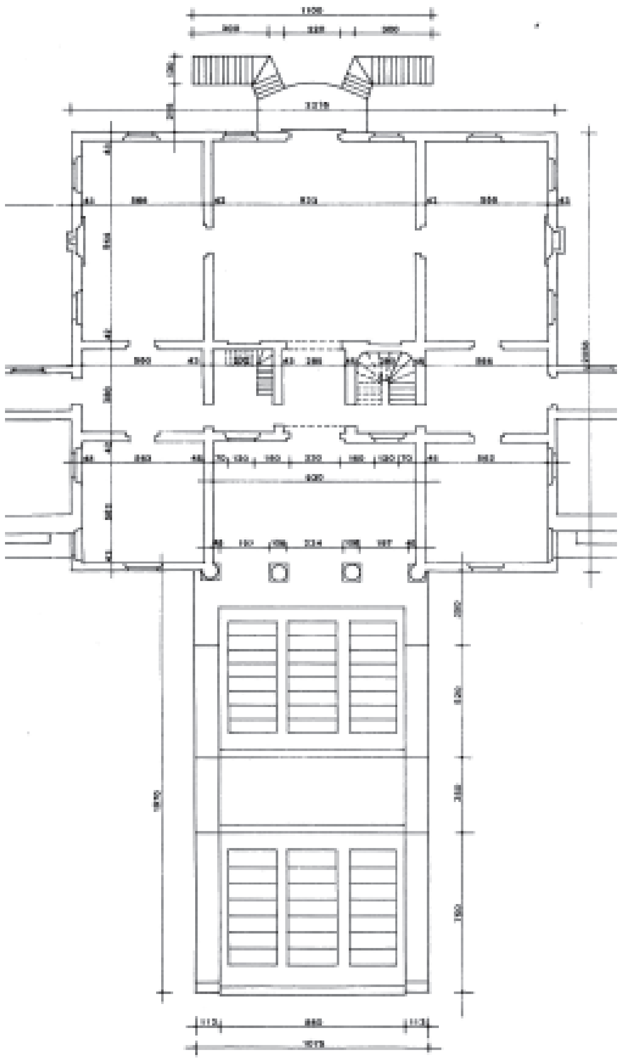
$$\text{Total length} = x + 27 + x + 12 + x + 16 + x = 55 + 4x.$$

Total width is calculated in similar fashion:

$$\text{Total width} = x + 16 + x + 27 + x + 16 + x = 59 + 4x.$$

Taking $x = 1$ as an initial choice for the wall thickness, following March, results in the ratio of total length to total width of 59:63, or 1:1.067.

Figure 2. Measured survey of Villa Emo by Mario Zocconi and Andrzej Pereswiet Soltan [*Rilievi* 1972: pl. XV].



How does this compare with the ratio of overall length to width in the villa's plan, as it was built (Figure 2)?

According to the C.I.S.A. survey, total length and total width are 20.56 meters and 22.35 meters, respectively. Therefore, the ratio of length to width is 20.56:22.35, or 1:1.087. A subtle difference from the 1:1.067 ratio of the published plan, but one that may be viewed at a glance.

Since the published plan does not provide a measure for the thickness of walls, we cannot assume it is equal to 1. But applying a variation of Lionel March's method, we can determine what the thickness must be for the published plan to match the built plan's

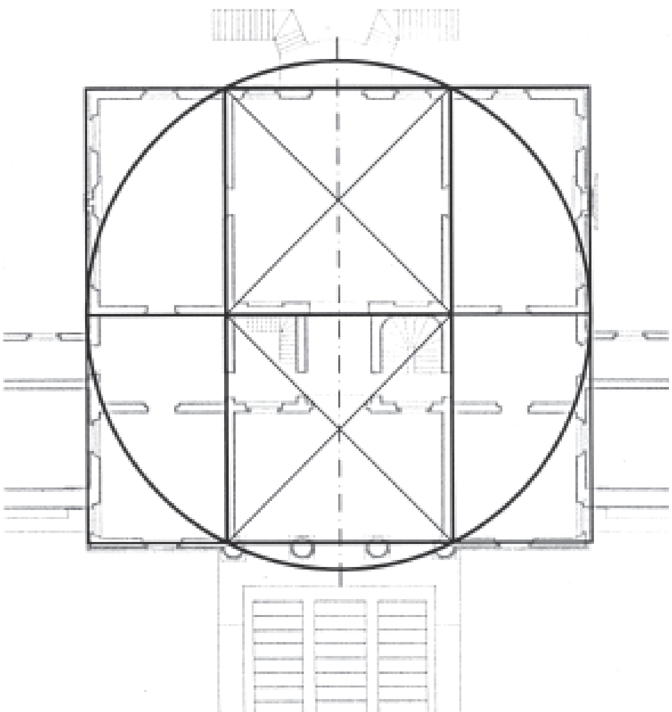


Figure 3. Plan of the central block, proportioned to a circle and its inscribed double square. Geometric overlay by Rachel Fletcher on surveyed drawing of Villa Emo by Mario Zocconi and Andrzej Pereswiet Soltan [*Rilievi* 1972: pl. III]. (This figure originally appeared as Figure 4 in [Fletcher 2000].)

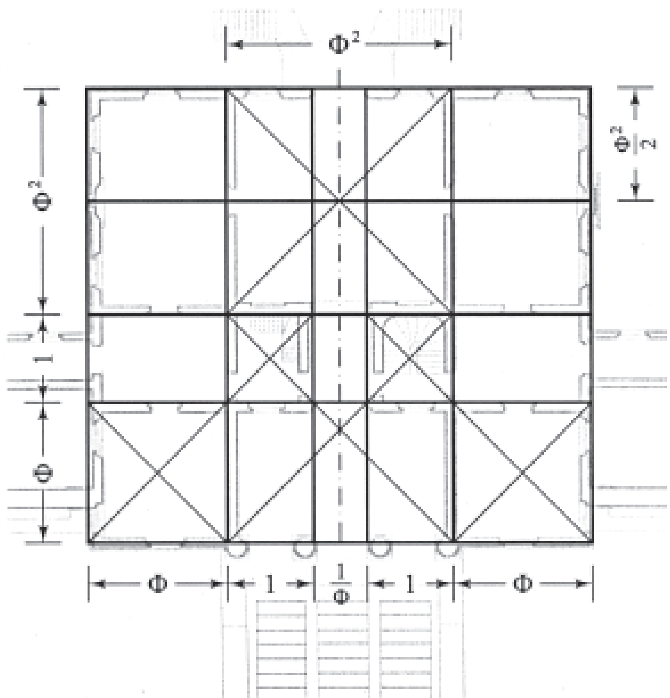
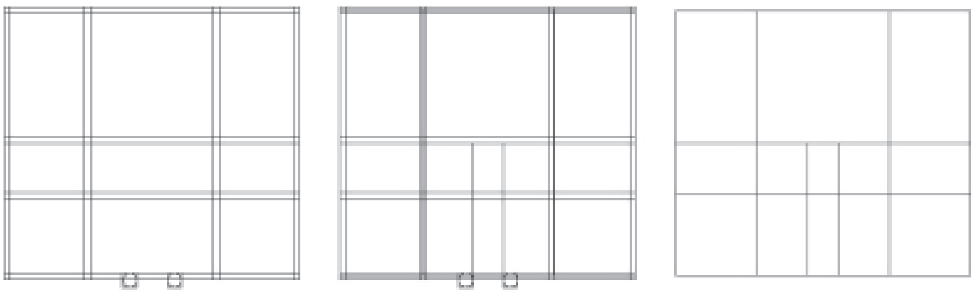


Figure 4. Progression of extreme and mean ratios in the main floor plan of the central block. Geometric overlay by Rachel Fletcher on surveyed drawing of Villa Emo by Mario Zocconi and Andrzej Pereswiet Soltan, [*Rilievi* 1972: pl. III]. (This figure originally appeared as Figure 9 in [Fletcher 2000].)

1:1.087 ratio of length to width. In other words: $(55 + 4x) : (59 + 4x) :: 1:1.087$. To satisfy the proportion, the thickness of a wall x must be approximately equal to -2.2557 feet. Not quite as impossible as the negative wall thickness of over five feet required for a Golden Mean scheme to match the plan's published measures, but impossible nevertheless.

Overall length and width of the geometric scheme. Did *I quattro libri* "correct" the measures of the villa as built, or was a different proportional scheme presented? Consider the proposed geometric scheme of Golden proportions, which is based on a rectangle that results from inscribing a double square within a circle (Figure 3).

The length of the rectangle equals the long edge of the double square. The width equals the diameter of the circle. The ratio of length to width is $2{:}\sqrt{5}$, or approximately 1:1.118. This 1:1.118 ratio is not the same as the 1:1.087 ratio of the overall plan, as built. A rectangle of ratio 1:1.118 may be accomplished, however, by subtracting 27 cm. of thickness each from the north- and south-facing exterior walls.[1] Once this adjustment is made, extreme and mean divisions align with one face or another of the remaining interior walls (Figures 4 and 5).[2]



a) **b)** **c)**

Figure 5. a) Key elements of the Villa Emo measured survey; b) Geometric analysis; c) Geometric analysis and measured survey, compared.

Length and width of individual rooms. To further illuminate the difference between the two plans, compare the dimensions of individual rooms. In meters, the central hall in the published plan is 9.38 x 9.38, but the hall that was built measures 9.44 x 9.33. The northeast and northwest bedrooms, as published, are each 9.38 m. x 5.56 m., whereas the rooms as built measure 9.44 m. x 5.66 m. The small southeast and southwest rooms, as published, are each 5.56 m. x 5.56 m., while the rooms as built measure 5.62 m. x 5.62 m. Excluding the thickness of the walls, the total width of the central block, as published, is 20.50 meters. The same, as built, is 20.66 meters [Favero 1972: 31].

This does not mean that the inside measurements of the rooms as built convey extreme and mean ratios, either within themselves or in relation to others. But when the thickness of walls is factored to one side or another, a scheme emerges in which the overall 2:√5 rectangle divides continuously in Golden proportion.

Given the evidence of the plan as it was built, perhaps Lionel March will reconsider whether "the visually gratifying result is so very wrong when tested by the numbers."

Historical precedent

The question remains: If Palladio designed with extreme and mean ratios, why didn't he publish a relevant construction in *I quattro libri*? Lionel March argues that Palladio never published a construction that produced the extreme and mean ratio. He grants that Alberti described an exact construction for a decagon and that in the 1540s, Serlio illustrated Dürer's exact construction for a pentagon. But as late as 1569, Barbaro presented only Dürer's approximate construction, even though an exact construction is required to produce the Golden Mean. And while Pacioli spiritualized the Divine Proportion and Kepler connected it to planetary motions, the extreme and mean ratio lay dormant essentially until the nineteenth century, when it was born again as the Golden Section.

Lionel March further cites the ancient theatres, which are based, Vitruvius tells us, on arrangements of squares and triangles and their inherent √2 and √3 proportions. Pentagons, however, are nowhere to be found. Never mind that the two sections of Epidaurus's theatron contain 21 and 34 rows and merely approximate a true extreme and mean division. I wouldn't consider it, either, were it not for a study by German scholars Gerkan and Müller-Wiener [1961: Pl. 3] that relates the theatre's *skene*, *orchestra* and *theatron* through a regular pentagon and its inscribed and circumscribing circles.[3]

Let's face it. From as early as Euclid through the Renaissance and beyond, the extreme and mean ratio was not unknown. Beside the examples already cited, as early as 1726, well before the nineteenth century, mathematicians, builders and architects published exact geometric constructions based on the Golden ratio. Peter Nicholson, Batty Langley and others illustrated its use for architects and builders [Nicholson 1827: Pl. 13 and problem XXII; Langley 1726: Pl. I, fig. XXVII and p. 41]. Mathematicians such as Sébastien Le Clerc demonstrated numerous constructions in elementary texts [Le Clerc 1742: 112-113, 180-181]. In at least one instance, Ephriam Chambers linked the extreme and mean ratio to the pentagon's exact construction [Chambers 1738: opp. 142].

None of this proves that Palladio favored the extreme and mean ratio. He did not publish an exact construction, but neither did he produce a book on geometry comparable to Serlio's Book I. Had he written such a book, might it contain an extreme and mean construction? Unfortunately, we probably will never know.

It is true that the extreme and mean division does not rank among Palladio's ratios for shapes for rooms. All but one, in fact, are comprised of ratios in whole numbers.[4] But these address individual rooms, not the plan as a whole, nor the rooms as they relate to one another.[5] The beauty of the Golden ratio, as it adorns the Villa Emo, is that it distinguishes the plan as a whole and persists through every level of subdivision. "Proportion" is defined conventionally as the relationship of parts to one another and to the greater whole. One would be hard pressed to find a better example.

Finally, Lionel March's most compelling argument is the practical one. "Buildings" he says, "have to be set out," and triangulation has been the method of choice "since time immemorial". Certainly, the Pythagorean 3:4:5 triangle is well suited to achieving the right angle, but surveyors may use triangles for many purposes. Consider the simple right triangle of sides one and one-half: it does not ensure the right angle, but its $\sqrt{5}/2$ hypotenuse leads directly to the Golden Section.

We base our understanding of the past on precious little evidence and so it is prudent, from time to time, to revisit what we know with a new and open mind. Without doubt, the Golden proportion hypothesis is filled with speculation, for we cannot prove that Palladio applied it with deliberate intent. And yet, given its persistence throughout the plan of Villa Emo, it may be time to consider if all the relevant evidence is in.

Lionel March is to be thanked for illuminating the many rich and wonderful mathematical techniques that grace the Villa Emo, from its 3:4:5 triangles to elaborate musical harmonies. Is it so hard to imagine that extreme and mean ratios occupied the Renaissance mind as well?

Notes:

1. One justification is that the columns along the south wall relate to the ramped entry, with the reduction repeated on the opposing north wall.

2. To be precise, the tolerance throughout is within 1 cm., with the exception of a single 9 cm. deviation.

3. The circumscribing circle traces the inside face of the *theatron*, or auditorium; the inscribed circle traces the inside edge of the orchestra perimeter; and the base of the pentagon locates the front edges of the *paraskenia*, or the *skene's* projected wings [Gerkan and W. Müller-Wiener 1961: Pl. 3]. Vitruvius's brief but evocative description tells but part of the actual story. Roman theatres, he says, emerged from a twelve-fold arrangement of four triangles, while the theatres of Greece followed a twelve-fold arrangement comprised of three squares. Both geometries are inscribed within the orchestra circle and locate elements of the different stage buildings. They also distinguish the half-round Roman theatron from the Greek auditorium's fuller expanse through eight of the circle's twelve divisions [Vitruvius 1999: 68-70, 247-248]. In fact, the Hellenistic Epidaurus appears to have adapted elements of both geometries to the situation at hand. The orchestra perimeter may be divided into twelve equal arcs, locating the apexes for a regular pattern of three inscribed squares. Eight of the twelve apexes roughly define the extent of the theatron, but the geometry isn't precise until axes taken from the center through the first and eighth apexes meet the outer edge of the lowest auditorium level. The remaining four apexes define the size of the *skene*, in the sense that axes taken from the center through the ninth and twelfth apexes mark the inside corners of the *paraskenia*. Meanwhile, the base of an equilateral triangle that is circumscribed by the orchestra circle locates the *theatron* at its innermost edge [Fletcher 1991: 100-103].

4. The one exception is a room in the ratio of $1{:}\sqrt{2}$ [Palladio 1997:59].

5. A simple whole number ratio may suffice for an individual room to express grace and harmony. But Jay Hambidge [1967] explains that incommensurable ratios such as the Golden Section permit a "dynamic symmetry" in which the same ratio persists through endless levels of subdivision.

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Rachel Fletcher is a theatre designer and geometer living in Massachusetts, with degrees from Hofstra University, SUNY Albany and Humboldt State University. She is the creator/curator of two museum exhibits on geometry, "Infinite Measure" and "Design By Nature". She is the co-curator of the exhibit "Harmony by Design: The Golden Mean" and author of its exhibition catalog. In conjunction with these exhibits, which have traveled to Chicago, Washington, and New York, she teaches geometry and proportion to design practitioners. She is an adjunct professor at the New York School of Interior Design. Her essays have appeared in numerous books and journals, including *Design Spirit*, *Parabola*, and *The Power of Place*. Her design/consulting credits include the outdoor mainstage for Shakespeare & Co. in Lenox, Massachusetts and the Marston Balch Theatre at Tufts University.