A Comparison Theorem in $\mathfrak{p}$-Adic Cohomology (*).

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Summary. – *We consider a 1-dimensional differential module* $(\mathcal{V}, \nabla)$ *over an algebraic variety* X. *We assume the singularities of* $(\mathcal{V}, \nabla)$ *at infinity to be separated and possibly irregular. We prove that the algebraic de Rham cohomology of* X *with coefficients in* $(\mathcal{V}, \nabla)$ *can be calculated by* $\mathfrak{p}$*-adic analytic methods.*

0. – Introduction.

In his two articles [1], [2] F. BALDASSARRI stated a conjecture about comparison of cohomology and proved it in some particular situations. Here, I would like to continue these efforts by considering the case of coefficients with irregular singularities.

Let X_0 be a non singular irreducible algebraic variety over the field $K_0 = \overline{\boldsymbol{Q}}^{\text{alg}}$ of algebraic numbers. Let $\mathcal{V}_0$ be a locally-free sheaf of $\mathcal{O}_{X_0}$-modules endowed with an integrable connection, that is a K_0-linear map:

$$(0.1) \qquad \nabla_0 : \mathcal{V}_0 \to \mathcal{V}_0 \otimes \Omega^1_{X_0/K_0}$$

satisfying Leibniz's rule and the usual integrability conditions ([10]).

Let K be a complete algebraically closed $\mathfrak{p}$-adic field (endowed with a valuation extending that of $\boldsymbol{Q}_p$), $K \supseteq K_0$.

We shall denote the extension of the preceding structures to K by ∇, $\mathcal{V}$, X, $\mathcal{O}_X$; in particular we have:

$$(0.2) \qquad \nabla : \mathcal{V} \to \mathcal{V} \otimes \Omega^1_{X/K} .$$

We can associate to an algebraic variety over K a rigid analytic space over the same field. Under our assumptions such an analytic space $(X_{\text{rig}}, \mathcal{O}_{X_{\text{rig}}})$ will be smooth. Similarly, we can associate to every locally-free $\mathcal{O}_X$-module $\mathcal{V}$, endowed with a connection, a locally free $\mathcal{O}_{X_{\text{rig}}}$-module $\mathcal{V}_{\text{rig}}$ and a connection:

$$(0.3) \qquad \nabla_{\text{rig}} : \mathcal{V}_{\text{rig}} \to \mathcal{V}_{\text{rig}} \otimes \Omega^1_{X_{\text{rig}}/K} .$$

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Using the operators (0.2), (0.3) we can construct the de Rham complexes:

$$\mathcal{DR}(X/K, (\mathcal{V}, \nabla)) : 0 \to \mathcal{V} \xrightarrow{\nabla} \mathcal{V} \otimes \Omega^1_{X/K} \to \mathcal{V} \otimes \Omega^2_{X/K} \to \dots \tag{0.4}$$

$$\mathcal{DR}(X_{\text{rig}}/K, (\mathcal{V}_{\text{rig}}, \nabla_{\text{rig}})) : 0 \to \mathcal{V}_{\text{rig}} \xrightarrow{\nabla_{\text{rig}}} \mathcal{V}_{\text{rig}} \otimes \Omega^1_{X_{\text{rig}}/K} \to \dots . \tag{0.5}$$

The conjecture, stated by BALDASSARRI in [1], asserts the existence of a natural isomorphism between the hypercohomology groups of the complexes (0.4) and (0.5) (under the essential hypothesis that (0.4) and (0.5) are derived from (0.1) i.e. from objects defined over $K_0 = \overline{\boldsymbol{Q}}^{\text{alg}}$).

Explicitly we put

$$\boldsymbol{H}^q\big(X, \mathcal{DR}(X/K, (\mathcal{V}, \nabla))\big) \underset{\text{def}}{=} H^q_{DR}(X/K, \mathcal{V}, \nabla) \tag{0.6}$$

$$\boldsymbol{H}^q\big(X_{\text{rig}}, \mathcal{DR}(X_{\text{rig}}/K, (\mathcal{V}_{\text{rig}}, \nabla_{\text{rig}}))\big) \underset{\text{def}}{=} H^q_{DR}(X/_{\text{rig}}K, \mathcal{V}_{\text{rig}}, \nabla_{\text{rig}}) . \tag{0.7}$$

The conjecture asserts that for $q \geqslant 0$, the natural morphisms

$$H^q_{DR}(X/K, \mathcal{V}, \nabla) \to H^q_{DR}(X_{\text{rig}}/K, \mathcal{V}_{\text{rig}}, \nabla_{\text{rig}}) \tag{0.8}$$

are isomorphisms.

In this section 1 of [1] this global problem of comparison of algebraic versus $\mathfrak{p}$-adic analytic cohomology was transformed into a local problem of comparison between cohomology with coefficients, respectively, meromorphic or essentially singular at infinity. The case in which X_0 is a curve was also proved in [1]. Later [2] (under the essential assumption that the locally free module $\mathcal{V}_0$ can be extended to a locally free one at infinity) established the isomorphism (0.8) in the case where $(\mathcal{V}, \nabla)$ has regular singularities at infinity, for any dimension of X_0. This gave a $\mathfrak{p}$-adic version of Deligne's theorem ([9], Chapt. 2, section 6). In this article we prove the conjecture when X_0 is a non-singular irreducible algebraic variety, the module is one dimensional and the connection has separated irregular singularities (Theorem 1.10).

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1. – Notation and statement of the Main Theorem (Theorem 1.10).

Let us recall the local situation of section 1 of [1], with the hypothesis that the module $\mathcal{V}_0$ has a locally-free extension at infinity. We put $S = SpA$, where A is an absolutely regular [4] affinoid K-algebra and an integral domain with $\Omega^1_{A/K}$ free

$$\Omega^1_{A/K} = \bigoplus_{i=1}^{r} A\,dt_i$$

$t_i \in A$ (for notations and terminology see [5] and [11]). We also put $B = A\langle x_1, ..., x_s\rangle$ and $X = SpB$.

N.B. Since A is an integral domain, it follows that all complete norms on it (as a K-algebra) are equivalent (in particular to the supremum norm [5] 6.2.4 Theorem 1). We shall use the supremum norm and indicate it by $|\cdot|_A$. (By our assumptions, there is an epimorphism $\alpha\colon T_n \to A$ for some n such that the residue norm $|\cdot|_\alpha$ coincides with the supremum norm [5] 6.4.3).

Furthermore, we can write

$$\Omega^1_{B/K} = B \otimes_A \Omega^1_{A/K} \oplus \bigoplus_{j=1}^{s} B\,dx_j\,.$$

Let Y be a divisor of X given by the equation $x_1 \ldots x_s = 0$. Let $\mathfrak{J}$ be the sheaf of $\mathcal{O}_X$-ideals defined by Y. Put $j\colon X' = X \setminus Y \hookrightarrow X$. For any sheaf of $\mathcal{O}_X$-modules $\mathcal{F}$ we shall define:

$$\mathcal{F}(*) = \varinjlim_{N \to +\infty} \mathcal{F} \otimes \mathfrak{J}^{-N} \quad \text{and } \mathcal{F}(-) = j_* j^{-1} \mathcal{F}\,. \tag{1.1}$$

By our reductions, $\mathcal{V}$ is a free finite-rank sheaf of $\mathcal{O}_X$-modules on X and it is associated to a free finite-rank B-module V, $\tilde{V} = \mathcal{V}$. We shall use the following convention: a formula containing the symbol $(\overset{*}{-})$ stands for two formulas, one containing only symbols of the type $(*)$, the other containing only symbols $(-)$. We then define:

$$V(\overset{*}{-}) = \Gamma(X, \mathcal{V}(\overset{*}{-})) = V \otimes B(\overset{*}{-}) = \bigoplus B(\overset{*}{-})\, e_i\,. \tag{1.2}$$

We notice that $B(*) = B[(x_1 \ldots x_s)^{-1}]$ and that $B(-)$ is the ring consisting of all powers series $\sum_{\alpha \in \mathbf{Z}^s} a_\alpha x^\alpha$, with $a_\alpha \in A$, such that, if $\alpha = (\alpha_1, \ldots, \alpha_s)$, $|\alpha| = \sum_{i=1}^{s} |\alpha_i|$, $2t(\alpha) = \sum_{i=1}^{s} (\alpha_i - |\alpha_i|)$ one has

$$\lim_{|\alpha| \to +\infty} |a_\alpha|_A \varepsilon^{t(\alpha)} = 0 \qquad \forall \varepsilon > 0\,, \quad \varepsilon \in |K|\,. \tag{1.3}$$

The sheaf $\tilde{V} = \mathcal{V}$ is endowed with an integrable X/K-connection with meromorphic singularities along Y. Namely, there exists a K-linear morphism

$$\nabla\colon \mathcal{V} \to \mathcal{V}(*) \otimes \Omega^1_{X/K} \tag{1.4}$$

satisfying Leibniz's rule. It is not restrictive to assume that ∇ comes from a K-linear map between the modules defining (1.4), namely from

$$\nabla\colon V \to V(*) \otimes \Omega^1_{B/K}\,. \tag{1.5}$$

Let us consider the de Rham complexes derived from (1.4):

$$(1.6) \qquad \mathcal{DR}_K(\mathcal{V}(\underline{*})) = \mathcal{DR}\big(X/K, (\mathcal{V}(\underline{*}), \nabla(\underline{*}))\big):$$
$$0 \to \mathcal{V}(\underline{*}) \xrightarrow{\nabla(\underline{*})} \mathcal{V}(\underline{*}) \otimes \Omega^1_{X/K} \to \mathcal{V}(\underline{*}) \otimes \Omega^2_{X/K} \to \dots$$

(by the $\nabla(\underline{*})$ formulas we indicate the obvious extension of (1.4) to $\mathcal{V}(\underline{*})$), and the corresponding complexes of global sections (with the connections derived from (1.5)):

$$(1.7) \qquad DR_K(V(\underline{*})) = DR\big(B/K, (V(\underline{*}), (\underline{*}))\big):$$
$$0 \to V(\underline{*}) \xrightarrow{(\underline{*})} V(\underline{*}) \otimes \Omega^1_{B/K} \to V(\underline{*}) \otimes \Omega^2_{B/K} \to .$$

In this situation, we have ([1] proof of proposition 2.14):

$$(1.8) \qquad H^q\big(X, \mathcal{DR}_K(\mathcal{V}(\underline{*}))\big) = H^q\big(DR_K(V(\underline{*}))\big) \qquad q \geqslant 0$$

thus, it is equivalent to think about sheaves or about their modules of global sections.

In this article we make the two assumptions that: (1) $\mathcal{V}$ *is one dimensional module*, $V \simeq B$, and (2) *the connection has separated singularities along* Y. By (2) we mean that, in a suitable B-base for $V \simeq B$, it is possible to write:

$$(1.9) \qquad \nabla = d_{B/K} + \sum_{j=1}^{s} h_j \, dx_j / x_j^{p_j+1} + \sum_{i=1}^{r} g_i \, dt_i$$

where $h_j, g_i \in B$, $p_j \in \boldsymbol{N}$ and $h_j \notin x_j B$.

We refer to [6] and [12] for the formal aspect of this notion, We shall see in section 2 that when developing every h_j as a power series in x_j, the first $p_j + 1$ coefficients of such a series which a priori are merely elements of $A\langle x_1, \dots, \hat{x}_j, \dots, x_s\rangle$, actually lie in K.

The aim of this article is to prove the following:

THEOREM 1.10. – *Under the previous hypotheses concerning the connection if the first $p_j + 1$ coefficients of each h_j as a power series in x_j (respectively), which are in K, are p-adically non-Liouville numbers, the natural morphism*

$$(1.11) \qquad DR_K(B(*)) \hookrightarrow DR_K(B(-))$$

is a quasi-isomorphism.

REMARK 1.12. – One should note that the hypotheses of the conjecture agree with those of the theorem. In fact algebraic numbers are $\mathfrak{p}$-adically non Liouville ([3], [8]).

2. – Reduction to the case a 1-variable relative differential operator.

In this paragraph we begin the proof of theorem 1.10 by reducing it to the statement of theorem 2.19 below. First we point out a property of the connection (1.9).

PROPOSITION 2.1. – *The first* p_j+1 *coefficients of each* $h_j \in B$ *in* (1.9) *as a power series in* x_j *with coefficients in* $A\langle x_1, \ldots, \hat{x}_j, \ldots, x_s\rangle$, *are in* K.

PROOF. – Since (1.9) is an integrable connections, we have:

$$\left[x_j^{p_j+1}\frac{\partial}{\partial x_j}+h_j, \frac{\partial}{\partial t_i}+g_i\right]=0 \qquad i=1,\ldots,r \tag{2.2}$$

thus

$$x_j^{p_j+1}\frac{\partial}{\partial x_j}(g_i)=\frac{\partial}{\partial t_i}(h_j) \qquad i=1,\ldots,r\,. \tag{2.3}$$

Developing h_j as a power series in x_j (and with coefficients in $A\langle x_1, \ldots, \hat{x}_j, \ldots, x_s\rangle$), since (2.3) holds for every $i=1,\ldots,r$ we can conclude that the first p_j+1 coefficients of h_j are in $K\langle x_1, \ldots, \hat{x}_j, \ldots, x_s\rangle$.

REMARK 2.4. – We are using the obvious fact that, under our assumptions, $\operatorname{Ker}(d_{A/K}\colon A \to \Omega^1_{A/K})=K$

Applying the integrability condition with respect to x_k $k=1,\ldots,\hat{j},\ldots,s$ we have

$$x_j^{p_j+1}\frac{\partial}{\partial x_j}(h_k)=x_k^{p_k+1}\frac{\partial}{\partial x_k}(h_j); \tag{2.5}$$

since for every choice of k the first part of (2.5) is divisible by $x_j^{p_j+1}$, we deduce that the first p_j+1 coefficients of h_j, as power series in x_j, are in K. So:

$$h_j=h_{j,0}+h_{j,1}x_j+h_{j,2}x_j^2+\ldots+h_{j,p_j+1}x_j^{p_j+1}+\ldots \qquad j=1,\ldots,s \tag{2.5.1}$$

where $h_{j,i}\in K$, $i=0,\ldots,p_j$, $j=1,..,s$; $h_{j,i}\in A<x_1,\ldots,\hat{x}_j,\ldots,x_s\rangle$ if $i\geqslant p_j+1$, $j=1,\ldots,s$. Q.E.D.

REMARK 2.6. – By our assumption (theorem 1.10) the elements $h_{j,i}$ $i=0,\ldots,p_j$; $j=1,\ldots,s$ are non Liouville.

We put:

$$p_j(x_j)=\sum_{i=0}^{p_j}h_{j,i}x_j^i \qquad \text{for } j=1,\ldots,s\,. \tag{2.7}$$

We can now begin the proof of theorem 1.10. As in section 3 of [2] we can reduce to a relative connection over A and need only to demonstrate that the natural inclusion map

$$DR_A(B(*)) \to DR_A(B(-)) \tag{2.8}$$

where, explicitly,

$$DR:(B(\overset{*}{-})): 0 \to B(\overset{*}{-}) \xrightarrow{\nabla_{B(\overset{*}{-})/A}} B(\overset{*}{-}) \otimes \Omega^1_{B/A} \to \dots \tag{2.8.1}$$

and

$$\nabla_{B(\overset{*}{-})/A} = d_{B(\overset{*}{-})/A} + \sum_{j=1}^{s} h_j\, dx_j / x_j^{p_j+1} \tag{2.8.2}$$

is a quasi-isomorphism.

Furthermore, we can assume, inductively, that the above fact is true when the relative dimension of B over A is less than s.

We also write

$$D_j = x_j^{p_j+1} \frac{\partial}{\partial x_j} + h_j \qquad j = 1, \dots, s\,.$$

We refer to D_j as the j^{th}-*component* of the connection (2.8.2) deduced from (1.9) and use the same symbol to denote the $(x_1, \dots, x_s)$-adically continuous extension of D_j to $\hat{B} = A[\![x_1, \dots, x_s]\!]$.

PROPOSITION 2.9. – *There exists a unit f of $\hat{B} = A[\![x_1, \dots, x_s]\!]$, such that*

$$D_j(f) = P_j(x_j) f \qquad j = 1, \dots, s\,.$$

$(P_j(x_j)$ as in $(2.7))$.

PROOF. – We put $p = \sum p_j =$ total irregularity of the connection. We carry out an induction on p. If $p = 0$ the singularities are logarithmic and the components of the connection will be

$$D_j = x_j \frac{\partial}{\partial x_j} + h_j \qquad j = 1, \dots, s$$

and

$$P_j(x_j) = h_{j,0} \in K \qquad j = 1, \dots, s\,.$$

By the proposition 2.1 the h_j have the form $h_j = h_{j,0} + x_j z_j$; $z_j \in A\langle x_1, \dots, x_s\rangle$ $j = 1, \dots, s$. We shall construct a unit $f \in \hat{B}$ such that

$$D_j(f) = h_{j,0} f \qquad j = 1, \dots, s$$

i.e.

$$\left(x_j \frac{\partial}{\partial x_j} + x_j z_j\right)(f) = 0 \qquad j = 1, \ldots, s\,. \tag{2.9.1}$$

Hence f has to satisfy

$$\left(\frac{\partial}{\partial x_j} + z_j\right)(f) = 0 \qquad j = 1, \ldots, s\,.$$

Now the operator has no singularities, and by a trivial generalization of proposition 8.9 of [10] such a f is well determined by its (arbitrary) value for $x_1 = \ldots = x_s = 0$. We have therefore proved the proposition when $p = 0$. By the induction hypothesis we may suppose the result to be proved if $p' < p$, $p \geqslant 1$. Now we prove it for p. By rearranging the variables we may assume that $p_s > 0$. We define:

$$\bar{D}_s\colon \hat{B}/(x_s) \to \hat{B}/(x_s) \tag{2.10}$$

as the map induced by D_s; $\bar{D}_s$ is an $A[\![x_1, \ldots, x_{s-1}]\!]$ linear map. According to proposition 2.1, $\bar{D}_s$ is the multiplication by $h_{s,0} \in K$. We consider $D_s^* = D_s - h_{s,0}$. The map induced by D_s^* on $\hat{B}/(x_s)$ is zero.

i.e. $$D_s^*(\hat{B}) \subseteq x_s \hat{B}\,.$$

So $x_s^{-1} D_s^*$ operates on $\hat{B}$ and we can endow $\hat{B}$ with an integrable connection given by the following components $(D_1, \ldots, D_{s-1}, x_s^{-1} D_s^*)$. To this module with connection we can apply the induction hypothesis: there exists $f \in \hat{B}$ such that

$$f\hat{B} = \hat{B}; \qquad D_j(f) = P_j(x_j) f \qquad j = 1, \ldots, s-1\,.$$

$$x_s^{-1} D_s^*(f) = b_s f \qquad \text{where } b_s = \big(P_s(x_s) - h_{s,0}\big)/x_s\,.$$

But $D_s^*(f) = x_s b_s f$ and $x_s b_s = P_s(x_s) - h_{s,0}$. Finally

$$D_s(f) = (h_{s,0} + x_s b_s) f = P_s(x_s) f\,. \qquad \text{Q.E.D.}$$

Hence, we can see that the new basis f satisfies the following differential equations

$$f^{-1} x_j^{p_j+1} \frac{\partial}{\partial x_j}(f) + h_j = P_j(x_j) \qquad j = 1, .., s\,. \tag{2.11}$$

i.e.

$$x_j^{p_j+1} \frac{\partial}{\partial x_j}(f) = \big(P_j(x_j) - h_j\big)(f) \qquad j = 1, \ldots, s\,. \tag{2.12}$$

By our definitions (2.7), the series $P_j(x_j) - h_j$ for $j = 1, \ldots, s$ belongs to $x_j^{p_j+1} A\langle x_1, \ldots, x_s\rangle$ so that:

$$(P_j(x_j) - h_j)/x_j^{p_j+1} \in A\langle x_1, \ldots, x_s\rangle \qquad j = 1, \ldots, s \,.$$

Using the same method as in section 5 of [2] it follows that the formal power series f is convergent for $|x_1|_A \leqslant \varrho, \ldots, |x_s|_A \leqslant \varrho$ for some $\varrho > 0$.

We now look more closely at the following complexes of sheaves whose global section are (2.8.1):

$$(2.13) \qquad 0 \to \mathcal{O}_X(\overset{*}{-}) \xrightarrow{\nabla_{B(\overset{*}{-})/A}} \mathcal{O}_X(\overset{*}{-}) \otimes \Omega^1_{X/S} \to \mathcal{O}_X(\overset{*}{-}) \otimes \Omega^2_{X/S} \to \ldots = \mathcal{DR}_S(\overset{*}{-}) \,.$$

Our aim is, therefore, to prove that the inclusion

$$(2.13.1) \qquad \mathcal{DR}_S(*) \hookrightarrow \mathcal{DR}_S(-)$$

induces isomorphisms of the relative hypercohomology groups

$$(2.13.2) \qquad \boldsymbol{H}^q(X, \mathcal{DR}_S(*)) \to \boldsymbol{H}^q(X, \mathcal{DR}_S(-)) \,, \qquad q \geqslant 0 \,.$$

There exists an admissible affinoid covering $\mathfrak{U}$ of X:

$$\mathfrak{U} = \{D_\varrho = \{P \in SpB, |x_j(P)| \leqslant \varrho, j = 1, \ldots, s\}\} \cup$$
$$\cup \{U_{j,\varrho} = \{P \in SpB, |x_j(P)| \geqslant \varrho\} \; j = 1, \ldots, s\}$$

which depends on ϱ and hence on the domain of convergence of f (proposition 2.9).

From the covering $\mathfrak{U}$ we get two convergent spectral sequences:

$$(2.13.3) \qquad E_2^{p,q} = H^p(\mathfrak{U}, \mathcal{H}^q(\mathcal{DR}_S(\overset{*}{-}))) \Rightarrow \boldsymbol{H}^{\bullet}(X, \mathcal{DR}_S(\overset{*}{-})) \,.$$

where $\mathcal{H}^q(\mathcal{DR}_S(\overset{*}{-}))$ stand for the presheaves $V \rightsquigarrow \boldsymbol{H}^q(V, \mathcal{DR}_S(\overset{*}{-})|_V)$ for every V affinoid subdomain in X (in fact by [1] proof of proposition 2.14 on every V affinoid subdomain of X we have

$$\boldsymbol{H}^q(V, \mathcal{DR}_S(\overset{*}{-})|_V) = H^q(\mathcal{DR}_S(\overset{*}{-})(V))) \,.$$

From the morphism (2.13.1) we get a homomorphism of the previous spectral sequances (2.13.3). So, in order to show that it induces an isomorphism at limit (2.13.2), we are reduced to prove we have isomorphisms for the terms E_2, which will follow if we know that the isomorphisms (2.13.2) hold when X is replaced by any element of $\mathfrak{U}$.

We observe that our inductive hypothesis on the relative dimension of B over A (i.e. on the number of variables in the polynomial which defines Y) implies that the isomorphisms (2.13.2) hold when X is replaced by $U_{j,\varrho}$ $j = 1, \dots, s$ and $\mathfrak{DR}_S(\stackrel{*}{-})$ are replaced by their restriction to $U_{j,\varrho}$. Finally, we are left to prove the isomorphisms (2.13.2) when X is replaced by D_ϱ and $\mathfrak{DR}_S(\stackrel{*}{-})$ by their restrictions to D_ϱ.

Since the hypercohomology of the restrictions of the complexes (2.13) to D_ϱ coincides with the cohomology of the complexes of global sections over D_ϱ ([1] proof of proposition 2.14), we are reduced to prove that

$$\mathfrak{DR}_S(*)(D_\varrho) \to \mathfrak{DR}_S(-)(D_\varrho)$$

is a quasi-isomorphism of complexes of modules.

On $D_\varrho = SpA\langle x_1/r, \dots, x_s/r\rangle = SpB_\varrho$, $r \in K$, $|e| = \varrho$, the formal series f (proposition 2.9) is converveng hence, putting $B_\varrho(*) = A\langle x_1/r, \dots, x_s/r\rangle[(x_1, \dots, x_s)^{-1}]$ and

$$B(-) = \left\{ \sum_{\alpha \in \mathbf{Z}^s} a_\alpha x^\alpha,\ a_\alpha \in A,\ \lim_{|\alpha| \to +\infty} |a_\alpha|_A\, \varepsilon^{t(\alpha)} \varrho^{t^+(\alpha)} = 0 \right\}$$

where $|\alpha| = \sum_{i=1}^{s} |\alpha_i|$, $2t(\alpha) = \sum_{i=1}^{s} (\alpha_i - |\alpha_i|)$, $t^+(\alpha) = |\alpha| - t(\alpha)$, on D_ϱ the connection will have the simplifyed form (proposition 2.9)

$$\nabla_{B_\varrho(\stackrel{*}{-})/A} = d_{B_\varrho(\stackrel{*}{-})/A} + \sum_{j=1}^{s} P_j(x_j)\, dx_j / x_j^{p_j+1} \tag{2.14}$$

where $P_j(x_j) \in K[x_j]$, $j = 1, \dots, s$ is the polynomial (2.7) and it has coefficients $\mathfrak{p}$-adically non-Liouville, $P_j(0) \neq 0$.

Moreover, since $D_\varrho = SpB_\varrho$ is an affinoid domain, the complexes, which we have to study, are:

$$0 \to B_\varrho(\stackrel{*}{-}) \xrightarrow{\nabla_{B_\varrho(\stackrel{*}{-})/A}} B_\varrho(\stackrel{*}{-}) \oplus_B \Omega^1_{B/A} \to B_\varrho(\stackrel{*}{-}) \otimes \Omega^2_{B/A} \to \dots = DR_A(B_\varrho(\stackrel{*}{-})) \tag{2.15}$$

REMARK 2.16. – We have obtained the simplified form (2.14) for the relative connection over $S = SpA$ on $D_\varrho = \{P \in SpB,\ |x_j(P)| \leqslant \varrho,\ j = 1, \dots, s\}$, for any $\varrho \geqslant 0$ such that the series f of proposition 2.9 converges on D_ϱ. We will need later to impose a further restraint on the size of ϱ of the type $\varrho < \varrho_0$, for a certain $\varrho_0 > 0$ depending only upon the coefficients $h_{j,i}$, $j = 1, \dots, s$, $i = 0, \dots, p_j$ of the connection (1.9) (see (2.5.1) and (2.7)).

In the situation of (2.8), (2.8.1), (2.8.2), from the canonical morphism (projection):

$$\varphi\colon \Omega^1_{B/A} \to \Omega^1_{B/A\langle x_1, \dots, x_h\rangle} \qquad 1 \leqslant h \leqslant s - 1$$

we get

$$B(\overset{*}{-}) \xrightarrow{\nabla_{B(\overset{*}{-})/A}} B(\overset{*}{-}) \otimes \Omega^1_{B/A} \xrightarrow{1\otimes\varphi} B(\overset{*}{-}) \otimes \Omega^1_{B/A\langle x_1,\dots,x_h\rangle}$$

$$B(\overset{*}{-}) \xrightarrow{\nabla_{B(\overset{*}{-})/A\langle x_1,\dots,x_h\rangle}} B(\overset{*}{-}) \otimes \Omega^1_{B/A\langle x_1,\dots,x_h\rangle}$$

(where $\nabla_{B(\overset{*}{-})/A\langle x_1,\dots,x_h\rangle} = d_{B(\overset{*}{-})/A\langle x_1,\dots,x_h\rangle} + \sum_{j=h+1}^{s} h_j\, dx_j/x_j^{p_j+1}$).

Hence we can define $DR_{A\langle x_1,\dots,x_h\rangle}(B(\overset{*}{-}))$ as the complexes:

$$0 \to B(\overset{*}{-}) \xrightarrow{\nabla_{B(\overset{*}{-})/A\langle x_1,\dots,x_h\rangle}} B(\overset{*}{-}) \otimes \Omega^1_{B/A\langle x_1,\dots,x_h\rangle} \to B(\overset{*}{-}) \otimes \Omega^2_{B/A\langle x_1,\dots,x_h\rangle} \to .$$

Using the particular case $h = s-1$ of the previous construction

$$\varphi\colon \Omega^1_{B/A} \to \Omega^1_{B/A\langle x_1,\dots,x_{s-1}\rangle}$$

we get a filtration of $DR_A(B(\overset{*}{-}))$ by the following subcomplexes:

$$F^i\big(DR_A(B(\overset{*}{-}))\big) = \operatorname{Im}\big(\Omega^i_{A\langle x_1,\dots,x_{s-1}\rangle/A} \otimes DR_A(B(\overset{*}{-}))^{[-i]} \to DR_A(B(\overset{*}{-}))\big)\,. \tag{2.17}$$

The sequence of B-modules:

$$0 \to B \otimes \Omega^1_{A\langle x_1,\dots,x_{s-1}\rangle/A} \to \Omega^1_{B/A} \to \Omega^1_{B/A\langle x_1,\dots,x_{s-1}\rangle} \to 0$$

is exact. We get, from (2.17), the graduations

$$gr^i\big(DR_A(B(\overset{*}{-}))\big) = \Omega^i_{A\langle x_1,\dots,x_{s-1}\rangle/A} \otimes DR_{A\langle x_1,\dots,x_{s-1}\rangle}(B(\overset{*}{-}))^{[-i]}\,.$$

From these, there exist spectral sequences, whose E_1-terms are:

$$E_1^{p,q}(\overset{*}{-}) = H^{p+q}\Big(gr^p\big(DR_A(B(\overset{*}{-}))\big)\Big) = \Omega^p_{A\langle x_1,\dots,x_{s-1}\rangle/A} \otimes H^q\big(DR_{A\langle x_1,\dots,x_{s-1}\rangle}(B(\overset{*}{-}))\big)\,.$$

abutting to $H^\bullet\big(DR_A(B(\overset{*}{-}))\big)$.

From the morphism (2.8) it is possible to construct a morphism between the above two spectral sequences. Hence, in order to show that this morphism induces an isomorphism at limit (i.e. that morphism (2.8) is a quasi-isomorphism, because $H^\bullet\big(DR_A(B(\overset{*}{-}))\big)$ are the limits of the spectral sequences), one can reduce oneself to verify that the morphism at the level E_1 is an isomorphism.

Thus it is enough to show that the natural morphism:

$$DR_{A\langle x_1,\dots,x_{s-1}\rangle}(B(*)) \to DR_{A\langle x_1,\dots,x_{s-1}\rangle}(B(-))$$

is a quasi-isomorphism.

By our assumptions $D_\varrho = SpB_\varrho = SpA\langle x_1/r, \ldots, x_s/r\rangle$ and $\Omega^1_{B_\varrho/A} = B_\varrho \otimes \Omega^1_{B/A}$: the structures on D_ϱ are analogous to those on $X = SpB$.

Hence, on the above construction, we can replace $DR_A(B(\overset{*}{-}))$ by $DR_A(B_\varrho(\overset{*}{-}))$ (2.15) and $\nabla_{B(\overset{*}{-})/A}$ by $\nabla_{B_\varrho(\overset{*}{-})/A}$ (2.14).

Thus, in order to verify a quasi-isomorphism between the complexes (2.15), endowed with the connections (2.14) (and so in (2.8) where ϱ does not appear), we need to show that the natural morphism

$$DR_{A\langle x_1/r, \ldots, x_{s-1}/r\rangle}(B_\varrho(*)) \hookrightarrow DR_{A\langle x_1/r, \ldots, x_{s-1}/r\rangle}(B_\varrho(-)) \tag{2.18}$$

is a quasi-isomorphism for ϱ sufficiently small.

We now remind the reader that if the singularities of (1.9) are logarithmic (that is, if $p = \sum p_j = 0$), theorem 1.10 is proved in [2]. We may therefore assume that the order of $x_1, \ldots, x_s$ is so chosen that $p_s \geqslant 1$.

The theorem 1.10 will then follow from the following statement which derives from (2.18):

THEOREM 2.19. – *Let*

$$L = \frac{\partial}{\partial x_s} + P_s(x_s)/x_s^{p_s+1}\,, \quad P_s(x_s) = h_{s,0} + \ldots + h_{s,p_s} x_s^{p_s}\,, \quad p_s \geqslant 1\,, \quad h_{s,i} \in K\,, \quad h_{s,0} \neq 0\,,$$

$h_{s,i}$ *numbers $\mathfrak{p}$-adically non-Liouville* (2.7). *The cohomology groups of the following complexes, endowed with L as differential*

$$0 \to B_\varrho(\overset{*}{-}) \xrightarrow{L} B_\varrho(\overset{*}{-}) \to 0 \tag{2.20}$$

all vanish for $\varrho > 0$ sufficiently small.

REMARK 2.21. – As in (2.14), $B_\varrho(\overset{*}{-})$ consist of power series in $x_1, \ldots, x_s$ with coefficients in A.

3. – End of Proof.

In this section we shall prove theorem 2.19.

We know that $B_\varrho(-)$ is the ring of analytic functions on $D_\varrho^* = \{P \in SpB, \varrho \geqslant |x_j(P)| > 0,\ j = 1, \ldots, s\}$ endowed with the topology of uniform convergence on all affinoid subdomains of D_ϱ^*. It coincides with the ring of the Laurent series $\varphi = \sum_{\beta \in \mathbf{Z}^s} b_\beta x^\beta$, $b_\beta \in A$, $x^\beta = x_1^{\beta_1} \ldots x_s^{\beta_s}$ such that

$$\lim_{|\beta| \to +\infty} |b_\beta|_A \varrho^{t^+(\beta)} \varepsilon^{t(\beta)} = 0 \qquad \forall \varepsilon > 0\,, \quad \varepsilon < \varrho \tag{3.0}$$

$\Big(2t(\beta) = \sum_{i=1}^{s} \beta_i - |\beta_i|,\ |\beta| = \sum_{i=1}^{s} |\beta_i|,\ t^+(\beta) = |\beta| - t(\beta)\Big)$, endowed with the topology given by the system $(\|\cdot\|_\varepsilon)_{\varepsilon\neq 0,\varepsilon\in K}$ of norms $\|\cdot\|_\varepsilon$ defined by

$$\|\varphi\|_\varepsilon = \sup_\beta |b_\beta|_A \varrho^{t^+(\beta)}\varepsilon^{t(\beta)} .$$

Let $\varphi \in B_\varrho(-)$, $\varphi = \sum_{\beta\in Z^s} b_\beta x^\beta$. It can be uniquely written, by rearranging the indexes, as

$$\varphi = \sum_{\alpha\in Z^{s-1}} a_\alpha(x_s)x^\alpha \qquad x^\alpha = x_1^{\alpha_1}\dots x_{s-1}^{\alpha_{s-1}}, \qquad \alpha_i\in Z \tag{3.0.1}$$

where $a_\alpha(x_s)$ are Laurent series, a priori formal, in x_s with coefficients in A.

By the above observations it follows that $a_\alpha(x_s)\in B_\varrho^s(-)$ where $B_\varrho^s(-)\subset B_\varrho(-)$ denotes the ring of analytic functions on $\{P\in SpA\langle x_s\rangle, \varrho \geqslant |x_s(P)| > 0\}$ i.e. the ring of alla Laurent series in x_s with coefficients in A, $\sum_{i\in Z} b_i x_s^i$ $b_i\in A$, such that

$$\lim_{i\to+\infty} |b_i|_A \varrho^i = 0 \qquad \lim_{i\to-\infty} |b_i|_A \varepsilon^i = 0 \qquad \forall\varepsilon > 0\,, \quad \varepsilon < \varrho\,.$$

The formal equality (3.0.1) is in fact an equality in $B_\varrho(-)$.

Suppose $L\varphi = 0$. Since $L\in K[x_s^{1}, x_s^{-1}, \partial/\partial x_s]$ is a continuos operator on $B_\varrho(-)$, this implies $L(a_\alpha(x_s)) = 0$ $\forall\alpha$; $a_\alpha(x_s) = \sum_{i\in Z} a_i^\alpha x_s^i$, $a_i^\alpha\in A$. Each $a_\alpha(x_s)$ can be evaluated at any $\mathcal{M}\in \operatorname{Max} A$ (maximal spectrum of A). Since A is an affinoid algebra over the algebraically closed field K, $A/\mathcal{M}\simeq K$ ([5] 6.1.2 cor. 3). Let $\bar a_\alpha(x_s) = \sum_{i\in Z}\bar a_i^\alpha x i$ be the reduction modulo $\mathcal{M}$ of $a_\alpha(x_s)$, so that $\bar a_i^\alpha$ is the projection of a_i^α on $A/\mathcal{M}\simeq K$.

The series $\bar a_\alpha(x_s)$ is an element of the ring of analytic functions on the set $\{x_s\in K,\ 0<|x_s|\leqslant\varrho\}$ (i.e. it is a Laurent series in x_s with coefficients in K, $\sum_{i\in Z}\bar a_i^\alpha x_s^i$ such that

$$\lim_{i\to+\infty} |\bar a_i^\alpha|\varrho^i = 0 \qquad \lim_{i\to-\infty} |\bar a_i^\alpha|\varepsilon^i = 0 \qquad \forall\varepsilon>0\,,\quad \varepsilon\leqslant\varrho\,.$$

We have $L(\bar a_\alpha(x_s)) = 0$: from the proof of lemma 3.14 in [1] it follows that $\bar a_\alpha(x_s) = 0$. We conclude that the coefficients of $a_\alpha(x_s)$ belong to $\mathcal{M}$ for every $\mathcal{M}\in\operatorname{Max} A$. Since A is an integral domain ([5] 6.2.1 prop. 4) we obtain $a_\alpha(x_s) = 0$ for every α and finally $\varphi = 0$.

We have shown:

$$\operatorname{Ker} L_{B_\varrho(*)} = \operatorname{Ker} L_{B_\varrho(-)} = 0\,. \tag{3.1}$$

We now prove the following identities (for sufficiently small $\varrho > 0$):

$$LB_\varrho(-)\cap B_\varrho(*) = LB_\varrho(*) \tag{3.2}$$

$$LB_\varrho(-) = B_\varrho(-)\,. \tag{3.3}$$

From which theorem 2.19 (and therefore theorem 1.10) follows directly. We prove (3.2) first.

Let $b \in B_\varrho(*)$ and $a \in B_\varrho(-)$ be such that

$$L(a) = b\,.$$

As above we can write

$$b = \sum_{\alpha \in \mathbf{Z}^{s-1}} b_\alpha(x_s) x^\alpha \in B_\varrho(*)\,, \qquad b_\alpha(x_s) \in B^s_\varrho(*) = A\langle x_s/r\rangle[x_s^{-1}]\,,$$

$$b_\alpha(x_s) = \sum_{i \in \mathbf{Z}} b_i^\alpha x_s^i\,, \qquad b_i^\alpha \in A;$$

$$a = \sum_{\alpha \in \mathbf{Z}^{s-1}} a_\alpha(x_s) x^\alpha \in B_\varrho(-) \qquad a_\alpha(x_s) \in B^s_\varrho(-)\,, \qquad a_\alpha(x_s) = \sum_{i \in \mathbf{Z}} a_i^\alpha x_s^i\,, \qquad a_i^\alpha \in A\,.$$

Since b is meromorphic along $x_1, \ldots, x_{s-1}$, $b_\alpha(x_s) = 0$ if $\alpha_i \ll 0$ for some $i = 1, \ldots, s-1$. We then deduce from $L(a_\alpha(x_s)) = b_\alpha(x_s) = 0$ and (3.1) that $a_\alpha(x_s) = 0$. Thus a is meromorphic along $x_1, \ldots, x_{s-1}$. Now we have to show that a is meromorphic along x_s, too. From $b_\alpha(x_s) = L(a_\alpha(x_s))$ ($b_\alpha(x_s)$ is meromorphic in x_s) it follows that for $j \leqslant -N_\alpha$, $N_\alpha =$ order of pole of $b_\alpha(x_s)$ at $x_s = 0$:

$$(3.4) \qquad (j + h_{s,p_s}) a_j^\alpha = -\sum_{i=0}^{p_s-1} h_{s,i} a^\alpha_{j-i+p_s} \qquad j \leqslant -N_\alpha\,.$$

Since $h_{s,0} \in K^*$, we can replace a_k^α for $k \geqslant -N_\alpha + p_s + 1$ by another element of A in such a way that (3.4) becomes valid for every j. Hence we build a Laurent series in x_s with coefficients in A, $a_\alpha^+(x_s)$, such that

$$L(a_\alpha^+(x_s)) = 0 \quad \text{ and } \quad a_\alpha^+(x_s) - a_\alpha(x_s) \in x_s^{-N_\alpha + p_s + 1} A[\![x_s]\!]\,.$$

It is also clear from (3.4) and [8] that $a_\alpha^+(x_s)$ is analytic on $\{P \in SpA\langle x_s\rangle\ 0 < |x_s(P)| < \varepsilon\}$ for some $\varepsilon > 0$. We can reduce $a_\alpha^+(x_s)$ modulo $\mathcal{M} \in \operatorname{Max} A$: we obtain a Laurent series in x_s with coefficients in K, $\bar{a}_\alpha^+(x_s)$ converging for $0 < |x_s| < \varepsilon$ and satisfying $L(\bar{a}_\alpha^+(x_s)) = 0$.

We deduce as before that $\bar{a}_\alpha^+(x_s) = 0$ and therefore $a_\alpha^+(x_s) = 0$. But since $a_\alpha^+(x_s) - a_\alpha(x_s) \in x_s^{-N_\alpha+p_s+1} A[\![x_s]\!]$, we conclude that $a_\alpha(x_s)$ is meromorphic along $x_s = 0$, the order of pole being at most $N_\alpha - p_s - 1$. Now, by hypothesis, b is meromorphic along $x_s = 0$, so that $\operatorname{Sup} N_\alpha = N < +\infty$. It follows that a is itself meromorphic at $x_s = 0$ (the order of its pole at $x_s = 0$ will be at most $N - p_s - 1$) and we have shown (3.2).

It remains only to show (3.3). To begin with, let us consider the case $A = T_n = K\langle y_1, \ldots, y_n\rangle$, $B = T_n\langle x_1, \ldots, x_s\rangle$.

Now we consider the following affinoid subdomain of SpB:

$$W_\varepsilon = \{P \in SpB: \varepsilon \leqslant |x_j(P)| \leqslant \varrho \;\; j = 1, \dots, s\} \qquad (\varepsilon < \varrho,\ \varepsilon, \varrho \in |K|)\,.$$

We denote T_ε the corresponding affinoid algebra

$$T_\varepsilon = T_n\langle x_1/r, \dots, x_s/r, e/x_1, \dots, e/x_s\rangle$$

$e, r \in K$, $|e| = \varepsilon < \varrho = |r|$, which is the ring of analytic functions on W_ε: T_ε is the ring of the Laurent series

$$(3.5) \qquad \varphi = \sum_{\substack{\alpha \in \boldsymbol{N}^n \\ \gamma \in \boldsymbol{Z}^s}} a_{\alpha\gamma} y^\alpha x^\gamma \qquad \begin{matrix} y^\alpha = y_1^{\alpha_1} \dots y_n^{\alpha_n} & \alpha_i \in \boldsymbol{N} \\ x^\gamma = x_1^{\gamma_1} \dots x_s^{\gamma_s} & \gamma_i \in \boldsymbol{Z} \end{matrix}$$

with $a_{\alpha\gamma} \in K$ satisfying

$$(3.5.1) \qquad \lim_{|\alpha| + |\gamma| \to +\infty} |a_{\alpha\gamma}| \varrho^{i^+(\gamma)} \varepsilon^{i(\gamma)} = 0\,.$$

By rearranging the coefficients of such a Laurent series, φ, we can write

$$(3.5.2) \qquad \varphi = \sum_{\substack{\alpha \in \boldsymbol{N}^n \\ \beta \in \boldsymbol{Z}^{s-1}}} a_{\alpha\beta}(x_s) y^\alpha x^\beta \qquad \begin{matrix} y^\alpha = y_1^{\alpha_1} \dots y_n^{\alpha_n} & \alpha_i \in \boldsymbol{N} \\ x^\beta = x_1^{\beta_1} \dots x_{s-1}^{\beta_{s-1}} & \beta_i \in \boldsymbol{Z} \end{matrix}$$

with $a_{\alpha\beta}(x_s)$ Laurent series in x_s with coefficients in K.

By the above condition (3.5.1) each $a_{\alpha\beta}(x_s)$ is an analytic function on

$$(3.6) \qquad \{P \in SpK\langle x_s\rangle,\ \varepsilon \leqslant |x_s(P)| \leqslant \varrho\} = \{x_s \in K \;\; \varepsilon \leqslant |x_s| \leqslant \varrho\}$$

i.e. it is an element of the ring of all Laurent series in x_s with coefficients in K, $\sum_{i \in \boldsymbol{Z}} b_i x_s^i$, such that

$$\lim_{i \to +\infty} |b_i| \varrho^i = 0\,, \qquad \lim_{i \to -\infty} |b_i| \varepsilon^i = 0$$

(hence, in particular, an element of T_ε). From the proof of lemma 3.14 of [1], it follows that L is bijective on the ring of analytic functions on (3.6) for sufficiently small ϱ (the choice depends on the $h_{s,i} \in K$ of L in theorem 2.19). This gives the further condition on ϱ which we have mentioned on remark 2.16. We fix such a ϱ, $0 < \varrho \in |K|$. Since $a_{\alpha\beta}(x_s)$ are analytic functions on (3.6), we can find analytic functions on the same region $b_{\alpha\beta}(x_s)$, such that

$$(3.7) \qquad L(b_{\alpha\beta}(x_s)) = a_{\alpha\beta}(x_s)\,.$$

PROPOSITION 3.8. – *Under the previous hypotheses, with $b_{\alpha\beta}(x_s)$ as in* (3.7), *the series of functions*

$$\sum_{\substack{\alpha\in\mathbf{N}^n\\ \beta\in\mathbf{Z}^{s-1}}} b_{\alpha\beta}(x_s)y^\alpha x^\beta \tag{3.9}$$

converges in T_ε to an element ψ, such that $L(\psi)=\varphi$ (φ as in (3.5)). *In particular we have shown that $L(T_\varepsilon)=T_\varepsilon$.*

PROOF. – T_ε is a K-affinoid algebra (hence complete) under the norm defined by (φ as in (3.5)):

$$\|\varphi\|_{T_\varepsilon}=\max_{\alpha,\gamma}\left(|a_{\alpha\gamma}|\varrho^{t^+(\gamma)}\varepsilon^{t(\gamma)|}\right).$$

So, to verify that the series (3.9) converges in T_ε, we have to show that:

$$\lim_{|\alpha|+|\beta|\to+\infty}\|b_{\alpha\beta}(x_s)y^\alpha x^\beta\|_{T_\varepsilon}=0\,.$$

By hypothesis, φ is an element of T_ε, it implies that (3.5.2):

$$\lim_{|\alpha|+|\beta|\to+\infty}\|a_{\alpha\beta}(x_s)y^\alpha x^\beta\|_{T_\varepsilon}=0\,. \tag{3.10}$$

Let us consider (3.6): it is an affinoid domain associated to the affinoid K-algebra

$$A_\varepsilon=K\langle x_s/r,\, e/x_s\rangle\qquad e,r\in K\,,\qquad |e|=\varepsilon<\varrho=|r|\,.$$

Such a K-affinoid algebra consists of all Laurent series $\sum_{i\in\mathbf{Z}}a_ix_s^i$, $a_i\in K$ such that:

$$\lim_{i\to-\infty}|a_i|\varepsilon^i=0\,,\qquad \lim_{i\to+\infty}|a_i|\varrho^i=0\,.$$

A_ε is contained in T_ε and it is an affinoid (hence complete) K-algebra under the norm defined from the restriction at A_ε of that one of T_ε i.e.

$$\|\sum a_ix_s^i\|_{T_\varepsilon}=\max\left(\max_{i\geqslant0}|a_i|\varrho^i;\ \max_{i<0}|a_i|\varepsilon^i\right)\ \ ([5]\ 9.7.1)\,.$$

Thus we can notice that:

$$\|b_{\alpha\beta}(x_s)y^\alpha x^\beta\|_{T_\varepsilon}=\|b_{\alpha\beta}(x_s)\|_{T_\varepsilon}\varrho^{t^+(\beta)}\varepsilon^{t(\beta)};\qquad \|a_{\alpha\beta}(x_s)y^\alpha x^\beta\|_{T_\varepsilon}=\|a_{\alpha\beta}(x_s)\|_{T_\varepsilon}\varrho^{t^+(\beta)}\varepsilon^{t(\beta)}\,.$$

The operator $L\colon A_\varepsilon\to A_\varepsilon$ is a continuos (hence bounded [5] 2.1.8 prop. 2 and cor. 3) K-linear operator between two Banach spaces. By our choice of ϱ L is bijective ([1] lemma 3.14). So, there exists L^{-1} by Banach's Theorem ([5] 2.8), and it is a bounded operator. Let c denote the operator norm of $L^{-1}\colon A_\varepsilon \to A_\varepsilon$. Being $L^{-1}(a_{\alpha\beta}(x_s)) = b_{\alpha\beta}(x_s)$ for every α, β we have

$$\|b_{\alpha\beta}(x_s)\|_{T_\varepsilon} \leqslant c\|a_{\alpha\beta}(x_s)\|_{T_\varepsilon} .$$

Thus:

$$\lim_{|\alpha|+|\beta|\to+\infty} \|b_{\alpha\beta}(x_s)\|_{T_\varepsilon}\varrho^{t^+(\beta)}\varepsilon^{t(\beta)} \leqslant \lim_{|\alpha|+|\beta|\to+\infty} c\|a_{\alpha\beta}(x_s)\|_{T_\varepsilon}\varrho^{t^+(\beta)}\varepsilon^{t(\beta)} = 0$$

by (3.10). We have shown that the series (3.9) satisfies the convergence condition and it rapresents an element ψ of T_ε such that $L(\psi) = \varphi$. Q.E.D.

Let us now return to a general A. An analytic function belong to $B_\varrho(-)$ if and only if it is analytic on every affinoid:

$$Sp A\langle x_1/r, \ldots, x_s/r, e/x_1, \ldots, e/x_s\rangle \quad \forall e \in K\setminus\{0\}\,, \qquad |r| = \varrho > \varepsilon = |e|$$

(in fact $B_\varrho(-)$ is the ring of analytic functions on D^*_ϱ).

In particular over every such an affinoid subdomain φ has a rapresentative in $T_n\langle x_1/r, \ldots, x_s|r, e/x_1, \ldots, e/x_s\rangle = T_\varepsilon$, because there exists a strict isomorphism:

$$(3.11) \qquad A\langle x_1/r, \ldots, x_s/r, e/x_1, \ldots, e/x_s\rangle \simeq T_n\langle x_1/r, \ldots, x_s/r, .., e/x_1, \ldots., e/x_s\rangle/(\mathcal{A})$$

([5] 6.1.1 prop. 11), if $A \simeq T_n/\mathcal{A}$ and $(\mathcal{A})$ is the ideal generated by $\mathcal{A}$.

Let φ_ε be such a rapresentative. By the proposition 3.8 we can find $\xi_\varepsilon \in T_\varepsilon$, such that $L(\xi_\varepsilon) = \varphi_\varepsilon$. But on every such an affinoid $L(\mathcal{A}) \subseteq (\mathcal{A})$, hence we write $L(\bar{\xi}_\varepsilon) = \bar{\varphi}_\varepsilon$ (on the right hand side of (3.11)). The morphism L commuts with the isomorphism (3.11).

PROPOSITION 3.12. – *In the previous notations and by our hypothesis about the choice of* ϱ*, the operator* L *is inijective on every affinoid algebra* $A\langle x_1/r, \ldots, x_s/r, e/x_1, \ldots, e/x_s\rangle$, $|r| = \varrho > \varepsilon = |e|$ $(e \neq 0)$.

PROOF. – The proof is analogous to that one for (3.1). In fact, by our choice of ϱ, L is injective on $\{x_s \in K\ \varepsilon \leqslant |x_s| \leqslant \varrho\}$ for every $\varepsilon \in K\ \varepsilon < \varrho$, $\varepsilon \neq 0$. Q.E.D

The functions $(\bar{\xi}_\varepsilon)_{\varepsilon\in|K|,\varepsilon\neq 0}$ $(\varepsilon < \varrho)$ may be pasted together, now, as elements of $A\langle x_1/r, \ldots, x_s/r, e/x_1, \ldots, e/x_s\rangle$, by the fact that L is injective on every such an affinoids.

Thus we can find an element $\xi \in B_\varrho(-)$ such that

$$L(\xi) = \varphi .$$

This concludes the proof of (3.3) and therefore that one of the theorem 2.19 and, hence, of the theorem 1.10.

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