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Autocorrelation Scaling and Fourier Transform of Non-Autonomous Systems

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Abstract. Upper bounds for the (strong) Fourier transform, of a rather general sequence of unitary operators, are related to the uniform α-Hölder continuity of its autocorrelation measure. It is a natural generalization of the "Dynamical Bombieri-Taylor Conjecture." Immediate applications include driven quantum systems, classical and quantum harmonic oscillators, and non-autonomous twisted generalized random walks in Hilbert spaces.

1 Introduction and Main Result

We are interested in asymptotic properties of the time evolution of a class of quantum systems, particularly in the non-autonomous case, in which the Hamiltonian H depends on time; we shall present upper bounds for the growth of the strong Fourier transform of sequences of unitary operators in terms of the α-Hölder continuity of the corresponding autocorrelation measures. We use the autonomous case to motivate our main result, mentioning that some works on the asymptotic properties of systems with non-trivial time dependence include [1–18].

Let $U(t,0), t \in \mathbb{R}$, be a strongly continuous one-parameter group of unitary operators on the (separable) Hilbert space $\mathbf{H}$. Denote its infinitesimal generator by H, i.e., $H : \text{dom } H \to \mathbf{H}$ is a self-adjoint operator such that $U(t,0) = e^{-iHt}, \forall t$. With respect to the time evolution in quantum mechanics H represents the Hamiltonian operator and $U(t,0)$ is called propagator. The Mean Ergodic Theorem [19] states that for each $\xi \in \mathbf{H}, \omega \in \mathbb{R}$,

$$\lim_{T\to\infty} \frac{1}{T^2} \left\| \int_0^T e^{i\omega t} U(t,0)\xi dt \right\|^2 = \|E_H(\omega)\xi\|^2 = \mu_\xi(\{\omega\}), \tag{1}$$

where $E_H(\omega)$ is the orthogonal projector onto $\text{Ker}(H - \omega I)$ and μ_ξ the spectral measure associated to the vector ξ. Thus, the left hand side of (1) can be used to recover the point spectrum of the Hamiltonian operator H. The mathematical formulation and adaptation of this result for a class of non-autonomous systems was discussed in [15]; now we briefly recall its principal result.

Let $\{U_n\}_{n=1}^\infty$ be a sequence of unitary operators on $\mathbf{H}$ and

$$\Lambda(n) = U_n \cdots U_2 U_1.$$

We set $\Lambda(0) = I$. This quantity can be seen as the time evolution operator of non-autonomous quantum systems for which the time-dependent law changes at every integer time. E.g., the time evolution associated to a family of kicked systems given by the Hamiltonian

$$H(t) = H_0 + \sum_{j=1}^{\infty} V_j \delta(t-j),$$

with $\{V_j\}$ being a sequence of potentials; in this case $U_n = e^{-iV_n} e^{-iH_0}$. Another possibility is a time-dependent quantum system built upon a sequence of autonomous potentials V_j, with V_j acting on the time interval $(j-1, j]$. A quantity that resembles the left hand side of the Mean Ergodic Theorem (1) for Λ was first suggested about ten years ago in [2], motivated by a similar relation in the context of diffraction by aperiodic structures studied by Bombieri and Taylor [20, 21]; by borrowing a conjecture from Bombieri-Taylor, in [2] it was proposed that *something like a point spectrum* for Λ would be present if the limit

$$\lim_{N\to\infty} \frac{1}{N^2} \varphi_N(\omega, \xi) \neq 0 \tag{2}$$

for some $\omega \in [0, 2\pi]\,, \xi \in \mathbf{H}$, where

$$\varphi_N(\omega, \xi) = \left\| \sum_{j=0}^{N-1} e^{i\omega j} \Lambda(j)\xi \right\|^2 . \tag{3}$$

This idea was also employed in [3, 9]. The precise statement appeared in [15], also clarifying the meaning of the *point spectrum for* Λ, i.e., it was proven that

$$\limsup_{N\to\infty} \frac{1}{N^2} \varphi_N(\omega, \xi) \leq 2\sigma_\xi(\{\omega\}), \tag{4}$$

where σ_ξ denotes the autocorrelation measure of $\{\Lambda(n)\xi\}$. Notice that the limit (2) is a kind of Fourier Transform of $\{\Lambda(n)\xi\}$. Recall that the autocorrelation measure σ_ξ is a (finite) Borel positive measure on $[0, 2\pi]$ (the unit circle), which is defined by Bochner-Herglotz theorem—the second equality below—via the autocorrelation functions

$$C_\xi(k) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=0}^{N-1} \langle \Lambda(j+k)\xi | \Lambda(j)\xi \rangle = \int e^{iks} d\sigma_\xi(s).$$

Recall also that the notion of autocorrelation measure is the natural generalization, for time-dependent systems, of spectral measures of propagators for autonomous quantum models. In this work the autocorrelation functions $C_\xi(k)$ are supposed to be well defined.

Relation (4) has been called the Dynamical Bombieri-Taylor conjecture, and it is a rigorous version of the Mean Ergodic Theorem (1) that holds for some systems with general time dependence. Notice, however, that there are examples [15] for which the left hand side of (4) vanishes while σ_ξ is pure point; this happens because φ_N is too sensible to phases variations, at least when compared to autocorrelations functions. In summary, φ_N can be used, even numerically, to derive properties about the point components of the autocorrelation measures.

Here it will be shown that φ_N can also be used to extract information on the continuous part of the autocorrelation measures; the point is to tune the growth rate of φ_N and consider $\varphi_N/N^{2-\alpha}$, with $\alpha \geq 0$. Our main result asks also for an average on ω, so we introduce the following notation for positive integrable functions $f : J \to \mathbb{R}$ on a closed interval $J \subset \mathbb{R}$:

$$\langle f(\omega)\rangle_J = \frac{1}{|J|}\int_J f(\omega)d\omega$$

($|\cdot|$ denotes Lebesgue measure). If $|J| = 0$ then $\langle f(\omega)\rangle_J \equiv 0$.

Definition 1. *[22, 23] A σ-finite positive Borel measure μ, on subsets of $\mathbb{R}$, is uniformly α-Hölder continuous (UαH) on the interval $J \subset \mathbb{R}$ if there is a positive constant C such that $\mu(J') \leq C|J'|^\alpha$, for any subinterval $J' \subset J$ with $|J'| < 1$.*

Remark 1.1. *UαH measures have been thought of as a kind of fractal measures among physicists. The most relevant property for quantum mechanics [22, 23] is that for each UαH measure μ in $\mathbb{R}$ there exists a constant $D < \infty$ such that $\frac{1}{T}\int_0^T |\hat{\mu}(t)|^2 dt \leq DT^{-\alpha}$, for all $T > 1$ ($\hat{\mu}$ denotes the Fourier Transform of μ).*

Theorem 1. *Suppose the autocorrelation functions exist for the sequence $\{\Lambda(n)\xi\}$ in the Hilbert space $\mathbf{H}$, with UαH autocorrelation measure σ_ξ, $0 \leq \alpha \leq 1$, in the closed interval $J \subset [0, 2\pi]$. Then there exists a constant $0 \leq K < \infty$ such that*

$$\limsup_{N\to\infty} \frac{1}{N^{2-\alpha}} \langle \varphi_N(\omega,\xi)\rangle_J \leq K. \tag{5}$$

Remark 1.2. *A clear corollary of this theorem is that if there is a subsequence of integers $\{N_r\}$ such that*

$$\lim_{r\to\infty} \frac{1}{N_r^{2-\alpha}}\langle\varphi_{N_r}(\omega,\xi)\rangle_J = \infty,$$

then σ_ξ is not UβH, for $\alpha \leq \beta$, in the closed interval J.

Remark 1.3. *By combining this theorem with Fatou's lemma, it follows that*

$$\liminf_{N\to\infty} \frac{\varphi_N(\omega,\xi)}{N^{2-\alpha}} < \infty, \tag{6}$$

for ω in a set of full Lebesgue measure in J.

Remark 1.4. *Although inserted in the context of quantum mechanics, the above theorem holds for rather general sequences of unitary operators on Hilbert spaces; it has potential applications in the studies of general (classical and quantum) driven dynamical systems. Relations (5) and (6) can, in principle, be a theoretical and numerical source of information on the Hölder properties of autocorrelation measures, which are in general very hard to be explicitly computed.*

Remark 1.5. *For the case $\alpha = 0$ there are more specific results, as discussed above, with a proper estimate for K—see (4). A measure may have a Hölder exponent α that depends on the point in its support; in this case it is the smallest value of α in J that is relevant for thm. 1.*

Remark 1.6. *If there is a unitary operator W such that $U_n = W, \forall n$—the autonomous case—then the autocorrelation measures are replaced by spectral measures of W, and Hof [24] has got an equivalence in the analogous of thm. 1 (see [24] for details). I was not able to prove that relation (5) implies that σ_ξ is UαH; based on the examples presented in [15] with strictly inequality in (4), we suspect that thm. 1 does not have a simple converse (see also the next two remarks).*

Remark 1.7. *The wide generality of thm. 1 with respect to the time dependence of Λ justifies, at least on intuitive grounds, the average over J in (5); it is a way to smear eventual wild oscillations of φ_N as function of ω. I do not know any non-trivial sufficient condition assuring that such average can be dropped out.*

Remark 1.8. *The Mean Ergodic Theorem (1) is a kind of rigorous formulation of the physical concept of "energy representation" in quantum mechanics (for pure point Hamiltonians). In case of non-autonomous systems such representation is actually expected to fail in general; this is a physical reason for the inequality and average over frequencies in (5).*

Remark 1.9. *The value $\alpha = 1$ is related to measures absolutely continuous with respect to Lebesgue measure (with continuous density, at least), while $\alpha = 0$ to point measures; but it is known that a UαH measure with $0 < \alpha < 1$ does not necessarily have a singular continuous component [23, 24].*

The remaining of this paper is organized as follows. In Section 2 the proof of the above theorem is presented, and the critical growth exponent for a sequence of real numbers is defined. In Section 3 thm. 1 is used to give upper bounds on the energy growth of a class of classical and quantum harmonic oscillators with general time dependence; that section finishes with a remark on non-autonomous twisted generalized random walks.

2 Proof of Theorem 1

In this Section we present the proof of thm. 1 and define the critical growth exponent for a sequence of positive real numbers.

The case $\alpha = 0$ was discussed in [15] and, since those results imply thm. 1 for this case, we suppose that $\alpha > 0$ and that σ_ξ is a continuous measure over J.

Each function $\varphi_N(\cdot, \xi)$ is continuous and bounded by $N^2\|\xi\|^2$, so it defines a measure $d\mu_N(\omega) = \varphi_N(\omega, \xi)/N d\omega$ absolutely continuous with respect to Lebesgue measure. Thm. 1 will follow, after additional manipulations, from the claim that, for each $\xi \in \mathbf{H}$, the sequence $\{\mu_N\}$ converges, in the weak* topology, to the autocorrelation measure σ_ξ. In fact, by expanding φ_N we get

$$\begin{aligned} d\mu_N(\omega) = & \\ = & \frac{1}{N}\left[\sum_{n,j=0}^{N-1} \exp(i(n-j)\omega)\langle\Lambda(n)\xi|\Lambda(j)\xi\rangle\right] d\omega \\ = & \frac{1}{N}\left[\sum_{j=0}^{N-1}\sum_{k=-j}^{N-1-j} e^{ik\omega}\langle\Lambda(j+k)\xi|\Lambda(j)\xi\rangle\right] d\omega. \end{aligned}$$

Its Fourier transform at $r \in \mathbb{Z}$ is given by (for $N > |r|$)

$$\begin{aligned} \hat{\mu}_N(r) = & \\ = & \frac{1}{N}\sum_{j=0}^{N-1}\sum_{k=-j}^{N-1-j} \delta_{k,r}\langle\Lambda(j+k)\xi|\Lambda(j)\xi\rangle \\ = & \frac{1}{N}\sum_{j=0}^{N-1}\langle\Lambda(j+r)\xi|\Lambda(j)\xi\rangle. \end{aligned}$$

Therefore, for $N \to \infty$ one gets that $\hat{\mu}_N(r) \to C_\xi(r) = \hat{\sigma}_\xi(r)$, for any $r \in \mathbb{Z}$, and so μ_N converges in the weak* topology to σ_ξ.

Since σ_ξ is UαH on the bounded interval J, there exists $0 < C < \infty$ such that $\sigma_\xi(J') \leq C|J'|^\alpha$ for any interval $J' \subset J$, and being σ_ξ a regular and continuous measure we have $\lim_{N\to\infty}\mu_N(J') = \sigma_\xi(J')$ (the border of any interval J' has zero σ_ξ measure) [25].

If $|J| = 0$ there is nothing to prove since thm. 1 becomes trivial, so we assume that $|J| > 0$. Pick $0 < \varepsilon < C|J|^\alpha$; by the above claim there exists $M > 0$ such that if $N \geq M$ then

$$\int_J \varphi_N(\omega, \xi)/N d\omega - \varepsilon \leq \sigma_\xi(J) \leq C|J|^\alpha,$$

and

$$\frac{1}{N}\langle\varphi_N(\omega,\xi)\rangle_J = \frac{1}{|J|}\int_J \varphi_N(\omega,\xi)/N d\omega \leq 2C|J|^{\alpha-1}.$$

Taking also M large enough so that $|J| \geq 1/M$, it follows that, for $N \geq M$, $|J|^{\alpha-1} \leq N^{1-\alpha}$. Thus

$$\frac{1}{N}\langle\varphi_N(\omega,\xi)\rangle_J \leq 2CN^{1-\alpha}.$$

From this relation it follows that

$$\limsup_{N\to\infty} \frac{1}{N^{2-\alpha}} \langle \varphi_N(\omega,\xi)\rangle_J \le K,$$

with $K = 2C$ and thm. 1 is proven. □

Definition 2. *Given a sequence $u = \{u_n\}_{n=1}^{\infty}$ of positive real numbers, its critical growth exponent $\beta(u)$ is the unique real number such that*

$$\limsup_{n\to\infty} \frac{u_n}{n^\gamma} = \begin{cases} \infty & \text{if } \gamma < \beta(u) \\ 0 & \text{if } \gamma > \beta(u). \end{cases}$$

Thm. 1 can be expressed in terms of the critical exponent β: If the autocorrelation measure σ_ξ is UαH, $0 \le \alpha \le 1$, in the closed interval J, then $\beta(\langle \varphi_N(\omega,\xi)\rangle_J) \le (2-\alpha)$.

3 Driven Harmonic Oscillators

Consider the Hamiltonian of a unidimensional harmonic oscillator, with natural frequency ω_o, under a time-dependent force

$$H(t) = H(\omega_o) + qF(t) \tag{7}$$

with $H(\omega_o) = (p^2 + \omega_o^2 q^2)/2$, and $F(t)$ being a piecewise continuous function. For example, given a sequence of continuous real functions F_n defined on $(0,1]$ set

$$F(t) = F_n(t-n), \quad t \in (n, n+1]. \tag{8}$$

Another interesting possibility is given by kicked oscillators with

$$F(t) = \varepsilon \sum_{j=1}^{\infty} \nu(n)\delta(t-n), \tag{9}$$

where $\{\nu(n)\}$ is a sequence (periodic, almost periodic or random) taking the values ± 1, and ε is the kick intensity. Both, the classical and quantum dynamics of (7) with forces (8) and (9) are well defined [6, 8]. We can apply thm. 1 to get

Corollary 1. *Let $U(t,s)$ be the quantum propagator of (7) with forces (8) or (9) [8]. Set*

$$U_n = s - \lim_{t\uparrow n, s\downarrow(n-1)} U(t,s), \quad \Lambda(n) = U_n \cdots U_0,$$

and suppose the autocorrelation functions for $\{\Lambda(n)\xi\}$ exist. If the corresponding autocorrelation measure σ_ξ is UαH, $0 \le \alpha \le 1$, in the closed interval $J \subset [0, 2\pi]$, then (5) holds in this case.

An interesting point about the harmonic oscillator (7,9) is that a variation of φ_N is directly related to the unperturbed (classical and quantum) energy growth. Let

$$z(N,\omega_o) \equiv \sum_{n=1}^{N} e^{in\omega_o}\nu(n).$$

For simplicity let's suppose that initially the classical oscillator is at rest at the origin, i.e., $p(0) = q(0) = 0$; in this case the value of the unperturbed energy $H(\omega_o)$ is given by [6, 8]

$$E_C(N,\omega_o) = \frac{\varepsilon^2}{2}\left|z(N,\omega_o)\right|^2. \tag{10}$$

The time dependence of the energy for general initial conditions, as well as the expectation of the quantum unperturbed energy for $\xi \in \mathrm{dom}H(\omega_o)$, differ from (10) by linear terms in $z(N,\omega_o)$. So, the long time behaviours of the classical and quantum unperturbed energies are essentially equivalent—see [6, 8, 26] for details, including applications to harmonic oscillators under perturbations modulated along random and substitution sequences.

Corollary 2. *Suppose the autocorrelation functions*

$$C_\nu(r) = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\nu(n+r)\nu(n)$$

exist, and denote the corresponding autocorrelation measure by η_ν. If η_ν is UαH, $0 \le \alpha \le 1$, in the closed interval J, then there exists a constant $0 \le K < \infty$ such that

$$\langle E_C(N,\omega_o)\rangle_J \le K\varepsilon^2 N^{2-\alpha} \tag{11}$$

for any $N \ge 1$, and the critical exponent $\beta\left(\langle E_C(N,\omega_o)\rangle_J\right) \le (2-\alpha)$ (here the average $\langle\cdot\rangle_J$ is with respect to ω_o).

Proof. Since $\nu(n)$ takes values on ± 1 it is a sequence of unitary operators on the Hilbert space $\mathbb{R}$. Set $\tilde{\Lambda}(n) = \nu(n)$, $\tilde{\Lambda}(0) = I$, and

$$\tilde{\varphi}_N(\omega_o, 1) = \left\|\sum_{j=0}^{N-1} e^{ij\omega_o}\tilde{\Lambda}(j)\right\|^2.$$

By noting that $\tilde{\varphi}_N(\omega_o,1) = |z(N,\omega_o)|^2$, corol. 2 is a simple consequence of thm. 1 and (10). □

Remark 3.1. *General initial conditions are reflected only on the value of the constant K in (11); therefore only properties of the numerical sequence $\nu(n)$ are relevant for the exponent ruling the energy growth in this case. In [8] there are more* *specific results on random, Thue-Morse and Rudin-Shapiro sequences; here we need the average since thm. 1 holds for very general sequences $\{\nu(n)\}$, and so it is expected to give weaker information than any specific analysis; e.g., the autocorrelation measures η_ν for Rudin-Shapiro sequence is Lebesgue measure, so (11) implies $\langle E_C(N,\omega_o)\rangle_J \le K\varepsilon^2 N$, a result that follows directly from Saffari inequality [8, 27].*

Remark 3.2. *The upper bound on the average energy growth $\langle E_C\rangle$ depends only on the behaviour of the autocorrelation measure η_ν of the perturbing sequence $\{\nu(n)\}$ near the natural frequency ω_o; it is a kind of resonance. In particular, if η_ν is a positive continuous function times Lebesgue measure in a neighbourhood $\mathcal{V}$ of ω_o, and pure point outside $\mathcal{V}$ (indicating a highly correlated sequence), we still get a linear upper bound for the average energy growth around ω_o.*

Remark 3.3. *In the case of substitution sequences $\nu(n)$ with pure point autocorrelation measures, we can only infer from thm. 1 that, for any interval J, $\langle E_C(N,\omega_o)\rangle_J \le K\varepsilon^2 N^2$. That is the case, for instance, of Fibonacci, paper-folding and period doubling sequences [28, 29].*

As a final remark we comment upon twisted non-autonomous random walks in Hilbert spaces. They are built upon a sequence of unitary operators $U_n : \mathbf{H} \hookleftarrow$ $\Lambda(n) = U_n \cdots U_1, \omega \in [0, 2\pi]$, and a vector $\xi \in \mathbf{H}$; each walk is defined by

$$S_N(\omega,\xi) = \sum_{j=1}^{N} e^{ij\omega}\Lambda(j)\xi.$$

For theoretical and numerical investigations of similar random walks see [30, 24] and references there in. A fundamental question is about the asymptotic behaviour of the mean square displacement

$$\varphi_N(\omega,\xi) = \|S_N(\omega,\xi)\|^2.$$

By thm. 1 it follows that if the autocorrelation measure σ_ξ for $\{\Lambda(n)\xi\}$ exists and is UαH in the closed interval J, then we have the following upper bound for the critical exponent of the average mean square displacement

$$\beta(\langle\|S_N(\omega,\xi)\|^2\rangle_J) \le (2-\alpha).$$

Notice that average superdiffusive behaviour is not possible if σ_ξ is absolutely continuous with respect to Lebesgue measure in J and with continuous density.

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