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Supersymmetric quantum chiral higher spin gravity

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Abstract: We study quantum properties of supersymmetric $\mathcal{N}=1$ and $\mathcal{N}=4$ extensions of the four dimensional bosonic Chiral Higher Spin Gravities (HiSGRAs). We discuss the spectra, the classical actions and define the Feynman rules in $\mathcal{N}=1$ and $\mathcal{N}=4$ superspaces in the light-front gauge. Using these Feynman rules, we compute tree and one-loop amplitudes for these systems. A dimensional reduction to a system with $\mathcal{N}=2$ supersymmetry and with massive higher spin fields is performed and quantum properties of this system are discussed.

Keywords: Higher Spin Symmetry, Superspaces

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1 Introduction

Chiral Higher Spin Gravity (Chiral HiSGRA) has several unique properties. Chiral HiSGRA was originally introduced in [1], where it was shown that one can consistently "truncate" a classical Hamiltonian of [2, 3], in such a way that it stays purely cubic without any higher order corrections. As a result, the action for the Chiral HiSGRA, which is written in the light-front gauge, has a simple local form, being in some sense a higher spin generalization of the action for the self-dual Yang-Mills theory [4].[1] The spectrum of the Chiral HiSGRA consists of an infinite tower of massless fields with integer spins, each spin being present only once. Among an infinite number of cubic interaction vertices present in the action, there are familiar cubic vertices for lower spin fields, such as three graviton chiral cubic vertex. All these features make Chiral HiSGRA the simplest higher spin extension of gravity and a candidate of being an essential building block of any consistent interacting massless higher spin theory in four $\mathcal{D}=4$ dimensions.

The Quantum Chiral HiSGRA was then studied in [6–8] where it was shown that despite it being naively non-renormalizable, the theory is consistent at the quantum level as well. The tree and loop amplitudes in the Chiral HiSGRA have a highly nontrivial structure, which nevertheless results in the S matrix being $S=1$, in accordance with various no-go theorems [9, 10] (see also [11–13] for reviews). The tree amplitudes vanish when putting all external momenta on shell, as a result of nontrivial cancellations between

[1] Further connections between Chiral HiSGRA and self-dual theories were studied in [5].

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individual Feynman diagrams. The loop amplitudes vanish due to the presence of an overall numerical factor ν_0, which can be regularized to zero according to the prescription given in [14]. An analogous situation, i.e. the presence of the overall factor ν_0, happens for the other loop amplitudes as well. The peculiar form of the tree and loop amplitudes is due to a specific choice of the coupling constants in the cubic vertex, the so-called "coupling constant conspiracy", which allowed for the existence of the classical Chiral HiSGRA in the first place.

From $\mathcal{D} = 4$ Chiral HiSGRA one can obtain some more consistent models with interacting higher spin fields, both on flat and on anti-de Sitter backgrounds. Namely, it is possible to consider its AdS_4 version and study its application to the AdS_4/CFT_3 correspondence [15–19]. Another interesting model can be obtained by making a specific kind of dimensional reduction to a three dimensional $\mathcal{D} = 3$ flat space [20, 21]. As it is common for three dimensional systems with higher spin fields (see for example [22–28] and references therein), the model consists of massive higher spin fields, with an extra requirement that the masses belong to a particular lattice. The latter condition, along with the "coupling constant conspiracy", ensures that the three dimensional action stays cubic, as it was in $\mathcal{D} = 4$.

Until now our discussion was concerned with the purely bosonic Chiral HiSGRA. The aim of the present paper is to include also fermionic fields into consideration. A natural way to proceed is to start with four dimensional supersymmetric cubic vertices constructed in [29–31],[2] then keep their chiral part and choose a particular form for the coupling constants, in order to make the Lagrangian purely cubic. After that, one can develop a supersymmetric perturbation theory for the Chiral Supersymmetric HiSGRAs in complete analogy with its bosonic counterpart.

In the present paper we consider Chiral HiSGRAs with $\mathcal{N} = 1$ and $\mathcal{N} = 4$ supersymmetries. Although it is possible to construct cubic interaction vertices for massless higher spin fields also for the systems with higher number of supersymmetries [29, 31] (see also [41]), and to develop the perturbation theory for such systems, here we restrict ourselves with more "conventional" cases.[3]

At this point we would like to mention two more features of the Chiral HiSGRA, which are heavily used throughout the paper. First, the Chiral HiSGRA allows for Chan-Paton-like gauging, with the same internal symmetry groups and the same symmetry of the representations as in the Open String Theory [44]. Taking a U(N) "colored" version of the Chiral HiSGRA greatly simplifies the computations since one can consider only color ordered amplitudes [6–8]. Second, since the classical Chiral HiSGRA is formulated in the light-front gauge,[4] for the computations of the amplitudes we extend the technique developed in [47, 48] for Yang-Mills theory to $\mathcal{N} = 1$ and $\mathcal{N} = 4$ light-front superspaces.

The paper is organised as follows. In section 2 we briefly describe $\mathcal{D} = 4$, $\mathcal{N} = 1$ and $\mathcal{N} = 4$ extensions of the classical bosonic Chiral HiSGRA. We give the field content,

[2] See [32–40] for Lorentz covariant constructions for supersymmetric vertices in various dimensions.

[3] Tree level scattering amplitudes for supersymmetric higher spin fields in the framework of higher spin-IKKT models were considered in [42]. Tree level supersymmetric scattering amplitudes with massive higher spin exchanges were recently discussed in [43].

[4] A covariant version of the equations of motion for the Chiral HiSGRA was recently obtained in [45, 46].

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the action and the Feynman rules for these models. Section 3 contains the results of the computation of n-point[5] tree amplitudes, and section 4 presents the results for loop amplitudes. In section 5 we describe a dimensional reduction to $\mathcal{D} = 3$, $\mathcal{N} = 2$ massive supersymmetric Chiral HiSGRA with central charges and a computation of tree and one loop diagrams. A brief summary of the derivation of cubic vertices for $\mathcal{D} = 4$, $\mathcal{N} = 1$ supersymmetric massless higher spin fields [30] and of a derivation of the corresponding chiral model is given in the appendix.

2 Classical $\mathcal{D} = 4$ supersymmetric chiral HiSGRA

2.1 $\mathcal{N} = 1$ chiral HiSGRA

The basic objects for the case of $\mathcal{N} = 1$ supersymmetric Chiral HiSGRA are superfields, which in general can be either singlets or belong to an algebra of matrices. The index λ denotes a "super helicity" i.e., a helicity of a superfield, and can be either integer or half-integer. The four-momentum $\boldsymbol{p}$ is split into longitudinal components $p^- \equiv \gamma$ and $p^+ \equiv \beta$ and to a pair of mutually complex conjugated transverse components p and $\bar{p}$. For the light-front superfield approach one introduces also Grassmann momenta p_θ.

On the level of components we have two sets of fields with integer and half-integer helicities, $\phi_\lambda(\boldsymbol{p})$ and $\psi_\lambda(\boldsymbol{p})$, combined into the superfields $\Theta_\lambda(\boldsymbol{p}, p_\theta)$ as follows

$$\Theta_{-s}(\boldsymbol{p}, p_\theta) = \phi_{-s}(\boldsymbol{p}) + \frac{p_\theta}{\beta}\phi_{-s+\frac{1}{2}}(\boldsymbol{p}), \quad \Theta_{s-\frac{1}{2}}(\boldsymbol{p}, p_\theta) = -\phi_{s-\frac{1}{2}}(\boldsymbol{p}) + p_\theta \phi_s(\boldsymbol{p}), \tag{2.1}$$

with $s = 1, 2, \dots, \infty$, and

$$\Theta_{s}(\boldsymbol{p}, p_\theta) = \psi_{s}(\boldsymbol{p}) + \frac{p_\theta}{\beta}\psi_{s+\frac{1}{2}}(\boldsymbol{p}), \quad \Theta_{-s-\frac{1}{2}}(\boldsymbol{p}, p_\theta) = -\psi_{-s-\frac{1}{2}}(\boldsymbol{p}) + p_\theta \psi_{-s}(\boldsymbol{p}), \tag{2.2}$$

with $s = 0, 1, 2, \dots, \infty$.

The hermitian conjugation rules for the component fields are defined as

$$\phi^\dagger_\lambda(\boldsymbol{p}) = \phi_{-\lambda}(-\boldsymbol{p}), \quad \psi^\dagger_\lambda(\boldsymbol{p}) = \psi_{-\lambda}(-\boldsymbol{p}). \tag{2.3}$$

The superfields obey the equal time Poisson brackets

$$\Big[\Theta^{AB}_\lambda(\boldsymbol{p}, p_\theta), \Theta^{CD}_{\lambda'}(\boldsymbol{p}', p'_\theta)\Big] = -\delta_{\lambda+\lambda', -\frac{1}{2}} \frac{\delta^3(\boldsymbol{p}+\boldsymbol{p}')\,\delta(p_\theta + p'_\theta)}{2\beta} \Pi^{AC,BD}_{(G)}, \tag{2.4}$$

where $\Pi^{AC,BD}$ is a group theoretic factor whose explicit form depends on the choice of gauge group G. The symmetry under the interchange of gauge indices also depends on the gauge group. Similarly to how it has been done in bosonic HiSGRA [3, 7], one can show

[5] See [49] for a recent developments for computation of n-point diagrams for higher spin gravity in the framework of AdS_4/CFT_3 correspondence.

that allowed gauge groups include U(N), SO(N), and USp(N), with[6]

$$\Pi_{(\mathrm{U}(N))}{}^{A,C}_{B,D} = (-)^{\lambda+\frac{1}{2}\epsilon_\lambda}\delta^C_B\delta^A_D \tag{2.5}$$

$$\Pi^{AC,BD}_{(\mathrm{SO}(N))} = \left[\delta^{AC}\delta^{BD} + (-)^{\lambda+\frac{1}{2}\epsilon_\lambda}\delta^{AD}\delta^{BC}\right], \qquad \Theta^{AB}_\lambda(\boldsymbol{p}, p_\theta) = (-)^{\lambda+\frac{1}{2}\epsilon_\lambda}\Theta^{BA}_\lambda(\boldsymbol{p}, p_\theta), \tag{2.6}$$

$$\Pi^{AC,BD}_{(\mathrm{USp}(N))} = \left[C^{AC}C^{BD} - (-)^{\lambda+\frac{1}{2}\epsilon_\lambda}C^{AD}C^{BC}\right], \ \Theta^{AB}_\lambda(\boldsymbol{p}, p_\theta) = -(-)^{\lambda+\frac{1}{2}\epsilon_\lambda}\Theta^{BA}_\lambda(\boldsymbol{p}, p_\theta). \tag{2.7}$$

In the equations above we used the symbol

$$\epsilon_\lambda \equiv \begin{cases} 0\,, & \lambda \in \mathbb{Z} \\ 1\,, & \lambda \in \mathbb{Z} + \frac{1}{2} \end{cases} \tag{2.8}$$

With this notation, the Grassmann parity of a (super)field of (super)helicity λ is $(-)^{\epsilon_\lambda}$.

The action for the $\mathcal{N} = 1$ supersymmetric chiral HiSGRA is

$$\begin{aligned} S = &-\frac{1}{2}\sum_{\lambda=-\infty}^{\infty}\int d^4p\, dp_\theta\; (-)^{\epsilon_\lambda}(\boldsymbol{p}^2)\,\mathrm{Tr}\left[\Theta_{\lambda-\frac{1}{2}}(\boldsymbol{p}, p_\theta)\,\Theta_{-\lambda}(-\boldsymbol{p}, -p_\theta)\right] \\ &+ \sum_{\lambda_{1,2,3}}\int\prod_{i=1}^{3}d^4p_i\prod_{j=1}^{3}dp_{\theta,j}\;\delta^4(\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3)\delta(p_{\theta_1}+p_{\theta_2}+p_{\theta_3})C_{\lambda_1,\lambda_2,\lambda_3}V(\boldsymbol{p}_i, p_{\theta,i}, \lambda_i), \end{aligned} \tag{2.9}$$

where

$$V = \frac{\overline{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3+1}}{\beta_1^{\lambda_1+\frac{1}{2}\epsilon_{\lambda_1}}\beta_2^{\lambda_2+\frac{1}{2}\epsilon_{\lambda_2}}\beta_3^{\lambda_3+\frac{1}{2}\epsilon_{\lambda_3}}}\mathrm{Tr}[\Theta_{\lambda_1}(\boldsymbol{p}_1, p_{\theta,1})\,\Theta_{\lambda_2}(\boldsymbol{p}_2, p_{\theta,2})\,\Theta_{\lambda_3}(\boldsymbol{p}_3, p_{\theta,3})]. \tag{2.10}$$

The trace in equations (2.9)–(2.10) is taken over the gauge group indices. The sum of helicities in the expression (2.10) is constrained to be a non-negative integer, with other coupling constants vanishing. The coupling constants $C_{\lambda_1,\lambda_2,\lambda_3}$ are chosen as

$$C_{\lambda_1,\lambda_2,\lambda_3} = \frac{(-)^{\epsilon_{\lambda_2}}(l_p)^{\lambda_1+\lambda_2+\lambda_3}}{\Gamma(\lambda_1+\lambda_2+\lambda_3+1)}\,, \tag{2.11}$$

in order to make the action purely cubic [30] (see the appendix for more details). The action is chiral in the sense that the complex conjugated expression to the interaction term is absent. Similarly to the bosonic chiral HiSGRA, the action contains chiral parts of the known low spin cubic vertices, along with an infinite number of vertices with higher spin fields. In particular, the choice $\lambda_1 = \lambda_2 = \frac{1}{2}$, $\lambda_3 = -1$ gives the chiral part of the cubic vertices for the $\mathcal{N} = 1$ Super Yang-Mills. Similarly, the choice $\lambda_1 = \lambda_2 = \frac{3}{2}$, $\lambda_3 = -2$ gives the chiral part of the cubic vertices for the $\mathcal{N} = 1$ Supergravity. The choice $\lambda_1 = \lambda_2 = \lambda_3 = 0$ corresponds to the chiral part of the cubic vertex in the Wess-Zumino model [50] in the light front gauge [51] (the antichiral cubic coupling being $\lambda_1 = \lambda_2 = \lambda_3 = -\frac{1}{2}$). This coupling

[6] The indices of the USp(N) group are raised and lowered in terms of antisymmetric matrices $C_{AB} = -C_{BA}$, $C_{AB}C^{CB} = \delta^C_A$ as $V^A = C^{AB}V_B$, $V^BC_{BA} = V_A$.

generates only interactions of a scalar with spin $\frac{1}{2}$ fermions. The three scalar coupling is absent, as in the purely bosonic case.

As one can see from the field content given in the equations (2.1)–(2.2), the $\mathcal{N}=1$ Chiral HiSGRA is not a straightforward supersymmetrization of the bosonic Chiral HiSGRA, since the spectrum of the former contains twice as many bosonic fields with nonzero helicities. The necessity for this "doubling" of the spectrum is dictated by the fact that taking only one set (say, that given in equation (2.2)), it would not have been possible to achieve the consistency at the level of quartic interactions and to obtain the chiral theory (see the discussion at the end of the appendix).

2.2 $\mathcal{N}=4$ chiral HiSGRA

The discussion of the previous subsection can be appropriately modified to describe $\mathcal{N}=4$ chiral HiSGRA. In this case one can consider higher spin superfields with only integer "super helicities" λ. These superfields have the form of the expansion in terms of Grassmann momenta $p_{\theta,\hat{i}}$, with $\hat{i}=1,\ldots,4$ [31]

$$\begin{aligned}\Theta_\lambda(\boldsymbol{p},p_\theta) &= \beta\phi_{\lambda-1}(\boldsymbol{p}) - \phi_{\lambda-\frac{1}{2};\hat{i}}(\boldsymbol{p})p_{\theta,\hat{i}} + \frac{1}{2}\phi_{\lambda;\hat{i}\hat{j}}(\boldsymbol{p})p_{\theta,\hat{i}}p_{\theta,\hat{j}} \\ &\quad - \frac{1}{3!}\beta^{-1}\phi_{\lambda+\frac{1}{2};\hat{i}}(\boldsymbol{p})\varepsilon^{\hat{i}\hat{j}\hat{k}\hat{l}}p_{\theta,\hat{j}}p_{\theta,\hat{k}}p_{\theta,\hat{l}} + \frac{1}{4!}\beta^{-1}\phi_{\lambda+1}(\boldsymbol{p})\varepsilon^{\hat{i}\hat{j}\hat{k}\hat{l}}p_{\theta,\hat{i}}p_{\theta,\hat{j}}p_{\theta,\hat{k}}p_{\theta,\hat{l}}.\end{aligned} \tag{2.12}$$

In particular, for $\lambda=0$ the expression (2.12) is the superfield for $\mathcal{N}=4$ Super Yang-Mills in the light-front gauge [51]. The component fields obey the hermitian conjugation properties

$$\phi^\dagger_{\lambda-1}(\boldsymbol{p}) = \phi_{-\lambda+1}(-\boldsymbol{p}), \quad \phi^\dagger_{\lambda-\frac{1}{2},\hat{i}}(\boldsymbol{p}) = \phi_{-\lambda-\frac{1}{2},\hat{i}}(-\boldsymbol{p}), \quad \phi^\dagger_{\lambda,\hat{i}\hat{j}}(\boldsymbol{p}) = \varepsilon^{\hat{i}\hat{j}\hat{k}\hat{l}}\phi_{-\lambda,\hat{k}\hat{l}}(-\boldsymbol{p}). \tag{2.13}$$

For the equal-time Poisson brackets one has

$$[\Theta_\lambda(\boldsymbol{p},p_\theta),\Theta_{\lambda'}(\boldsymbol{p}',p'_\theta)] = -\delta_{\lambda,\lambda'}\frac{\delta^3(\boldsymbol{p}+\boldsymbol{p}')\,\delta^4(p_\theta+p'_\theta)}{2}\,\Pi^{AC,BD}_{(G)} \tag{2.14}$$

where the expressions for $\Pi^{AC,BD}_{(G)}$ are as in (2.5)–(2.7). The action for the Chiral $\mathcal{N}=4$ HiSGRA is similar to the one for the Chiral $\mathcal{N}=1$ HiSGRA considered in the previous subsection,

$$\begin{aligned}S &= -\frac{1}{2}\sum_{\lambda=-\infty}^{\infty}\int d^4p\,d^4p_\theta\;(\boldsymbol{p}^2)\,\mathrm{Tr}[\Theta_\lambda(\boldsymbol{p},p_{\theta,\hat{i}})\,\Theta_{-\lambda}(-\boldsymbol{p},-p_{\theta,\hat{i}})] \\ &\quad + \sum_{\lambda_{1,2,3}}\int\prod_{i=1}^{3}d^4p_i\prod_{j=1}^{3}d^4p_{\theta,j}\,\delta^4(\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3)\delta^4(p_{\theta_1}+p_{\theta_2}+p_{\theta_3})C_{\lambda_1,\lambda_2,\lambda_3}V(\boldsymbol{p}_i,p_{\theta,i},\lambda_i),\end{aligned} \tag{2.15}$$

where the cubic vertex and the coupling constants are given in (2.10) and in (2.11), respectively, with $\epsilon_{\lambda_i}=0$. The action (2.15) contains an infinite number of cubic vertices, along with the chiral part of the $\mathcal{N}=4$ Super Yang-Mills cubic interactions [51, 52]. The latter can be obtained from the action (2.15) by choosing $\lambda_1=\lambda_2=\lambda_3=0$.

2.3 Feynman rules

Before moving to computation of tree and loop amplitudes, we set up the corresponding Feynman rules for supersymmetric Chiral HiSGRAs.

From the Lagrangians (2.9) and (2.15) the propagators are found to be

$$\langle \Theta^{AB}_{\lambda_i}(\boldsymbol{p}_i, p_{\theta,i}), \Theta^{CD}_{\lambda_j}(\boldsymbol{p}_j, p_{\theta,j})\rangle = \frac{\delta^{\lambda_i+\lambda_j,-\frac{1}{2}}\delta^4(\boldsymbol{p}_i+\boldsymbol{p}_j)\delta(p_{\theta,i}+p_{\theta,j})}{\boldsymbol{p}_i^2}\,\Pi^{AB,CD}_{(G)} \quad \text{for} \quad \mathcal{N}=1, \tag{2.16}$$

$$\langle \Theta^{AB}_{\lambda_i}(\boldsymbol{p}_i, p_{\theta,i}), \Theta^{CD}_{\lambda_j}(\boldsymbol{p}_j, p_{\theta,j})\rangle = \frac{\delta^{\lambda_i+\lambda_j,0}\delta^4(\boldsymbol{p}_i+\boldsymbol{p}_j)\delta^4(p_{\theta,i}+p_{\theta,j})}{\boldsymbol{p}_i^2}\,\Pi^{AB,CD}_{(G)} \quad \text{for} \quad \mathcal{N}=4, \tag{2.17}$$

where the expressions for $\Pi^{AB,CD}_{(G)}$ are given in (2.5)–(2.7).

From the interaction terms in (2.9) and (2.15) we get for the vertex function

$$\begin{aligned}\mathcal{V}(\boldsymbol{p}_i, p_{\theta,i}, \lambda_i) = \delta^4(\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3)C_{\lambda_1,\lambda_2,\lambda_3}\frac{\overline{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3+1}}{\beta_1^{\lambda_1+\frac{1}{2}\epsilon_{\lambda_1}}\beta_2^{\lambda_2+\frac{1}{2}\epsilon_{\lambda_2}}\beta_3^{\lambda_3+\frac{1}{2}\epsilon_{\lambda_3}}} \\ \times\int\prod_{l=1}^{3}dp^{\mathcal{N}}_{\theta,l}\;\delta^{\mathcal{N}}(p_{\theta,1}+p_{\theta,2}+p_{\theta,3}),\end{aligned} \tag{2.18}$$

where the coupling constants are given by (2.11). The value of $\mathcal{N}$ is either 1 or 4, with $\epsilon_{\lambda_i}=0$ for $\mathcal{N}=4$. The integration goes over all bosonic momenta, which are not fixed by the momentum conservation. Besides the vertex function (2.18) is multiplied by wavefunctions of superfields which correspond to external legs.

For the sake of completeness let us note that the Feynman rules given above can be applied for non-chiral $\mathcal{D}=4$ $\mathcal{N}=1$ and $\mathcal{N}=4$ theories in the light-front gauge, by including the hermitian conjugate vertex to the one given in (2.18),

$$\overline{\mathcal{V}}(\boldsymbol{p}_i, p_{\theta,i}, \lambda_i) = \delta^4(\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3)\int\prod_{l=1}^{3}dp^{\mathcal{N}}_{\theta,l}\;\delta^{\mathcal{N}}(p_{\theta,1}+p_{\theta,2}+p_{\theta,3})\mathcal{F}\,, \tag{2.19}$$

with

$$\begin{aligned}\mathcal{F} &= \overline{C}_{\lambda_1,\lambda_2\lambda_3}\frac{\mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3-\frac{1}{2}}}{\beta_1^{-\lambda_1+\frac{1}{2}\epsilon_{\lambda_1}}\beta_2^{-\lambda_2+\frac{1}{2}\epsilon_{\lambda_2}}\beta_3^{-\lambda_3+\frac{1}{2}\epsilon_{\lambda_3}}}\mathbb{P}_\theta, \\ \overline{C}_{-\lambda_1-\frac{1}{2},-\lambda_2-\frac{1}{2},-\lambda_3-\frac{1}{2}} &= (-)^{\lambda_1+\lambda_2+\lambda_3+\epsilon_{\lambda_2}+1}C^\star_{\lambda_1,\lambda_2,\lambda_3} \quad \text{for} \quad \mathcal{N}=1\end{aligned} \tag{2.20}$$

and

$$\mathcal{F} = (-)^{\lambda_1+\lambda_2+\lambda_3}C^\star_{-\lambda_1,-\lambda_2,-\lambda_3}\frac{\mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3+1}}{\beta_1^{-\lambda_1+2}\beta_2^{-\lambda_2+2}\beta_3^{-\lambda_3+2}}\frac{\varepsilon_{\hat{i}_1,\ldots,\hat{i}_4}}{4!}\mathbb{P}_\theta^{\hat{i}_1},\ldots,\mathbb{P}_\theta^{\hat{i}_4} \quad \text{for} \quad \mathcal{N}=4 \tag{2.21}$$

The definition of $\mathbb{P}_\theta$ is given in (A.4).

The sums of helicities in (2.18) and in (2.21) are restricted to be non-negative and non-positive integers, respectively. The sum of the helicities in (2.20) is restricted to be a half-integer less or equal to $-\frac{3}{2}$.

In the present approach we use a perturbation theory in superspaces, where the space-time coordinates are extended with Grassmann momenta [53], rather than more commonly used approach, when the superspace contains Grassmann coordinates [54–56]. One can reformulate the Feynman rules in the Grassmann coordinate space by performing a Fourier transform. The choice of the momentum space representation, as well as the choice of the light-front superspace approach for the perturbation theory (see [57] for computations of correlation functions in $\mathcal{N}=4$ super Yang-Mills), comes naturally, since the cubic interactions in the vertices depend on Grassmann momenta.

3 Tree amplitudes

3.1 $\mathcal{N}=1$

In this section we consider the tree level diagrams for $\mathrm{U}(N)$ colored supersymmetric HiSGRA. The computations are much simpler than in the other versions of HiSGRA, since one has to consider only a particular cyclic ordering of external fields. Consequently, for the four point tree level amplitude one gets two diagrams

The calculations of tree level amplitudes are nearly identical to the bosonic Chiral HiSGRA [6]. One modification is due to the fact that the wave functions Θ_{λ_i}, which are placed on each vertex connected to an external leg, have Grassmann parity $(-)^{\epsilon_{\lambda_i}}$. In addition, in the case of $\mathcal{N}=1$, integration measures $dp_{\theta,i}$ and the propagator (2.16), are Grassmann-odd. Therefore, one must pay particular attention to their ordering in order to get the correct sign for each diagram. A general rule for a diagram with n external legs is that the sum of superhelicities for the external legs

$$\Lambda_n \equiv \sum_{i=1}^{n} \lambda_i \tag{3.1}$$

should be half-integer for even n, and integer for odd n, else the diagram is zero trivially because of the requirements that superhelicities add up to an integer at each cubic vertex.

Taking into account explicitly the sign in the vertices (2.11), we have for the first diagram in the four point amplitude

$$\begin{aligned} A_4(12\to 34) &= \int dp_{\theta_1} dp_{\theta_2} dp_{\theta_\omega} \delta(p_{\theta_1}+p_{\theta_2}+p_{\theta_\omega})(-)^{\epsilon_{\lambda_2}}\Theta_{\lambda_1}\Theta_{\lambda_2}\delta(p_{\theta_\omega}+p_{\theta_{\omega'}}) \\ &\quad \times dp_{\theta_{\omega'}} dp_{\theta_3} dp_{\theta_4} \delta(p_{\theta_{\omega'}}+p_{\theta_3}+p_{\theta_4})(-)^{\epsilon_{\lambda_3}}\Theta_{\lambda_3}\Theta_{\lambda_4}\tilde{A}_4(12\to 34) \\ &= (-)^{1+\epsilon_{\lambda_1}+\epsilon_{\lambda_3}} \int \prod_{i=1}^{4} dp_{\theta,i}\, \delta(p_{\theta_1}+p_{\theta_2}+p_{\theta_3}+p_{\theta_4})\Theta_{\lambda_1}\Theta_{\lambda_2}\Theta_{\lambda_3}\Theta_{\lambda_4}\tilde{A}_4(12\to 34) \end{aligned} \tag{3.2}$$

where $\tilde{A}_4(12\to 34)$ is essentially the contribution one gets for the purely bosonic case

$$\tilde{A}_4(12\to 34) = \frac{\overline{\mathbb{P}}_{12}\overline{\mathbb{P}}_{34}(\overline{\mathbb{P}}_{12}+\overline{\mathbb{P}}_{34})^{\Lambda_4-\frac{1}{2}}}{4\Gamma\left(\Lambda_4+\frac{1}{2}\right)\prod_{i=1}^{4}\beta_i^{\lambda_i+\frac{1}{2}\epsilon_{\lambda_i}}(\boldsymbol{p}_1+\boldsymbol{p}_2)^2}\delta^4\left(\sum_{i=1}^{4}\boldsymbol{p}_i\right). \tag{3.3}$$

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For the second diagram we permute the indices cyclically, and then rearrange the integration measure and wave functions to bring them to the same ordering as for the first diagram:

$$
\begin{aligned}
A_4(23 \shortrightarrow 41) &= (-)^{1+\epsilon_{\lambda_2}+\epsilon_{\lambda_4}} \int dp_{\theta_2} dp_{\theta_3} dp_{\theta_4} dp_{\theta_1} \delta(p_{\theta_2}+p_{\theta_3}+p_{\theta_4}+p_{\theta_1}) \\
&\quad \times \Theta_{\lambda_2}\Theta_{\lambda_3}\Theta_{\lambda_4}\Theta_{\lambda_1} \tilde{A}_4(23 \shortrightarrow 41) \\
&= (-)^{\epsilon_{\lambda_2}+\epsilon_{\lambda_4}} \int \prod_{i=1}^{4} dp_{\theta,i}\, \delta(p_{\theta_1}+p_{\theta_2}+p_{\theta_3}+p_{\theta_4}) \Theta_{\lambda_1}\Theta_{\lambda_2}\Theta_{\lambda_3}\Theta_{\lambda_4} \tilde{A}_4(23 \shortrightarrow 41),
\end{aligned}
\tag{3.4}
$$

with

$$
\tilde{A}_4(23 \shortrightarrow 41) = \frac{\overline{\mathbb{P}}_{23}\overline{\mathbb{P}}_{41}(\overline{\mathbb{P}}_{23}+\overline{\mathbb{P}}_{41})^{\Lambda_4-\frac{1}{2}}}{4\Gamma\left(\Lambda_4+\frac{1}{2}\right)\prod_{i=1}^{4}\beta_i^{\lambda_i+\frac{1}{2}\epsilon_{\lambda_i}}(\boldsymbol{p}_2+\boldsymbol{p}_3)^2}\delta^4\left(\sum_{i=1}^{4}\boldsymbol{p}_i\right). \tag{3.5}
$$

Since Λ_4 is a half-integer, one obtains that $\sum_i \epsilon_{\lambda_i}$ is odd. Therefore, the signs of both diagrams are the same and they add up as in the purely bosonic case [6].

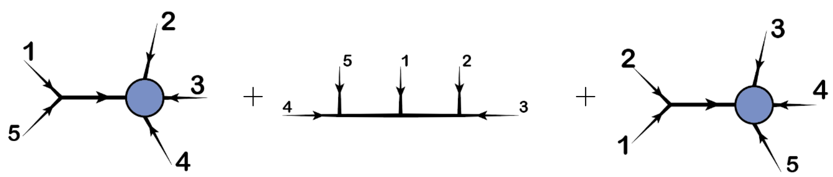
Summing both diagrams, and keeping the four-momentum of the first particle off-shell,

$$
\begin{aligned}
A_4(1234) = &\frac{(-)^{\epsilon_{\lambda_2}+\epsilon_{\lambda_4}}\alpha_4^{\Lambda_4-\frac{1}{2}}\beta_3\,\boldsymbol{p}_1^2}{4\Gamma\left(\Lambda_4+\frac{1}{2}\right)\prod_{i=1}^{4}\beta_i^{\lambda_i+\frac{1}{2}\epsilon_{\lambda_i}-1}\beta_1\mathbb{P}_{23}\mathbb{P}_{34}} \\
&\times \delta^4\left(\sum_{i=1}^{4}\boldsymbol{p}_i\right)\int\prod_{j=1}^{4}dp_{\theta,j}\,\delta\left(\sum_{k=1}^{4}p_{\theta,k}\right),\prod_{l=1}^{4}\Theta_{\lambda_l}(p_l,p_{\theta_l})
\end{aligned}
\tag{3.6}
$$

where

$$
\alpha_4 = \overline{\mathbb{P}}_{12}+\overline{\mathbb{P}}_{34}. \tag{3.7}
$$

After computing the four point tree amplitude, one can compute n-point diagrams recursively, using the method of [58]. For example, the five-point function can be computed by using four- and three-point functions as follows



where the external particles in the four-and three-point functions which are used as propagators, are kept off-shell. Note that in the computation of the sign factor which comes from the four point amplitudes in (3.2)–(3.4), one now has to remove the external wave function Θ_{λ_1} to get the correct result for the amplitude with the first particle being off-shell. Alternatively, from the same considerations as for the four-point amplitude one can conclude that the n-point comb diagram will come with a factor of $A_n \propto (-)^{\sum_k \epsilon_{\lambda_{2k}}}$ for any cyclic permutation of $123\ldots n$. Then one can sum all the n-point diagrams by noting that they have the same structure in the bosonic case.

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In this way one can prove the following expression for a tree level n-point function

$$A_n(1\dots n) = \frac{(\prod_{k=1}^{\lfloor n/2\rfloor}(-)^{\epsilon_{\lambda_{2k}}})(-)^n\,\alpha_n^{\Lambda_n-\frac{n-3}{2}}\beta_3\dots\beta_{n-1}\,\boldsymbol{p}_1^2}{2^{n-2}\Gamma\left(\Lambda_n-\frac{n-3}{2}+1\right)\prod_{i=1}^n\beta_i^{\lambda_i+\frac{1}{2}\epsilon_{\lambda_i}-1}\beta_1\mathbb{P}_{23}\dots\mathbb{P}_{n-1,n}} \times\delta^4\left(\sum_{i=1}^n\boldsymbol{p}_i\right)\int\prod_{j=1}^n dp_{\theta,j}\,\delta\left(\sum_{k=1}^n p_{\theta,k}\right)\prod_{l=1}^n\Theta_{\lambda_l}(p_l,p_{\theta_l}), \tag{3.8}$$

where

$$\alpha_n = \sum_{i<j}^{n-2}\overline{\mathbb{P}}_{ij}+\overline{\mathbb{P}}_{n-1,n}, \tag{3.9}$$

and the four-momentum of the first particle is taken off-shell. When taken on shell, the amplitude vanishes.

3.2 $\mathcal{N}=4$

The computations for tree level n-point amplitudes for the Chiral $\mathcal{N}=4$ HiSGRA can be performed in a similar way. They are however, simpler then for the case of $\mathcal{N}=1$ since all superfields, propagators and integration measures are even. As a result the expression for the n-point tree level amplitude obtained in terms of $\mathcal{N}=4$ superfields, is almost identical to the case of the bosonic Chiral HiSGRA and reads [6, 7]

$$A_n(1\dots n) = \frac{(-)^n\,\alpha_n^{\Lambda_n}\beta_3\dots\beta_{n-1}\,\boldsymbol{p}_1^2}{2^{n-2}\Gamma(\Lambda_n)\prod_{i=1}^n\beta_i^{\lambda_i-1}\beta_1\mathbb{P}_{23}\dots\mathbb{P}_{n-1,n}} \times\delta^4\left(\sum_{i=1}^n\boldsymbol{p}_i\right)\int\prod_{j=1}^n d^4p_{\theta,j}\delta^4\left(\sum_{j=1}^n p_{\theta,j}\right)\prod_{l=1}^n\Theta_{\lambda_l}(p_l,p_{\theta_l}), \tag{3.10}$$

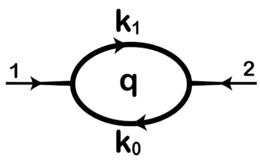
with Λ_n and α_n are defined as in (3.1) and (3.9), respectively.

4 Loop amplitudes

As usually happens in supersymmetric field theories, the tadpole diagrams vanish both for $\mathcal{N}=1$ and $\mathcal{N}=4$ Chiral HiSGRAs due to the property $\delta(0)=0$ of the Grassmann δ-function, which is present in the propagators (2.16)–(2.17).

4.1 Self-energy

The simplest one loop diagram corresponds to the self-energy amplitude



This diagram can be evaluated by introducing of dual momenta $\boldsymbol{k}_1, \boldsymbol{k}_0, \boldsymbol{q}$, related to the external momentum as $\boldsymbol{p}_1 = \boldsymbol{k}_1 - \boldsymbol{k}_0$. The loop momentum is $\boldsymbol{p} = \boldsymbol{q} - \boldsymbol{k}_0$. Using the Feynman rules, given in section 2.3, one can compute for the self-energy diagram

$$\Gamma_{\text{self}} = [\delta(p_{1,\theta} + p_{2,\theta})]^2 \tilde{\Gamma}_{\text{self}}, \tag{4.1}$$

where, performing a summation over internal helicities ω, one has

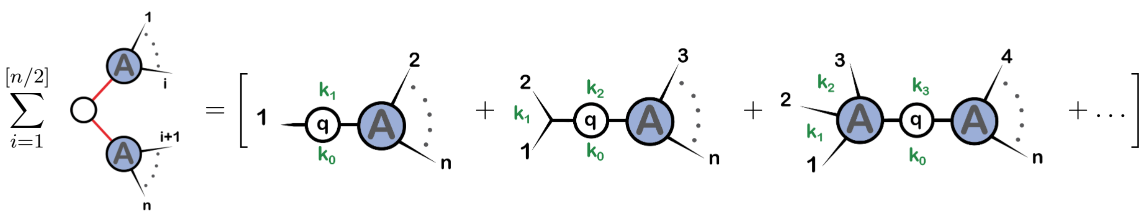
$$\begin{aligned} \tilde{\Gamma}_{\text{self}} &= N \sum_{\omega} \frac{(l_p)^{\Lambda_2}}{\beta_1^{\lambda_1 + \frac{1}{2}\epsilon_{\lambda_1}} \beta_2^{\lambda_2 + \frac{1}{2}\epsilon_{\lambda_2}} \Gamma(\Lambda_2)} \int \frac{d^4 q}{(2\pi)^4} \frac{\overline{\mathbb{P}}^2_{q-k_0,p_1} \delta_{\Lambda_2,0}}{(\boldsymbol{q} - \boldsymbol{k}_0)^2 (\boldsymbol{q} - \boldsymbol{k}_1)^2} \\ &= N\nu_0 \left(\bar{k}_0^2 + \bar{k}_0 \bar{k}_1 + \bar{k}_1^2\right) \frac{(l_p)^{\Lambda_2} \delta_{\Lambda_2,0}}{\beta_1^{\lambda_1 + \frac{1}{2}\epsilon_{\lambda_1} - 1} \beta_2^{\lambda_2 + \frac{1}{2}\epsilon_{\lambda_2} - 1} \Gamma(\Lambda_2)}. \end{aligned} \tag{4.2}$$

The expression Γ_{self} corresponds to the self-energy amplitude for the purely bosonic HiSGRA [6, 7]. It is proportional to a finite expression, times the total number of polarizations ν_0. In the bosonic case it is an infinite sum $\nu_0 = 1 + 2\sum_{\lambda=1}^{\infty} 1$, where the first "1" stands for the scalar and "2" per each massless higher spin field. According to the prescription of [14], this sum is regularized to zero, $\nu_0 = 1 + 2\zeta_R(0) = 0$, by using the Riemann zeta function regularization.

In the $\mathcal{N} = 1$ supersymmetric case, each superfield carries twice as many degrees of freedom as the bosonic field. Besides, one has a "doubling" of the spectrum, discussed in subsection 2. Similar considerations can be applied for $\mathcal{N} = 4$ Chiral HiSGRA, where each superfield carries four times as many degrees of freedom as $\mathcal{N} = 1$ superfield. In any case, supersymmetry provides further cancellations between bosonic and fermionic loops, reflected by the presence of the square of the Grassmann δ-function in (4.1). As long as the number of degrees of freedom can be regularized to a finite value, this makes the amplitude vanish and therefore, supersymmetry is an additional "source" of finiteness of the Chiral HiSGRA at the one loop level.

4.2 General argument for one loop amplitudes

As it has been proven in [8], a general n-point one loop amplitude in the bosonic Chiral HiSGRA can be obtained by combining $A_i(1, \ldots, i)$ and $A_{n-i}(i + 1., , , n)$ tree amplitudes with the self-energy amplitude and taking cyclic permutations.



This procedure leads in the purely bosonic case to the general structure of one-loop amplitudes in the Chiral HiSGRA

$$\Gamma_{\text{1-loop}} = \Gamma^{++.,,.+}_{\text{1-loop, QCD}} \times D^{\text{HSGR}}_{\lambda_1,\ldots\lambda_n} \times \nu_0, \tag{4.3}$$

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where $D^{\rm HSGR}_{\lambda_1,\dots\lambda_n}$ is a kinematical higher spin dressing factor. Again, the total amplitude vanishes because the number ν_0 is regularized to zero. Repeating the same procedure for the supersymmetric case one can see that supersymmetry provides another cancellation mechanism, because of the vanishing of individual self-energy amplitudes that was discussed in the previous subsection.

5 Massive $\mathcal{D}=3$, $\mathcal{N}=2$ chiral HiSGRA

As was mentioned in the Introduction, three dimensional massive bosonic Chiral HiSGRA can be obtained from the four dimensional theory by dimensional reduction [20, 21]. In what follows we describe a $\mathcal{N}=2$ supersymmetric extension of this model, by making a dimensional reduction of the $\mathcal{D}=4$, $\mathcal{N}=1$ Chiral HiSGRA, considered in the subsection 2.1. We start, by making a Fourier transformation with respect to the p^2 component of the four momentum and then making the corresponding x^2 coordinate compact. Then, we expand $\mathcal{D}=4$ superfields $\Theta_\lambda(\boldsymbol{p},p_\theta)$ as

$$\Theta_\lambda(\vec{p},x^2,p_\theta)=\sum_k \exp\left(ikmx^2\,\varepsilon\right)\Theta_{\lambda,k}(\vec{p},x^2,p_\theta), \qquad \varepsilon\equiv \text{sign}(\lambda) \tag{5.1}$$

with $\vec{p}=(\rho,\beta,\gamma)$ being a three-momentum. The mass scale m is determined by the compactification radius. From the form of the compactification (5.1) it follows, that the masses of the $\mathcal{D}=3$ superfields have the form $m_k=\varepsilon mk$, for some integer k. In this way, one obtains $\mathcal{D}=3$, $\mathcal{N}=2$ supersymmetry with central charges, as can be seen for example from the supersymmetry transformations (A.10).

Inserting the expression (5.1) into the action (2.9) and integrating over the compact coordinate, one gets

$$\begin{aligned} S=&-\sum_{\lambda\geq 0,k}\int d^3p\,dp_\theta\,(-)^{\epsilon_\lambda}((\vec{p})^2+m^2k^2)\,\text{Tr}\left[\Theta_{\lambda-\frac{1}{2},k}(\vec{p},p_\theta)\,\Theta_{-\lambda,k}(-\vec{p},-p_\theta)\right] \\ &+\sum_{\lambda_{1,2,3};k_{1,2,3}}\int\prod_{i=1}^3 d^3p_i\prod_{j=1}^3 dp_{\theta,j}\,\delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3)\delta(p_{\theta_1}+p_{\theta_2}+p_{\theta_3})C(k_i,\lambda_i)V(k_i,\lambda_i,\vec{p}_i,p_{\theta,i}), \end{aligned} \tag{5.2}$$

with the cubic vertex

$$V=\frac{(\mathbb{P}+\mathbb{P}_\lambda)^{\lambda_1+\lambda_2+\lambda_3+1}}{\beta_1^{\lambda_1+\frac{1}{2}\epsilon_{\lambda_1}}\beta_2^{\lambda_2+\frac{1}{2}\epsilon_{\lambda_2}}\beta_3^{\lambda_3+\frac{1}{2}\epsilon_{\lambda_3}}}\text{Tr}[\Theta_{\lambda_1,k_1}(\vec{p}_1,p_{\theta,1})\,\Theta_{\lambda_2,k_2}(\vec{p}_2,p_{\theta,2})\,\Theta_{\lambda_3,k_3}(\vec{p}_3,p_{\theta,3})], \tag{5.3}$$

where $\mathbb{P}$ and $\mathbb{P}_\lambda$ are given by (A.3) and (A.5) respectively, with the momenta p having only a real component ρ, as it can be seen by performing a decomposition of the four dimensional complex transverse momenta according to $\bar{p}=p_1-ip_2=\rho-im\varepsilon k$. The expression for the coupling constants in (5.3)

$$C_{\lambda_1,\lambda_2,\lambda_3}=\frac{(-)^{\epsilon_{\lambda_2}}(l_p)^{\lambda_1+\lambda_2+\lambda_3}}{\Gamma(\lambda_1+\lambda_2+\lambda_3+1)}\,\delta_{k_i\varepsilon_i,0}, \qquad \varepsilon_i\equiv\text{sign}(\lambda_i) \tag{5.4}$$

includes a condition, which implies that the masses of the fields present in the cubic vertex belong to a lattice

$$m_1\varepsilon_1+m_2\varepsilon_2+m_3\varepsilon_3=0. \tag{5.5}$$

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Alternatively, cubic vertices for $\mathcal{D}=3$, $\mathcal{N}=2$ supersymmetric massive higher spins without central charges can be constructed, using the method described in the appendix [59].

Before proceeding further, let us note, that the quantum consistency of the higher dimensional theory is not a priori preserved by the dimensional reduction, and therefore has to be checked separately [60]. To this end, we set up the corresponding Feynman rules in $\mathcal{D}=3$ and then compute tree and loop amplitudes, as we did in the previous sections.

The propagator and the vertex functions are

$$\langle \Theta^{AB}_{\lambda_i,k_i}(\vec{p}_i,p_{\theta,i}),\Theta^{CD}_{\lambda_j,k_j}(\vec{p}_j,p_{\theta,j})\rangle = \frac{\delta^{\lambda_i+\lambda_j,\frac{1}{2}}\delta^{k_i,k_j}\delta^3(\vec{p}_i+\vec{p}_j)\delta(p_{\theta,i}+p_{\theta,j})}{\vec{p}_i^{\,2}+m^2k_i^2}\,\Pi^{AB,CD}_{(G)}, \tag{5.6}$$

$$\begin{aligned}\mathcal{V}(\vec{p}_i,p_{\theta,i}\lambda_i,k_i) = \delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3)C_{\lambda_1,\lambda_2,\lambda_3}&\frac{(\mathbb{P}+\mathbb{P}_\lambda)^{\lambda_1+\lambda_2+\lambda_3+1}}{\beta_1^{\lambda_1+\frac{1}{2}\epsilon_{\lambda_1}}\beta_2^{\lambda_2+\frac{1}{2}\epsilon_{\lambda_2}}\beta_3^{\lambda_3+\frac{1}{2}\epsilon_{\lambda_3}}}\\ &\times\int\prod_{l=1}^{3}dp_{\theta,l}\,\delta(p_{\theta,1}+p_{\theta,2}+p_{\theta,3}).\end{aligned} \tag{5.7}$$

Using the Feynman rules one can show, that tree level n-point functions vanish in a complete analogy of the purely bosonic case [21] and with four dimensional supersymmetric models, considered in section 3.

Finally, let us consider the self-energy diagram. Using the Feynman rules (5.6)–(5.7), and proceeding in the same way as for the four dimensional case, one obtains the expression (4.1) with

$$\begin{aligned}\tilde{\Gamma}_{\text{self}} = N\sum_{\omega}\sum_{l}&\frac{(l_p)^{\Lambda_2}}{\beta_1^{\lambda_1+\frac{1}{2}\epsilon_{\lambda_1}}\beta_2^{\lambda_2+\frac{1}{2}\epsilon_{\lambda_2}}\Gamma(\Lambda_2)}\\ \times\int\frac{d^3q}{(2\pi)^3}&\frac{\mathbb{P}^2_{q-k_0,p_1}\delta_{\Lambda_2,0}}{((\vec{q}-\vec{k}_0)^2+(l-r_0)^2m^2)((\vec{q}-\vec{k}_1)^2+(l-r_1)^2m^2)}.\end{aligned} \tag{5.8}$$

The sum over the integer l and the appropriate regularization can be performed using the approach developed for Kaluza-Klein compactifications [61–63], whereas the integral over the non-compact component of the momentum can be treated similarly to the four dimensional case. The main conclusion is that supersymmetry makes the entire contribution equal to zero, similarly to how it happened for the analogous diagrams in $\mathcal{D}=4$ Chiral HiSGRAs.

Acknowledgments

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A Cubic vertices for $\mathcal{N}=1$

In the light-front formulation one chooses the four dimensional coordinates as

$$x^{\pm} = \frac{1}{\sqrt{2}}(x^3\pm x^0), \quad z=\frac{1}{\sqrt{2}}(x^1+ix^2), \quad \bar{z}=\frac{1}{\sqrt{2}}(x^1-ix^2), \tag{A.1}$$

the corresponding components of the four momentum being denoted as β, γ, p and $\bar{p}$. The combinations of momenta that appear in the cubic interaction vertices and scattering amplitudes have the form

$$\mathbb{P}_{km} = p_k\beta_m - p_m\beta_k\,, \qquad \overline{\mathbb{P}}_{km} = \bar{p}_k\beta_m - \bar{p}_m\beta_k\,. \qquad \mathbb{P}_{km,\theta} = p_{k,\theta}\beta_m - p_{m,\theta}\beta_k, \tag{A.2}$$

where k and m are numbers of the fields. For n-point amplitudes only $n-2$ combinations $\mathbb{P}_{km}$ and $\overline{\mathbb{P}}_{ij}$ are independent due to the momentum conservation $\sum p_k = \sum \bar{p}_k = \sum \beta_k = \sum p_{\theta,k} = 0$. For a cubic vertex the only independent combinations are

$$\mathbb{P} = \frac{1}{3}\left[(\beta_1-\beta_2)p_3 + (\beta_2-\beta_3)p_1 + (\beta_3-\beta_1)p_2\right], \tag{A.3}$$

$$\mathbb{P}_\theta = \frac{1}{3}\left[(\beta_1-\beta_2)p_{\theta,3} + (\beta_2-\beta_3)p_{\theta,1} + (\beta_3-\beta_1)p_{\theta,2}\right], \tag{A.4}$$

and the complex conjugate to (A.3). In $\mathcal{D}=3$ we use also a combination

$$\mathbb{P}_\lambda = \frac{i}{3}\left[(\beta_1-\beta_2)\varepsilon_3 k_3 + (\beta_2-\beta_3)\varepsilon_1 k_1 + (\beta_3-\beta_1)\varepsilon_2 k_2\right], \tag{A.5}$$

as well as (A.3) with the complex momenta p_k having only a real component ρ_k.

Let us move to the construction of the cubic vertices for $\mathcal{N}=1$ following [30] and refer to [31] for analogous construction for $\mathcal{N}=4$. Recall, that in four dimensions $\mathcal{N}=1$ super Poincaré algebra without central charges contains generators of Lorentz transformations $J^{\mu\nu}$, generators of translations P^μ, and generators of Supersymmetry transformations Q^α, and $\bar{Q}^{\dot{\alpha}}$. In order to construct cubic and higher order vertices, one splits the generators of the super Poincaré algebra into kinematical and dynamical ones

$$\text{kinematical}: \qquad P^+, P^z, P^{\bar{z}}, J^{z+}, J^{\bar{z}+}, J^{+-}, J^{z\bar{z}}, Q^+, \bar{Q}^+, \tag{A.6}$$

$$\text{dynamical}: \qquad P^-, J^{z-}, J^{\bar{z}-}, Q^-, \bar{Q}^-. \tag{A.7}$$

The coordinate x^+ is treated as time and P^- as a Hamiltonian. The kinematical generators at the surface $x^+=0$ are realized in terms of differential operators as follows

$$\begin{aligned}
&P^+ = \beta\,, \quad P^z = p, \quad P^{\bar{z}} = \bar{p}, \quad J^{z+} = -\beta\frac{\partial}{\partial\bar{p}}, \quad J^{\bar{z}+} = -\beta\frac{\partial}{\partial p}, \\
&J^{-+} = -\frac{\partial}{\partial\beta}\beta - \frac{1}{2}p_\theta\frac{\partial}{\partial p_\theta} + \frac{1}{2}\epsilon_\lambda, \quad J^{z\bar{z}} = p\partial_p - \bar{p}\frac{\partial}{\partial\bar{p}} + \lambda - \frac{1}{2}p_\theta\frac{\partial}{\partial p_\theta} \\
&Q^+ = (-)^{\epsilon_\lambda}\beta\frac{\partial}{\partial p_\theta}, \qquad \bar{Q}^+ = (-)^{\epsilon_\lambda}p_\theta
\end{aligned} \tag{A.8}$$

For the dynamical operators at $x^+=0$ one has

$$\begin{aligned}
H &= -\frac{p\bar{p}}{\beta}\,, \\
J^{z-} &= -\frac{\partial}{\partial\bar{p}}\frac{p\bar{p}}{\beta} + p\frac{\partial}{\partial\beta} - \left(\lambda - \frac{1}{2}p_\theta\frac{\partial}{\partial p_\theta}\right)\frac{p}{\beta} + \left(\frac{1}{2}p_\theta\frac{\partial}{\partial p_\theta} - \frac{1}{2}\epsilon_\lambda\right)\frac{p}{\beta}, \\
J^{\bar{z}-} &= -\frac{\partial}{\partial p}\frac{p\bar{p}}{\beta} + \bar{p}\frac{\partial}{\partial\beta} + \left(\lambda - \frac{1}{2}p_\theta\frac{\partial}{\partial p_\theta}\right)\frac{\bar{p}}{\beta} + \left(\frac{1}{2}p_\theta\frac{\partial}{\partial p_\theta} - \frac{1}{2}\epsilon_\lambda\right)\frac{p}{\beta}, \\
Q^- &= (-)^{\epsilon_\lambda}\frac{p}{\beta}p_\theta, \qquad \bar{Q}^- = (-)^{\epsilon_\lambda}\bar{p}\frac{\partial}{\partial p_\theta}.
\end{aligned} \tag{A.9}$$

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From the explicit realization of the generators (A.8)–(A.9) and the explicit form of the higher spin superfields (2.1), (2.2), (2.12) one can find transformation rules for the component fields. For example for supersymmetry transformations we have

$$\delta\phi_s = \left(\bar{\epsilon}^- + \frac{p}{\beta}\,\epsilon^+\right)\phi_{s-\frac{1}{2}}\,, \qquad \delta\phi_{s-\frac{1}{2}} = -(\bar{\epsilon}^-\beta + \epsilon^+\bar{p})\phi_s \tag{A.10}$$

as well as similar expressions of the component fields in (2.2).

For the subsequent calculations it is simpler to perform a partial Fourier transform with respect to γ and consider the fields on the surface $x^+ = 0$. To quadratic order in superfields, the Poincaré algebra is realised by the expressions

$$G_2 = \sum_{\lambda=-\infty}^{\infty} \int \beta\, d^3p\; dp_\theta (-)^{\epsilon_\lambda} \mathrm{Tr}\left[\Theta_{\lambda-\frac{1}{2}}(\boldsymbol{p}, p_\theta)\, G\, \Theta_{-\lambda}(-\boldsymbol{p}, -p_\theta)\right], \tag{A.11}$$

where $d^3p = d\beta\, dp\, d\bar{p}$ and G collectively denotes differential operators given in (A.8)–(A.9).

In the chiral theory one keeps the operators $\bar{Q}^-$ and $J^{\bar{z}-}$ quadratic in the fields also at the interaction level, and modifies the other dynamical generators as

$$H_3 = H_2 + \int d\Gamma_{[3]}\, \Theta^{\lambda_1\lambda_2\lambda_3}_{q_1q_2q_3}\, h^{q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}\,, \tag{A.12}$$

$$Q_3^- = Q_2^- + \int d\Gamma_{[3]}\, \Theta^{\lambda_1\lambda_2\lambda_3}_{q_1q_2q_3}\, q^{q_1q_2q_3,}_{\lambda_1\lambda_2\lambda_3}\,, \tag{A.13}$$

$$\begin{aligned} J_3^{z-} = J_2^{z-} + \int d\Gamma_{[3]} \\ \times\left[\Theta^{\lambda_1\lambda_2\lambda_3}_{q_1q_2q_3}\, j^{q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3} - \frac{1}{3}\left(\sum_{k=1}^{3}\frac{\partial\Theta^{\lambda_1\lambda_2\lambda_3}_{q_1q_2q_3}}{\partial\bar{q}_k}\right) h^{q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3} - \frac{1}{3}\left(\sum_{k=1}^{3}\frac{\partial\Theta^{\lambda_1\lambda_2\lambda_3}_{q_1q_2q_3}}{\partial q_{\theta,k}}\right) q^{q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}\right]. \end{aligned} \tag{A.14}$$

Here $\Theta^{\lambda_1\lambda_2\lambda_3}_{q_1q_2q_3} \equiv \Theta_{\lambda_1}(\boldsymbol{q}_1, q_{\theta,1})\Theta_{\lambda_2}(\boldsymbol{q}_2, q_{\theta,2})\Theta_{\lambda_3}(\boldsymbol{q}_3, q_{\theta,3})$ and

$$d\Gamma_{[3]} = d\Gamma_{[3,q]}\cdot d\Gamma_{[3,\theta]} = (2\pi)^3 \prod_{k=1}^{3}\frac{d^3q_k}{(2\pi)^{\frac{3}{2}}}\,\delta^3\left(\sum_{i=1}^{3} q_i\right)\cdot \prod_{l=1}^{3} dq^{\mathcal{N}}_{\theta,l}\,\delta^{\mathcal{N}}\left(\sum_{j=1}^{3} q_{\theta,j}\right) \tag{A.15}$$

is an integration measure, written for a generic $\mathcal{N}$. The vertices are determined from the requirement of preservation of the super Poincaré algebra at the cubic level in the superfields [30, 31]. Their explicit form is found to be

$$h^{q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3} = C^{\lambda_1\lambda_2\lambda_3}(\overline{\mathbb{P}})^{\lambda_1+\lambda_2+\lambda_3+1}\prod_{i=1}^{3}\beta_i^{-\lambda_i-\frac{1}{2}\epsilon_{\lambda_i}}\,, \tag{A.16}$$

$$q^{q_1q_2q_3,}_{\lambda_1\lambda_2\lambda_3} = -C^{\lambda_1\lambda_2\lambda_3}(\overline{\mathbb{P}})^{\lambda_1+\lambda_2+\lambda_3}\mathbb{P}_\theta\prod_{i=1}^{3}\beta_i^{-\lambda_i-\frac{1}{2}\epsilon_{\lambda_i}}\,, \tag{A.17}$$

$$j^{q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3} = 2C^{\lambda_1\lambda_2\lambda_3}(\overline{\mathbb{P}})^{\lambda_1+\lambda_2+\lambda_3}\,\chi\prod_{i=1}^{3}\beta_i^{-\lambda_i-\frac{1}{2}\epsilon_{\lambda_i}}\,, \tag{A.18}$$

where

$$\chi = \beta_1(\lambda_2-\lambda_3) + \beta_2(\lambda_3-\lambda_1) + \beta_3(\lambda_1-\lambda_2)\,. \tag{A.19}$$

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As usual, the explicit form of the coupling constants $C^{\lambda_1\lambda_2\lambda_3}$ is not determined at the level of cubic interactions. The restriction on the coupling constants comes from the further requirement, that the (super)Poincaré algebra is preserved at all orders in (super)fields, without adding of quartic and higher order vertices to the dynamical generators. In other words, one has to find an expression for coupling constants, that keeps the equations

$$[Q_3^-, P_3^-] = 0, \quad [J_3^{z,-}, P_3^-] = 0 \tag{A.20}$$

intact also at the quartic level. Let us consider the first equation. Using the expressions (A.16) and (A.17), one can see that

$$[Q_3^-, P_3^-] \sim \overline{\mathbb{P}} \sum_{\lambda_1\lambda_2\lambda_3} \sum_{\tau_1\tau_2\tau_3} C^{\lambda_1\lambda_2\lambda_3} C^{\tau_1\tau_2\tau_3} \times \left[\prod_{i=1}^{3} \beta_i^{-\lambda_i - \frac{1}{2}\epsilon_{\lambda_i}} \Theta^{\lambda_1\lambda_2\lambda_3} \mathbb{P}^{\lambda_1+\lambda_2+\lambda_3}, \prod_{i=1}^{3} \beta_i^{-\tau_i - \frac{1}{2}\epsilon_{\tau_i}} \Theta^{\tau_1\tau_2\tau_3} \mathbb{P}^{\tau_1+\tau_2+\tau_3} \right] \tag{A.21}$$

which is zero due to the antisymmetry of the Poisson bracket.

Now let us consider the second equation in (A.20). In the same way as for the bosonic Chiral HiSGRA [7], one can show that the sum of the first two terms in the nonlinear part of J_3^{z-} gives zero Poisson bracket with P_3^-, provided the coupling constants have the form (2.11). After integrating by parts in the third term of the nonlinear part of J_3^{z-}, one can see, that its Poisson bracket with P_3^- is zero by the same argument as in (A.21).

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