Gravitational waves from the phase transition in the B-LSSM

Xing-Xing Dong,^{a,b} Tai-Fu Feng,^{a,b,c} Hai-Bin Zhang,^{a,b} Shu-Min Zhao^{a,b}
and Jin-Lei Yang^{d,e}

^aDepartment of Physics, Hebei University,
Baoding 071002, China

^bKey Laboratory of High-precision Computation and Application of Quantum Field Theory
of Hebei Province, Baoding, China

^cDepartment of Physics, Chongqing University,
Chongqing 401331, China

^dCAS Key Laboratory of Theoretical Physics,
Institute of Theoretical Physics, Chinese Academy of Sciences,
Beijing 100190, China

^eSchool of Physical Sciences, University of Chinese Academy of Sciences,
Beijing 100049, China

E-mail: dx_x_0304@163.com, fengtf@hbu.edu.cn, hbzhang@hbu.edu.cn,
zhaosm@hbu.edu.cn, yangjinlei@itp.ac.cn

ABSTRACT: Based on the gauge symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$, the minimal supersymmetric extension of the SM with local B-L gauge symmetry (B-LSSM) has been introduced. In this model, we study the Higgs masses with the one-loop zero temperature effective potential corrections. Besides, the finite temperature effective potentials connected with two $U(1)_{B-L}$ Higgs singlets are deduced specifically. Then we can obtain the gravitational wave spectrums generated from the strong first-order phase transition. In the B-LSSM, with the fine-tuned parameter regions, we can obtain the strength parameter $\alpha_\theta \sim 0.14$ and the ratio of speed to Hubble rate $\beta/H_n \sim 5$ at nucleation temperature, and then obtain observable gravitational wave signals. The gravitational wave signals can be as strong as $h^2\Omega_{GW} \sim 10^{-9}$, which may be detectable in the future experiments.

KEYWORDS: Supersymmetry Phenomenology

ARXIV EPRINT: [2106.11084](https://arxiv.org/abs/2106.11084)

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1 Introduction

Based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, the standard model(SM) has been successfully established. Nevertheless, the SM possesses some limitation and additional $U(1)_{B-L}$ gauge interaction will be a promising extension to the SM. Therefore, the minimal supersymmetric extension of the SM(MSSM) [1–4] with local B-L gauge symmetry (B-LSSM) is introduced, where B represents baryon number and L stands for lepton number [5–9]. In this model, the invariance under $U(1)_{B-L}$ gauge group imposes the R-parity

conservation to avoid proton decay [10]. Besides, the right-handed neutrino superfields have been imported in the B-LSSM to obtain the tiny neutrino masses through type-I seesaw mechanism. Thus, B-LSSM provides an elegant solution to the existence of the light left-handed neutrino. Then the possibility of baryogenesis via leptogenesis will be well explained. Furthermore, due to the introduction of the additional singlet Higgs states and right-handed (s)neutrinos, B-LSSM alleviates the hierarchy problem through additional parameter space released from the LEP, Tevatron and LHC constraints [11, 12]. Other than this, the model can also provide much more dark matter candidates [13–16] than the MSSM.

The gravitational wave(GW) signals have been detected at the Laser Interferometer Gravitational Wave Observer (LIGO) [17, 18], which urges physicists to explore the various universe mysteries. The sources of GW signals can be sensitive to arise from the first-order phase transition(PT) in the early universe [19–23]. The symmetric broken phase is produced in the form of bubbles. When the universe cools down to the nucleation temperature, bubbles of broken phase may nucleate in the background of symmetric phase. Bubbles expand, collide, merge and finally fill the whole universe to finish the first-order PT. Then, the stochastic GW signals can be generated through the bubble collisions, sound waves after the bubble collision and magnetohydrodynamic turbulent of the surrounding plasma [22, 24, 25].

Unfortunately, the electroweak PT in the SM turns out to be too weak to obtain GW signals [26–28]. In order to realize strong first-order PT and finally obtain the GW spectrums, physicists have studied various extensions of the SM, such as the model with a dimension-six operator [29–31], NMSSM [32–34] and other studies [35–37]. In this paper, we hope to investigate the GW spectrums produced from the first-order PT of the B-LSSM. We find that the strength of GW signals will be up to $h^2\Omega_{GW} \sim 10^{-9}$, which can be detected by the future experiments, such as the LISA with N2A5M5L6 design configurations, the Big Bang Observer(BBO), DECI-hertz Interferometer Observatory(DECIGO) and Ultimate-DECIGO [38, 39].

In this paper, after discussing the Higgs mass, we mainly study the GW spectrums generated from the strong first-order PT of the B-LSSM. We first introduce characteristics of the B-LSSM, then we discuss the Higgs masses corrected by the one-loop zero temperature effective potential in section II. In section III, we compute the concrete finite temperature effective potential associated with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$. Meanwhile, we derive the GW generation by cosmological first-order PT in section IV. The GW spectrums are comprised by the bubble collision, sound wave and turbulence contributions. The numerical results of the GW spectrums that depend on the model parameters will be further illustrated in section V. Last but not least, we summarize the conclusion in section VI. The anomaly free of B-LSSM will be demonstrated in appendix A. The vacuum of eq.2 is indeed the global one and there is no charge breaking nor CP violating deeper minima, which will be demonstrated in appendix B. The value of parameter g_{YB} will be discussed in appendix C. The neutralino mass matrix will be searched in appendix D. The one-loop correction through the one-loop zero temperature effective potential $A_{\text{one-loop}} \equiv \frac{B_\eta + \Delta_{B\eta}}{\sqrt{(m_\eta^2 + \mu_\eta^2 + \Delta_{11})(m_\eta^2 + \mu_\eta^2 + \Delta_{22})}}$ will be deduced in appendix E.

2 The B-LSSM and the Higgs mass

2.1 The B-LSSM

In 1967, Sakharov proposed three necessary conditions for dynamics to produce asymmetry between matter and anti-matter in the universe [40]: (1) baryon number(B) non-conservation; (2) charge conjugation(C) transformation and charge conjugate-parity(CP) joint transformation non-conservation; (3) The system deviation from thermal equilibrium. Through a lot of research, people realize that the SM can satisfy Sakharov's three conditions. However, the quantitative calculation shows that the CP violation in CKM matrix is very small and far from producing the observed matter-antimatter asymmetry in the present SM. The literature [41] has pointed out that the CP violation from the CKM matrix only could be sufficient in models where the SM Yukawa couplings are field-dependent and evolve during the electroweak PT. In addition, in order to produce strong first-order electroweak PT, the mass of Higgs particle in the SM must be less than 66 GeV [42–44]. We study the ref. [45], and find a lower bound of the mass of the SM Higgs boson $m_h^0 > 114.4 \text{ GeV}$ by the experimental LEP II. So the possibility of a first-order electroweak PT within the SM has been ruled out by Higgs searches at the LEP experiment. In 2012, the mass of physical Higgs particle was found to be around 125 GeV by the European Large Hadron Collider (LHC) [46, 47], which directly denied the possibility of realizing the electroweak baryon number production mechanism (baryogenesis) in the SM. Therefore, the current SM can not explain the matter and anti-matter asymmetries in the universe.

In any models with a non-anomalous global $U(1)_{B-L}$ symmetry, which includes the B-LSSM we studied, the baryon number and lepton number are broken by Sphaleron process respectively, but the difference between the two is conserved. This links the change of the baryon number with the change of the lepton number, and the asymmetry of the baryon number can be converted from the asymmetry of the lepton number through the sphaleron process. In general, the mechanism of lepton number asymmetry (leptogenesis) requires the lepton number violating process, the C and CP destruction of the lepton part and the realization of the non-equilibrium state. These conditions can be achieved in the general models within the mass neutrino. The B-LSSM introduces the right-handed neutrinos. Besides, neutrinos are Majorana type and gain mass through the type-I seesaw mechanism, which breaks the lepton number symmetry. In B-LSSM, the C or CP symmetry is broken. In addition, the decoupling of heavy right-handed neutrinos provides non-equilibrium conditions. Sphaleron process only acts directly on left-handed fermions, and can partially convert the lepton number of left-handed leptons to the baryon number, thus explains the baryon number asymmetry. Therefore, the possibility of baryogenesis via leptogenesis can be realized within the B-LSSM.

In the B-LSSM, the local gauge group is defined as $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$. The specification of the quantum numbers of fields in the B-LSSM will be discussed in the table 1. Besides, B-LSSM is assumed the gauged one, and we have proved carefully that this model is anomaly free. The concrete demonstrations will be given out in the appendix A. B-LSSM introduces $U(1)_{B-L}$ gauge field, right-handed neutrinos ν_i^c and their superpartners. The masses of right-handed neutrinos will be constructed by $Y_{x,ij} \hat{\nu}_i^c \hat{\eta} \hat{\nu}_j^c$. Then we can

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(U(1)_Y \otimes SU(2) \otimes SU(3) \otimes U(1)_{B-L})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, 0)$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1}, 0)$
$\hat{Q}$	$\tilde{Q}$	Q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3}, \frac{1}{6})$
$\hat{L}$	$\tilde{L}$	L	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, -\frac{1}{2})$
$\hat{D}$	$\tilde{D}_R^*$	D_R^*	3	$(\frac{1}{3}, \mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{6})$
$\hat{U}$	$\tilde{U}_R^*$	U_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{6})$
$\hat{R}$	$\tilde{R}^*$	R^*	3	$(1, \mathbf{1}, \mathbf{1}, \frac{1}{2})$
$\hat{\nu}$	$\tilde{\nu}_R^*$	ν_R^*	3	$(0, \mathbf{1}, \mathbf{1}, \frac{1}{2})$
$\hat{\eta}$	η	$\tilde{\eta}$	1	$(0, \mathbf{1}, \mathbf{1}, -1)$
$\hat{\bar{\eta}}$	$\bar{\eta}$	$\tilde{\bar{\eta}}$	1	$(0, \mathbf{1}, \mathbf{1}, 1)$

Table 1. The quantum numbers of fields in the B-LSSM.

obtain the tiny neutrinos masses through the type-I seesaw mechanism after the right-handed neutrinos and left-handed neutrinos mixing together. The B-LSSM superpotential is deduced as

$$W_{B-L} = \mathcal{W}_{MSSM} - \mu_\eta \hat{\eta} \hat{\bar{\eta}} + Y_{x,ij} \hat{\nu}_i^c \hat{\eta} \hat{\nu}_j^c + Y_{\nu,ij} \hat{L}_i \hat{H}_u \hat{\nu}_j^c, \quad (2.1)$$

where $\mathcal{W}_{MSSM}$ is the superpotential of MSSM. i, j represent the generation indices, $Y_{x,ij}$ and $Y_{\nu,ij}$ correspond to the Yukawa coupling coefficients. μ_η , considered as a parameter with mass dimension, is the supersymmetric mass between $U(1)_{B-L}$ Higgs singlets η and $\bar{\eta}$.

In the B-LSSM, the Higgs doublets and Higgs singlets obtain the nonzero vacuum expectation values, which break the gauge group $SU(3) \otimes U(1)_Y \otimes U(1)_{B-L}$ down to $U(1)_{em}$ at zero temperature. For the model's parameters values considered in our analyses, the vacuum of eq. 2.2 is indeed the global one and that there is no charge breaking nor CP violating deeper minima. The concrete discussion will be shown in appendix B.

$$\begin{aligned} H_d^0 &= \frac{1}{\sqrt{2}}(\phi_d + v_d + i\sigma_d), & H_u^0 &= \frac{1}{\sqrt{2}}(\phi_u + v_u + i\sigma_u), \\ \eta &= \frac{1}{\sqrt{2}}(\phi_\eta + v_\eta + i\sigma_\eta), & \bar{\eta} &= \frac{1}{\sqrt{2}}(\phi_{\bar{\eta}} + v_{\bar{\eta}} + i\sigma_{\bar{\eta}}), \end{aligned} \quad (2.2)$$

where $\phi_d, \phi_u, \phi_\eta, \phi_{\bar{\eta}}$ represent the CP-even Higgs components and $\sigma_d, \sigma_u, \sigma_\eta, \sigma_{\bar{\eta}}$ correspond to the CP-odd Higgs components. The VEVs of the $U(1)_{B-L}$ Higgs singlets $\hat{\eta}$ and $\hat{\bar{\eta}}$ satisfy $u = \sqrt{v_\eta^2 + v_{\bar{\eta}}^2}$. While the VEVs of the Higgs doublets $\hat{H}_d$ and $\hat{H}_u$ are v_d and v_u , which satisfy $v = \sqrt{v_d^2 + v_u^2}$. We take $tb' = \frac{v_{\bar{\eta}}}{v_\eta}$ by analogy to the definition $tb = \frac{v_u}{v_d}$ in the MSSM.

2.2 Higgs mass in the B-LSSM

First, in the base $(\phi_d, \phi_u, \phi_\eta, \phi_{\bar{\eta}})$, the tree level mass squared matrix for neutral CP-even scalars M_h^2 is deduced as:

$$M_h^2 = u^2 \times \begin{pmatrix} \frac{1}{4} \frac{g^2 x^2}{1+tb^2} + n^2 tb & -\frac{1}{4} g^2 \frac{x^2 tb}{1+tb^2} - n^2 & \frac{1}{2} g_B g_{YB} \frac{x}{T} & -\frac{1}{2} g_B g_{YB} \frac{x \times tb'}{T} \\ -\frac{1}{4} g^2 \frac{x^2 tb}{1+tb^2} - n^2 & \frac{1}{4} \frac{g^2 x^2 tb^2}{1+tb^2} + \frac{n^2}{tb} & \frac{1}{2} g_B g_{YB} \frac{x \times tb}{T} & \frac{1}{2} g_B g_{YB} \frac{x \times tb \times tb'}{T} \\ \frac{1}{2} g_B g_{YB} \frac{x}{T} & \frac{1}{2} g_B g_{YB} \frac{x \times tb}{T} & \frac{g_B^2}{1+tb'^2} + tb' N^2 & -g_B^2 \frac{tb'}{1+tb'^2} - N^2 \\ -\frac{1}{2} g_B g_{YB} \frac{x \times tb'}{T} & \frac{1}{2} g_B g_{YB} \frac{x \times tb \times tb'}{T} & -g_B^2 \frac{tb'}{1+tb'^2} - N^2 & g_B^2 \frac{tb'^2}{1+tb'^2} + \frac{N^2}{tb'} \end{pmatrix}. \quad (2.3)$$

Here, $g^2 = g_1^2 + g_2^2 + g_{YB}^2$, $T = \sqrt{1+tb^2} \sqrt{1+tb'^2}$, $n^2 = \frac{\text{Re}(B\mu)}{u^2}$ and $N^2 = \frac{\text{Re}(B_\eta)}{u^2}$.

Then, we will consider the radiative corrections $\Delta\Pi$ from the one-loop zero temperature effective potential to the tree level Higgs mass squared matrix.

$$m_h^2 = M_h^2 + \Delta\Pi, \quad \Delta\Pi = \begin{pmatrix} \Delta\Pi_{11} & \Delta\Pi_{12} & \Delta\Pi_{13} & \Delta\Pi_{14} \\ \Delta\Pi_{21} & \Delta\Pi_{22} & \Delta\Pi_{23} & \Delta\Pi_{24} \\ \Delta\Pi_{31} & \Delta\Pi_{32} & \Delta\Pi_{33} & \Delta\Pi_{34} \\ \Delta\Pi_{41} & \Delta\Pi_{42} & \Delta\Pi_{43} & \Delta\Pi_{44} \end{pmatrix}, \quad (2.4)$$

with $\Delta\Pi_{ii} = [-\frac{1}{\phi_i} \frac{\partial \Delta V_1}{\partial \phi_i} + \frac{\partial^2 \Delta V_1}{\partial \phi_i^2}]_{\phi_i=v_i}$ and $\Delta\Pi_{ij} = [\frac{\partial^2 \Delta V_1}{\partial \phi_i \partial \phi_j}]_{\phi_i=v_i, \phi_j=v_j}$, $v_i, v_j \in (v_d, v_u, v_\eta, v_{\bar{\eta}})$ and $\phi_{i,j}$ are the CP-even Higgs components $\phi_d, \phi_u, \phi_\eta, \phi_{\bar{\eta}}$. ΔV_1 represents the one-loop zero temperature effective potential whose full form has been discussed in the literature [48]. In principle, the radiative correction is dominated by the contributions of top quark, bottom quark, stop quarks and sbottom quarks.

$$\Delta V_1 = -\frac{3}{(4\pi)^2} \left[\frac{m_t^4}{4} \left(\ln \left(\frac{m_t^2}{\Lambda^2} \right) - \frac{3}{2} \right) - \frac{1}{2} \frac{m_t^4}{4} \left(\ln \left(\frac{m_t^2}{\Lambda^2} \right) - \frac{3}{2} \right) \right] \\ - \frac{3}{(4\pi)^2} \left[\frac{m_b^4}{4} \left(\ln \left(\frac{m_b^2}{\Lambda^2} \right) - \frac{3}{2} \right) - \frac{1}{2} \frac{m_b^4}{4} \left(\ln \left(\frac{m_b^2}{\Lambda^2} \right) - \frac{3}{2} \right) \right]. \quad (2.5)$$

Here, the masses of top quark and bottom quark are respectively $m_t = \frac{1}{\sqrt{2}} Y_t \phi_u$ and $m_b = \frac{1}{\sqrt{2}} Y_b \phi_d$. Additionally, the mass squared matrix of stop quark and sbottom quark will be deduced respectively in the basis $(\tilde{U}_L, \tilde{U}_R)$ and $(\tilde{D}_L, \tilde{D}_R)$.

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 & m_{\tilde{t}_{LR}}^2 \\ m_{\tilde{t}_{RL}}^2 & m_{\tilde{t}_R}^2 \end{pmatrix}, \quad m_{\tilde{b}}^2 = \begin{pmatrix} m_{\tilde{b}_L}^2 & m_{\tilde{b}_{LR}}^2 \\ m_{\tilde{b}_{RL}}^2 & m_{\tilde{b}_R}^2 \end{pmatrix}, \quad (2.6)$$

where,

$$\begin{aligned}
 m_{\tilde{t}_L}^2 &= +\frac{1}{24} \left[3g_2^2 \left(-\phi_u^2 + \phi_d^2 \right) + \left(g_1^2 + g_{YB}^2 \right) \left(-\phi_d^2 + \phi_u^2 \right) - 2g_B^2 \left(-\phi_{\tilde{\eta}}^2 + \phi_{\eta}^2 \right) \right. \\
 &\quad \left. + g_{YB}g_B \left(2\phi_{\tilde{\eta}}^2 - 2\phi_{\eta}^2 - \phi_d^2 + \phi_u^2 \right) \right] + \frac{1}{2} \left(2m_{\tilde{Q}_3}^2 + \phi_u^2 Y_t^\dagger Y_t \right), \\
 m_{\tilde{t}_R}^2 &= +\frac{1}{24} \left[2g_B^2 \left(-\phi_{\tilde{\eta}}^2 + \phi_{\eta}^2 \right) + 4 \left(g_1^2 + g_{YB}^2 \right) \left(-\phi_u^2 + \phi_d^2 \right) \right. \\
 &\quad \left. + g_{YB}g_B \left(-8\phi_{\tilde{\eta}}^2 + 8\phi_{\eta}^2 - \phi_u^2 + \phi_d^2 \right) \right] + \frac{1}{2} \left(2m_{\tilde{U}_3}^2 + \phi_u^2 Y_t Y_t^\dagger \right), \\
 m_{\tilde{t}_{RL}}^2 &= (m_{\tilde{t}_{LR}}^2)^\dagger = \frac{1}{\sqrt{2}} (\phi_u T_t - \phi_d \mu^* Y_t). \tag{2.7}
 \end{aligned}$$

Similarly, the elements in the sbottom mass squared matrix are

$$\begin{aligned}
 m_{\tilde{b}_L}^2 &= +\frac{1}{24} \left[-2g_B^2 \left(-\phi_{\tilde{\eta}}^2 + \phi_{\eta}^2 \right) + \left(3g_2^2 + g_1^2 + g_{YB}^2 \right) \left(-\phi_d^2 + \phi_u^2 \right) \right. \\
 &\quad \left. + g_{YB}g_B \left(2\phi_{\tilde{\eta}}^2 - 2\phi_{\eta}^2 - \phi_d^2 + \phi_u^2 \right) \right] + \frac{1}{2} \left(2m_{\tilde{Q}_3}^2 + \phi_d^2 Y_b^\dagger Y_b \right), \\
 m_{\tilde{b}_R}^2 &= +\frac{1}{24} \left[2 \left(\left(g_1^2 + g_{YB}^2 \right) \left(-\phi_d^2 + \phi_u^2 \right) + g_B^2 \left(-\phi_{\tilde{\eta}}^2 + \phi_{\eta}^2 \right) \right) \right. \\
 &\quad \left. + g_{YB}g_B \left(4\phi_{\tilde{\eta}}^2 - 4\phi_{\eta}^2 - \phi_u^2 + \phi_d^2 \right) \right] + \frac{1}{2} \left(2m_{\tilde{D}_3}^2 + \phi_d^2 Y_b Y_b^\dagger \right), \\
 m_{\tilde{b}_{RL}}^2 &= (m_{\tilde{b}_{LR}}^2)^\dagger = \frac{1}{\sqrt{2}} \left(\phi_d T_b - \phi_u Y_b \mu^* \right). \tag{2.8}
 \end{aligned}$$

Then, the mass eigenvalues of stop and sbottom quarks will be given by

$$\begin{aligned}
 m_{\tilde{t}_{1,2}}^2 &= \frac{1}{2} (m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2) \pm \frac{1}{2} [(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)^2 + 4m_{\tilde{t}_{LR}}^2 m_{\tilde{t}_{RL}}^2]^{\frac{1}{2}}, \\
 m_{\tilde{b}_{1,2}}^2 &= \frac{1}{2} (m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2) \pm \frac{1}{2} [(m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2)^2 + 4m_{\tilde{b}_{LR}}^2 m_{\tilde{b}_{RL}}^2]^{\frac{1}{2}}. \tag{2.9}
 \end{aligned}$$

Therefore, the radiative corrections $\Delta\Pi$ from the one-loop zero temperature effective potential will be deduced as:

$$\begin{aligned}
 \Delta\Pi_{ii} &= \frac{3}{(4\pi)^2} \left[\sum_{k=\tilde{t},b} \left(\frac{2}{\phi_i} f(m_k^2) \frac{\partial m_k^2}{\partial \phi_i} - 2f(m_k^2) \frac{\partial^2 m_k^2}{\partial \phi_i^2} - \frac{1}{2} \ln \left(\frac{m_k^2}{\Lambda^2} \right) \frac{\partial m_k^2}{\partial \phi_i} \frac{\partial m_k^2}{\partial \phi_i} \right) \right. \\
 &\quad \left. + \sum_{k=\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2} \left(-\frac{1}{\phi_i} f(m_k^2) \frac{\partial m_k^2}{\partial \phi_i} + f(m_k^2) \frac{\partial^2 m_k^2}{\partial \phi_i^2} + \frac{1}{4} \ln \left(\frac{m_k^2}{\Lambda^2} \right) \frac{\partial m_k^2}{\partial \phi_i} \frac{\partial m_k^2}{\partial \phi_i} \right) \right], \\
 \Delta\Pi_{ij} &= \frac{3}{(4\pi)^2} \sum_{k=\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2} \left(f(m_k^2) \frac{\partial^2 m_k^2}{\partial \phi_i \partial \phi_j} + \frac{1}{4} \ln \left(\frac{m_k^2}{\Lambda^2} \right) \frac{\partial m_k^2}{\partial \phi_i} \frac{\partial m_k^2}{\partial \phi_j} \right). \tag{2.10}
 \end{aligned}$$

Here, $f(m_k^2) = \frac{1}{4} m_k^2 [\ln(\frac{m_k^2}{\Lambda^2}) - 1]$, Λ is the renormalization scale and we take $\Lambda = 1$ TeV in the following numerical discussion. m_k represent the corresponding particle masses. The square matrix m_h^2 will be diagonalized to the mass eigenstate by the unitary matrix Z_{h_i} .

2.3 Numerical discussion of the Higgs mass in the B-LSSM

The mass of physical Higgs boson reads $m_{h_0} = 125.1 \pm 0.14$ GeV by the latest LHC experiments [49]. We consider the impact of LHC constraints from the light top and bottom squarks. Limits are considered in the supersymmetric models at the 95% confidence level, and bottom squarks with mass up to 1.5 TeV are excluded [50]. The results are interpreted in simplified models featuring direct production of the light top squark, excluding light top squark masses up to about 1.22 TeV at 95% confidence level [51]. In our recently published article [52], we had studied the CP-even Higgs boson mass through the fine-tuning in both the MSSM and B-LSSM. We found the fine-tuning measure was $\Delta_{FT} \gtrsim 35$ ($\lesssim 2.9\%$ fine tuning) according to $m_{\tilde{t}_1} \gtrsim 1.5$ TeV in the MSSM. However, the fine-tuning measure in the B-LSSM was $\Delta_{FT} \gtrsim 22$ ($\lesssim 4.5\%$ fine tuning) according to $m_{\tilde{t}_1} \gtrsim 1.5$ TeV. We prefer to a small fine-tuning measure Δ_{FT} , which can be realized in B-LSSM. Therefore, the physical Higgs boson mass, which is mainly affected by stop quark masses, will be more naturally obtained in the B-LSSM.

The updated experimental data from run II searches at 13 TeV with 139/fb statistics indicates the mass of Z' boson satisfies $M'_Z \geq 5.1$ TeV with 95% confidence level (CL) [53]. Refs. [54, 55] give us an upper bound on the ratio between the mass of Z' boson and its gauge coupling at 99% CL as $\frac{M'_Z}{g_B} \geq 6$ TeV. We choose $M'_Z = 5.1$ TeV in our numerical calculation, so the value of parameter g_B is restricted in the region of $0 < g_B \leq 0.85$. The Yukawa coupling Y_b , determined by the parameter tb , is defined as $Y_b = \sqrt{2(tb^2 + 1)}m_b/v$. In general, the value of Y_b is smaller than 1, $m_b \simeq 4.18$ GeV and $v \simeq 246$ GeV, so the parameter tb should be approximatively smaller than 40. Besides, the large tb has been excluded by the $\bar{B} \rightarrow X_s \gamma$ experiment [56, 57]. The coupling parameter g_{YB} will be taken around $-0.45 \sim -0.05$, and the reason why constant g_{YB} is negative has been discussed specifically in appendix C [58]. In addition, LHC searches constrain $tb' < 1.5$ [58, 59].

We reanalyze the influence of free parameters on Higgs mass. Let the free parameters be randomly valued within the reasonable intervals: $tb \in (5, 40)$, $tb' \in (1.02, 1.5)$, $g_B \in (0.1, 0.9)$, $g_{YB} \in (-0.8, 0.05)$, $B_\eta \in (0.405, 1.6)$ TeV², $m_{\tilde{t}}, m_{\tilde{b}} \in (1.6, 4)$ TeV and $\mu_\eta, A_t, A_b \in (0.2, 2)$ TeV, we analyze the effect of parameters on the mass of Higgs boson. We consider the following constraints on the numerical results: $B \rightarrow X_s \gamma$, $(g-2)_\mu$ and Higgs decay ratio all satisfy the experimental deviation of 2σ , physical Higgs boson mass satisfies the experimental deviation of 3σ , and the second light Higgs boson mass is greater than 350 GeV. At this time, we find some sensitive free parameters. As shown in figure 1, we first analyze the changes of parameters tb versus tb' and g_B versus g_{YB} . It is not difficult to find that when the corresponding experimental limitations are met, $14 < tb < 40$ and $1.3 < tb' < 1.5$. Besides, the range of g_{YB} is not limited by the constraints considered above when $g_B > 0.6$. When g_{YB} approaches to bigger than -0.4, the allowed range of g_B reads $0.1 < g_B < 0.8$.

Then, we analyze the influence on the mass of the second light Higgs boson versus the parameters tb' and B_η respectively in figure 2. Parameters tb' and B_η both affect the second light Higgs boson mass smartly. The larger tb' , the larger mass of the second light Higgs boson we can obtain. Parameter B_η possesses the same character as parameter tb' .

Figure 1. The changes of parameters tb versus tb' and g_B versus g_{YB} .

Figure 2. The second light Higgs boson masses change with the parameters tb' and B_η respectively.

3 Finite temperature effective potential in the B-LSSM

In the B-LSSM, in the basis $(B_\mu, W_\mu^3, B_\mu'')$, we can obtain the corresponding gauge boson mass matrix. Then we deduce the exact eigenvalues of Z' boson: $m_{Z'}^2 = \frac{1}{8} \left((g_1^2 + g_2^2 + g_{YB}^2) v^2 + 4g_B^2 u^2 + \sqrt{(g_1^2 + g_2^2 + g_{YB}^2)^2 v^4 + 8(g_{YB}^2 - g_1^2 - g_2^2)g_B^2 v^2 u^2 + 16g_B^4 u^4} \right)$. The mass of Z' boson will be reduced as $m_{Z'}^2 \simeq \frac{1}{4} g_{YB}^2 v^2 + g_B^2 u^2$. $v \simeq 246$ GeV, $-0.45 < g_{YB} < -0.05$, $m_{Z'} > 5.1$ TeV is restricted by the run II searches at 13TeV with 139/fb statistics and $0 < g_B < 0.85$ is limited through refs. [54, 55]. The value of $U(1)_{B-L}$ breaking Higgs singlets VEV u is $u \simeq m_{Z'}/g_B \simeq 10$ TeV, which is much larger than the Higgs doublets VEV $v \simeq 246$ GeV. When the temperature of the universe is higher than the scale of the new physics, both $U(1)_{B-L}$ Higgs singlets and Higgs doublets are supposed to be trapped at the origins of their potential. As the temperature drops down to the scale of the new physics, the first-order PT with the two Higgs singlets may occur because the scale of Higgs potential becomes the new physics one. The scale of new physics is much higher than the electroweak scale, so we ignore two Higgs doublets part and their interaction terms in the following theoretical analyses and numerical discussions. Then, we consider the tree-level

scalar potential including the two singlet superfields $\hat{\eta}$ and $\hat{\bar{\eta}}$, which will be written as

$$V_0 = \frac{1}{2} \left[g_B \left(\frac{1}{6} Q^{I*} Q^I - \frac{1}{6} D^{I*} D^I - \frac{1}{6} U^{I*} U^I - \frac{1}{2} L^{I*} L^I + \frac{1}{2} E^{I*} E^I + \frac{1}{2} \nu^{I*} \nu^I - \eta^\dagger \eta + \bar{\eta}^\dagger \bar{\eta} \right) \right. \\ \left. + \frac{1}{2} g_{YB} \left(\frac{1}{3} Q^{I*} Q^I + \frac{2}{3} D^{I*} D^I - \frac{4}{3} U^{I*} U^I - L^{I*} L^I + 2 E^{I*} E^I \right) \right]^2 + |\mu_\eta|^2 \eta^\dagger \eta + |\mu_{\bar{\eta}}|^2 \bar{\eta}^\dagger \bar{\eta} \\ - (\mu_\eta Y_x \bar{\eta}^\dagger \nu^I \nu^I + \text{H.c.}) + Y_x^2 |\nu^I \nu^I|^2 + 4 Y_x^2 |\eta \nu^I|^2 + m_\eta^2 \eta^\dagger \eta + m_{\bar{\eta}}^2 \bar{\eta}^\dagger \bar{\eta} - (B_\eta \eta \bar{\eta} + \text{H.c.}). \quad (3.1)$$

Here, the contribution of $-B_\eta \eta \bar{\eta}$ is deduced from the softbreaking terms, and parameter B_η possesses mass square dimension.

3.1 Finite temperature effective potential

The GW spectrum is generated by the first-order PT, which is determined by the finite temperature effective potential. In the B-LSSM, the finite temperature effective potential for both zero and finite temperatures are essential for realizing the first-order PT. We are actually interested in the $\text{Re}(\eta) \equiv \frac{1}{\sqrt{2}} \phi_\eta$ and $\text{Re}(\bar{\eta}) \equiv \frac{1}{\sqrt{2}} \phi_{\bar{\eta}}$. Therefore, the finite temperature effective potential with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$ can be expressed as

$$V_{\text{eff}}(\phi_\eta, \phi_{\bar{\eta}}, T) = V_0(\phi_\eta, \phi_{\bar{\eta}}) + \Delta V_1(\phi_\eta, \phi_{\bar{\eta}}, 0) + \Delta V_1(\phi_\eta, \phi_{\bar{\eta}}, T) + \Delta V_{\text{daisy}}(\phi_\eta, \phi_{\bar{\eta}}, T), \quad (3.2)$$

where $V_0(\phi_\eta, \phi_{\bar{\eta}})$ and $\Delta V_1(\phi_\eta, \phi_{\bar{\eta}}, 0)$ represent the tree-level and one-loop zero temperature effective potential respectively. $\Delta V_1(\phi_\eta, \phi_{\bar{\eta}}, T)$ are the one-loop finite temperature contributions, and $\Delta V_{\text{daisy}}(\phi_\eta, \phi_{\bar{\eta}}, T)$ are the daisy corrections. The concrete expressions will be concluded as [60, 61]

$$V_0(\phi_\eta, \phi_{\bar{\eta}}) = \frac{1}{2} \left[\frac{1}{4} g_B^2 (\phi_\eta^* \phi_\eta - \phi_{\bar{\eta}}^* \phi_{\bar{\eta}})^2 + (|\mu_\eta|^2 + m_\eta^2) \phi_\eta^* \phi_\eta \right. \\ \left. + (|\mu_{\bar{\eta}}|^2 + m_{\bar{\eta}}^2) \phi_{\bar{\eta}}^* \phi_{\bar{\eta}} + (-B_\eta \phi_\eta \phi_{\bar{\eta}} + \text{H.c.}) \right], \\ \Delta V_1(\phi_\eta, \phi_{\bar{\eta}}, 0) = \sum_i (-1)^k \frac{n_i}{64\pi^2} m_i^4(\phi_\eta, \phi_{\bar{\eta}}) \left[\log \frac{m_i^2(\phi_\eta, \phi_{\bar{\eta}})}{\Lambda^2} - c_i \right], \\ \Delta V_1(\phi_\eta, \phi_{\bar{\eta}}, T) = \frac{T^4}{2\pi^2} \sum_i (-1)^k n_i J_i \left[\frac{m_i^2(\phi_\eta, \phi_{\bar{\eta}})}{T^2} \right], \\ \Delta V_{\text{daisy}}(\phi_\eta, \phi_{\bar{\eta}}, T) = -\frac{T}{12\pi} \sum_{i=\text{boson}} n_i [\mathcal{M}_i^3(\phi_\eta, \phi_{\bar{\eta}}, T) - m_i^3(\phi_\eta, \phi_{\bar{\eta}})]. \quad (3.3)$$

m_i^2 are the particle masses square of fermions and bosons with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$. n_i are the degrees of freedom corresponding to the particles. $k = 1(0)$ and $c_i = \frac{3}{2}(\frac{5}{6})$ correspond to fermions (bosons). The thermal function corresponding to boson and fermion particles can be written as $J_{\text{boson}}(x^2) = \int_0^\infty dy y^2 \log(1 - \exp[y^2 + x^2])$ and $J_{\text{fermion}}(x^2) = \int_0^\infty dy y^2 \log(1 + \exp[y^2 + x^2])$. The masses square $\mathcal{M}_i^2$ are obtained from the m_i^2 by adding the T-dependent self-energy corrections [62–65]. We will discuss the $(\phi_\eta, \phi_{\bar{\eta}})$ -dependent particle masses square m_i^2 and $\mathcal{M}_i^2$ specifically as follows.

The $(\phi_\eta, \phi_{\bar\eta})$ -dependent masses square of the CP-even Higgs and CP-odd Higgs are shown as

$$\begin{aligned} m_{\phi_\eta\phi_\eta}^2 &= g_B^2\phi_\eta^2 + B_\eta\frac{\phi_{\bar\eta}}{\phi_\eta}, & m_{\phi_{\bar\eta}\phi_{\bar\eta}}^2 &= g_B^2\phi_{\bar\eta}^2 + B_\eta\frac{\phi_\eta}{\phi_{\bar\eta}}, \\ m_{\sigma_\eta\sigma_\eta}^2 &= B_\eta\frac{\phi_{\bar\eta}}{\phi_\eta}, & m_{\sigma_{\bar\eta}\sigma_{\bar\eta}}^2 &= B_\eta\frac{\phi_\eta}{\phi_{\bar\eta}}. \end{aligned} \quad (3.4)$$

The $(\phi_\eta, \phi_{\bar\eta})$ -dependent masses square of the squarks and sleptons, CP-even and CP-odd sneutrinos can be written as

$$\begin{aligned} m_{\tilde{Q}^I}^2 &= -\frac{1}{12}(g_B^2 + g_B g_{YB})(\phi_\eta^2 - \phi_{\bar\eta}^2), & m_{\tilde{U}^I}^2 &= \frac{1}{12}(g_B^2 + 4g_B g_{YB})(\phi_\eta^2 - \phi_{\bar\eta}^2), \\ m_{\tilde{D}^I}^2 &= \frac{1}{12}(g_B^2 - 2g_B g_{YB})(\phi_\eta^2 - \phi_{\bar\eta}^2); \end{aligned} \quad (3.5)$$

$$\begin{aligned} m_{\tilde{L}^I}^2 &= \frac{1}{4}(g_B^2 + g_B g_{YB})(\phi_\eta^2 - \phi_{\bar\eta}^2), & m_{\tilde{R}^I}^2 &= -\frac{1}{4}(g_B^2 + 2g_B g_{YB})(\phi_\eta^2 - \phi_{\bar\eta}^2), \\ m_{\tilde{\nu}_i}^2 &= m_{\tilde{\nu}_i^R}^2 = -\frac{1}{4}g_B^2(\phi_\eta^2 - \phi_{\bar\eta}^2). \end{aligned} \quad (3.6)$$

In the B-LSSM, there is a $(\phi_\eta, \phi_{\bar\eta})$ -dependent $U(1)_{B-L}$ gauge boson, whose mass square is given by

$$m_{Z'}^2 = g_B^2(\phi_\eta^2 + \phi_{\bar\eta}^2). \quad (3.7)$$

In the basis $(\tilde{B}', \tilde{\eta}, \tilde{\bar\eta})$, we obtain the neutralino mass matrix, which is a Majorana fermionic component.

$$m_{\tilde{\chi}^0} = \begin{pmatrix} 0 & -g_B\phi_\eta & g_B\phi_{\bar\eta} \\ -g_B\phi_\eta & 0 & 0 \\ g_B\phi_{\bar\eta} & 0 & 0 \end{pmatrix}. \quad (3.8)$$

Diagonalizing the $m_{\tilde{\chi}^0}^\dagger m_{\tilde{\chi}^0}$, we can obtain the corresponding mass eigenvalues, which are $g_B^2(\phi_\eta^2 + \phi_{\bar\eta}^2)$, $g_B^2(\phi_\eta^2 + \phi_{\bar\eta}^2)$ and 0.

Then, considering the T^2 -proportional part of the one-loop corrections, we adopt the mass square $\mathcal{M}_i^2$ for particles such as the Higgs boson, squark, slepton, sneutrino and Z' boson. Here, the temperature corrections for the third squarks consider the Yukawa couplings Y_t and Y_b , which are both determined by the parameter tb . Besides, we only

consider the temperature contributions to the longitudinal component of Z' boson.

$$\begin{aligned}\mathcal{M}_{\phi_\eta\phi_\eta}^2 &= m_{\phi_\eta\phi_\eta}^2 + \frac{T^2}{2}g_B^2, \quad \mathcal{M}_{\phi_{\bar{\eta}}\phi_{\bar{\eta}}}^2 = m_{\phi_{\bar{\eta}}\phi_{\bar{\eta}}}^2 + \frac{T^2}{2}g_B^2; \\ \mathcal{M}_{\sigma_\eta\sigma_\eta}^2 &= m_{\sigma_\eta\sigma_\eta}^2 + \frac{T^2}{2}g_B^2, \quad \mathcal{M}_{\sigma_{\bar{\eta}}\sigma_{\bar{\eta}}}^2 = m_{\sigma_{\bar{\eta}}\sigma_{\bar{\eta}}}^2 + \frac{T^2}{2}g_B^2;\end{aligned}\tag{3.9}$$

$$\begin{aligned}\mathcal{M}_{\tilde{Q}^{1,2}}^2 &= m_{\tilde{Q}^{1,2}}^2 + \frac{T^2}{8} \left(\frac{16}{3}g_s^2 + 3g_2^2 + \frac{1}{3}g_1^2 + \frac{1}{36}(4g_B^2 + 4g_{YB}^2 + 7g_Bg_{YB}) \right), \\ \mathcal{M}_{\tilde{Q}^3}^2 &= m_{\tilde{Q}^3}^2 + \frac{T^2}{8} \left(\frac{16}{3}g_s^2 + 3g_2^2 + \frac{1}{3}g_1^2 + 2(Y_t^2 + Y_b^2) + \frac{1}{36}(4g_B^2 + 4g_{YB}^2 + 7g_Bg_{YB}) \right), \\ \mathcal{M}_{\tilde{U}^{1,2}}^2 &= m_{\tilde{U}^{1,2}}^2 + \frac{T^2}{8} \left(\frac{16}{3}g_s^2 + \frac{16}{9}g_1^2 + \frac{1}{9}(g_B^2 + 16g_{YB}^2 + 7g_Bg_{YB}) \right), \\ \mathcal{M}_{\tilde{U}^3}^2 &= m_{\tilde{U}^3}^2 + \frac{T^2}{8} \left(\frac{16}{3}g_s^2 + \frac{16}{9}g_1^2 + 4Y_t^2 + \frac{1}{9}(g_B^2 + 16g_{YB}^2 + 7g_Bg_{YB}) \right), \\ \mathcal{M}_{\tilde{D}^{1,2}}^2 &= m_{\tilde{D}^{1,2}}^2 + \frac{T^2}{8} \left(\frac{16}{3}g_s^2 + \frac{1}{3}g_1^2 + \frac{1}{9} \left(g_B^2 + 4g_{YB}^2 + \frac{5}{2}g_Bg_{YB} \right) \right), \\ \mathcal{M}_{\tilde{D}^3}^2 &= m_{\tilde{D}^3}^2 + \frac{T^2}{8} \left(\frac{16}{3}g_s^2 + \frac{1}{3}g_1^2 + 4Y_b^2 + \frac{1}{9} \left(g_B^2 + 4g_{YB}^2 + \frac{5}{2}g_Bg_{YB} \right) \right);\end{aligned}\tag{3.10}$$

$$\begin{aligned}\mathcal{M}_{L^I}^2 &= m_{L^I}^2 + \frac{T^2}{8} \left(3g_2^2 + g_1^2 + \frac{1}{4}(4g_B^2 + 4g_{YB}^2 + 7g_Bg_{YB}) \right), \\ \mathcal{M}_{R^I}^2 &= m_{R^I}^2 + \frac{T^2}{8} \left(4g_1^2 + \frac{1}{4}(4g_B^2 + 7g_{YB}^2 + 14g_Bg_{YB}) \right), \\ \mathcal{M}_{\tilde{\nu}_i}^2 &= \mathcal{M}_{\tilde{\nu}_i^R}^2 = m_{\tilde{\nu}_i}^2 + \frac{T^2}{8}g_B^2;\end{aligned}\tag{3.11}$$

$$\mathcal{M}_{Z'}^2 = m_{Z'}^2 + \frac{T^2}{2}(2g_B^2 + g_{YB}^2).\tag{3.12}$$

3.2 The PT from the finite temperature effective potential with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$

We analyze the PT from the finite temperature effective potential with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$. In order to discuss the PT easier, we make the approximate assumption as follows: $\frac{\phi_{\bar{\eta}}}{\phi_\eta} = tb'$ and $\phi = \sqrt{\phi_\eta^2 + \phi_{\bar{\eta}}^2}$. Then the finite temperature effective potential $V_{\text{eff}}(\phi_\eta, \phi_{\bar{\eta}}, T)$ will be deduced as $V_{\text{eff}}(\phi, T)$.

With the parameters $tb' = 1.35$, $g_B = 0.5$, $g_{YB} = -0.2$, $\mu_\eta = 600$ GeV, $B_\eta = 1.3$ TeV², $m_\eta = 1.3$ TeV and $m_{\bar{\eta}} = 300$ GeV, we plot the finite temperature effective potential $V_{\text{eff}}(\phi, T)$ changing with ϕ in figure 3 at critical temperature $T_c \simeq 1.65$ TeV. We will easily find that the effective potential $V_{\text{eff}}(\phi, T)$ possesses the minimum both at $\phi = 0$ GeV and $\phi = v_c \simeq 1.75$ TeV when $T_c \simeq 1.65$ TeV. Besides, $v_c/T_c > 1$, which can satisfy the strong first-order PT. The critical temperature is around 1.65 TeV, which is much larger than the electroweak scale, so we mainly discuss the GW from the strong first-order PT with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$. With the temperature continue decreasing and around the electroweak scale, the electroweak symmetry breaks down.

Figure 3. The finite temperature effective potential $V_{\text{eff}}(\phi, T)$ changes with ϕ at $T_c \simeq 1.65\text{TeV}$.

4 GW generation by cosmological first-order PT

The breaking of B-L symmetry not only induces first-order PT, but also induces second-order PT. The first-order PT is from a false vacuum state(symmetry phase) to a true one(symmetry broken phase), while the second-order PT is the process from an unstable state to a vacuum state.

The baryon asymmetry of the universe can be generated via the baryogenesis mechanism, which requires a strong first-order PT to provide a system deviation from thermal equilibrium. First-order PT will generate bubbles, inside which are symmetry broken phases, their externals are the symmetry phases. The universe begins with the symmetry phase, which is a metastable one. With the expansion of the bubbles, the collision and then fusion, the universe will reside at the symmetry broken phase. The GW will be generated along with the collision between bubbles, the plasma movement near the bubble wall disturbed by the expanding bubbles, and the magnetofluid turbulence caused by the plasma. The relative height of potential between false vacuum state and true one will determine the strength of GW. The GW can be detectable with the large enough relative height of potential, and the corresponding PT is called as the strong first-order PT.

The second-order PT can also generate bubbles initially. However, the background is unstable and will quickly roll to the vacuum state, so the bubbles and background will be exactly the same soon. The second-order PT will be more synchronous than the first-order one. Therefore, there are almost no bubble collision, plasma turbulence even the GW in the second-order PT. Therefore, we pay attention to the strong first-order PT to study the GW signals within the B-LSSM.

4.1 Scalar potential parameters related to the GW spectrum

In this section, we briefly discuss the properties of the GWs, which critically depend on two quantities: the strength parameter α_θ , which will be calculated using the trace of energy momentum tensor; β is the speed of the PT at the nucleation temperature T_n . The transition from the symmetric phase to the symmetric broken phase takes place via thermal

tunneling at the finite temperature. First-order PT pushes the bubbles of the symmetric broken phase to be nucleated, then they expand and eventually fill up the entire universe. The bubble nucleation rate per unit volume at the finite temperature is given by

$$\Gamma \sim A(T)e^{-S} = A(T)e^{-S_E/T}. \quad (4.1)$$

$A(T)$ is a factor that is roughly proportional to T^4 . S represents the action in the four-dimensional Minkowski space, while S_E is the three-dimensional Euclidean action [66, 67]. Parameter β can be defined as:

$$\beta \equiv -\frac{dS}{dt}\Big|_{t=t_n} \simeq H_n T \frac{dS}{dT}\Big|_{T=T_n} = H_n T \frac{d(S_E/T)}{dT}\Big|_{T=T_n}, \quad (4.2)$$

where H_n denotes the Hubble rate at the nucleation temperature T_n . The key parameter that controls the GW signals is β/H_n . And the smaller β/H_n will lead to the stronger PT and consequently the more sensitive GW signals.

Then, referencing [68] and [69], we use the trace of the energy momentum tensor to calculate the strength parameter α_θ . The hydrodynamics in the plasma will be described by the energy density $\mathcal{E}$, the pressure density p and the enthalpy ω . The pressure p is determined by the free energy $\mathcal{F}$, which is the finite temperature effective potential V_{eff} : $p = -\mathcal{F} = -V_{\text{eff}}$. Then the energy $\mathcal{E}$ and enthalpy ω will be obtained respectively from the pressure density: $\mathcal{E} = T \frac{\partial p}{\partial T} - p$ and $\omega = T \frac{\partial p}{\partial T} = p + \mathcal{E}$.

The energy-momentum tensor of the plasma will be given by $T^{\mu\nu} = u^\mu u^\nu \omega + g^{\mu\nu} p$ with u^μ being the four-velocity of the fluid and $g^{\mu\nu}$ being the inverse Minkowski metric. Then the strength parameter α_θ will be deduced using the trace of the energy momentum tensor ($\theta = g_{\mu\nu} T^{\mu\nu}$) [70–73]: $\alpha_\theta = \frac{\Delta\theta}{3\omega_+(T_n)}$. Here, $\Delta\theta = \Delta\mathcal{E} - 3\Delta p$, Δ denotes the difference between symmetric and broken phase and $\omega_+(T_n)$ corresponds to the enthalpy of the symmetric phase at nucleation temperature T_n . We prefer to a larger value of α_θ , which will produce a much stronger PT, even a stronger GW spectrum.

In figure 4, we will discuss the β/H_n changing with parameters B_η , g_B , m_η and $m_{\bar{\eta}}$ respectively. The values of β/H_n decrease with the enlarging B_η , while increase with the enlarging g_B , m_η and $m_{\bar{\eta}}$. The strength of PT α_θ versus the parameters B_η , g_B , m_η and $m_{\bar{\eta}}$ will be researched in figure 5 respectively. With the enlarging g_B and m_η , the values of α_θ decrease. The values of α_θ are around 0.14 as $m_{\bar{\eta}}$ in the region $300 \sim 400$ GeV. Besides, the values of α_θ will almost do not change when B_η is in the region $(1.2 \sim 1.4)$ TeV². We prefer to the smaller β/H_n and larger α_θ to obtain the more suitable GW signals. So, we will take $\mu_\eta = 400$ GeV, $B_\eta = 1.3$ TeV², $g_B = 0.5$, $m_\eta = 1.6$ TeV and $m_{\bar{\eta}} = 300$ GeV in the following numerical discussion.

4.2 GW spectrum

In this section, we will discuss the GW spectrums specifically. There are three sources that generate the GW spectrums from the first-order PT: (1) The initial collision of scalar field and relevant shocks in the plasma. The technique of ‘envelope approximation’ has been widely used to model GW power spectrum from bubble collisions, which can be denoted by $h^2\Omega_{\text{coll}}$. (2) After the bubbles merging, the fluid kinetic energy waves in the plasma go

Figure 4. The values of β/H_n change with the parameters B_η , g_B , m_η and $m_{\bar{\eta}}$ respectively.

Figure 5. The strengths of PT α_θ change with the parameters B_η , g_B , m_η and $m_{\bar{\eta}}$ respectively.

on propagating outward into the broken phase. These waves spread at the sound speed in the plasma without the tractive force of scalar field bubble wall. Sound waves $h^2\Omega_{\text{sw}}$ are produced after the bubble collisions but before expansion. (3) The magnetohydrodynamic turbulent $h^2\Omega_{\text{trub}}$ of the surrounding plasma will be formed after the bubble collisions. These three contributions linearly combine, then we can obtain the corresponding GW spectrums, which will be expressed as

$$h^2\Omega_{\text{GW}} \approx h^2\Omega_{\text{coll}} + h^2\Omega_{\text{sw}} + h^2\Omega_{\text{trub}}. \quad (4.3)$$

4.2.1 Bubble collisions

Using the technique of ‘envelope approximation’, the contribution of the GWs generated from the bubble collisions is [22, 74]

$$h^2\Omega_{\text{coll}}(f) = 1.65 \times 10^{-5} \kappa^2 \Delta \left(\frac{\beta}{H_n} \right)^{-2} \left(\frac{\alpha_\theta}{1 + \alpha_\theta} \right)^2 \left(\frac{g_*}{100} \right)^{-1/3} \frac{(a+b)(f/f_{\text{coll}})^a}{b + a(f/f_{\text{coll}})^{(a+b)}}, \quad (4.4)$$

where $a = 2.8$ and $b = 1.0$. κ denotes the efficiency factor of the latent heat deposited into a thin shell with $A = 0.715$. g_* corresponds to the relativistic degree of freedom in the thermal plasma at T_n . The concrete expression of κ and Δ will be discussed as follows [22]

$$\kappa = \frac{1}{1 + A\alpha_\theta} \left(A\alpha_\theta + \frac{4}{27} \sqrt{\frac{3\alpha_\theta}{2}} \right), \quad \Delta = \frac{0.11v_b^3}{0.42 + v_b^2}, \quad (4.5)$$

here, v_b characterizes the bubble wall velocity. We choose $v_b \simeq 1.0$, since scenarios with nearly luminal wall velocities are more promising from the standpoint of observable gravitational radiation [33]. The peak frequency f_{coll} will be determined by the characteristic time-scale of the PT. From simulations, the peak frequency at T_n is approximately given by $\frac{f_n}{\beta} = \frac{0.62}{1.8 - 0.1v_b + v_b^2}$, which is then red-shifted to yield the peak frequency today:

$$f_{\text{coll}} = 1.67 \times 10^{-5} \left(\frac{f_n}{\beta} \right) \left(\frac{\beta}{H_n} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}.$$

4.2.2 Sound waves

As a more significant and long-lasting source of GW, sound wave is produced by expanding sound shells in the fluid kinetic energy after the bubble collisions. The concrete sound wave contribution to the GWs is given by [71, 75]

$$h^2\Omega_{\text{sw}}(f) = h^2\Omega_{\text{sw}}(f_{\text{sw}}) \left(\frac{f}{f_{\text{sw}}} \right)^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2}, \quad (4.6)$$

where the concrete peak amplitude $h^2\Omega_{\text{sw}}(f_{\text{sw}})$ and the corresponding peak frequency f_{sw} from sound waves can be shown as

$$h^2\Omega_{\text{sw}}(f_{\text{sw}}) = 2.65 \times 10^{-6} \kappa_v^2 v_b \left(\frac{\beta}{H_n} \right)^{-1} \left(\frac{\alpha_\theta}{1 + \alpha_\theta} \right)^2 \left(\frac{g_*}{100} \right)^{-1/3},$$

$$f_{\text{sw}} = 1.9 \times 10^{-5} \frac{1}{v_b} \left(\frac{\beta}{H_n} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}. \quad (4.7)$$

We take $\kappa_v \simeq \frac{\alpha_\theta}{0.73 + 0.083\sqrt{\alpha_\theta + \alpha_\theta}}$ [70], which denotes the fraction of the latent heat transforming into the bulk motion of the fluid. Besides, efficiency κ_v will be determined by the bubble expansion mode.

4.2.3 Turbulence

Since the plasma is ionized fully, percolation can also induce magnetohydrodynamic turbulence in the plasma. Then, the GW contribution from the turbulence can be written as [76, 77]

$$h^2\Omega_{\text{turb}}(f) = 3.35 \times 10^{-4} v_b \left(\frac{\beta}{H_n} \right)^{-1} \left(\frac{\kappa_{\text{turb}} \alpha_\theta}{1 + \alpha_\theta} \right)^{3/2} \left(\frac{g_*}{100} \right)^{-1/3} \times \frac{\left(f/f_{\text{turb}} \right)^3}{\left(1 + f/f_{\text{turb}} \right)^{11/3} \left(1 + 8\pi f/h_n \right)}, \quad (4.8)$$

where $h_n = 1.65 \times 10^{-5} \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6}$ Hz is the Hubble parameter today. We set the efficiency factor of the latent heat for turbulence to be $\kappa_{\text{turb}} \simeq 0.05 \kappa_v$ [24]. The peak frequency f_{turb} of the GWs produced from the turbulence will be as follows

$$f_{\text{turb}} = 2.7 \times 10^{-5} \frac{1}{v_b} \left(\frac{\beta}{H_n} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}. \quad (4.9)$$

5 The numerical results of GW spectrum in the B-LSSM

In our numerical calculation, we consider the constraints of parameter space related to the future experiment within B-LSSM. In section II, we study the Higgs masses with the one-loop zero temperature effective potential corrections. Parameters g_B , g_{YB} , tb and tb' have been limited within small regions, which will affect the numerical results of GW spectrum. As well as, we have studied Higgs decay modes, B meson rare decay $\bar{B} \rightarrow X_s \gamma$ and the muon anomalous magnetic dipole moment in the B-LSSM [78]. The corresponding parameter constraints will be considered in our numerical discussion. In the following numerical discussion, we use the numerical package **CosmoTransitions** [79] for analyzing the corresponding PT. After calculating, we obtain the bounce solutions and discover the nucleation temperature T_n . The value of S_E/T_n is around 140 below certain nucleation temperature T_n . Then we can obtain the important results of parameters α_θ and β , which finally determine the GW spectrums.

In this part, we study the dependence of the GW spectrums on the parameters tb , g_{YB} , g_B , μ_η , B_η , m_η and $m_{\bar{\eta}}$. First, the GW spectrums versus parameter g_B are plotted in figure 6 with $tb = 20$, $g_{YB} = -0.15$, $\mu_\eta = 600 \text{ GeV}$, $B_\eta = 1.3 \text{ TeV}^2$, $m_\eta = 1.6 \text{ TeV}$ and $m_{\bar{\eta}} = 300 \text{ GeV}$. The color lines show the sensitivities of the future GW experiments, which include the LISA(N2A5M5L6), BBO, DECIGO and Ultimate-DECIGO. We can discover that the present and future GW observations could probe a broad parameter space for g_B . Besides, the GWs signal will be as large as $h^2\Omega_{GW} \sim 10^{-9}$, which can be detected by the LISA, BBO, DECIGO and Ultimate-DECIGO with g_B in the region $(0.49 \sim 0.79)$. Then, the GW spectrums $h^2\Omega_{GW}$ versus frequency f will be further researched in the right of figure 6 with $g_B = 0.49$, which is preferred by the space-based experiments. The contributions to bubble collision, sound waves and turbulence are respectively plotted by

Figure 6. The contributions of GW spectrum vary with parameters g_B in the B-LSSM.

Figure 7. The contributions of GW spectrum vary with parameters tb and g_{YB} in the B-LSSM.

dashed, dotted and dot dashed lines. The contributions of GW spectrum, coming from the sum of the aforementioned three contributions, will be denoted by the solid black line. We can easily find that the contribution to sound wave is dominant and almost indistinguishable from the solid black line when f is around $10^{-4} \sim 0.005$ Hz.

Then, the GW spectrums versus parameters tb and g_{YB} are plotted in figure 7 with $g_B = 0.5$, $\mu_\eta = 600$ GeV, $B_\eta = 1.3$ TeV², $m_\eta = 1.6$ TeV and $m_{\bar{\eta}} = 300$ GeV. Parameter tb presents in the Yukawa couplings Y_t and Y_b , which affects the finite temperature effective potential through the temperature corrections of the third squark masses, so that influence the GWs. We can discover that the GW spectrums can be detected by the LISA(N2A5M5L6), BBO, DECIGO and Ultimate-DECIGO as $19 < tb < 31$ and $-0.21 < g_{YB} < -0.10$. Besides, with the decreasing tb but enlarging g_{YB} , the GW signal strength will increase slowly. In our following discussion, we make $tb = 20$ and $g_{YB} = -0.15$.

Parameters m_η and $m_{\bar{\eta}}$ both exist in the tree-level zero temperature effective potential. When $tb = 20$, $g_{YB} = -0.15$, $g_B = 0.5$, $\mu_\eta = 600$ GeV and $B_\eta = 1.3$ TeV², the GW spectrums versus frequency f with parameters m_η and $m_{\bar{\eta}}$ will be shown in figure 8. The smaller m_η or $m_{\bar{\eta}}$, the larger GW spectrum with relatively smaller peak frequency we can obtain. Other than this, the value of parameter m_η is around 1.6 TeV, while the value of parameter $m_{\bar{\eta}}$ is around 300 GeV. Our numerical simulations reveal that the parameters m_η and $m_{\bar{\eta}}$ possess tiny parameter spaces to acquire suitable GW spectrums.

Figure 8. The contributions of GW spectrum vary with parameters m_η and $m_{\bar{\eta}}$ in the B-LSSM.

Figure 9. The contributions of GW spectrum vary with parameters μ_η and B_η in the B-LSSM.

After fixing $tb = 20$, $g_{YB} = -0.15$, $g_B = 0.5$, $m_\eta = 1.6$ TeV and $m_{\bar{\eta}} = 300$ GeV in the B-LSSM, the GW spectrums versus parameters μ_η and B_η are demonstrated through the figure 9. As μ_η is in the region of $590 \sim 640$ GeV, a larger value of μ_η tends to produce a larger GW spectrum with a relatively smaller peak frequency. It is worth noting that GW spectrum can be detected by the LISA(N2A5M5L6), BBO, DECIGO and Ultimate-DECIGO when μ_η is in the region of $590 \sim 640$ GeV. Then the GW spectrum versus parameter B_η will be studied. With the increasing value of B_η , the GW signal gradually strengthens. As $1.28 \text{ TeV}^2 \leq B_\eta \leq 1.36 \text{ TeV}^2$, GW spectrum will be detected by LISA(N2A5M5L6), BBO, DECIGO and Ultimate-DECIGO.

Through the above discussion in figure 8 and figure 9, we find that parameters m_η , $m_{\bar{\eta}}$, μ_η and B_η fluctuate within a small region. We expect to find the relationship between these parameters to explain these relatively special parameter values. So, we random scan the parameter space as $14 < tb < 40$, $-0.45 < g_{YB} < -0.01$, $0.1 < g_B < 0.8$, $200 \text{ GeV} < \mu_\eta < 700 \text{ GeV}$, $0.8 \text{ TeV}^2 < B_\eta < 1.6 \text{ TeV}^2$, $1.2 \text{ TeV} < m_\eta < 2.0 \text{ TeV}$ and $100 \text{ GeV} < m_{\bar{\eta}} < 600 \text{ GeV}$. When the numerical results satisfy the strong first-order PT and can obtain the suitable GW signals, we plot the ratio of random parameter B_η to $\sqrt{(m_\eta^2 + \mu_\eta^2)(m_{\bar{\eta}}^2 + \mu_{\bar{\eta}}^2)}$ in figure 10(a). We conclude that these parameters satisfy the approximate relation: $A_{\text{tree}} \equiv \frac{B_\eta}{\sqrt{(m_\eta^2 + \mu_\eta^2)(m_{\bar{\eta}}^2 + \mu_{\bar{\eta}}^2)}} \simeq 1.25$. Then, we research the total contributions after considering

Figure 10. The tree-level and one-loop corrections of $\frac{B_\eta}{\sqrt{(m_\eta^2 + \mu_\eta^2)(m_{\bar{\eta}}^2 + \mu_{\bar{\eta}}^2)}}$ versus i .

the one-loop correction through the one-loop zero temperature effective potential with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$: $A_{\text{one-loop}} \equiv \frac{B_\eta + \Delta_{B_\eta}}{\sqrt{(m_\eta^2 + \mu_\eta^2 + \Delta_{11})(m_{\bar{\eta}}^2 + \mu_{\bar{\eta}}^2 + \Delta_{22})}}$. The corresponding theoretical analyses will be given out in appendix E. Besides, we plot the influence of the one-loop corrections in the total contributions $\frac{A_{\text{one-loop}} - A_{\text{tree}}}{A_{\text{one-loop}}}$ in figure 10(b), which indicates that the contributions of one-loop corrections are small and only around 2% $\sim$ 6% of the total contributions.

6 Conclusion

In the B-LSSM, we first study the Higgs masses with the one-loop zero temperature effective potential corrections. When the physical Higgs mass satisfies 3σ experimental interval, parameters tb and tb' are limited within small regions: $14 < tb < 40$ and $1.3 < tb' < 1.5$. In additional, the larger parameter tb' and B_η , the larger second-light Higgs mass we can obtain. Second, the strength parameter will be calculated using the trace of the energy momentum tensor. Then, we study the GW spectrums generated by the strong first-order PT with the two $U(1)_{B-L}$ singlet Higgs superfields. Our numerical calculations reveal that the contribution from sound wave is dominated to generate the GW signals when the frequency f is around $10^{-4} \sim 0.005$ Hz. In the B-LSSM, parameters g_B , tb and g_{YB} affect the GW signals within the regions: $0.49 < g_B < 0.8$, $19 < tb < 31$ and $-0.21 < g_{YB} < -0.1$. Besides, we can easily find that parameters m_η , $m_{\bar{\eta}}$, μ_η and B_η fluctuate within small regions and exist relationship: $\frac{B_\eta}{\sqrt{(m_\eta^2 + \mu_\eta^2)(m_{\bar{\eta}}^2 + \mu_{\bar{\eta}}^2)}} \simeq 1.25$. Therefore, the parameter regions in the B-LSSM are very fine-tuned to obtain the strength parameter $\alpha_\theta \sim 0.14$ and the ratio of speed to the Hubble rate $\beta/H_n \sim 5$, and then obtain observable GW signals. Then, we research the total contributions for the relationship of parameters m_η , $m_{\bar{\eta}}$, μ_η and B_η after considering the one-loop correction through the one-loop zero temperature effective potential with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$: $A_{\text{one-loop}} \equiv \frac{B_\eta + \Delta_{B_\eta}}{\sqrt{(m_\eta^2 + \mu_\eta^2 + \Delta_{11})(m_{\bar{\eta}}^2 + \mu_{\bar{\eta}}^2 + \Delta_{22})}}$. The contribution of one-loop corrections are small and only around 2% $\sim$ 6% of the total contributions. As the parameters take the suitable values, the strength of the GW spectrum will be as large as $h^2 \Omega_{GW} \sim 10^{-9}$, which may be detected by the future GW detection experiments such as LISA, BBO, DECIGO and Ultimate-DECIGO.

Acknowledgments

We are very grateful to Li-Gong Bian the teacher of Chongqing University and Wu-Long Xu the Dr. of Beijing University of Technology, for giving us some useful discussions. This work is supported by the Major Project of National Natural Science Foundation of China (NNSFC) No. 12075074, No. 11535002 and No. 11705045, Natural Science Foundation of Hebei Province No. A2020201002 and the youth top-notch talent support program of the Hebei Province.

A The anomaly free of B-LSSM

B-LSSM is assumed the gauged one, and we have proved carefully that this model is anomaly free. We use Y^Y and Y^{B-L} respectively representing the charge of $U(1)_Y$ and $U(1)_{B-L}$. The quantum field theory written by M. E. Peskin has introduced the anomaly free of standard model(SM) in page 705-707. So, we show the demonstration of anomaly free in the B-LSSM.

1. In the B-LSSM, the anomaly of three $SU(2)$ bosons banishes, as well as the anomaly of three $SU(3)$ boson. These anomalies are same as the SM.
 2. In the B-LSSM, the anomalies containing one $SU(3)$ boson or one $SU(2)$ boson are proportional to $Tr[t^a] = 0$ or $Tr[\tau^a] = 0$.
 3. The remaining nontrivial anomalies are: the anomaly of one $U(1)$ boson with two $SU(3)$ bosons, the anomaly of one $U(1)$ boson with two $SU(2)$ bosons, the anomaly of three $U(1)$ bosons and the gravitational anomaly with one $U(1)$ boson. In the B-LSSM, there are $U(1)_Y$ and $U(1)_{B-L}$ gauge groups, which are more complicated than the SM.
- (1) The anomaly of one $U(1)_Y$ or $U(1)_{B-L}$ boson with two $SU(3)$ bosons is proportional to the group theory factor $Tr[t^a t^b Y^Y] = \frac{1}{2} \delta^{ab} \sum_q Y_q^Y$ or $Tr[t^a t^b Y^{B-L}] = \frac{1}{2} \delta^{ab} \sum_q Y_q^{B-L}$.
 - (2) Similarly, the anomaly of one $U(1)_Y$ or $U(1)_{B-L}$ boson with two $SU(2)$ bosons is also proportional to the group theory factor $Tr[\tau^a \tau^b Y^Y] = \frac{1}{2} \delta^{ab} \sum_{fL} Y_{fL}^Y$ or $Tr[\tau^a \tau^b Y^{B-L}] = \frac{1}{2} \delta^{ab} \sum_{fL} Y_{fL}^{B-L}$.
 - (3) The anomalies of three $U(1)$ gauge bosons are divided into four types with the B-LSSM: $Tr[Y^Y Y^Y Y^Y] = \sum_n (Y_n^Y)^3$, $Tr[Y^{B-L} Y^{B-L} Y^{B-L}] = \sum_n (Y_n^{B-L})^3$, $Tr[Y^Y Y^{B-L} Y^{B-L}] = \sum_n (Y_n^Y)(Y_n^{B-L})^2$ and $Tr[Y^Y Y^Y Y^{B-L}] = \sum_n (Y_n^Y)^2 (Y_n^{B-L})$.
 - (4) The gravitational anomaly with one $U(1)$ gauge boson $U(1)_Y$ or $U(1)_{B-L}$ is proportional to $Tr[Y^Y] = \sum_n (Y_n^Y)$ or $Tr[Y^{B-L}] = \sum_n (Y_n^{B-L})$.

The anomalies that do not relate with $U(1)_{B-L}$ are proved free and they are very similar as the SM condition.

$$\begin{aligned}
 \sum_q Y_q^Y &= \left[\frac{1}{6} \times 2 + \frac{1}{3} + \left(-\frac{2}{3} \right) \right] \times 3 \times 3 = 0, \\
 \sum_{fL} Y_{fL}^Y &= \frac{1}{6} \times 3 \times 3 + \left(-\frac{1}{2} \right) \times 3 + \frac{1}{2} + \left(-\frac{1}{2} \right) = 0, \\
 \sum_n (Y_n^Y)^3 &= \left[\left(\frac{1}{6} \right)^3 \times 2 + \left(\frac{1}{3} \right)^3 + \left(-\frac{2}{3} \right)^3 \right] \times 3 \times 3 + \left[\left(-\frac{1}{2} \right)^3 \times 2 + 1^3 \right] \times 3 \\
 &\quad + \left(\frac{1}{2} \right)^3 \times 2 + \left(-\frac{1}{2} \right)^3 \times 2 = 0, \\
 \sum_n Y_n^Y &= \left[\frac{1}{6} \times 2 + \frac{1}{3} + \left(-\frac{2}{3} \right) \right] \times 3 \times 3 + \left[\left(-\frac{1}{2} \right) \times 2 + 1 \right] \times 3 + \left(\frac{1}{2} \right) \times 2 + \left(-\frac{1}{2} \right) \times 2 = 0.
 \end{aligned}$$

The anomalies including $U(1)_{B-L}$ are also proved free, which are more complicated than the calculation of SM.

$$\begin{aligned}
 \sum_q Y_q^{B-L} &= \left[\frac{1}{6} \times 2 + \left(-\frac{1}{6} \right) + \left(-\frac{1}{6} \right) \right] \times 3 \times 3 = 0, \\
 \sum_{fL} Y_{fL}^{B-L} &= \frac{1}{6} \times 3 \times 3 + \left(-\frac{1}{2} \right) \times 3 = 0, \\
 \sum_n (Y_n^{B-L})^3 &= \left[\left(\frac{1}{6} \right)^3 \times 2 + \left(-\frac{1}{6} \right)^3 + \left(-\frac{1}{6} \right)^3 \right] \times 3 \times 3 + \left[\left(-\frac{1}{2} \right)^3 \times 2 + \left(\frac{1}{2} \right)^3 + \left(\frac{1}{2} \right)^3 \right] \times 3 \\
 &\quad + 1^3 + (-1)^3 = 0, \\
 \sum_n Y_n^Y (Y_n^{B-L})^2 &= \left[\frac{1}{6} \times \left(\frac{1}{6} \right)^2 \times 2 + \frac{1}{3} \times \left(-\frac{1}{6} \right)^2 + \left(-\frac{2}{3} \right)^2 \times \left(-\frac{1}{6} \right) \right] \times 3 \times 3 \\
 &\quad + \left[\left(-\frac{1}{2} \right)^2 \times \left(-\frac{1}{2} \right) \times 2 + 1 \times \left(\frac{1}{2} \right)^2 \right] \times 3 = 0, \\
 \sum_n (Y_n^Y)^2 (Y_n^{B-L}) &= \left[\left(\frac{1}{6} \right)^2 \times \frac{1}{6} \times 2 + \left(\frac{1}{3} \right)^2 \times \left(-\frac{1}{6} \right) + \left(-\frac{2}{3} \right)^2 \times \left(-\frac{1}{6} \right) \right] \times 3 \times 3 \\
 &\quad + \left[\left(-\frac{1}{2} \right) \times \left(-\frac{1}{2} \right)^2 \times 2 + 1^2 \times \frac{1}{2} \right] \times 3 = 0, \\
 \sum_n Y_n^{B-L} &= \left[\frac{1}{6} \times 2 + \left(-\frac{1}{6} \right) + \left(-\frac{1}{6} \right) \right] \times 3 \times 3 + \left[\left(-\frac{1}{2} \right) \times 2 + \frac{1}{2} + \frac{1}{2} \right] \times 3 + 1 + (-1) = 0.
 \end{aligned}$$

B The vacuum of Higgs doublets and Higgs singlets

In the B-LSSM, the Higgs doublets and Higgs singlets obtain the nonzero vacuum expectation values (VEVs), which break the gauge group $SU(3) \otimes U(1)_Y \otimes U(1)_{B-L}$ down to $U(1)_{em}$ at zero temperature. For the model's parameter values considered in our analysis, the vacuum of eq.2 is indeed the global one and there is no charge breaking nor CP violating deeper minima.

First, in the MSSM, there are two Higgs doublets H_d and H_u . So the tree-level effective potential can be written as $V_{MSSM} = \frac{1}{8}(g_1^2 + g_2^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2}g_2^2|H_u^0 H_d^{*-} + H_u^+ H_d^{0*}|^2 + (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) + (B_\mu(H_u^+ H_d^- - H_u^0 H_d^0) + \text{H.c.})$. On the premise of not losing generality, the potential energy minimum point $H_u^+ = 0$ is selected, and $H_d^- = 0$ is required. Therefore, the charged component of the Higgs scalar potential cannot obtain the vacuum expected value, and the electromagnetic action at the potential energy minimum point must not be broken. When $H_u^+ = H_d^- = 0$, the scalar potential of MSSM is $V'_{MSSM} = \frac{1}{8}(g_1^2 + g_2^2)(|H_u^0|^2 - |H_d^0|^2)^2 + (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 + (-B_\mu H_u^0 H_d^0 + \text{H.c.})$. Only the parameter B_μ in this effective potential depends on the field phase. The phase of H_u^0 and H_d^0 will be defined to absorb the phase of B_μ , which takes to be real and positive. The minimum point of the effective potential requires that $H_u^0 H_d^0$ is also real and positive, so that $\langle H_d^0 \rangle$ and $\langle H_u^0 \rangle$ must have opposite phases. H_u and H_d have opposite weak supercharges ($\pm \frac{1}{2}$), so the $U(1)_Y$ gauge transformation is used to make $\langle H_d^0 \rangle$ and $\langle H_u^0 \rangle$ real and positive without losing generality. The convention that the vacuum expectation values and B_μ are selected as real at the same time leads to the spontaneous breaking of Higgs scalar potential in MSSM without CP violating. In order to make the scalar potential feasible, it is necessary to satisfy: $2B_\mu < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2$ and $B_\mu^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2)$. At this time, there exists electroweak symmetry spontaneous breaking and the corresponding vacuum expectation values are obtained.

In the B-LSSM, $U(1)_{B-L}$ gauge group will be added to the MSSM gauge group, and two singlet Higgs fields η and $\bar{\eta}$ will be added. Therefore, in the B-LSSM, the tree-level scalar potential is written as $V_{B-L} = V'_{MSSM} + \frac{1}{8}g_{YB}^2(|H_u^0|^2 - |H_d^0|^2)^2 + \frac{1}{2}g_B g_{YB}(|H_u^0|^2 |\bar{\eta}|^2 + |H_d^0|^2 |\eta|^2 - |H_u^0|^2 |\eta|^2 - |H_d^0|^2 |\bar{\eta}|^2) + \frac{1}{2}g_B^2(|\eta|^2 - |\bar{\eta}|^2)^2 + (|\mu_\eta|^2 + m_\eta^2)|\eta|^2 + (|\mu_\eta|^2 + m_\eta^2)|\bar{\eta}|^2 + (-B_\eta \eta \bar{\eta} + \text{H.c.})$. The parameter B_η in the effective potential also depends on the field phase. The phase of η and $\bar{\eta}$ will be defined to absorb the phase of B_η , which takes as real and positive. The minimum point of the effective potential requires that $\eta \bar{\eta}$ is also real and positive, so that $\langle \eta \rangle$ and $\langle \bar{\eta} \rangle$ must have opposite phases. η and $\bar{\eta}$ have opposite weak supercharges (± 1), so the $U(1)_{B-L}$ gauge transformation is used to make η and $\bar{\eta}$ real and positive without losing generality. The convention that the vacuum expectation values and B_η are selected as real simultaneously causes the spontaneous breaking of Higgs scalar potential in the B-LSSM without CP violating. In order to make the scalar potential feasible, it also requires that: $2B_\eta < 2|\mu_\eta|^2 + m_\eta^2 + m_\eta^2$ and $B_\eta^2 > (|\mu_\eta|^2 + m_\eta^2)(|\mu|^2 + m_\eta^2)$. At this time, there exists electroweak symmetry spontaneous breaking and the corresponding vacuum expectation values are obtained in the B-LSSM. The parameter space we chosen satisfies these conditions.

C The value of parameter g_{YB}

The range $[-0.45, -0.05]$ assumed for g_{YB} applies to the total contribution including one-loop effects from fields carrying both Y and B-L charges. The gauge groups $U(1)_Y$ and $U(1)_{B-L}$ mix together, and the coupling matrix is: $\begin{pmatrix} g_{YY} & g'_{YB} \\ g'_{BY} & g_{BB} \end{pmatrix}$. Through the orthogo-

nal matrix R , the coupling matrix can be rewritten as: $\begin{pmatrix} g_{YY} & g'_{YB} \\ g'_{BY} & g_{BB} \end{pmatrix} R = \begin{pmatrix} g_1 & g_{YB} \\ 0 & g_B \end{pmatrix}$.

Then, we can redefine the U(1) gauge fields: $R^T \begin{pmatrix} B_\mu^{Y'} \\ B_\mu^{BL} \end{pmatrix} = \begin{pmatrix} B_\mu \\ B_\mu'' \end{pmatrix}$. In ref. [58],

the β -functions of eqs. (B.16) and (B.17) are respectively $\beta_{g_{YB}}^{(1)} = \frac{3}{5}[g_{YY}g'_{BY}(15g_{BB} + 2\sqrt{10}g'_{YB}) + g_{YY}^2(11g'_{YB} + 2\sqrt{10}g_{BB}) + g'_{YB}(11g_{YB}^2 + 15g_{BB}^2 + 4\sqrt{10}g'_{YB}g_{BB})]$ and $\beta_{g_{BY}}^{(1)} = \frac{3}{5}[11g_{YY}^2g'_{BY} + g_{YY}(11g'_{YB}g_{BB} + 2\sqrt{10}(2g_{YB}^2 + g_{BB}^2)) + g'_{BY}(15(g_{BB}^2 + g_{YB}^2) + 2\sqrt{10}g'_{YB}g_{BB})]$. For vanishing off-diagonal gauge couplings, the β -functions of eqs. (B.16) and (B.17) will always be positive. As parameters g_{YY} and g_{BB} are positive, parameters g'_{YB} and g'_{BY} can be driven negative. Besides, at the electroweak scale, $g_{YB} = \frac{g'_{YB}g_{BB} + g'_{BY}g_{YY}}{\sqrt{g_{BB}^2 + g_{YB}^2}}$, which indicates that parameter g_{YB} is driven negative. In ref. [59], the author also indicates that $g_{YB} \simeq -0.147$ at SUSY scale. Therefore, the value of parameter g_{YB} will be taken around $[-0.45, -0.05]$ in our numerical calculation.

D The neutralino mass matrix

In the basis $(\lambda_Y, \lambda_W^3, \psi_{H_d}^1, \psi_{H_u}^2, \lambda_{B'}, \psi_\eta, \psi_{\bar{\eta}})$, we can obtain a 7×7 neutralino mass matrix:

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1v_d & \frac{1}{2}g_1v_u & M_{BB'} & 0 & 0 \\ 0 & M_2 & \frac{1}{2}g_2v_d & -\frac{1}{2}g_2v_u & 0 & 0 & 0 \\ -\frac{1}{2}g_1v_d & \frac{1}{2}g_2v_d & 0 & -\mu & -\frac{1}{2}g_{YB}v_d & 0 & 0 \\ \frac{1}{2}g_1v_u & -\frac{1}{2}g_2v_u & -\mu & 0 & \frac{1}{2}g_{YB}v_u & 0 & 0 \\ M_{BB'} & 0 & -\frac{1}{2}g_{YB}v_d & \frac{1}{2}g_{YB}v_u & M_{BL} & -g_Bv_\eta & g_Bv_{\bar{\eta}} \\ 0 & 0 & 0 & 0 & -g_Bv_\eta & 0 & -\mu_\eta \\ 0 & 0 & 0 & 0 & g_Bv_{\bar{\eta}} & -\mu_\eta & 0 \end{pmatrix} \quad (D.1)$$

The value of $U(1)_{B-L}$ breaking Higgs singlets VEV u is around $u \simeq 10$ TeV, which is much larger than the Higgs doublets VEV $v \simeq 246$ GeV. Besides, we only consider the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$ to study the strong first-order PT, so the components from $SU(2)_L \times U(1)_Y$ gauginos are not included.

E The one-loop correction $A_{\text{one-loop}}$

We consider the one-loop correction through the one-loop zero temperature effective potential with the CP-even Higgs components ϕ_η and $\phi_{\bar{\eta}}$: $A_{\text{one-loop}} \equiv \frac{B_\eta + \Delta_{B_\eta}}{\sqrt{(m_\eta^2 + \mu_\eta^2 + \Delta_{11})(m_{\bar{\eta}}^2 + \mu_{\bar{\eta}}^2 + \Delta_{22})}}$. The corresponding theoretical analyses will be given out as follows: $\Delta_{11} = [-\frac{1}{\phi_\eta} \frac{\partial \Delta V_1}{\partial \phi_\eta} + \frac{\partial^2 \Delta V_1}{\partial \phi_\eta^2}]_{\phi_\eta=v_\eta}$, $\Delta_{22} = [-\frac{1}{\phi_{\bar{\eta}}} \frac{\partial \Delta V_1}{\partial \phi_{\bar{\eta}}} + \frac{\partial^2 \Delta V_1}{\partial \phi_{\bar{\eta}}^2}]_{\phi_{\bar{\eta}}=v_{\bar{\eta}}}$ and $\Delta_{B_\eta} = [\frac{\partial^2 \Delta V_1}{\partial \phi_\eta \partial \phi_{\bar{\eta}}}]_{\phi_\eta=v_\eta, \phi_{\bar{\eta}}=v_{\bar{\eta}}}$. ΔV_1 represents the one-loop zero temperature effective potential:

$$\Delta V_1(\phi_\eta, \phi_{\bar{\eta}}, 0) = \sum_i (-1)^k \frac{n_i}{64\pi^2} m_i^4(\phi_\eta, \phi_{\bar{\eta}}) \left[\log \frac{m_i^2(\phi_\eta, \phi_{\bar{\eta}})}{\Lambda^2} - c_i \right]. \quad (E.1)$$

Therefore, the radiative corrections Δ_{ii} from the one-loop zero temperature effective potential will be deduced as:

$$\begin{aligned}\Delta_{ii} &= \frac{n_i}{(4\pi)^2} \left[\sum_{k=\chi_i^0} \left(\frac{2}{\phi_i} f(m_k^2) \frac{\partial m_k^2}{\partial \phi_i} - 2f(m_k^2) \frac{\partial^2 m_k^2}{\partial \phi_i^2} - \frac{1}{2} \ln \left(\frac{m_k^2}{\Lambda^2} \right) \frac{\partial m_k^2}{\partial \phi_i} \frac{\partial m_k^2}{\partial \phi_i} \right) \right. \\ &\quad \left. + \sum_{k=\tilde{S}} \left(-\frac{1}{\phi_i} f(m_k^2) \frac{\partial m_k^2}{\partial \phi_i} + f(m_k^2) \frac{\partial^2 m_k^2}{\partial \phi_i^2} + \frac{1}{4} \ln \left(\frac{m_k^2}{\Lambda^2} \right) \frac{\partial m_k^2}{\partial \phi_i} \frac{\partial m_k^2}{\partial \phi_i} \right) \right], \\ \Delta_{ij} &= \frac{n_i}{(4\pi)^2} \sum_{k=\phi_\eta, \phi_{\tilde{\eta}}, \sigma_\eta, \sigma_{\tilde{\eta}}} \left(f(m_k^2) \frac{\partial^2 m_k^2}{\partial \phi_i \partial \phi_j} + \frac{1}{4} \ln \left(\frac{m_k^2}{\Lambda^2} \right) \frac{\partial m_k^2}{\partial \phi_i} \frac{\partial m_k^2}{\partial \phi_j} \right).\end{aligned}\quad (\text{E.2})$$

Here, $\phi_{i,j}$ are the CP-even Higgs components $\phi_\eta, \phi_{\tilde{\eta}}, \tilde{S} \in (Z', \tilde{Q}^I, \tilde{U}^I, \tilde{D}^I, \tilde{L}^I, \tilde{R}^I, \tilde{\nu}_i, \tilde{\nu}_i^R, \phi_\eta, \phi_{\tilde{\eta}}, \sigma_\eta, \sigma_{\tilde{\eta}})$. $f(m_k^2) = \frac{1}{4} m_k^2 [\ln(\frac{m_k^2}{\Lambda^2}) - 1]$, Λ is the renormalization scale and we take $\Lambda = 1$ TeV in the numerical discussion. m_k represent the corresponding particle masses.

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